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Alejandro Perez

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Statistical and entanglement entropy for black holes in quantum geometry

Alejandro Perez[1]

[1] *Aix Marseille Universit, CNRS, CPT, UMR 7332, 13288 Marseille, and Universit de Toulon, CNRS, CPT, UMR 7332, 83957 La Garde, France.*

We analyse the relationship between entanglement (or geometric) entropy with statistical mechanical entropy of horizon degrees of freedom when described in the framework of isolated horizons in loop quantum gravity. We show that, once the relevant degrees of freedom are identified, the two notions coincide. The key ingredient linking the two notions is the structure of quantum geometry at Planck scale implied by loop quantum gravity, where correlations between the *inside* and *outside* of the black hole are mediated by eigenstates of the horizon area operator.

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In the semiclassical regime where an approximate notion of black hole (BH) makes sense, horizon area is quantized in loop quantum gravity (LQG) [1–3]. As a consequence, the dimensionality of the surface Hilbert space, compatible with a maximum macroscopic area, is finite and grows exponentially with the area in the large area limit [4, 5]. In other words, if one treats the horizon itself as a thermodynamical system in equilibrium—justified in the case of large semiclassical black holes—then the micro canonical entropy, in the ensemble of surface states, is proportional to the macroscopic black hole area. The relevant ensemble is the one defined by the horizon surface states.

The standard loop quantum gravity counting yields [6, 7]

$$S_{stat} = \frac{\gamma_0}{\gamma \frac{G}{G_N}} \frac{A}{4 G_N \hbar} \tag{1}$$

where G is the UV value of the gravitational constant at the fundamental scale, G_N is the low energy Newton's constant, γ is the Immirzi parameter, and γ_0 is a numerical constant that appears in the asymptotic expression of the number of states.

The above result is compatible with Bekenstein-Hawking's entropy only if one fixes

$$\gamma = \gamma_0 \frac{G_N}{G}. \tag{2}$$

The necessity of fixing γ in order to achieve consistency with the semiclassical regime is puzzling. The reason is that the Immirzi parameter is a topological coupling in the gravity first order action not affecting the classical equations of motion (like the θ parameter of Yang-Mills theories [8, 9]). Therefore, in contrast with expression (1), one would expect to find γG only in quantum corrections of a leading term matching exactly the Bekenstein-Hawking entropy (as the Immirzi parameter plays an important role in the quantum theory where geometric operators have discrete spectra in the fundamental area $\ell^2_{LQG} = \gamma G \hbar$).

One logical possibility [10] is to interpret equation (2) as a renormalization condition fixing the relationship between G_N and G. If that were the case, then the numerical value γ_0 would have to have a clear interpretation in the renormalization group flow of gravity. However, due to the completely combinatorial way in which γ_0 arises (which does not make reference to any dynamical notion) it is so far unclear how such scenario could be realized.

There is however a recent new perspective that might be helpful in resolving this question. The idea is that vacuum fluctuation in the non geometric sector (including matter degrees of freedom) should be appropriately taken into account in the computation of black hole entropy (as we will argue in Section I B all degrees of freedom close to the horizon must be accounted for). This is not so in the treatment leading to (4) as only pure-geometry area eigenvalues are counted while ignoring the degeneracy associated to other degrees of freedom (including matter). Indeed by taking these degrees of freedom suitably into account one obtains[11], for arbitrary values of γ,

$$S_{stat} = \frac{A}{4 G_N \hbar} + \eta \frac{\sqrt{A}}{\sqrt{\gamma G \hbar}} \tag{3}$$

where η a dimensionless constant that depends on the punctures statistics. As expected the fundamental scale $\gamma G \hbar$ appears only in the quantum corrections to the Bekenstein-Hawking entropy.

The fundamental surface degrees of freedom associated to matter fields are hard to describe in LQG. Thus vacuum fluctuations in the analysis leading to the previous equation are accounted for by using (qualitative) information provided by standard QFT on the black hole background (in a spirit analogous to what is done in cosmological situations [12]). In particular there is an expected contribution to black hole entropy from the entanglement entropy of quantum fields across the black hole horizon [13]. In standard QFTs the result is

$$S_{ent} = \lambda \frac{A}{G_N \hbar} + corrections. \tag{4}$$

with λ left undetermined due to UV divergences and other ambiguities such as the species problem (concretely, $\lambda = \lambda_0 G_N \hbar \epsilon^{-2}$ for λ_0 a regularization dependent dimensionless constant and ϵ a UV cut-off length).

In the analysis of [11] equation (3) follows from the assumption that the degeneracy contribution to the area

spectrum coming from vacuum fluctuations of non geometric degrees of freedom is given by the exponential of (4) with an undetermined λ (this justifies the term *qualitative* used above [1]). In the perturbative quantum gravity framework there are indications that the ambiguities encoded in the value of λ disappear if the gravitational degrees of freedom are correctly taken into account [18]. In the non perturbative framework, such possibility is reinforced in a completely independent way by the results of [11] where consistency fixes $\lambda = 1/4$ up to quantum corrections.

However, as just mentioned, in the argument that leads to (3) one assumes that one can interpret the (exponential of the) entanglement entropy of quantum fields across the horizon as a measure of the degeneracy of the area spectrum of the horizon (including all the UV degrees of freedom). This assumption necessitates a suitable relationship between entanglement and statistical entropy. Exploring and establishing the degree to which such relationship holds in the LQG treatment of BHs is one of the aims of this work. Some aspects concerning this question have been considered in previous literature (for a recent discussion see [19]). Entanglement of apparent horizons in toy theories have been considered in [20], regions of spin network states have been studied in [21], and it is an important part of the argument of [17].

Finally, we would like to point out in this introduction that there is a well know relationship between entanglement entropy and Wald's entropy [22] (for a review see [23]). The present analysis deals with a similar question but at the more fundamental Planckian level where a statistical mechanical account of the thermodynamical properties of BHs becomes available (including the fully quantum treatment of gravitational contributions).

In Section I we will argue that the relevant degrees of freedom both for entanglement and statistical mechanical approaches to BH entropy are to be found on the local Planckian vicinity of the BH horizon. In making this case, we will revisit the results of [18], and gain new insights by discussing their generalisation to eternal static black holes in Section I A. In Section II we will briefly review the quantum description of the BH horizon quantum degrees of freedom in LQG. With this framework at hand we will establish the sense in which entanglement and statistical entropy are equivalent in quantum geometry in Section III.

[1] This type of holographic degeneracy appears naturally when considering the analytic continuation of the pure geometric degeneracy from real the Immirzi parameter γ to $\gamma \to \pm i$ [14, 15]. The same holographic behaviour of the number of degrees of freedom available at the horizon surface is found from a conformal field theoretical perspective for $\gamma = \pm i$ [16]. A relationship between the termal nature of BH horizons and self dual variables seems also valid according to similar analytic continuation arguments [17].

I. WHY SURFACE DEGREES OF FREEDOM?

In this section we will discuss a common feature between the entanglement approach to black hole entropy and the statistical mechanical approach followed in loop quantum gravity: that the relevant correlations are the UV correlations across the BH horizon in the first case; that the relevant microstates are local excitation of the horizon geometry and matter on the horizon in the second case. In both cases a separation of scales is necessary to isolate the relevant physics behind the notion of BH entropy. Once this ingredient is incorporated to the analysis we will show that both approaches actually compute the same thing: namely, the Boltzmann-Gibbs entropy of the ensemble of surface microstates. Therefore, from the appropriate perspective, BHs are standard thermodynamical objects where the horizon plays the role of the statistical mechanical system.

A. Long range correlation in entanglement entropy

Let us start with some discussion of the entanglement approach. In particular we want to point out the influence of global features of the quantum state considered when trying to compute the entanglement entropy associated to the presence of a horizon.

The Rindler wedge with its family of uniformly accelerated observers and associated Rindler horizon captures some of the physical aspects of black hole systems in their infinite area limit. Entanglement entropy of the Minkowski vacuum for the wedge is ill defined for the UV problems mentioned above or exactly zero if one uses the regularization prescription that is consistent with the semiclassical Einstein's equations compatible with the flat background (i.e. for which $\langle T_{\mu\nu}\rangle = 0$ [24]). We will come back to this important point in Section I B. However, the changes in the entanglement entropy (relative entropy [25]) under perturbations of the reduced density matrix $\rho = \rho_0 + \delta\rho$ turn out to be well defined [18] and give

$$\begin{aligned} \delta S_{ent} &= -\mathrm{Tr}[(\rho_0 + \delta\rho)\log(\rho + \delta\rho)] \\ &= 2\pi \int_\Sigma \delta\langle T_{\mu\nu}\rangle \chi^\mu d\Sigma^\nu \\ &= \frac{\delta A}{4G_N\hbar} \end{aligned} \tag{5}$$

where χ is the corresponding boost killing vector field (to which accelerated observers are tangent), Σ is an arbitrary Cauchy surface of the Rindler wedge, and δA is the change in the Rindler horizon area produced by the back reaction of the perturbation. The second line follows from the fact that, formally, the reduced density matrix ρ_0 obtained from the Minkowski pure state by tracing out the degrees of freedom inaccesible to the Rindler ac-

celerated observers takes the form

$$\rho_0 = \frac{\exp(-2\pi \int_\Sigma \hat{T}_{\mu\nu}\chi^\mu d\Sigma^\nu)}{Z}. \tag{6}$$

Conservation of the current $\delta\langle T_{\mu\nu}\rangle\chi^\mu$ can be used to express the second line as the energy momentum flux of the perturbation across the horizon. The Raychaudhuri equations for the horizon generators—which are a consequence of semiclassical Einsteins equations—imply the final result. In this sense the above equation is semiclassical and hence insensitive to the UV quantum gravity physics (as expected for a relative entanglement entropy).

One might have naively expected an additive term δS_∞ coming from the energy momentum flux to infinity in the direction normal to the horizon. However, as this corresponds to (a piece of) a single generator of $\mathscr{I}^+$ (future null infinity) the associated energy flux vanishes for a regular perturbation (e.g. compact support on Σ). A proof of such statement for massless scalar fields is given in [24], section 5.1. On physical grounds one expects this to be valid in general. However, we will see below that this potential entropy flux at infinity becomes non trivial when equation (5) is generalised to the black hole setting.

While it is true that a finite slab of proper width ℓ outside a stationary BH horizon is locally isometric with the corresponding slab of width ℓ of Rindler space-time in the limit where the BH area $A \to \infty$ while keeping ℓ fixed [26]; the two space times are clearly not the same globally in that limit. The difference is apparent when one considers the spacetime conformal compactification as in Figure 1. This is a key limitation of modelling black hole physics with a Rindler spacetime in quantum field theory where long range correlations can be important.

As mentioned, there is no entanglement entropy flow term at infinity in the Rindler analysis: due to the global structure of the Rindler horizon regular perturbations cannot avoid crossing the horizon and register their entropy content by an area change as in (5). However, the situation completely changes in the context of BHs. Notice first that the argument leading to (5) can be generalized in order to calculate perturbations to the entanglement entropy of the eternal static black hole in the Hartle-Hawking vacuum. This is achieved by replacing (6) with $\rho_0 = \exp(-2\pi\kappa^{-1}\int_\Sigma \hat{T}_{\mu\nu}\chi^\mu d\Sigma^\nu)$ where χ^u is the Killing field generating inertial time translations at infinity (such simple generalization might not be available for general stationary black holes where the analog of the Hartle-Hawking vacuum does not exist [27]). However, the BH generalization of (5) will contain a non vanishing extra term accounting for the energy-momentum flow of the perturbation at infinity, namely

$$\delta S^{BH}_{ent} = \frac{\delta A}{4G_N\hbar} + \delta S_\infty, \tag{7}$$

where δS_∞ is the entropy flow across future null and timelike infinity $\mathscr{I}^+ \cup i^+$.

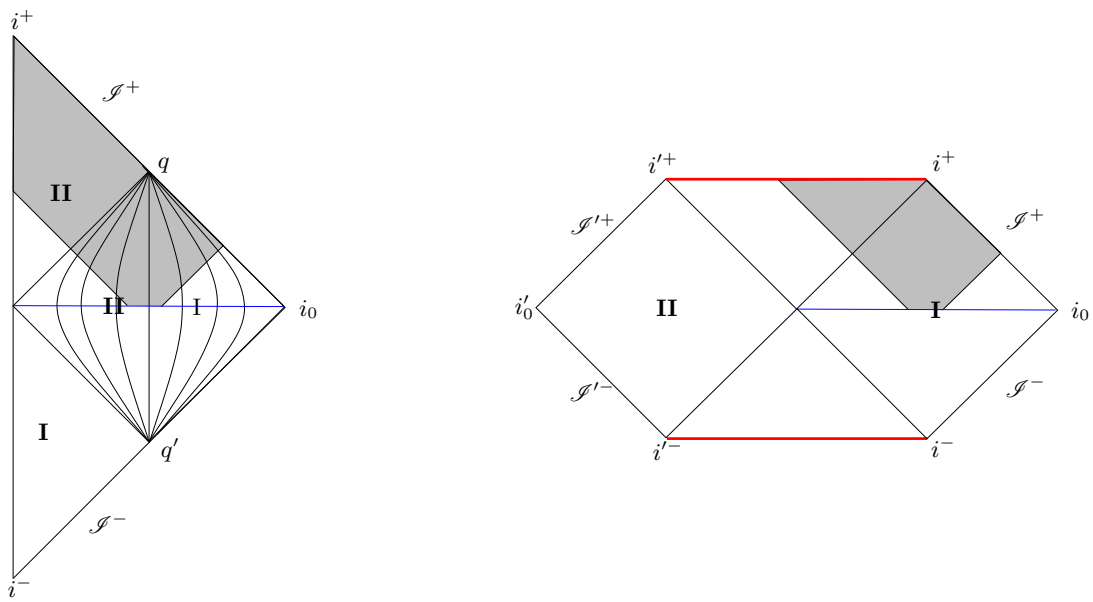

FIG. 1: **On the left:** Penrose diagram of Minkowski spacetime exhibiting a Rindler horizon. In the conformal compactification the Rindler horizon is the past light cone of a point $q \in \mathscr{I}^+$ and the future light cone of a point $q' \in \mathscr{I}^-$ minus a single generator of $\mathscr{I}^+$ and $\mathscr{I}^-$ respectively. This global picture implies that, for suitable fields and in contrast with a BH horizon, the Rindler horizon is a good initial value surface. The Rindler wedge corresponds here to region I. In the Rindler wedge portion of the diagram, bold face letters **II** in contrast to simply I tell us that I is separated from the readers 'view' by the Rindler horizon. **On the right:** The Penrose diagram of the eternal Schwarzschild BH, where radiation can escape to infinity without being registered on the BH horizon. The shaded regions represent a perturbation that has compact support at a Cauchy surface of region **I**: on the left, the energy flow of the perturbation cannot avoid crossing the Rindler horizon.

Thus equation (7) tells us two important pieces of information. The first one is that long range correla-

tions contribute non trivially to entanglement entropy but that these contributions are not related to BH entropy. Notice that one can have $\delta S_{ent} \neq 0$ with a perturbation that does not affect the horizon surface (the entanglement boundary) by arranging $\delta\langle T_{\mu\nu}\rangle$ to completely flow into $\mathscr{I}^+$ without crossing the horizon. Long range correlations—see equation (10)—which are present in massless fields can produce entropy flow to null infinity as well as to future timelike infinity (the classical counterpart of these are the so-called tails). Massive fields cannot produce entropy flow out to $\mathscr{I}^+$; however, for suitable perturbations (unbounded states of the perturbation) there are non trivial contributions to δS_∞ from the flow through i^+. The term at infinity in (7) can be written as[2]

$$\delta S_\infty = \frac{\delta E_+}{T_H}, \tag{8}$$

where δE_+ is the amount of energy flow to future infinity and $T_H = \kappa/(2\pi)$ is Hawking temperature. One can also write (7) as

$$\delta S_{ent}^{BH} = \frac{\delta M}{T_H} = \frac{\delta A}{4G_N\hbar} + \frac{\delta E_+}{T_H}, \tag{9}$$

where δM is the change in the ADM mass of the system due to the perturbation. Thus, even thought it is well known that bulk corrections exist in (4), the present analysis makes their geometric origin and thermodynamical meaning transparent.

All this implies that if one wants to describe black hole entropy from the entanglement idea one has to have the means of separating the bulk contributions to entanglement from the genuine horizon contributions. The criterion is locality: the term arising from the UV physics close to the entanglement boundary are the ones to be taken into account. Concretely, in standard QFT these are the modes that produce a UV diverging contribution or the ones that lead to a term proportional to the area. As we will discuss further below, the same separation of degrees of freedom is made in the statistical mechanical account of BH entropy in LQG.

[2] More generally, the flow of entropy across $\mathscr{I}^+$ can also be given a thermodynamical interpretation for stationary local observers inside as follows: assume that one gives initial conditions for the perturbation $\delta\rho$ on a Cauchy surface Σ for the exterior of the static BH space-time. Assume that $\langle\delta T_{\mu\nu}\rangle = \mathrm{Tr}[\hat{T}_{\mu\nu}\delta\rho]$ has compact support on Σ. Take a timelike hyper surface W defined by $r = r_0$ =constant intersecting Σ on a sphere with r_0 sufficiently large to contain the region where $\langle\delta T_{\mu\nu}\rangle \neq 0$. W represents a family of stationary observers surrounding the BH. Then one has that $\delta S_\infty^{\mathscr{I}^+} = -\delta E/T$ where δE is the energy-momentum flow across W, and $T = T_H/\|\xi\|$ is the local temperature of the Hartle-Hawking state as measured by the stationary observers at W (with $\|\xi\|$ the norm of the stationarity Killing field normalised at infinity). This is the the standard thermodynamical entropy flow of the perturbation across W as measured by these local observers.

The second indication from the perturbative result (5)—the main message of [18]—is that the inclusion of the gravitational degrees of freedom renders entanglement entropy finite and free from its usual regularization ambiguities and the species problem. This second conclusion is validated in the non perturbative by the analysis of [11] (given the relationship between entanglement and the number of surface microstates advocated here). The argument shows how the equations of state of the quantum horizon imply that the undetermined (UV divergent) constant λ (of equation (4)) is finite an takes the value $\lambda = 1/4$ (up to quantum corrections) in the semiclassical regime. As suggested in [18], gravitational effects resolve the ambiguity of the standard QFT entanglement account. However, as mentioned before, one key assumption in [11] is that the equation (4) can be directly interpreted as degeneracy of the area spectrum of the quantum horizon $D(A) = \exp(\lambda A/(G_N\hbar))$ due to non-geometric degrees of freedom. Before arguing that this is indeed correct let us discuss the question of the localisation of the relevant degrees of freedom accounting for BH entropy.

B. From the statistical mechanics perspective the relevant degrees of freedom are localised at the horizon

Mixed states do not exist in nature. However, the concept is the key mathematical tool for the description of systems in statistical mechanics which in turn is the fundamental basis of thermodynamics. One of its keystones in the construction is the introduction of a notion of coarse graining producing exact physical laws for macroscopic variables by treating microscopic details probabilistically. Statistical mechanics becomes useful in those situations where one can concentrate on suitable average quantities assumed to take well defined values up to (typically) small fluctuations whose exact details are ignored. The states of such systems are mathematically modelled by a statistical mixture of microstates (in a suitably defined ensemble) in quantum statistical mechanics. It is in terms of the latter that the notion of thermodynamic entropy becomes non trivial and meaningful.

Stationary black holes are, in a suitable sense, equilibrium statistical mechanical systems. As stationary space times they represent the final stage of gravitational collapse which is completely described, according to the no-hair theorem, by members of the Kerr-Newman family labelled by only three macroscopic parameters: the mass M, the angular momentum J, and the electromagnetic charge Q. These three macroscopic parameters are the coarse graining variables for the thermodynamical description of BHs.

The microstates are to be found in the local Planckian details of fields at the BH horizon. Stationary observers *outside* of the BH have causal access to the local degrees of freedom of matter and space-time as they crossed the

forming BH horizon. These degrees of freedom are entirely registered on the horizon; as a null surface whose domain of dependence is the interior of the BH. However, they are (infinitely) redshifted to late stationary *outside* observers who are, in this way, only sensitive to their UV or Planckian structure. The many different initial conditions leading to the formation of a black with the same M, J and Q are ironed out by this infinite redshift and become inaccessible in practice to macroscopic observers (much like the phase space position of molecules in a gas in thermal equilibrium). Moreover, because of the redshift factor vanishing at the horizon, these excitations appear as horizon *surface degrees of freedom* for (late) stationary *outside* observers. The latter include both the Planckian geometric as well as matter excitations ('vacuum fluctuations'). These are the micro states in the statistical mechanical description of BHs; they are, by the above argument, fundamentally quantum in nature, and they are localised at the BH horizon.

The thermal system is the horizon itself. This view is complemented by the fact that Hawking's effect of BH radiation is entirely due to the near horizon geometry in stationary BHs (for a concrete example see [28]).

1. *A perspective on information loss in BH systems*

According to the perspective advocated here (aiming at the statistical mechanical description of BH thermodynamics), a system producing a BH by gravitational collapse is to be described by a mixed state initially. The coarse graining variables are the final parameters M, J, and Q of a stationary BH, [3] while the large number of different initial conditions leading to it at late times are the microstates. As argued the latter are Planckian in nature and involve both matter as well as gravitational degrees of freedom.

In this context, dynamical evolution from the initially mixed-state need not to purify it in the late future. Even if the standard semiclassical expectation is correct and one ends up with a final state made purely of the thermal radiation emitted during BH evaporation, this will not represent by itself a paradox. In fact if the evolution is unitary one would expect the initial entropy of the mixed state on $\mathscr{I}_-$ simply to match that of the final state at $\mathscr{I}^+$.

However, this is not yet satisfactory to clarify the fate of information 'falling behind the BH event horizon'. This is so because one can set up a thought scenario where the initial state is indeed pure (one of the microstates leading to the formation of the macroscopic BH in late times). However, as mentioned above, this would necessitate the specification of Planckian degrees of freedom; therefore, the use of a quantum dynamical evolution that can only be described in a fully background independent manner (semiclassical arguments are just not the right language to describe the question of unitarity). If evolution is unitary [4] the end result would look—from the semiclassical perspective—as a thermal state for continuum field excitations (particles of QFT) but it would remain pure due to the correlations with the Planckian gravitational degrees of freedom that remain hidden to low energy (coarse) observers. If evolution is unitary a detailed account on how the correlations leak out of 'the BH region' during the long history of BH formation and subsequent evaporation would have to be given in the framework of the quantum theory. However, we have seen in the discussion of the present paper that non-local correlations are relevant in the discussion of entanglement between the *inside* and the *outside* of the horizon. It seems to us that here is no need to invoke other sources of non-locality such as the constraint structure of gravity as advocated in a proposal aiming at preserving unitarity in the much stronger sense of the ADS-CFT framework see [30].

The scenario described here seems quite plausible in the framework LQG, where even flat space time is expected to arise as very intricate linear combinations of polymer like quantum excitations. In such framework, the very notion of Minkowski is only approximate, i.e., arising in a suitable coarse grained sense [31–33]. Thus many different microstates will be classified as flat spacetime by low energy (standard QFT) observers for which Planckian details and correlations with the fundamental degrees of freedom remain hidden.

The picture proposed here is quite the analog of that of burning an encyclopaedia: information before and after is the same due to unitarity. Before burning we have the books in the atmosphere in some initial state. After burning the information remains in the correlations of the individual molecules of hot gas produced by the combustion. If we wait even longer, then the air in the atmosphere seems to recover its initial state which of course is not the same due to the everlasting correlations of the microscopic degrees of freedom. Observers that are not capable of reading these correlations must accept with resignation that information is (practically) lost.

[3] As, in the context of the information loss paradox discussion, the system is analyzed until complete evaporation of the BH to Hawking radiation; here M, J, and Q are the values of these parameters long time after the collapse but before back reaction of the hawking radiation starts being important.

[4] The possibility that information is lost to outside observers by quantum gravity in the context of gravitational collapse cannot be ruled out and does not represent any inconsistency with local unitarity [29]. In order to a quantum gravity dynamical description of the fate of the classical BH singularity is mandatory.

C. Black hole entropy as entanglement entropy: the relevant correlations are also the local ones across the horizon

In quantum field theory vacuum correlations at spacelike separated points in 3+1 dimensions diverge quadratically in the geodesic distance $d(x,y)$ separating them, namely

$$\langle\phi(\vec{x})\phi(\vec{y})\rangle \approx \frac{1}{|d(x,y)|^2}. \tag{10}$$

As a consequence of the local nature of correlations entanglement entropy in $3+1$ dimension (when calculated in standard QFT) diverges quadratically with the cut-off scale ϵ and is hence proportional to the boundary surface area A:

$$S_{ent} = \lambda_0 \frac{A}{\epsilon^2} + corrections, \tag{11}$$

where λ_0 is a regularisation dependent factor. The previous equation is the main motivation to view entanglement entropy as a possible candidate for an account BH entropy [13]. It can be proved rigorously for ground states of lattice systems with suitable interactions mimicking QFT systems [34]. Despite of the fact that entanglement entropy is not really well defined in the standard QFT scenario, entropy difference between two quantum states (relative entropy) is well defined and satisfies important properties [25, 35]. However, a description of entanglement entropy across the horizon in view of a fundamental explanation of BH entropy cannot be based on the concept of relative entropy in a background independent approach as the one of LQG. This is thus because relative entropy is insensitive to the UV physics that we have argued encodes the relevant degrees of freedom behind BH entropy. Thus, for the problem at hand, one necessitates a full quantum gravity formulation, or UV completion, that eliminates the ambiguities of entanglement entropy referred to above [5] . In what follows we describe what quantum geometry and LQG tell us about entanglement from the UV structure close to the BH horizon.

II. ACCOUNTING FOR SURFACE DEGREES OF FREEDOM IN LOOP QUANTUM GRAVITY

In this section we describe the isolated horizon model for the black hole horizon surface degrees of freedom. We will also show that the entanglement entropy of these degrees of freedom coincides in a suitable sense with the micro canonical entropy of the ensemble defined by the surface micro states.

Statistical mechanics of the Horizon degrees of freedom

Isolated horizons [40] is the standard mathematical framework for the statistical mechanical treatment of quantum black holes in LQG. The notion is an idealisation that allows to represent a suitable quasi-local definition of BH horizons in equilibrium by an infinite dimensional phase space to be quantized. While such framework might not capture all the physics associated to BHs in quantum gravity (in particular due to its local nature it is unclear how to encode in the formalism the long range correlations that are important in the description of certain quantum processes), it is expected to describe the local microscopic degrees of freedom necessary for the statistical mechanical account of semiclassical black holes.

The presentation of the basic geometric idea will be enough for the purposes of this letter. The basic idea behind isolated horizons can be understood from the perspective of the initial value problem of general relativity in terms of characteristic data on null surfaces. One considers a null surface Δ with topology $S^2 \times [0,1] \subset \mathbb{R}$ and a transversal null surface δ as shown in Figure 3. For simplicity here we concentrate on the spherically symmetric case (distortion and rotation can be included [41] but this is not central to the argument presented here).

Concretely, if one puts the characteristic data corresponding to the Schwarzschild horizon with area A on Δ and then completes the characteristic data with suitable Schwarzschild data on δ, then one recovers the Schwarzschild geometry as the unique solution of Einsteins equations (up to diffeomorphisms) in the domain of dependence of $\Delta\cup\delta$. This is a single point in the phase space Γ_A of spherically symmetric isolated horizons. The infinite dimensional phase space Γ_A is given by the set of solutions of Einstein's equations obtained by keeping the previous data fixed on Δ while setting arbitrary free characteristic data on δ. By fixing Schwarzschild data with (macroscopic) horizon area A on Δ the definition introduces the coarse graining that is the cornerstone of the thermodynamical description.

The phase space Γ_A can be quantised according to the standard canonical prescription once appropriate fundamental extended field variables are chosen [42]. The Hilbert space $\mathscr{H}$ of the system splits into two factors according to $\mathscr{H} = \mathscr{H}_\Delta \otimes \mathscr{H}_{out}$, where $\mathscr{H}_\Delta$ is the a boundary Hilbert space (describing the degrees of freedom of the isolated horizon), and $\mathscr{H}_{out}$ is the exterior volume Hilbert space (describing the physical degrees of freedom in the exterior of the BH horizon). The latter is defined by solutions of the dynamical constraints in the bulk which

[5] One would expect that gravitational effects would modify correlations at the microscopic scales. Interestingly if one bounds the correlations (10) by ℓ_{Pl}^{-2} then the Hawking effect is not affected in the large BH limit [36]. Such bound for correlations are confirmed by spherically symmetric model calculations in LQG [37] [6]. Here we are assuming that the fundamental discreteness predicted by LQG must be compatible with Lorentz invariance as enforced on physical grounds by the constraints raised in [38] and rediscussed from an independent perspective in [39].

are assumed to be expressible in terms of suitable combinations of spin network states (including matter field configurations supported on them). The Hilbert space $\mathscr{H}_\Delta$ can be shown to be given by an $SU(2)$ Chern-Simons Hilbert space of a sphere S^2 with an arbitrary number of sources labelled by spins [43][7]. Physical states satisfy a boundary constraint imposing the spins of the Chern-Simons sources to match the spins of the open links of bulk spin network states in $\mathscr{H}_{out}$ [5].

The physical states described above are eigenstates of the IH horizon area operator $\hat{A}_\Delta$ whose eigenvalues (being $\hat{A}_\Delta$ local) depend only on the spins labelling the Chern-Simons states (or equivalently the puncturing links of the bulk spinnetworks). More precisely, if $\{j_p\}$ denote the ensemble of spins at punctures labelled by an index p then

$$\widehat{A}_\Delta \, |\psi_{out}^{\{j_p\}}\rangle = 8\pi\gamma\ell_{Pl}^2 \sum_p \sqrt{j_p(j_p+1)} \, |\psi_{out}^{\{j_p\}}\rangle \qquad (12)$$

where $|\psi_{out}^{\{j_p\}}\rangle \in \mathscr{H}_\Delta \otimes \mathscr{H}_{out}$ and the subindex *out* schematically denotes all the quantum numbers corresponding to labels of a complete basis of local observables in the *outside*. Notice that these include the quantum numbers describing not only the space-time geometry but also the matter degrees of freedom. The parameter γ is the Immirzi parameter. Entropy of the isolated Horizon is defined (in the microcanonical) approach as

$$S_{BH} = \log(\mathscr{N}_A) \qquad (13)$$

where $\mathscr{N}_A$ spin configurations satisfying the restriction.

$$A - \delta \leq 8\pi\gamma\ell_{Pl}^2 \sum_p \sqrt{j_p(j_p+1)} \leq A + \delta \, . \qquad (14)$$

Notice that one is only counting horizon surface degrees of freedom that are compatible with the coarse graining condition defined by the macroscopic area A that defines the isolated horizon.

Correlations across isolated horizons

A key point is that one could follow a similar procedure and construct the *inside* physical states. Starting from the phase space Γ'_A of the *inside* isolated horizon—defined now in terms of the same characteristic data on Δ and independent free data on δ' (see Figure 3). One can thus define a Hilbert space $\mathscr{H}' = \mathscr{H}_{in} \otimes \mathscr{H}_\Delta$ and, going through the very same prescription, obtain the physical states describing the fields on the *inside* of the isolated horizon. Such states can be denoted $|\psi_{in}^{\{j_p\}}\rangle \in \mathscr{H}_{in} \otimes \mathscr{H}_\Delta$ in analogy with the previous notation for *outside* physical states. The horizon constraints imply that the number of punctures with their spins arriving at Δ from the *inside* match the number of punctures with their spin arriving at Δ from the *outside*. This implies that the physical states describing a neighbouring four dimensional region of an isolated horizon Δ can be graphically represented as in Figure 2, where the links connecting the *inside* with the *outside* are spin network edges labelled by spins that are not explicitly written for notational simplicity.

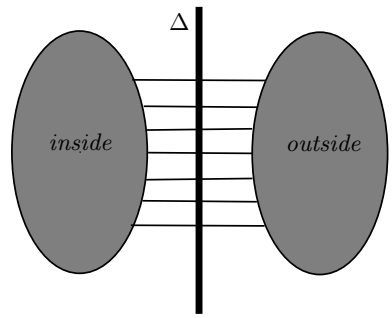

FIG. 2: The isolated horizon quantisation allows for correlations to be mediated only through the links of spin-networks crossing the horizon. This implies the equivalence between entanglement and statistical mechanical entropy.

At this stage it will be convenient to adopt a more compact notation

$$\begin{aligned} |\psi_{in}^{\{j_p\}}\rangle &\longrightarrow |\psi_{in}^{a}\rangle \\ |\psi_{out}^{\{j_p\}}\rangle &\longrightarrow |\psi_{out}^{a}\rangle \end{aligned} \qquad (15)$$

where the single multy-index a labels eigenvalues of the area and replaces the ensemble $\{j_p\}$. Then a state describing a four dimensional region of the space-time containing the horizon compatible with the coarse graining condition (14) can be written as follows

$$|\Psi\rangle = \sum_a \alpha_a \, |\psi_{in}^{a}\rangle \, |\psi_{out}^{a}\rangle, \qquad (16)$$

where $|\psi_{in}^{a}\rangle$ and $|\psi_{out}^{a}\rangle$ denote physical states compatible with the boundary data a, and describing interior and in the exterior state of matter and geometry of the BH respectively and the α_a are coefficients.

The expression (16) of the physical state follows directly from the definition of the isolated horizon quantization. The form of this equation implies that correlations between the *outside* and the *inside* at Planckian scales are mediated by the spin-network links puncturing the separating boundary. The isolated horizon treatment restricts the vacuum correlations to be ultra-local at Planck scale[8].

[7] For a proposal of an intrinsic derivation of the BH physical states from the full theory see [44].

[8] The heart of the point raised here might be related to properties

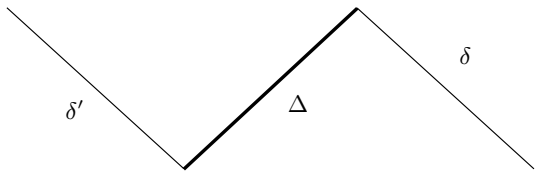

FIG. 3: A set of null surfaces with a non trivial domain of dependence containing the isolated horizon Δ.

As described above, the bulk degrees of freedom in the isolated horizon framework consist of entirely outgoing modes of gravitational and matter radiation. By bulk degrees of freedom here we mean those that are controlled by the Hamiltonian and diffeomorphism constraints on the interior of δ. These bulk degrees of freedom are ignored in the computation of BH entropy; such prescription is well justified physically by the discussion of the physics of gravitational collapse. However, notice that they are exactly of the same nature of those excitations leading to long range correlation contributions to the entanglement entropy in (7). The separation of degrees of freedom necessary in the entanglement account of BH entropy and the one provided by the IH prescription are in this way compatible with each other.

III. ENTANGLEMENT ENTROPY VS. STATISTICAL MECHANICAL ENTROPY

Now we can compute the entanglement entropy associated to the local surface correlations across the horizon. We will do this first in the case of a pure state of the form (16). We will show that it is bounded by the micro canonical entropy (13); however, its precise value depends (as should be expected) on the values of the coefficients α_a. As argued above the nature of the BH collapse is such that the α_a encode details of the microphysics that becomes unavailable in the coarse grained setting where the notion of BH entropy makes sense. Such ignorance of the microphysics leading to the macroscopic BH requires the introduction of an statistical mixture or mixed state. When this is done then the entanglement entropy of the physically appropriate mixed state is equal to the micro canonical entropy in LQG.

1. *Pure state entanglement entropy*

We start from (16) and assume states to be normalized as follows: $\langle\psi^a_{out}|\psi^a_{out}\rangle = 1$, $\langle\psi^a_{in}|\psi^a_{in}\rangle = 1$, and $\langle\Psi|\Psi\rangle = 1$. The pure state density matrix is

$$\rho = |\Psi\rangle\langle\Psi| = \sum_{a,a'} \alpha_a \bar{\alpha}'_a \, |\psi^a_{in}\rangle \, |\psi^a_{out}\rangle \, \langle\psi^{a'}_{in}| \, \langle\psi^{a'}_{out}| \qquad (17)$$

The reduced density matrix, obtained by tracing over the *inside* degrees of freedom, becomes

$$\begin{aligned} \rho_{out} &\equiv \sum_{i,a} \langle a,i|\Psi\rangle\langle\Psi|a,i\rangle \\ &= \sum_{a,i} |\alpha_a|^2 \beta_{ia}\bar{\beta}_{ia} \, |\psi^a_{out}\rangle\langle\psi^a_{out}| \end{aligned} \qquad (18)$$

where $\beta_{ia} \equiv \langle a,i|\psi^a_{in}\rangle$. From this one gets

$$\rho_{out} = \sum_a p_a |\psi^a_{out}\rangle\langle\psi^a_{out}|,$$

with $p_a = |\alpha_a|^2$. The question now is what are the correct physical values of the probabilities p_a? In the present context where we started from a pure state (17) the only thing we can say is that the (pure state) entanglement entropy

$$S_{out} \equiv -\mathrm{Tr}[\rho_{out}\log(\rho_{out})] \leq \log(\mathscr{N}_A) = S_{BH}, \qquad (19)$$

i.e. it is bounded by the micro-canonical entropy (13). The equivalence can be achieved if the physical states (pure states) describing a semiclassical black hole would have nearly to equal values of the probabilities $p_a = |\alpha_a|^2$ in the set of a satisfying (14).

The approximate equiprobability condition is certainly plausible considering that the recovering of semiclassical states compatible with the low energy continuum limit would require the contribution of the vast ensemble of eigenstates of the horizon area operator. Contributions might turn out to be approximately homogeneous in the range (14). Unfortunately, details on the way in which the continuum limit is to be recovered in the LQG framework are hard to quantify at present in the framework. However, we will argue in the following subsection that this perhaps not necessary.

2. *Mixed states entanglement entropy*

As we have argued in the introduction the coarse grained physics of the horizon is insensitive to the Planckian details that distinguish the different pure (micro) states producing black holes with a definite final value of their extensive parameters. In such context the idea of reproducing *the* quantum state (as a pure state) reflecting the low energy properties of the suitable vacuum is unrealistic (as it would be to established the phase space coordinates of 10^{23} particles in a gas in equilibrium inside a box). As discussed before the only available information on the state producing the black hole is coarse grained (low resolution) information. In such context the best one can do is to define a density matrix describing

of tensor network states and related notions where area law for entanglement entropy holds by construction. For a review and references see [34].

a mixed state corresponding to the statistical mixture of all the micro states—each of which would correspond to a distinguished initial pure state of matter and fields before collapse—satisfying our coarse graining criterion.

Consequently instead of (17) we take

$$\rho = \sum_{a,a'} p_{aa'} \left|\psi^a_{in}\right\rangle \left|\psi^a_{out}\right\rangle \left\langle\psi^{a'}_{in}\right| \left\langle\psi^{a'}_{out}\right| \tag{20}$$

as our starting point. Here $p_{aa'}$ is a positive definite matrix of coefficients describing the statistical mixture of states of the form (16). The reduced density matrix becomes

$$\begin{aligned} \rho_{out} &\equiv \sum_{i,a} \langle a,i|\rho|a,i\rangle \\ &= \sum_{a,i} p_{aa} \left|\psi^a_{out}\right\rangle\left\langle\psi^a_{out}\right| \end{aligned} \tag{21}$$

which, as a result of the partial trace, only depends on the diagonal elements p_{aa}. The assumption of ergodicity (expected in a system with such huge number of microscopic degrees of freedom in equilibrium) together with the idea that horizon area is the correct coarse graining observable naturally selects a p_{aa} independent of the particular eigenvalue within the range (14), hence $p_{aa} = \mathscr{N}_A^{-1}$. From this is follows that

$$S_{out} = S_{BH}. \tag{22}$$

Moreover, if a notion of local energy is available for those observers that are stationary with respect to the horizon one can find the equivalence with a canonical ensemble description. Assuming that the horizon is in thermal equilibrium with the *outside* for such observers, then the previous equiprobability criterion leads to the Boltzman probabilities $p_a = Z^{-1}\exp(-\beta E_a)$ where β is inverse temperature and E_a is the corresponding energy eigenvalue. The Hamiltonian is the one generating a time flow with respect to which the system is in local equilibrium [45]. This is precisely the area Hamiltonian introduced in [26]. In such case we recover the canonical ensemble standard expression of the entropy. The structure of physical states at the Planck scale and the semiclassical input of the previous section imply that entanglement entropy is just the same as statistical mechanical entropy in LQG.

Long range correlations

Equation (16) is central for establishing the relationship between statistical and entanglement entropy in the framework of loop quantum gravity. As argued, this is a consequence of the IH boundary condition (notice that the restriction imposed here on short range correlations are similar to those imposed in recent treatments [46] or [47], yet they are weaker in the sense that correlations in the UV are defined at a single horizon puncture). The question that remains is wether such condition is compatible with the form of *inside-outside* correlations expected in semiclassical states describing the near horizon geometry.

Even thought it would be necessary to leave the framework of quantum isolated horizons in order to describe situations where the bulk physics is relevant and *in-out* exchanges are allowed, it seems reasonable to expect that the dominant contributions to BH entropy (coming from the UV correlations close to the horizon) should be correctly captured by the present treatment. As argued in Section I, long range correlations contribute to changes in entanglement entropy that can correctly be accounted for by the notion of relative entropy which is insensitive to the UV structure of spacetime.

The description of BH entropy from the entanglement perspective necessarily requires a separation of scales where UV correlations close to the horizon dominate over IR long range correlations. Such separation of scales is natural from the classical physics of gravitational collapse, and the definition of the coarse graining that associates micro states to Planckian details of the horizon physics. Quantum isolated horizons are designed to model these fundamental excitations of the horizon degrees of freedom. They provide at the same time the natural separation of scales and the UV structure of geometry near the horizon.

IV. DISCUSSION

We have illustrated how, by generalising the discussion of [18] to static black hole backgrounds, without the appropriate separation of degrees of freedom, entanglement entropy contains bulk terms which in perturbation theory represent leakage of entropy at infinity. Even when these flow terms can be interpreted thermodynamically in a suitable sense (see eq. (9) and footnote 2) they are not associated to BH entropy. This is not surprising as it is well know that entanglement across the horizon has bulk contributions which strongly depend on the nature of the quantum state considered due to long range correlations which are unrestricted in QFT. For the entanglement approach to black hole entropy is essential to have the means to separate the wheat of local UV contributions across the BH horizon from the straw of long range correlations.

Another important aspect is that relative entropy (for perturbations of the vacuum state) is by definition insensitive to the UV microphysics that we expect to be at the root of a fundamental account for BH entropy. A counterpart of this is the fact that changes of entanglement entropy and changes in the horizon law are related by the use of the semiclassical Einsteins equations where geometry remains classical (low energy physics governs the relationship (5)). The fact that the inclusion of the (semiclassical) gravitational coupling eliminates the species problem in a thermodynamical sense. This is an

encouraging indication which needs to be realized at the fundamental level (statistically) by the description of the would-be-divergent UV contributions of matter and its background independent coupling to geometry in quantum gravity.

The framework of quantum isolated horizon accounts for the UV excitations of the horizon geometric and non geometric degrees of freedom (including matter). These are the micro states in an ensemble defined by the macroscopic parameters associated to a stationary semiclassical horizon. Once the appropriate physical UV degrees of freedom are identified, entanglement entropy and statistical mechanical entropy are the same. Conversely, the physical states selected by the isolated horizon boundary condition present *inside-outside* correlations leading to the maximum entanglement of the geometric degrees of freedom (under the standard assumption of equal probability distribution for states satisfying (14)). In such situation entanglement entropy coincides with the logarithm of the number of micro states compatible with the coarse graining macroscopic parameters that define the statistical ensemble.

The present discussion naturally leads to a possibly clarifying perspective on the information loss discussion. In Section I B 1 we described this perspective based on the possibility of having purifying correlations between the low energy degrees of freedom of continuous fields and the UV discrete gravitational degrees of freedom that are hidden to standard low energy semiclassical observers. Such possibility is allowed by quantum gravity theories where continuum physics is only recovered under appropriate coarse graining of local observables. This view is complementary to the one expressed in [48] where the fate of classical singularities in quantum gravity is a key element of the discussion.

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