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Application of a simple viscous model to the cyclic behavior of clays at small strains

Gilberto F. Alexandre, D.Sc.*; Ian S. M. Martins, D.Sc.†

July 2, 2014

Abstract

This paper presents the analytical development of the cyclic behavior of clays at small strains using the model proposed by Martins (1992) as modified by Alexandre (2006). Initially, discussions about the influence of the rate of strain, creep and stress relaxation are presented, which are followed by the development of differential equations of simple cases (and their respective solutions either in closed form or numerical) and predictions for two tests carried out by Mortezaie (2012) in a fabricated clay. It is shown that satisfactory predictions were achieved both qualitatively and quantitatively for the presented tests.

1 Introduction

In a presentation at NGI in June 2014 about the cyclic behavior of soils, professor Mladen Vucetic showed that rate of loading and rate of straining can have profound effects. According to him it is possible to notice creep and stress relaxation in cyclic tests and even G_{max} and damping are influenced by rate effects.

Considering as a working hypothesis that these effects are significant, an attempt to describe physically and mathematically the behavior of clays under cyclic loading at small strains was carried out using the model proposed by Martins (1992) as modified by Alexandre (2006). Although this model was not intended for explaining the cyclic behavior of clays, it accounts for the understanding of viscous effects such as creep and stress relaxation, therefore justifying its use.

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2 The model proposed by Martins (1992)

According to Terzaghi (1941), the contact between clay particles would be of two types. Terzaghi called the contact types “solid bonds” and “film bonds”. In his view, both contacts would be able to transmit effective stresses and would result from the adsorbed water layers that surround the clay particles. The “solid bonds” would result from the contact between the adsorbed water layers in the immediate vicinity of the clay particle, which, according to Terzaghi, would be in the solid state. The “film bonds” would result from the contact between adsorbed water layers which would not be in the solid state but which would possess a viscosity higher than of the viscosity of the free water (by free water it should be understood the water that flows out of the pores of the soil during seepage or consolidation).

Having this picture in mind, Martins (1992) assumes as a hypothesis that the shear strength of a saturated normally consolidated clay has two components, the frictional resistance and the viscous resistance.

The frictional resistance would develop between Terzaghi’s “solid bonds” and it would be a function of the shear strain. The viscous resistance would develop between Terzaghi’s “film bonds” and it would be a function of the strain rate. The equation for the shear strength would then be:

$$\tau = \tau_f + \tau_v = \sigma' \cdot \tan \phi'_{mob} + \eta(e) \cdot \dot{\epsilon} \quad (1)$$

Where:

τ_f is the frictional resistance (the solid component of the shear stresses);

τ_v is the viscous resistance (the viscous component of the shear stresses);

ϕ'_{mob} is the mobilized effective angle of internal friction;

$\eta(e)$ is the coefficient of viscosity of the adsorbed water layer that surrounds the clay particles (a function of the void ratio for a normally consolidated clay);

and

$\dot{\epsilon}$ is the strain rate

The normal effective stress, σ' , is taken as the difference between the normal total stress, σ , and pore-pressure, u .

One problem in the model is that, because Equation (1) is written in terms of shear stresses, rate/time effects related to the normal effective stress such as the hydrostatic relaxation observed by Arulanandan et al. (1971), secondary consolidation under hydrostatic conditions and other effects cannot be macroscopically explained.

To overcome this problem it was suggested by Martins that Equation (1) could be generalized for the normal effective stresses. In this regard, Thomasi (2000) carried out similar tests as Arulanandan et al. (1971), confirming the possibility of generalizing the normal effective stress equations as proposed by Martins. The generalized Equation (1) is:

$$\sigma' = \sigma'_s + \sigma'_v \quad (2)$$

Where:

σ' is the normal effective stress;

σ'_s is the normal effective stress due to the “solid” bonds; and

σ'_v is the normal effective stress due to the “film” bonds.

A state of stress that combines Equations (1) and (2) was proposed by Alexandre (2006) for the case of axial-symmetry and is reproduced below:

$$\sigma'_s = \left(\frac{\sigma'_{1s} + \sigma'_{3s}}{2} \right) + \left(\frac{\sigma'_{1s} - \sigma'_{3s}}{2} \right) \cdot \cos(2 \cdot \alpha) \quad (3)$$

$$\tau_s = \left(\frac{\sigma'_{1s} - \sigma'_{3s}}{2} \right) \cdot \sin(2 \cdot \alpha) \quad (4)$$

$$\sigma'_v = \left(\frac{\sigma'_{1v} + \sigma'_{3v}}{2} \right) + \left(\frac{\sigma'_{1v} - \sigma'_{3v}}{2} \right) \cdot \cos(2 \cdot \alpha) \quad (5)$$

$$\tau_v = \left(\frac{\sigma'_{1v} - \sigma'_{3v}}{2} \right) \cdot \sin(2 \cdot \alpha) \quad (6)$$

Where the indexes “s” and “v” represent the “solid” and the “viscous” components of the effective stresses and the indexes “1” and “3” represent the major and minor principle stresses. In the above equations and for a given plane, α is the angle between the direction perpendicular to the plane considered and the plane where σ'_1 acts.

It can be seen that Equations (3) and (4) describe a circle for the solid effective stress state as well as Equations (5) and (6) constitute a circle for the viscous effective stress state. These two circles combined give the effective stress state at a point.

An important remark about this model is that when the strain rate is made equal to zero, the effective stress state decreases to a lower stationary value greater than zero, which is the solid effective stress state. If a real laboratory test could be carried-out with a strain rate equal to zero then the results of such a test would comprise only the solid component of the effective stress state. However, as a test with a strain rate equal to zero is impossible to perform there will always be an additional component of the effective stress state, which is the viscous effective stress component.

The concepts introduced in this section were intended to provide the minimum information for the development of the study of cyclic behavior at small

strains, which will be presented in the next section. For a better understanding of this model the reader is referred to Martins (1992), Alexandre (2006), and in Alexandre and Martins (2013a, 2013b, 2014).

3 Interpretation of cyclic behavior of clays at small strains considering viscous effects

The concepts of strain rate as well as creep and stress relaxation mentioned in the introduction can be used to understand the behavior of clays under cyclic conditions at small strains. As strain rate effects are more easily understood than creep and stress relaxation, the first interpretation of cyclic behavior mechanism to be presented in this section will account exactly for it. In addition, for all the mechanisms presented below, it is assumed that no plastic strains or failure will ever occur and that inertial effects and degradation can be disregarded.

Considering that viscous stresses are generally described by means of strain rate, every time a clay experience deformation over time an additional stress component will exist. In addition, regardless of the test type, stress or strain controlled, if the strain rate vary over time then so the viscous stresses. Because the stress controlled test involves a transient phase (as it will be shown later), the analysis of the influence of the viscous stress for this mechanism is better presented using a strain controlled test.

In a strain controlled test, the strains are cycled over time according to a prescribed known function, being the most commonly used a sinusoidal function. Representing this function as $A_\gamma \cdot \sin(\omega \cdot t)$, where A_γ is the strain amplitude, ω is the frequency and t is the time, then the strain rate is also cycled over time in such a test and is given by $\omega \cdot A_\gamma \cdot \cos(\omega \cdot t)$. As viscous resistance can either be considered in compression as well as in tension, as anyone who has at least once chewed gum can attest, then, at a given shear strain, γ , during loading or unloading the soil will present, respectively, a greater compression resistance and a smaller compressive resistance than a test carried out with a zero strain rate (provided such a test could be carried out). A schematic representation of the stress-strain relationship during cyclic strain condition is the following.

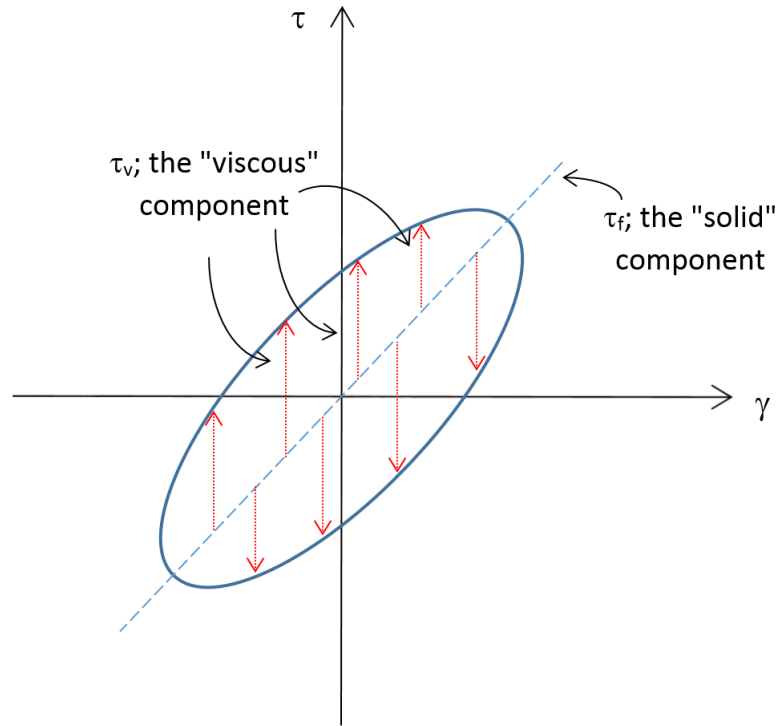


Figure 1: Influence of rate of strain in the cyclic behavior of clays at small strains.

Clays with higher viscosity or subjected to higher frequencies (and therefore higher strain rates) will present higher viscous stresses and the hysteretic loop will be more pronounced. On the other hand, a soil that does not present significant viscous effects will present a less pronounced hysteretic effect. If a soil does not present viscous effects at all then in the case where the soil is considered linear-elastic, no hysteretic behavior will occur with the closed loop degenerating to a straight line (the blue dashed line in figure 1).

The next interpretation of cyclic behavior to be explained deals with creep. As creep can be defined as the variation of strain over time under a constant state of stress and during cyclic loading the stress are varying all the time one might not expect that creep could play a role. However, as any loading over time can be approximated by a succession of constant stress loadings, than creep can used for this matter. Considering the following figures where a sinusoidal stress is cycled over time (represented by a continuous line), then the creep approximation to this loading can be represented by the rectangles where the average stress, $\bar{\tau}$ can be considered constant over a short period of time, Δt . Better approximations can always be achieved by making the time interval approximating zero.

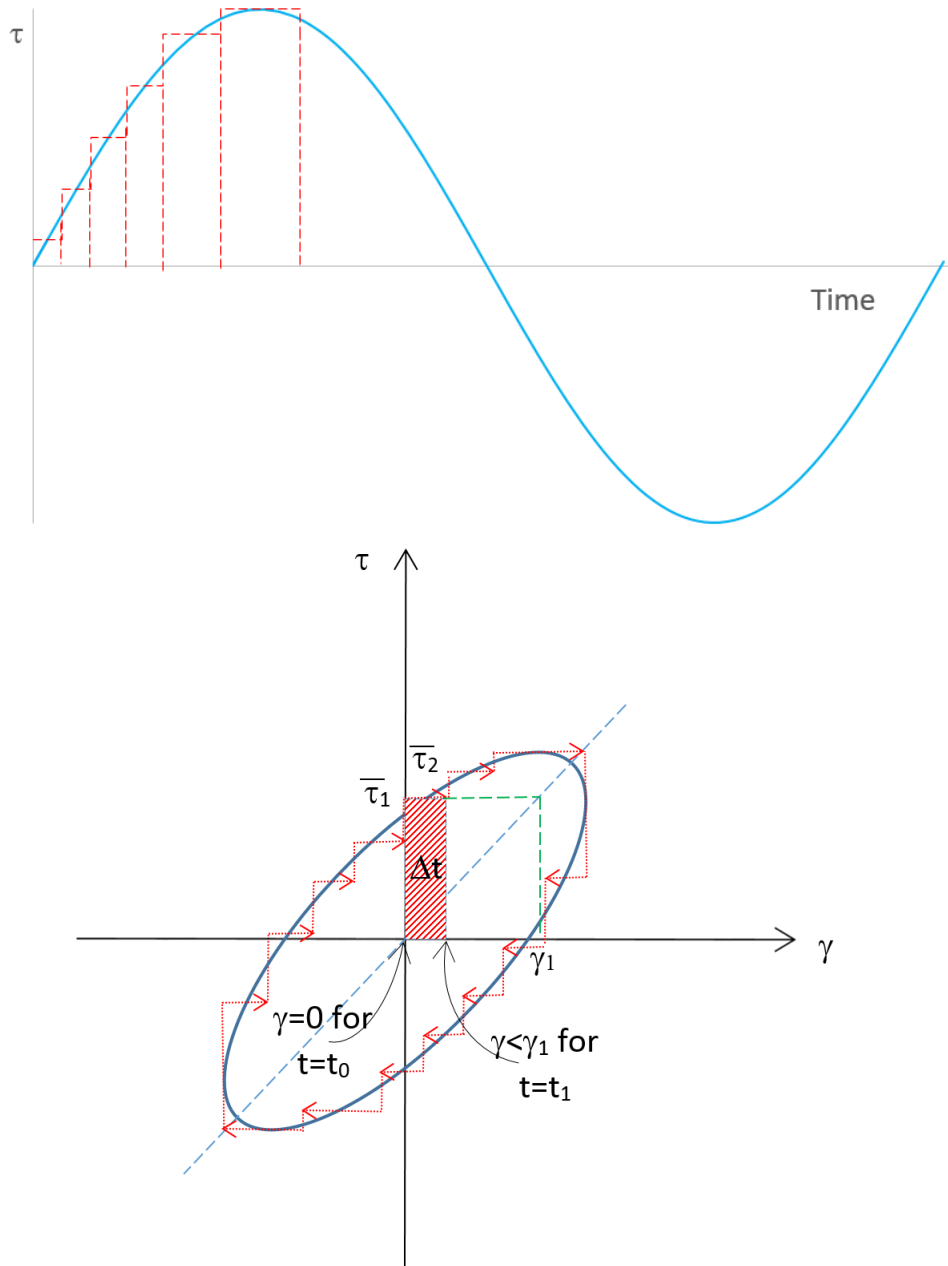


Figure 2: Influence of creep in the cyclic behavior of clays at small strains. Fig. 2A (top) : Shear stress x time sinusoidal function and step creep approximation. Fig 2B (below): Shear stress x shear strain curve and step creep approximation.

Consider the first creep loading where the stress is kept constant from $t = 0$ to $t = t_1$. During this time interval, the strains will increase over time as in any creep test. Considering again, that no failure is involved, then, if sufficient time is allowed, the strain would reach a final stationary value, γ_1 . However, as this is not the case, the strain at the end of this time interval will be less than that. Considering that, at $t = t_1^+$, the load is instantaneously increased to the level of the second “step” creep, $\bar{\tau}_2$, then, during this new time interval strains will accumulate and increase over time until the next step and so on and so forth. As this mechanism is valid for loading and unloading, the strains for a given stress will always be different than their respective “final stationary” values, or in other words, the state of strain will always be delayed in relation to the state of stress.

If instead of considering creep, stress relaxation is used, then the appropriate figures for discussion are the ones in the sequence.

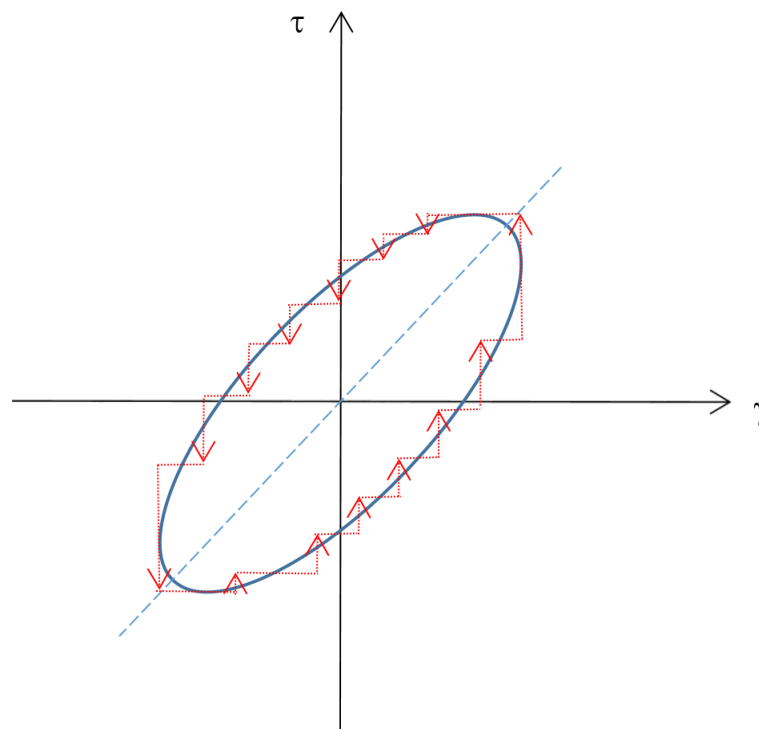
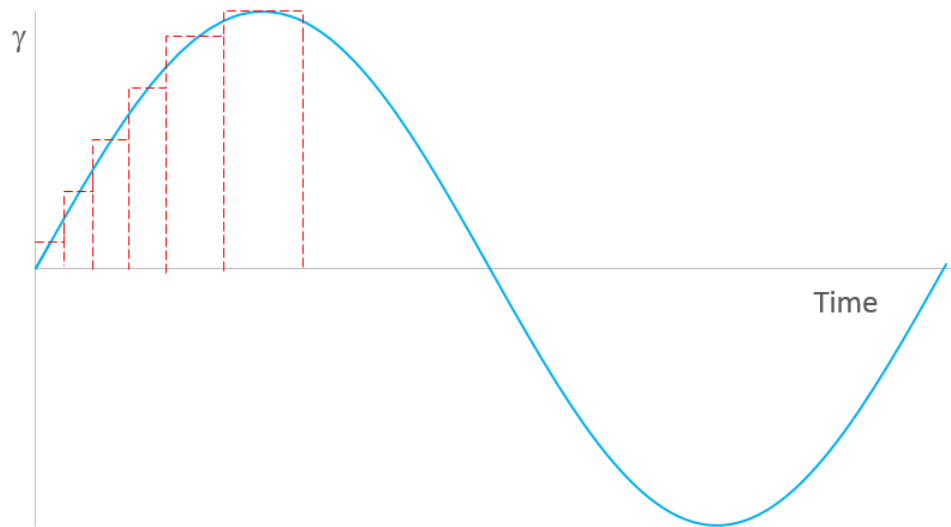


Figure 3: Influence of stress relaxation in the cyclic behavior of clays at small strains. Fig. 3A (top) : Shear strain x time sinusoidal function and step stress relaxation approximation. Fig 3B (below): Shear stress x shear strain curve and step stress relaxation approximation.

Considering that, instead of cycling stress, the strains are cycled over time in the shape of a sinusoidal curve and using the same approach mentioned before, the curve can be represented by small step stress relaxation tests in succession. In this case though, the stress relaxation that occur in one of these steps is interrupted by the next step in such a way that the final stress equilibrium cannot be reached.

Physically, the viscous component of the stress delays either the stress or the strain from reaching the final equilibrium state.

4 Analytical Development

4.1 Strain-controlled tests

Although it is possible to represent the cyclic behavior of clays at small strains under strain controlled conditions by predicting stress relaxations in consecutive steps this method is cumbersome and unnecessary. As the strain is cycled according to a know function, which in general is a sinusoidal function, then, the following equation describes the development of shear stresses over time:

$$\tau = G \cdot \gamma + \eta \cdot \dot{\gamma} = G \cdot A_\gamma \cdot \sin(\omega \cdot t) + \eta \cdot \omega \cdot A_\gamma \cdot \cos(\omega \cdot t) \quad (7)$$

Where:

A_γ is the strain amplitude;

G is the shear modulus;

γ and $\dot{\gamma}$ are the shear strain and shear strain rate, respectively; and

η is coefficient of viscosity of the soil.

In the above equation both the solid and the viscous components were considered as linear functions. It can be seen that equation (7) is a special case of equation (1). A better approximation to the actual behavior can be achieved by considering non-linear functions. However, as this work is concerned with small strains there is need to adopt a non-linear function for the solid component. On the other hand, a better approximation for the viscous component would be to adopt a power law viscous function ($K \cdot \dot{\gamma}^n$, where K and n are constants) as considered by Alexandre and Martins (2013a,b). An schematic plot of equation (7) and of an equation equivalent but considering a power law viscous function, as mentioned above, is the following:

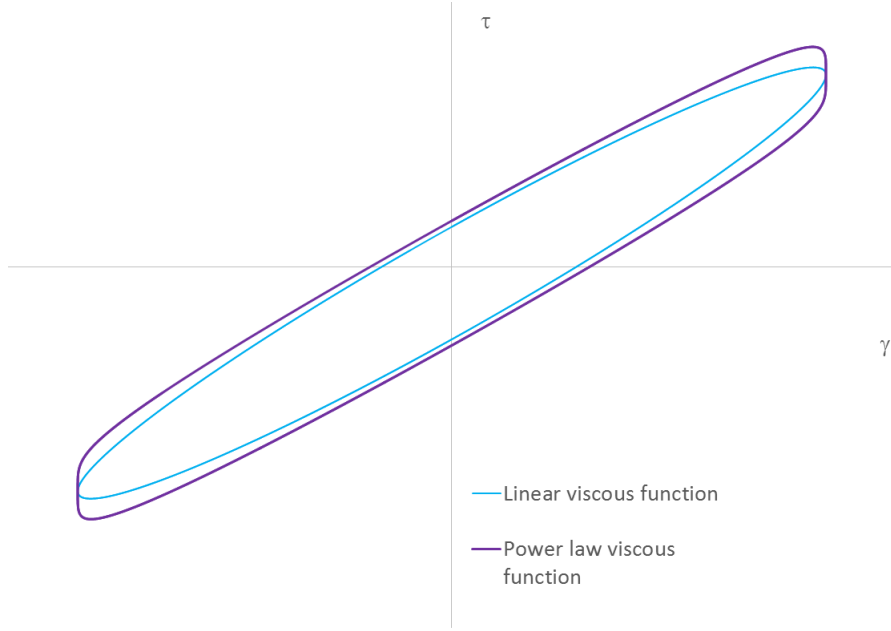


Figure 4: Cyclic behavior of clays at small strains in a strain controlled test considering two different viscous functions.

4.2 Stress controlled tests

For these tests, the shear stress is cycled over time in general in accordance to a sinusoidal function. In this case, considering linear functions for the solid and viscous components of the shear stress, the differential equation of the cyclic behavior of clays at small strains is:

$$\tau = A_\tau \cdot \sin(\omega \cdot t) = G \cdot \gamma + \eta \cdot \dot{\gamma} \quad (8)$$

Where A_τ is the shear stress amplitude;

As this is a non-homogeneous linear ordinary differential equation, then the general solution is the sum of the solution of the associated homogeneous equation and the particular solution of the non-homogeneous equation itself.

For the associated homogenous equation it can be shown that the solution, γ_h is of the form $\gamma_h = C_1 \cdot e^{C_2 \cdot t}$, where C_1 and C_2 are constants. On the other hand, it can be shown that the particular solution, γ_p , of the non-homogeneous equation is of the form $\gamma_p = C_3 \cdot \sin(\omega \cdot t) + C_4 \cdot \cos(\omega \cdot t)$. Therefore the general solution of equation (8) is:

$$\gamma = C_1 \cdot e^{C_2 \cdot t} + C_3 \cdot \sin(\omega \cdot t) + C_4 \cdot \cos(\omega \cdot t) \quad (9)$$

Considering that the initial conditions for $t = 0$ are $\gamma = 0$ and $\tau = 0$, then the general solution is:

$$\gamma = \frac{A_\tau \cdot \eta \cdot \omega}{G^2 + \eta^2 \cdot \omega^2} \cdot e^{\frac{-G \cdot t}{\eta}} + \frac{G \cdot A_\tau}{G^2 + \eta^2 \cdot \omega^2} \cdot \sin(\omega \cdot t) - \frac{A_\tau \cdot \eta \cdot \omega}{G^2 + \eta^2 \cdot \omega^2} \cdot \cos(\omega \cdot t) \quad (10)$$

The first term to the right of the equal sign represents a transient phase and the other two terms represent the cyclic behavior after the transient function “vanishes”. Figure 5 illustrates a τ versus γ plot according to the solution of the differential equation.

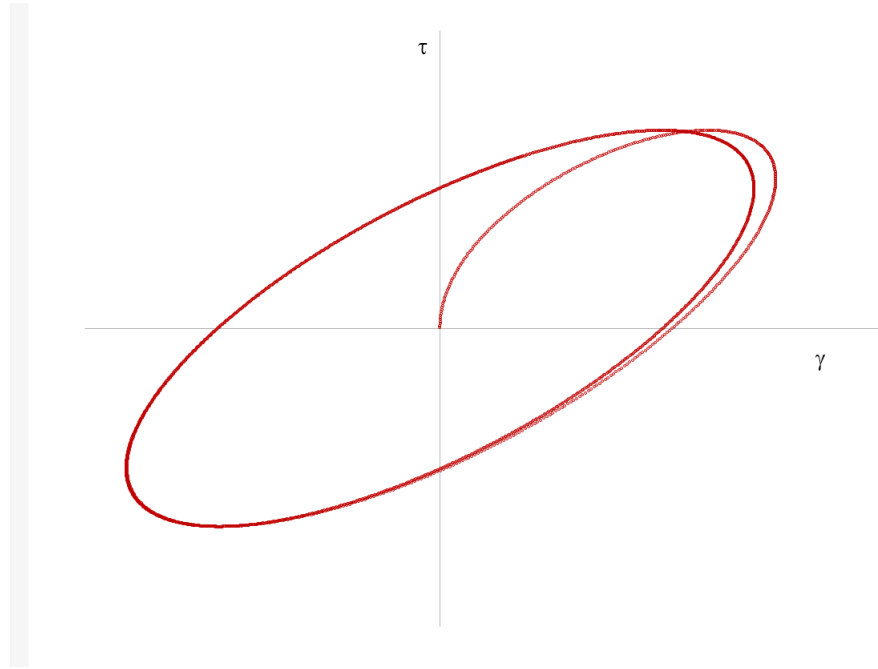


Figure 5: Cyclic behavior of clays at small strains in a stress controlled test considering a linear viscous function.

If stresses are cycled around an average shear stress, τ_{ave} , than equation (8) becomes:

$$\tau = \tau_{ave} + A \cdot \sin(\omega \cdot t) = G \cdot \gamma + \eta \cdot \dot{\gamma} \quad (11)$$

For this case the appropriate initial conditions are $\gamma = 0$ and $\tau = \tau_{ave}$ for $t = 0$ and the solution is:

$$\gamma = \left[\left(\frac{A_\tau \cdot \eta \cdot \omega}{G^2 + \eta^2 \cdot \omega^2} \right) - \frac{\tau_{ave}}{G} \right] e^{\frac{-G \cdot t}{\eta}} + \frac{G \cdot A_\tau}{G^2 + \eta^2 \cdot \omega^2} \cdot \sin(\omega \cdot t) - \left(\frac{A_\tau \cdot \eta \cdot \omega}{G^2 + \eta^2 \cdot \omega^2} \right) \cdot \cos(\omega \cdot t) + \frac{\tau_{ave}}{G} \quad (12)$$

Again, the first term to the right of the equal sign is the transient phase and the remaining terms are the steady-state phase of the cyclic behavior around an average shear stress, τ_{ave} . Figures 6A and 6B illustrate the solution of this differential equation.

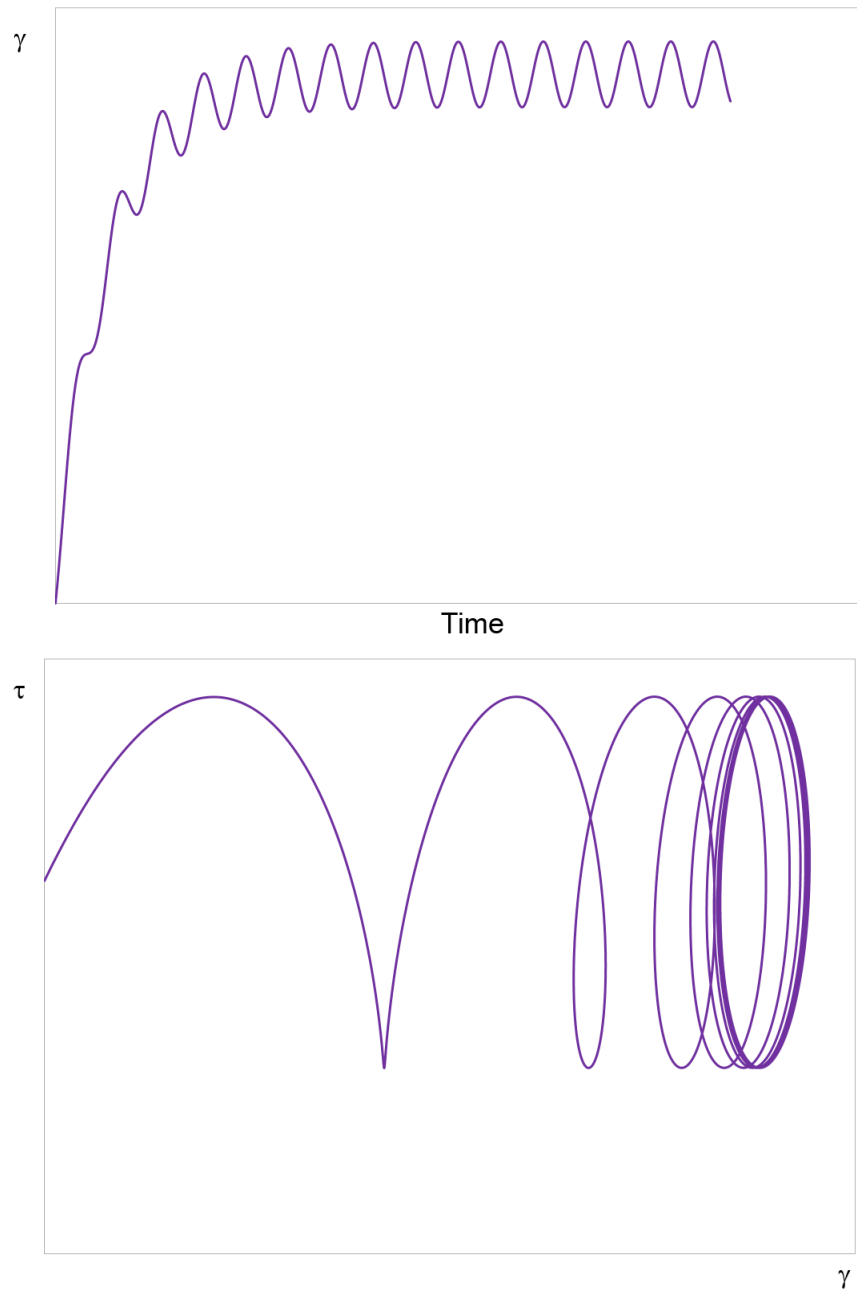


Figure 6: Cyclic behavior of clays at small strains in a stress controlled test considering linear functions and an average shear stress, τ_{ave} greater than zero. Figure 6A (top): Shear strain x time plot. Figure 6B (bottom): Shear stress x shear strain plot.

As can be seen on figures 6A and 6B when stress is cycled around an average shear stress, the result is a combination of creep due to the average shear stress and the cyclic behavior itself. If the amplitude of the cycled stress is made equal to zero then creep results. On the other hand, if the average shear stress is made equal to zero, the cyclic behavior around a zero shear stress is obtained. Making , $\tau_{ave} = 0$ in equations (11) and (12) lead to equations (8) and (10), which is the mathematical representation of the explanation given before.

In this interpretation of the cyclic behavior at small strains, in the case the stresses (or strains) are cycled around an average shear stress of zero (or zero shear strain) the shear modulus can be obtained as the slope of the shear stress-shear strain line that passes thru zero which coincides with the major axis of the ellipse. In the case the stresses or strains are cycled around an average given value, then G can be assessed by discounting from τ the viscous component, τ_v . A better procedure however exists and is explained below. As, according to the model, the viscous component of the stresses assumes a zero value for strain rate equal to zero all is required is identify the zero strain rate points in the stress strain plot to assess G . This condition is mathematically satisfied by deriving equation (12) with respect to time and making it equal to zero. For times such the transient phase of equation (12) can be disregarded one gets:

$$\cos(\omega \cdot t) = -\frac{\eta \cdot \omega}{G} \cdot \sin(\omega \cdot t) \quad (13)$$

It is important to note that inertia effects were not accounted for in the present work. However this can be achieved by making use of Newton's second law of movement. In addition no strength degradation, plastic strains or anisotropy of the solid or viscous components were taking into account.

5 Illustrative examples

Mortezaie (2012) carried out strain cycled tests on a kaolinite clay fabricated in laboratory in the range of small strains. This soil, which is classified as MH according to the USCS classification system, has a liquid limit of about 61%, a plastic limit of about 33% and therefore a plasticity index of about 28%. A linear prediction using equation (7) and a prediction using a similar equation (a power law function, $K \cdot \dot{\gamma}^n$, where K and n are constants was used instead of the linear viscous function) were made for Tests Kao31A and Kao31B. The following figures show the plots of the comparisons between predictions and experimental results.

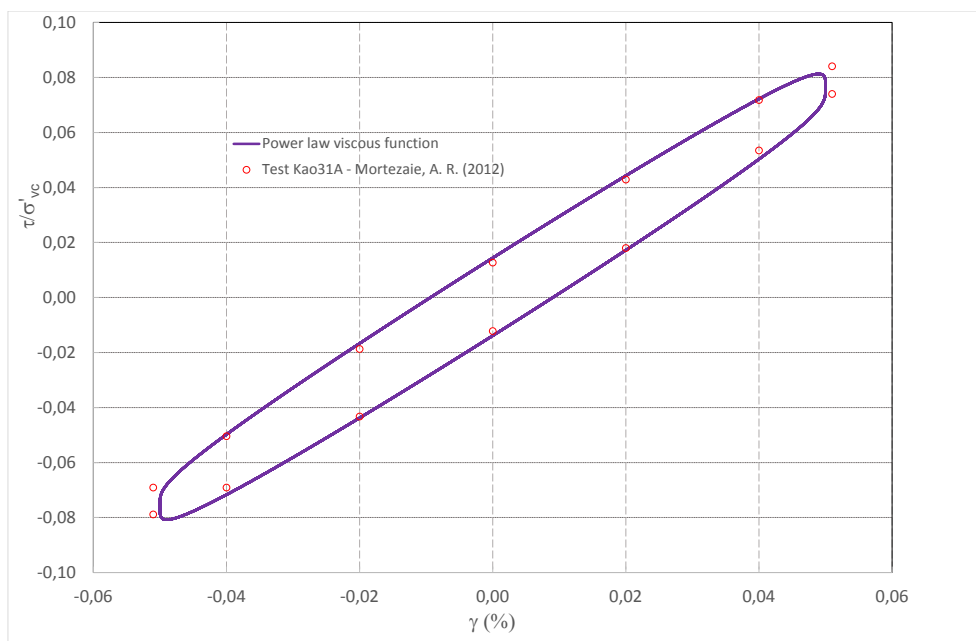
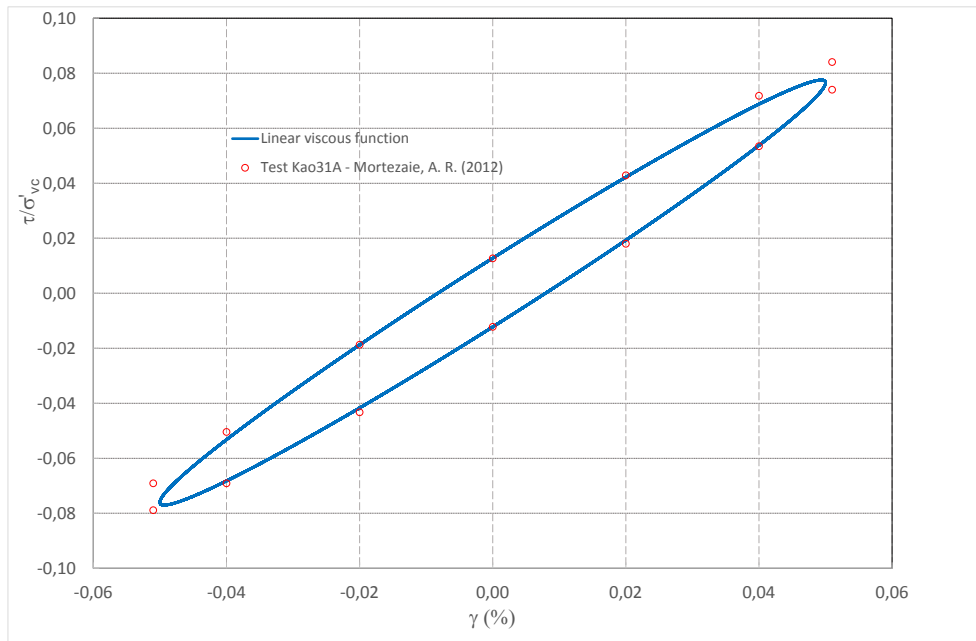


Figure 7: Test Kao31A carried out by Mortezaie (2012).

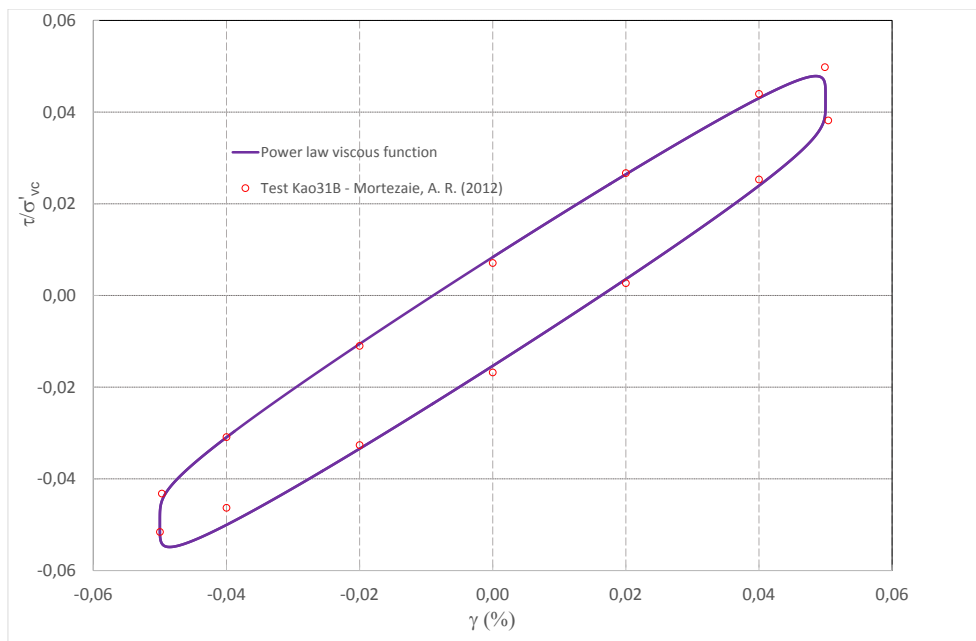
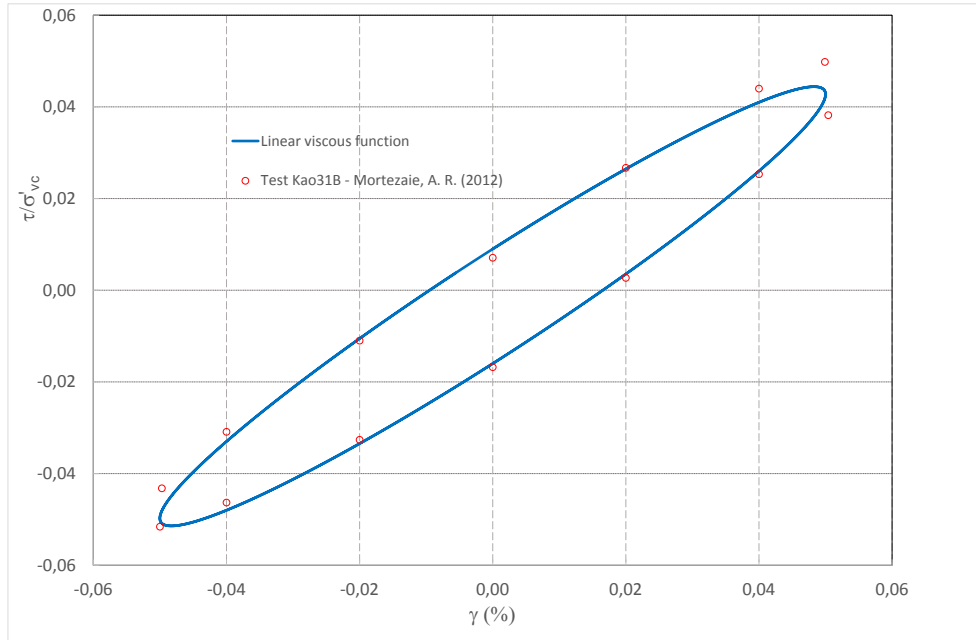


Figure 8: Test Kao31B carried out by Mortezaie (2012).

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7 Conclusions

The model proposed by Martins (1992) as modified by Alexandre (2006) was applied to the interpretation of the cyclic behavior of clays at small strains. Comparison of predictions and two tests results obtained by Mortezaie (2012) seems to indicate that the proposed model has a potential for explaining the cyclic behavior of clays.

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