



# Takagi-Sugeno Observers: Experimental Application for Vehicle Lateral Dynamics Estimation

Zedjiga Yacine, Dalil Ichalal, Naima Ait Oufroukh, Saïd Mammar, Said Djennoune

## ► To cite this version:

Zedjiga Yacine, Dalil Ichalal, Naima Ait Oufroukh, Saïd Mammar, Said Djennoune. Takagi-Sugeno Observers: Experimental Application for Vehicle Lateral Dynamics Estimation. IEEE Transactions on Control Systems Technology, 2015, 23 (2), pp.754-761. 10.1109/TCST.2014.2327592 . hal-01018944

**HAL Id: hal-01018944**

**<https://hal.science/hal-01018944>**

Submitted on 18 Aug 2022

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



Distributed under a Creative Commons Attribution 4.0 International License

# Takagi-Sugeno Observers: Experimental Application for Vehicle Lateral Dynamics Estimation

Zedjiga Yacine, Dalil Ichalal, Naïma Ait-Oufroukh, Saïd Mammar, and Saïd Djennoune

**Abstract**—This brief presents a contribution to nonlinear observer design for both the estimation of vehicle lateral dynamics and road curvature. The latter is recovered by a simple algebraic technique and high-order sliding mode differentiator, which allows to estimate exactly the time derivatives of measured signals. For the lateral dynamics of the vehicle, a new approach is developed by the use of a Takagi-Sugeno (TS) model representing exactly, with no loss of information, the nonlinear vehicle behavior in a compact set of the state space. The TS model involves unmeasured premise variables. The proposed new observer starts with the estimation of these premise variables. Second, the design of an observer with weighting functions depending on these estimated premise variables is considered. Theoretically, the proposed observer ensures exponential convergence of the state estimation error toward zero. This convergence is studied by the Lyapunov theory and the obtained stability conditions are expressed in terms of linear matrix inequalities. Finally, experimental results are given for vehicle lateral dynamics and road curvature estimation with real data.

**Index Terms**—Algebraic approach, high-order sliding mode differentiators (HOSMDs), nonlinear observers, Takagi-Sugeno (TS) systems.

## I. INTRODUCTION

THE advanced progress in technology has allowed the elaboration of sophisticated vehicle safety systems in terms of driving assistance. From passive to active driving assistance, the objectives vary but all have one mission that is to ensure the safety of the road users. Driving assistance systems have seen their deployment widen these last years to become indispensable vehicle equipments, such as anti-lock braking system, electronic stability program, and adaptive cruise control. The design and the implementation of these equipments requires the availability of certain vehicle information, such as the lateral dynamics parameters (sideslip angle and lateral velocity). Extension of these systems requires lane attributes perception and in particular the road curvature. This information, however, is often unavailable either for technological reasons or economical ones (high cost of the sensors

required and implementation of measuring instruments). To overcome this lack of unavailable measures, their estimation proves to be a promising alternative.

Observer design for linear systems is a subject of an extensive literature. Knowing that a linear model is valid around a specific operating point, and thus only locally, this alters the quality of the estimations, in terms of accuracy. On the other side, while nonlinear models are more accurate in representing the real system's behavior, the design of observers for such systems, is still a field of intensive researches. However, there are many approaches to solve the problem of state estimation for nonlinear systems, but they are specific to certain categories of nonlinear systems for which they are developed, which is somehow restrictive. Namely, geometric approaches [13], high-gain methods [9], and sliding mode techniques [5], [16]. In algebraic framework, a nonlinear system is algebraically observable if it is possible to express the state vector as a mapping with respect to the measured inputs, the outputs and their successive time derivatives. The problem of state estimation is then turned into a problem of time derivatives estimation of both the inputs and the outputs of the system. The interest of such a result is in the formulation of the states as algebraic equations. The time derivatives can be estimated by different approaches such as sliding mode differentiators (super twisting, higher order sliding mode [16]), which provides an exact finite time derivatives estimation or by nonasymptotic algebraic techniques [8]. These techniques are very interesting and proved their efficiency in estimating the time derivatives of measured signals. However, the algebraic technique is sensitive to parameter uncertainties because there is no tuning term that could render the state estimation robust to these uncertainties. More recently, Takagi-Sugeno (TS) approaches have been highly considered in control and design observer theories, allowing to extend some tools developed in the linear case to nonlinear systems. They allow combining the accuracy of the modeling and the availability of designing tools [11], [15], [18]. They basically transform the nonlinear system into a polytopic form, represented by a convex sum of local linear submodels. A large class of systems are then expressed in terms of polytopic systems as linear parameter varying (LPV) or quasi-LPV systems that coincide with the so-called TS systems where the weighting functions are deterministic [10], [22].

Vehicle lateral dynamics are described by a nonlinear model affected by intrinsic parameters and influenced by many external others, such as the cross wind, road friction coefficient, and hence forth, and many other unknown effects that cannot be measured, which are identified as unknown inputs

Z. Yacine is with the Laboratory of Computing, Integrative Biology and Complex Systems, University of Evry Val d'Essonne, Évry 91000, France, and also with the University of Tizi Ouzou, Tizi Ouzou 15000, Algeria (e-mail: yacine.zedjiga@yahoo.fr).

D. Ichalal, N. Ait-Oufroukh, and S. Mammar are with the Laboratory of Computing, Integrative Biology and Complex Systems, University of Evry Val d'Essonne, Évry 91000, France (e-mail: dalil.ichalal@ibisc.univ-evry.fr; naïma.aitoufroukh@ibisc.univ-evry.fr; said.mammar@ibisc.univ-evry.fr).

S. Djennoune is with the University of Tizi Ouzou, Tizi Ouzou 15000, Algeria (e-mail: s\_djennoune@yahoo.fr).

(perturbations, noise, and modeling uncertainties). Thus, the vehicle model can be represented by a nonlinear model subject to unknown inputs.

In the field of lateral dynamics studies, the linear single track vehicle model is usually used, neglecting roll and pitch dynamics [1], [7], named yaw-drift model. The design of observers for linear systems with unknown inputs has been undertaken extensively these last decades [12], [23]. Observer design are proposed for estimating these lateral dynamics in normal driving zone (mainly, linear part of the tire forces) with linear Luenberger, PI or unknown input observers [17]. But if we speak about detection of dangerous driving situations (sliding situation, etc.) the model describing this behavior is nonlinear. For such situation, the extended Kalman filter is often used [2]. Recently, a technique using the bounded Jacobian approach is proposed by Phanomchoeng *et al.* [21], inspired from the approach of Zemouche *et al.* [25] for Lipschitz systems. TS observers are proposed in [19] by approximating the nonlinear model by a TS one [4]. A more interesting TS model obtained by sector nonlinear transformations is proposed in [24] for more accurate representation of the nonlinear behavior. This model is applied to lateral velocity and road curvature estimation.

In this brief, we propose to consider the problem of designing nonlinear observers with unknown inputs from the nonlinear point of view using the TS formalism. This brief is organized as follows. Section II presents the nonlinear model of the vehicle lateral dynamics and its positioning on the traffic lane with its reformulation in terms of two cascaded interconnected systems. In Section III, the second subsystem is used in an algebraic framework and high-order sliding mode differentiator (HOSMD) to estimate the road curvature and the unknown premise variables, which will be used in the construction of the TS observer with estimated premise variables. Then, the transformation of the second subsystem under TS structure into a polytopic TS model with unmeasurable premise variables is performed, representing exactly the nonlinear one in a compact set of the state space including the nonlinear region of the lateral forces and considering the longitudinal velocity varying. The observer is then designed based on the obtained TS model. Section IV presents some validations with real data and discussions about the proposed state observation strategy.

## II. VEHICLE LATERAL MODEL AND POSITIONING RELATIVE TO ROAD

This section is dedicated to the presentation of the vehicle model used for the observer synthesis. First, a nonlinear yaw-drift model is considered. Then, the positioning of the vehicle according to the road section is established. The dynamics of the lateral forces are also considered. The overall model including the lateral dynamics, the position relative to the road and the tire force dynamics are then given as a model with the road curvature acting as an unknown input.

### A. Lateral-Drift Model

Most of the work that has dealt with vehicle lateral dynamics use linear models. They are based on simplification

assumptions, which limit the evolution domain to the linear region only [7]. Instead, we propose to use a nonlinear yaw-drift model to overcome this limitation. Furthermore, we consider the longitudinal velocity time-varying, to reflect a more realistic behavior of the vehicle system. From [1] and [7], the lateral model is described by

$$\begin{cases} m\dot{v}_y = (F_f + F_r) - mv_x \dot{\psi} \\ I_z \ddot{\psi} = a_f F_f - a_r F_r. \end{cases} \quad (1)$$

The lateral front and rear forces  $F_f$  and  $F_r$  are expressed by the Pacejka's magic formula [3], [20]

$$F_i = D_i \sin(C_i \tan^{-1}(B_i(1 - E_i)\alpha_i + \tan^{-1}(B_i\alpha_i))) \quad (2)$$

$i = \{f, r\}$ . The front and rear sideslip angles of the tires  $\alpha_f$  and  $\alpha_r$  are given by

$$\begin{cases} \alpha_f = \delta_f - \beta - \tan^{-1}\left(\frac{a_f}{v_x} \dot{\psi} \cos(\beta)\right) \\ \alpha_r = -\beta + \tan^{-1}\left(\frac{a_r}{v_x} \dot{\psi} \cos(\beta)\right). \end{cases} \quad (3)$$

For small variations of the sideslip angles, corresponding to the rational driving, between the normal and pseudosliding regions, not exceeding  $8^\circ$ , the sideslip angles may be simplified as follows ( $\beta \approx v_y/v_x$ ):

$$\begin{cases} \alpha_f \approx \delta_f - \frac{v_y}{v_x} - \frac{a_f}{v_x} \dot{\psi} \\ \alpha_r \approx -\frac{v_y}{v_x} + \frac{a_r}{v_x} \dot{\psi}. \end{cases} \quad (4)$$

### B. Positioning of the Vehicle Relative to the Track

To establish the lateral dynamics of the model (1) related to the traffic lane, we add the differential equations of the lateral deviation  $y_L$  at a target distance  $l_s$  from the center of gravity and the relative heading angle  $\psi_L$  with the road curvature  $\rho$  [7]

$$\begin{cases} \dot{y}_L = v_x(\beta + \psi_L) + l_s(\dot{\psi} - v_x\rho) \\ \dot{\psi}_L = \dot{\psi} - v_x\rho. \end{cases} \quad (5)$$

### C. Dynamic Lateral Forces

In practical situations, due to the characteristics of the tires, the forces  $F_f$  and  $F_r$  are generated by dynamic systems given by

$$\begin{cases} \tau_f \dot{F}_f + F_f = F_f^S \\ \tau_r \dot{F}_r + F_r = F_r^S. \end{cases} \quad (6)$$

For more details, the reader can refer to [20, ch. 5]. The inputs of these two systems are  $F_f^S$  and  $F_r^S$  denoting the steady values of the lateral forces, which can be represented by different models as Pacejka magic formula (2) [20], Dugoff's model, and hence forth. In this brief, the Pacejka's model is considered

$$F_i^S = D_i \sin(C_i \tan^{-1}(B_i(1 - E_i)\alpha_i + \tan^{-1}(B_i\alpha_i))). \quad (7)$$

The parameters  $\tau_f$  and  $\tau_r$  are given by  $\tau_i = r_i/r_D|\Omega|$ ,  $i = \{f, r\}$ , where  $r_D|\Omega|$  denotes the longitudinal velocity ( $r_D$  and  $\Omega$  represent the dynamic rolling radius and the angular velocity),  $r_i$  are the relaxation lengths.

The global nonlinear model is then given by the following two cascaded nonlinear models:

$$S_1 : \begin{cases} \dot{v}_y = \frac{1}{m}(F_f + F_r) - v_x \dot{\psi} \\ \ddot{\psi} = \frac{1}{I_z}(a_f F_f - a_r F_r) \\ \dot{F}_f = -\frac{v_x}{r_f} F_f + \frac{v_x}{r_f} F_f^S \\ \dot{F}_r = -\frac{v_x}{r_r} F_r + \frac{v_x}{r_r} F_r^S \end{cases} \quad (8)$$

$$S_2 : \begin{cases} \dot{\psi}_L = \dot{\psi} - v_x \rho \\ \dot{y}_L = (v_y + v_x \psi_L) + l_s(\dot{\psi} - v_x \rho). \end{cases} \quad (9)$$

The aim of this decomposition is to reduce the complexity of the system, so the first model  $S_1$  will be used to estimate the lateral velocity  $v_y$  and the lateral tire forces  $F_f$  and  $F_r$  and the second one  $S_2$  will be used to estimate the road curvature  $\rho$  since  $\psi_L$  and  $y_L$  are available to measure with adequate sensors (vision system).

### III. OBSERVERS DESIGN FOR LATERAL DYNAMICS ESTIMATION AND ROAD CURVATURE

#### A. Road Curvature Estimation

From the vision system  $S_2$ , it is also possible to estimate exactly the road curvature from the knowledge of the angle  $\psi_L$ , the yaw rate  $\dot{\psi}$  and the longitudinal velocity  $v_x$ . Consider

$$\dot{\psi}_L = \dot{\psi} - v_x \rho \quad (10)$$

it is easy to express the road curvature by the algebraic equation  $\rho = \dot{\psi} - \dot{\psi}_L / v_x$ , which is valid for all  $v_x \neq 0$ . The time derivative of  $\psi_L$  is obtained by a HOSMD noted  $\dot{\psi}_{Le}$ . Finally, the equation allowing the estimation of the road curvature is

$$\hat{\rho} = \frac{\dot{\psi} - \dot{\psi}_{Le}}{v_x}. \quad (11)$$

#### B. Lateral Velocity and Tire Forces Estimation

Before designing the observer that estimates the lateral dynamics of a vehicle, the model (8) is transformed, by the following state transformation:

$$\begin{cases} x_1(t) = v_y \\ x_2(t) = \dot{\psi} \\ x_3(t) = \frac{1}{m}(F_f + F_r) \\ x_4(t) = \frac{1}{I_z}(a_f F_f - a_r F_r). \end{cases} \quad (12)$$

Let us assume that the relaxation parameters  $r_i$ ,  $i \in \{f, r\}$  have the same value, i.e.,  $r_f = r_r = r$ . The system (8) becomes

$$\begin{cases} \dot{x}_1(t) = -v_x x_2(t) + x_3(t) \\ \dot{x}_2(t) = x_4(t) \\ \dot{x}_3(t) = -\frac{v_x}{r} x_3(t) + \frac{v_x}{mr}(F_f^S + F_r^S) \\ \dot{x}_4(t) = -\frac{v_x}{r} x_4(t) + \frac{v_x}{I_z r}(a_f F_f^S - a_r F_r^S). \end{cases} \quad (13)$$

In this section, an observer design strategy is proposed by using the system presented in (13).

1) *TS Formulation of the Lateral Dynamics Model:* In this section, we aim to rewrite the system (13) into a polytopic TS model, using the sector nonlinearity approach. For this, we proceed by expressing the nonlinearities in the model into a TS formulation. Note that the transformation is exact in a compact set of the state space, so there is no loss of information (the two models are equivalent in that set).

a) *TS formulation of the lateral forces  $F_f^S$  and  $F_r^S$ :* We aim to rewrite the nonlinear functions defining the lateral forces model  $F_f^S$  and  $F_r^S$  in a TS formulation. First, let us consider the expressions of Pacejka's forces (7). The objective is to express (7) in TS form as follows:

$$F_f^S = \sum_{i=1}^2 \mu_{fi}(\alpha_f) M_{fi} \alpha_f \quad (14)$$

with simple mathematical manipulations, it is easy to write

$$F_f^S = f(\alpha_f) \alpha_f \quad (15)$$

where

$$f(\alpha_f) = \mathcal{A}_f \frac{\sin(S_{3f}) \tan^{-1}(S_{2f})}{S_{3f} S_{2f}} + \mathcal{B}_f \frac{\sin(S_{3f}) \tan^{-1}(S_{2f}) \tan^{-1}(S_{1f})}{S_{3f} S_{2f} S_{1f}} \quad (16)$$

$$S_1 = B \alpha_f, \quad S_2 = B(1 - E) \alpha_f + E \tan^{-1}(S_1)$$

$$S_3 = C \tan^{-1}(S_2), \quad \mathcal{A}_f = BCD(1 - E)$$

$$\mathcal{B}_f = BCDE.$$

It is known that  $\sin(x)/x$  is defined on  $\mathbb{R}$  and when  $x \rightarrow 0$   $\sin(x)/x \rightarrow 1$ . It is the case also for the function  $\tan^{-1}(x)/x$ . It is obvious that  $\lim_{x \rightarrow 0} (\tan^{-1}(x)/x) = 1$ . Knowing that  $\alpha_f$  is a bounded angle, the function  $\tan^{-1}(x)/x$  is also bounded. The function  $f(\alpha_f)$  is bounded  $\forall \alpha_f$ :  $f_{\min} \leq f(\alpha_f) \leq f_{\max}$ . Let us define the  $f(\alpha_f)$  as a premise variable, it follows:

$$\mu_{f1}(\alpha_f) = \frac{f(\alpha_f) - f_{\min}}{f_{\max} - f_{\min}}, \quad \mu_{f2}(\alpha_f) = \frac{f_{\max} - f(\alpha_f)}{f_{\max} - f_{\min}}. \quad (17)$$

Thus, the exact T-S model is given by  $F_f^S = \sum_{i=1}^2 \mu_i(\alpha_f) M_i \alpha_f$ , where the parameters  $M_i$ ,  $i = 1, 2$ , are defined by  $M_1 = f_{\max}$  and  $M_2 = f_{\min}$ , describing the tire stiffness. The same reasoning is followed to establish the TS formulation of the rear lateral force  $F_r^S$ , so that one obtains:

$$F_r^S = \sum_{i=1}^2 \mu_{ri}(\alpha_r) M_{ri} \alpha_r. \quad (18)$$

Next, both  $F_f^S$  and  $F_r^S$  will be expressed in exact T-S formulations using the above formulas, the new expressions of these forces are as follows:

$$\begin{cases} F_f^S = \sum_{i=1}^2 \mu_{fi} \left( \left( \frac{-M_{fi}}{v_x} \frac{-a_f M_{fi}}{v_x} 0 0 \right) x + M_{fi} \delta_f(t) \right) \\ F_r^S = \sum_{j=1}^2 \mu_{rj} \left( \left( \frac{-M_{rj}}{v_x} \frac{a_r M_{rj}}{v_x} 0 0 \right) x \right) \end{cases} \quad (19)$$

where  $x^T(t) = [x_1(t) \ x_2(t) \ x_3(t) \ x_4(t)]^T$  is the state vector, the functions  $\mu_{if}$  and  $\mu_{jr}$  satisfy the convex sum property, i.e.,  $\sum_{i=1}^2 \mu_{fi} = 1$ ,  $\sum_{j=1}^2 \mu_{rj} = 1$  and  $0 \leq \mu_{fi}, \mu_{rj} \leq 1$ ,

$i, j = 1, 2$ . Using (19) in the vehicle lateral-drift model, its dynamics can be expressed in a T-S formulation

$$\dot{x}(t) = \sum_{i=1}^4 \mu_i(x(t))(A_i x(t) + B_i u(t)) \quad (20)$$

where  $u(t)$  is the steering angle  $\delta_f$ .  $0 \leq \mu_i \leq 1$ ,  $i = 1, 4$ , and  $\sum_{i=1}^4 \mu_i = 1$ . The matrices of the model are defined by  $A_1 = A_{11}$ ,  $A_2 = A_{12}$ ,  $A_3 = A_{21}$ , and  $A_4 = A_{22}$  and  $B_1 = B_{11}$ ,  $B_2 = B_{12}$ ,  $B_3 = B_{21}$ , and  $B_4 = B_{22}$ , where

$$A_{jk} = \begin{pmatrix} 0 & -v_x & 1 & 0 \\ 0 & 0 & 0 & 1 \\ a_{1jk} & a_{2jk} & -\frac{v_x}{r} & 0 \\ a_{3jk} & a_{4jk} & 0 & -\frac{v_x}{r} \end{pmatrix}, \quad B_{jk} = \begin{pmatrix} 0 \\ 0 \\ b_{1jk} \\ b_{2jk} \end{pmatrix} \quad (21)$$

where

$$\begin{aligned} a_{1jk} &= -\frac{1}{mr}(M_{fj} + M_{rk}) \\ a_{2jk} &= -\frac{1}{mr}(a_f M_{fj} - a_r M_{rk}) \\ a_{3jk} &= -\frac{1}{I_z r}(a_f M_{fj} - a_r M_{rk}) \\ a_{4jk} &= -\frac{1}{I_z r}(a_f^2 M_{fj} + a_r^2 M_{rk}) \\ b_{1jk} &= \frac{v_x}{mr} M_{fj}, \quad b_{2jk} = \frac{a_f v_x}{I_z r} M_{fj}. \end{aligned}$$

The weighting functions  $\mu_i$  are defined by

$$\begin{aligned} \mu_1 &= \mu_{f1} \times \mu_{r1}, \quad \mu_2 = \mu_{f2} \times \mu_{r1} \\ \mu_3 &= \mu_{f1} \times \mu_{r2}, \quad \mu_4 = \mu_{f2} \times \mu_{r2}. \end{aligned} \quad (22)$$

Note that the developed TS model (20) with four submodels assumes that the longitudinal velocity  $v_x$  is constant as commonly used in the literature for lateral dynamics representation. In this brief, an extension to time-varying velocity is considered, then the TS system (20) becomes

$$\dot{x}(t) = \sum_{i=1}^4 \mu_i(x(t)) (A_i(v_x(t))x(t) + B_i(v_x(t))u(t)) \quad (23)$$

the matrices  $A_i(v_x(t))$  and  $B_i(v_x(t))$  are defined in (21), where  $v_x$  is time-varying and replaced by  $v_x(t)$ . Since  $v_x(t)$  is measured and bounded as follows  $v_x^{\min} \leq v_x(t) \leq v_x^{\max}$ ,  $\forall t$ , which is a realistic and nonrestrictive assumption in practical situations, the time-varying parameter  $v_x$  in the matrices  $A_i(v_x(t))$  and  $B_i(v_x(t))$  can be rewritten in TS formulation by applying the sector nonlinearity approach. One obtains  $A_i(v_x(t)) = \sum_{k=1}^2 \mu_{vk}(v_x(t)) A_i^k$  and  $B_i(v_x(t)) = \sum_{k=1}^2 \mu_{vk}(v_x(t)) B_i^k$ . The global TS model of the considered subsystem  $S_1$  is then given by (24) with eight submodels, where the weighting functions depend on the system's state variables, which are not totally measurable

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^8 h_i(z(t)) (A_i x(t) + B_i u(t)) \\ y(t) = Cx(t). \end{cases} \quad (24)$$

The output of the system is the measured yaw rate  $\dot{\psi}(t)$ , which leads to the following observation matrix  $C =$

$(0 \ 1 \ 0 \ 0)$ , and the vector of premise variables is  $z(t) = (v_x(t) \ v_y(t) \ \dot{\psi}(t) \ \delta_f)^T$ , note that this vector is partially measured due to the presence of  $v_y(t)$ . Next, the obtained TS model of the vehicle is used to design a nonlinear observer to estimate the lateral vehicle velocity  $v_y(t)$  and the lateral forces  $F_f$  and  $F_r$  acting on the vehicle wheels with accessible measurements  $\delta_f(t)$ ,  $v_x(t)$ , and  $\dot{\psi}$ . For that purpose, the proposed approach follows the steps:

- 1) use the vision system to estimate exactly the premise variables by algebraic and sliding mode techniques;
- 2) transform the TS system with unmeasurable premise variables into an equivalent TS system with measured premise variables.

2) *Premise Variables Estimation*: As presented previously, the premise variables vector  $z(t)$  depends on the measured variables  $\dot{\psi}$ ,  $v_x$ , and  $\delta_f$  and the unmeasured variables  $v_y$ . By examining the vision system, giving the equation of the lateral offset  $y_L$

$$\dot{y}_L = v_y + v_x \psi_L + l_s \dot{\psi}_L. \quad (25)$$

From (25), it is possible to express  $v_y$  with respect to the measured variables and their time derivatives as follows:

$$v_y = \dot{y}_L - v_x \psi_L - l_s \dot{\psi}_L. \quad (26)$$

The TS system becomes a model with premise variables depending only on measured variables  $v_x$ ,  $\psi$ , and  $\psi_L$  and the time derivatives  $\dot{y}_L$  and  $\dot{\psi}_L$  of  $y_L$  and  $\psi_L$ , respectively. Then, the TS system is equivalent to the following:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^8 h_i(s(t)) (A_i x(t) + B_i u(t)) \\ y(t) = Cx(t) \end{cases} \quad (27)$$

where  $s(t) = (v_x \ \dot{\psi} \ \psi_L \ \dot{y}_L \ \dot{\psi}_L \ \delta_f)^T$ .

*Remark 1*: Note that the system (27) is equivalent to the first one if the time derivatives of  $\psi_L$  and  $y_L$  are computed exactly  $\forall t \geq 0$ . However, this is not true in practical situation, for example, if a HOSMD is used, the time derivatives are obtained exactly after a given finite time  $T$ .

In the light of Remark 1, the transient phase of the differentiators is taken into account by defining the vector  $s_e(t)$ , which contains the measured variables and the outputs of the differentiators as follows:

$$s(t) = (v_x \ \dot{\psi} \ \psi_L \ \dot{y}_{Le} \ \dot{\psi}_{Le} \ \delta_f)^T \quad (28)$$

where  $\dot{y}_{Le}$  and  $\dot{\psi}_{Le}$  are the time derivatives of  $y_L$  and  $\psi_L$  obtained from HOSMDs. The interest of such an approach is that  $\forall t > T$ , the vector  $s_e(t) = s(t)$  (finite time exact convergence). To consider Remark 1, the system (27) is equivalent to

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^8 h_i(s_e(t)) (A_i x(t) + B_i u(t)) + \Gamma \delta(t) \\ y(t) = Cx(t) \end{cases} \quad (29)$$

where

$$\delta(t) = \begin{cases} \sum_{i=1}^8 (h_i(s(t)) - h_i(s_e(t))) (\mathcal{A}_i x(t) + \mathcal{B}_i u(t)) & t \leq T \\ 0 & t > T \end{cases} \quad (30)$$

and

$$\Gamma = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}^T.$$

Then, the proposed observer takes the form

$$\begin{cases} \dot{\hat{x}}(t) = \sum_{i=1}^8 h_i(s_e(t)) (\mathcal{A}_i \hat{x}(t) + \mathcal{B}_i u(t) + L_i (y(t) - \hat{y}(t))) \\ \hat{y}(t) = \mathcal{C} \hat{x}(t). \end{cases} \quad (31)$$

Let us consider the state estimation error  $e(t) = x(t) - \hat{x}(t)$ . It obeys to the following differential equation:

$$\dot{e}(t) = \sum_{i=1}^8 h_i(\hat{x}(t)) ((\mathcal{A}_i - L_i \mathcal{C}) e(t) + \Gamma \delta(t)). \quad (32)$$

At this stage, the objective is to compute the gains  $L_i$  of the observer to ensure exponential stability of the system (32). The following theorem provides sufficient linear matrix inequality (LMI) constraints to design the gains  $L_i$  guaranteeing the exponential convergence of the state estimation error.

*Theorem 1:* Given a positive scalar  $\alpha$ . If there exist a positive definite matrix  $P \in \mathbb{R}^{n \times n}$ , gain matrices  $M_i \in \mathbb{R}^{n \times n_y}$  and a positive scalar  $c$  solution to the following LMI constraints:

$$\begin{pmatrix} \mathcal{A}_i^T P + P \mathcal{A}_i - M_i \mathcal{C} - \mathcal{C}^T M_i^T + \alpha P & P \Gamma \\ \Gamma^T P & -c I_2 \end{pmatrix} < 0, \quad i = 1, \dots, 8 \quad (33)$$

then the state estimation error converges exponentially to zero according to the following inequality:

$$\|e(t)\|_2 < \begin{cases} \sqrt{\frac{\alpha_2}{\alpha_1}} \|e(0)\|_2 e^{-\frac{\alpha}{2}t} + \sqrt{\frac{c}{\alpha \alpha_1}} \max_{\tau \in [0, T]} \|\delta(\tau)\|_2 & t \in [0, T] \\ \sqrt{\frac{\alpha_2}{\alpha_1}} \left( \|e(0)\|_2^2 + \frac{c}{\alpha \alpha_1} \max_{\tau \in [0, T]} \|\delta(\tau)\|_2^2 e^{\alpha T} \right) e^{-\frac{\alpha}{2}t} & t \in [T, \infty) \end{cases} \quad (34)$$

where  $\alpha_1$  and  $\alpha_2$  are given by the lower and the upper eigenvalues of the matrix  $P$  (i.e.,  $\alpha_1 = \lambda_{\min}(P)$  and  $\alpha_2 = \lambda_{\max}(P)$ ). The gains of the observer are derived from  $L_i = P^{-1} M_i$  and the decay rate of the exponential convergence is given by the scalar  $\alpha/2$ .

*Proof:* The proof of the theorem is divided into two parts: the first one demonstrates the exponential convergence of the observer and the second one provides sufficient LMI conditions to ease the design of the observer ensuring exponential stability.

*a) Exponential convergence proof:* Let us consider the Lyapunov function

$$V(e(t)) = e^T(t) P e(t), \quad P = P^T > 0. \quad (35)$$

Its time derivative along the trajectory of  $e(t)$  is

$$\begin{aligned} \dot{V}(e(t)) &= e^T(t) \left( \sum_{i=1}^8 \mu_i(s_e(t)) (\Phi_i^T P + P \Phi_i) \right) \\ &\quad \times e(t) + e^T(t) P \Gamma \delta(t) + \delta^T(t) \Gamma^T P e(t) \end{aligned} \quad (36)$$

where  $\Phi_i = \mathcal{A}_i - L_i \mathcal{C}$ . By adding and subtracting the term  $\alpha e^T(t) P e(t) - c \delta^T(t) \delta(t)$ , where  $\alpha$  and  $c$  are the positive scalars, one obtains

$$\dot{V}(e(t)) = \sum_{i=1}^8 \mu_i(s_e(t)) \zeta^T(t) \Xi_i \zeta(t) - \alpha e^T(t) P e(t) + c \delta^T(t) \delta(t) \quad (37)$$

where  $\zeta(t) = [e^T(t) \ \delta^T(t)]^T$  and

$$\Xi_i = \begin{pmatrix} \Phi_i^T P + P \Phi_i + \alpha P & P \Gamma \\ \Gamma^T P & -c I_2 \end{pmatrix}, \quad i = 1, \dots, 8. \quad (38)$$

If there exist matrices  $P$  and  $L_i$  and positive scalars  $\alpha$  and  $c$  such that  $\sum_{i=1}^8 \mu_i(s_e(t)) \zeta^T(t) \Xi_i \zeta(t) < 0$ , then, the time derivative of the Lyapunov functions (37) can be bounded as follows:

$$\dot{V}(e(t)) < -\alpha e^T(t) P e(t) + c \delta^T(t) \delta(t). \quad (39)$$

The solution of this differential inequality is bounded as follows:

$$V(e(t)) < V(e(0)) e^{-\alpha t} + \frac{c}{\alpha} \max_{\tau \in [0, t]} \|\delta(\tau)\|_2^2. \quad (40)$$

From this inequality, and since the term  $\max_{\tau \in [0, t]} \|\delta(\tau)\|_2^2$  is bounded in the interval  $[0, t]$ , the ISS is then ensured. But if we analyze this inequality in two time intervals  $t \in [0, T]$  and  $t \in [T, \infty)$ , one can conclude that the state estimation error converges exponentially to zero. To prove this claim, let us begin by the first interval  $t \in [0, T]$ , the state estimation error norm is bounded by the inequality (40) and at time  $T$ , it follows:

$$V(e(T)) < V(e(0)) e^{-\alpha T} + \frac{c}{\alpha} \max_{\tau \in [0, T]} \|\delta(\tau)\|_2^2. \quad (41)$$

Now, considering the second interval time, it leads to

$$V(e(t)) < V(e(T)) e^{-\alpha(t-T)} + \frac{c}{\alpha} \max_{\tau \in [T, t]} \|\delta(\tau)\|_2^2 \quad (42)$$

and knowing that after  $t = T$ , the term  $\delta(t) = 0$ , then,  $\max_{\tau \in [T, t]} \|\delta(\tau)\|_2^2 = 0$ , which leads to

$$V(e(t)) < \left( V(e(0)) e^{-\alpha T} + \frac{c}{\alpha} \max_{\tau \in [0, T]} \|\delta(\tau)\|_2^2 \right) e^{-\alpha(t-T)} \quad (43)$$

$$< \left( V(e(0)) + \frac{c}{\alpha} \max_{\tau \in [0, T]} \|\delta(\tau)\|_2^2 e^{\alpha T} \right) e^{-\alpha t} \quad (44)$$

which proves the exponential convergence of the state estimation error to zero after time  $T$ . The decay rate is given by  $\alpha/2$

as follows:

$$\|e(t)\|_2 < \sqrt{\left(\frac{\alpha_2}{\alpha_1}\|e(0)\|_2^2 + \frac{c}{\alpha\alpha_1} \max_{\tau \in [0, T]} \|\delta(\tau)\|_2^2 e^{\alpha T}\right) e^{-\frac{\alpha}{2}t}}, \quad t \in [T, \infty). \quad (45)$$

Therefore, the state estimation error is bounded in the interval  $[0, T]$  (transient phase of the differentiators) and converges exponentially to zero after time  $T$ , which corresponds to the time of exact convergence of all the differentiators, then, the inequality (34) in Theorem 1 is obtained, which proves the exponential convergence of the state estimation error toward zero.

*b) Sufficient LMI conditions:* As pointed out in the previous section, the exponential convergence is obtained if  $\sum_{i=1}^r \mu_i(s_e(t))\zeta^T(t)\Xi_i\zeta(t) < 0$ . Due to the convex sum property of the weighting functions, sufficient conditions guaranteeing  $\sum_{i=1}^r \mu_i(s_e(t))\zeta^T(t)\Xi_i\zeta(t) < 0$  can be expressed by  $\Xi_i < 0$ ,  $i = 1, \dots, 8$ . With the change of variables  $M_i = PL_i$ , the LMIs given in Theorem 1 are easily obtained, which ends the proof. ■

After estimating the states of the subsystem  $S_1$  in the new coordinates (12), the estimated forces can be obtained easily from the algebraic equations

$$\hat{F}_f = \frac{ma_r\hat{x}_3(t) + I_z\hat{x}_4(t)}{a_f + a_r}, \quad \hat{F}_r = \frac{ma_f\hat{x}_3(t) - I_z\hat{x}_4(t)}{a_f + a_r}. \quad (46)$$

#### IV. EXPERIMENTAL RESULTS

In this section, the proposed observation strategy is implemented with real data obtained by a scenario realized in the track of Versailles (France). The track is 3.5-km length with different profiles. The lookahead lateral offset and the relative yaw angle are measured using clustering of a video camera mounted under the mirror of the vehicle and vision algorithms [6], [14]. An inertial unit provides measurement of the yaw rate  $\dot{\psi}(t)$  and an optical encoder provides the steering angle  $\delta_f(t)$ . The vehicle longitudinal velocity is measured by an odometer and the lateral one is obtained by a CORREVIT sensor. Note that the proposed strategy uses the time-varying longitudinal velocity. Finally, the measured forces are generated from the validated nonlinear Pacejka's model [20] with the measured signals:  $v_y$ ,  $v_x$ ,  $\dot{\psi}$ , and  $\delta_f$ .

The road curvature  $\rho(t)$  and the premise variable  $v_y(t)$  are estimated by the algebraic technique and by the use of a third-order sliding mode differentiator [16]. After that, the estimated premise variable is used in the proposed TS observer to reestimate the lateral velocity due to the inaccuracies of the initial estimate obtained from the algebraic technique. In addition, the TS observer provides an estimation of the lateral forces. The gains of the observer are computed by solving the LMIs in Theorem 1, with  $\alpha = 0.001$ . The observer is initialized to  $\hat{x}(0) = [0 \ 0 \ 0 \ 0]$  and implemented with the sampling period 0.001. In our scenario, the constants are fixed to  $M_{f1} = 4.2 \times 10^4$ ,  $M_{f2} = 3.3 \times 10^4$ ,  $M_{r1} = 4.9 \times 10^4$ , and  $M_{r2} = 3.3 \times 10^4$ , and the bounds related to the longitudinal

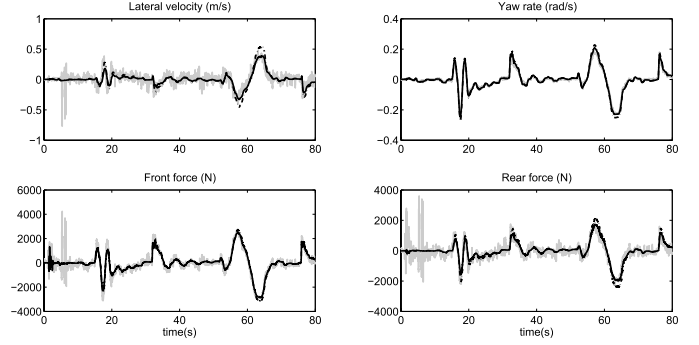


Fig. 1. TS model validation (gray solid line: real data, black dashed line: nonlinear model without force relaxations, and black solid line: TS model with force relaxations).

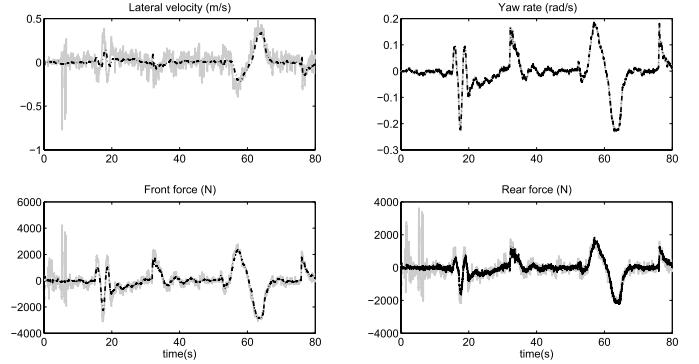


Fig. 2. Lateral dynamics estimations compared with real data (gray solid line: real data and black dashed line: TS observer).

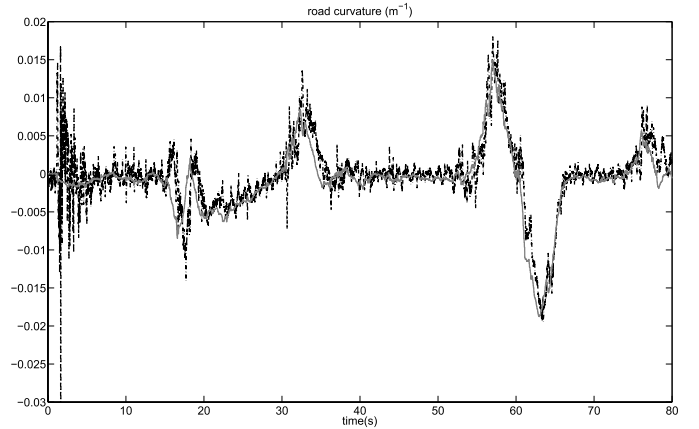


Fig. 3. Road curvature estimation (gray solid line: real data and black dashed line: estimation).

velocity are fixed to  $v_x^{\min} = 0.2$  m/s and  $v_x^{\max} = 17$  m/s. Note that these parameters can be chosen in such a way to enlarge the domain of the validity of the TS model and hence considers extreme nonlinear behaviors. First, the developed TS model is compared with the nonlinear one. Fig. 1 shows the comparison between the TS model and the measurements. Fig. 1 shows that the model fits perfectly the measurements, which reflects the very accurate representation of the nonlinear model by the TS model even when the steering angle input values vary highly to include all the nonlinear dynamics. Second, the proposed observation strategy is exploited and

TABLE I  
USED PARAMETERS

Time varying variables	
$y_L, \psi_L$	lateral deviation ( $m$ ) and heading angle ( $rad$ ) at a look ahead distance
$\rho, \dot{\psi}$	road curvature ( $m^{-1}$ ) and yaw rate ( $rad/s$ )
$v_x, v_y$	longitudinal and lateral velocities ( $m/s$ )
$F_f^S, F_r^S$	front and rear steady state lateral efforts ( $N$ )
$F_f, F_r$	front and rear lateral efforts ( $N$ )
Constant parameters	
$l_s = 0.5m, r_f = 0.1, r_r = 0.1$	look ahead distance ( $m$ ) and the relaxation length of the tires
$m = 1500Kg, I_z = 2454Kg.m^2$	mass of the vehicle ( $Kg$ ) and the yaw Inertia ( $Kg.m^2$ )
$a_f = 1.1m, a_r = 1.44m$	distances from the front and rear axle to the center of gravity ( $m$ )
$B_i, C_i, D_i$ and $E_i, i = \{f, r\}$	describe the characteristics of the tires in the Pacejka's model

the results are shown in Figs. 2 and 3. One can observe that the TS observer provides satisfactory estimations of the lateral velocity and the lateral forces. In addition, the simple algebraic technique provides an acceptable estimation of the road curvature (Fig. 3).

## V. CONCLUSION

In this brief, an observer design strategy is proposed to estimate the lateral dynamics of a vehicle and the road curvature. The nonlinear model of the vehicle dynamics is used and transformed into an exact TS model with unmeasured state-dependent weighting functions. The proposed TS observer is decomposed into two steps: the first step aims to estimate, because of the vision system, the premise variables of the TS model. This is performed by the algebraic techniques and the HOSMDs. In the second step, the estimated premise variables are exploited to design an observer that converges exponentially toward zero. The stability of the observer is proved using the Lyapunov theory and a quadratic Lyapunov function. The established exponential stability conditions are then expressed in terms of LMIs to ease the computation of the gains of the observer. Finally, the road curvature is recovered by a simple algebraic equation provided by the vision system. This estimation uses the time derivatives of the measured signals, which have been obtained from a HOSMD. The observation strategy is implemented successfully with real data.

## APPENDIX

See Table I shown at the top of this page.

## REFERENCES

- [1] J. Ackermann, T. Bunte, W. Sienel, H. Jeebe, and K. Naab, "Driving safety by robust steering control," in *Proc. Int. Symp. Adv. Veh. Control*, 1996.
- [2] G. Baffet, A. Charara, and J. Stephant, "Lateral vehicle-dynamic observers: Simulations and experiments," *Int. J. Veh. Auto. Syst.*, vol. 5, nos. 3–4, pp. 184–203, 2007.
- [3] E. Bakker, H. B. Pacejka, and L. Lidner, "A new tire model with an application in vehicle dynamics studies," *SAE Trans.*, vol. 98, no. 6, pp. 83–93, 1989.
- [4] H. Dahmani, M. Chadli, A. Rabhi, and A. El Hajjaji, "Design of unknown inputs robust fuzzy observer for lane departure detection," *Int. J. Veh. Des.*, vol. 56, nos. 1–2, pp. 186–202, 2011.
- [5] J. Davila, L. Fridman, A. Pisano, and E. Usai, "Finite-time state observation for non-linear uncertain systems via higher-order sliding modes," *Int. J. Control*, vol. 82, no. 8, pp. 1564–1574, 2009.
- [6] N. M. Enache, S. Mammar, B. Lusetti, and Y. Sebsadji, "Active steering assistance for lane keeping and lane departure prevention," *J. Dyn. Syst., Meas., Control*, vol. 133, no. 6, p. 061003, Sep. 2011.
- [7] N. M. Enache, S. Mammar, M. Netto, and B. Lusetti, "Driver steering assistance for lane-departure avoidance based on hybrid automata and composite Lyapunov function," *IEEE Trans. Intell. Transp. Syst.*, vol. 11, no. 1, pp. 28–39, Mar. 2010.
- [8] M. Fliess, C. Join, and H. Sira-Ramirez, "Non-linear estimation is easy," *Int. J. Model., Identificat. Control*, vol. 4, no. 1, pp. 12–27, Jan. 2008.
- [9] J.-P. Gauthier and I. Kupka, *Deterministic Observation Theory and Applications*. Cambridge, U.K.: Cambridge Univ. Press, 2001.
- [10] T.-M. Guerra, A. Kruszewski, L. Vermeiren, and H. Tirmant, "Conditions of output stabilization for nonlinear models in the Takagi–Sugeno's form," *Fuzzy Sets Syst.*, vol. 157, no. 9, pp. 1248–1259, May 2006.
- [11] D. Ichalal, B. Marx, J. Ragot, and D. Maquin, "Brief paper: State estimation of Takagi–Sugeno systems with unmeasurable premise variables," *IET Control Theory Appl.*, vol. 4, no. 5, pp. 897–908, 2010.
- [12] D. Koenig, "Unknown input proportional multiple-integral observer design for linear descriptor systems: Application to state and fault estimation," *IEEE Trans. Autom. Control*, vol. 50, no. 2, pp. 212–217, Feb. 2005.
- [13] A. J. Krener and W. Respondek, "Nonlinear observers with linearizable error dynamics," *J. Control Optim.*, vol. 23, pp. 197–216, 1985.
- [14] R. Labayrade, J. Douret, J. Laneurit, and R. Chapuis, "A reliable and robust lane detection system based on the parallel use of three algorithms for driving safety assistance," *IEICE Trans. Inform. Syst.*, vol. E89-D, no. 7, pp. 2092–2100, Jul. 2006.
- [15] Z. Lendek, R. Babuska, and B. De Schutter, "Stability of cascaded fuzzy systems and observers," *IEEE Trans. Fuzzy Syst.*, vol. 17, no. 3, pp. 641–653, Jun. 2009.
- [16] A. Levant, "Higher-order sliding modes, differentiation and output-feedback control," *Int. J. Control*, vol. 76, nos. 9–10, pp. 924–941, 2003.
- [17] S. Mammar, S. Glaser, and M. Netto, "Vehicle lateral dynamics estimation using unknown input proportional-integral observers," in *Proc. Amer. Control Conf.*, Minnesota, MN, USA, Jun. 2006.
- [18] B. Marx, D. Koenig, and J. Ragot, "Design of observers for Takagi–Sugeno descriptor systems with unknown inputs and application to fault diagnosis," *IET Control Theory Appl.*, vol. 1, no. 5, pp. 1487–1495, Sep. 2007.
- [19] M. Oudghiri, M. Chadli, and A. El Hajjaji, "Robust observer-based fault-tolerant control for vehicle lateral dynamics," *Int. J. Veh. Design*, vol. 48, nos. 3–4, pp. 173–189, 2008.



- [20] H. B. Pacejka, *Tire and Vehicle Dynamics*. Warrendale, PA, USA: SAE International, 2005.
- [21] G. Phanomchoeng, R. Rajamani, and D. Piyabongkarn, "Nonlinear observer for bounded Jacobian systems, with applications to automotive slip angle estimation," *IEEE Trans. Autom. Control*, vol. 56, no. 5, pp. 1163–1170, May 2011.
- [22] K. Tanaka and H. O. Wang, *Fuzzy Control Systems Design and Analysis: A Linear Matrix Inequality Approach*. New York, NY, USA: Wiley, 2001.
- [23] B. Wojciechowski, "Analysis and synthesis of proportional-integral observers for single-input-single-output time-invariant continuous systems," Ph.D. dissertation, Technical Univ Gliwice, Poland, 1978.
- [24] Z. Yacine, D. Ichalal, N. A. Oufroukh, S. Mammar, and S. Djennoune, "Nonlinear vehicle lateral dynamics estimation with unmeasurable premise variable Takagi–Sugeno approach," in *Proc. 20th MED*, Jul. 2012, pp. 1117–1122.
- [25] A. Zemouche, M. Boutayeb, and G. I. Bara, "Observers for a class of Lipschitz systems with extension to  $H_\infty$  performance analysis," *Syst. Control Lett.*, vol. 57, no. 1, pp. 18–27, Jan. 2008.