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► **To cite this version:**

Olivier Bareille, Mohamed Chabchoub, Sébastien Besset, Mohamed Ichchou. An inverse mid-high frequency energy method: Formulation and applications. 17th International Congress on Sound and Vibration, Jul 2010, Le Caire, Egypt. pp.1240-1247. hal-01011800

HAL Id: hal-01011800

<https://hal.science/hal-01011800>

Submitted on 24 Jun 2014

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AN INVERSE MID-HIGH FREQUENCY ENERGY METHOD : FORMULATION AND APPLICATIONS

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An energy method called Simplified Energy Method (MES) has already been proposed to predict energy densities repartition for structural-acoustic problems in the mid-high frequency range. In order to illustrate but also to present one of the applications of this method, this latter is used here to solve inverse structural problems. The main application, in this paper, is the structural forces identification thanks to energy densities calculation and analysis. The injected forces estimation and localization are hence obtained in the mid-high frequency range. Internal as well as boundary sources are here detected through 2D Kirchhoff-plate numerical tests.

1. Introduction

Identification of forces features exciting structural components is an important concern in structural engineering. Indeed, localization and quantification of input loads is of great interest and often the direct measurements of sources reveals complicated to perform. Indirect means to define the excitation is often an alternative. A number of work have been done and published around that question [1, 2, 3, 4, 5, 6, 7, 8, 9]. The problem considered here can have also some applications in structural health monitoring. The damage, whose features are often unknown, leads to modifications of the vibration energy distribution of structural members. The damage size and position detection is then a key issue.

By and large, two ways of thinking are available in the open literature concerning input forces identifications issues. The first way of thinking is roughly inspired from identification issues of black-box like behavior of dynamical systems and the use of estimation theories [1, 2]. The second way of thinking consists in defining the input force that allows the measured set of inputs to verify the equilibrium equations given either in a continuous or discretised format [4, 5, 6]. The problem of vibration input force localization was also dealt with in structural intensity community. Moreover, instead of using analytical model [4] or modal modal [7] in the inversion process, discretized finite element (FE) or boundary element (BE) can be introduced in the identification [9]. Ultimately, in [8]

intensity vectors indicating the energy flow and the localization of the sources were used.

It should be however noticed that most of what was proposed in the literature concerns a limited frequency band of interest; precisely, the so-called low frequency region where modal overlap is weak and the resonant behavior of the structure still dominant. Energy methods are often used as alternative methods in high and medium frequency range when numerical methods like finite element or boundary equation based formulations run to important limitations. Among these energy methods, the most recognized remains *Statistical Energy Analysis* (SEA) [10] Many energy methods have been based on SEA, attempting to enhance the Statistical Energy Analysis robustness and predictivity , leading to the *Simplified Energy Method* (MES) [11, 12, 13], as a “direct” prediction tools [13, 16].

In this paper, the main concern is the use of MES to predict energy sources [14]. Such an issue needs the inversion of the direct formulation. The paper is structured as follows. For a sake of clarity, the direct MES formulation is reminded in section 2. Then the formulation of the inverse MES is given. A two dimensional case is considered and the algebraic matrix format of the inversion problem is offered in section 3. In order to validate the proposed method, numerical simulations are processed section 4, using estimated direct energies or a finite-element simulated input energy.

2. Overview of the direct energy flow method MES

2.1 Assumptions

Simplified Energy Method is based on the description of two local energy quantities. The first one is the total energy density W defined as the sum of the potential energy density and the kinetic energy density. The second energy variable \vec{I} is the energy flow. The energy balance of a system can be written as follows:

$$\vec{\nabla} \cdot \vec{I} + \pi_{diss} = \pi_{inp} \quad (1)$$

where $\vec{\nabla}$ is the gradient operator, π_{diss} is the dissipated power density and π_{inp} is the input power density. MES adopts the same damping model as SEA. It has already been discussed in the literature [10] and can be expressed as follow:

$$\pi_{diss} = \eta\omega W \quad (2)$$

where η is the damping loss factor and ω the circular frequency. The propagating waves considered in the MES formulation are made of partial energy quantities corresponding to direct and reverberated fields. As the considered fields are quadratic variables, the superposition principle can be applied:

$$\mathcal{L} = \sum_i \mathcal{L}_i \quad (3)$$

where $(\mathcal{L}, \mathcal{L}_i)$ are quadratic variables corresponding to global energy or a partial energy associated to wave fields (W, W_i) or (\vec{I}, \vec{I}_i) . In the following, an intrinsic energy law is used, which is often introduced to define the wave velocity c :

$$\vec{I}^i = c \cdot W^i \vec{n} \quad (4)$$

This expression simply defines the energy velocity, which is in some cases the group velocity, as the ratio of the energy flow and convected energy density. This expression was demonstrated in a number of textbooks and is respected for any kind of linear elastic propagative wave.

2.2 Direct formulation

In the following, the derivation of the MES formulation is offered considering a n Dimensional space (nD) space. Considering a symmetrical waves field and equations (4) and (1), the MES approach [15] leads to the following relation:

$$\frac{1}{r^{n-1}} \frac{\partial}{\partial r} (r^{n-1} \vec{I}) + \eta\omega W \vec{n} = 0 \quad (5)$$

which can also be written using equation (4):

$$-c^2 \frac{1}{r^{n-1}} \frac{\partial}{\partial r} \left(r^{n-1} W \vec{n} \right) = \eta \omega \vec{I} \quad (6)$$

Finally, the equation can be rewritten considering only the W field and provides a *local energy equation* for symmetrical waves energies.

$$\frac{1}{r^{n-1}} \frac{\partial}{\partial r} \left(r^{n-1} W \right) + \frac{\eta \omega}{c} W = 0 \quad (7)$$

Energy variables are defined as the superposition of a direct field and a reverberated field which can be added to obtain the total field.

The elementary solutions in terms of energy density and active intensity are denoted G and \vec{H} .

$$G(S, M) = \frac{1}{SM} e^{-\frac{\eta \omega}{c} SM} \quad \vec{H}(S, M) = \frac{c}{SM} e^{-\frac{\eta \omega}{c} SM} \vec{u}_{SM} \quad (8)$$

The kernels are determined in each point M of the considered space and created by a source located in S . Hence, using a superposition principle the energy solution is then expressed using the contribution of the primary source ρ (direct field) and fictitious sources σ . The fictitious sources are energy unknown parameters representing the contribution of the reverberant field. Precisely:

$$W(M) = \int_{\Omega} \rho(S) G(S, M) dS + \int_{\partial\Omega} \sigma(P) \vec{u}_{SM} \vec{n}_P G(P, M) dP \quad (9)$$

Ω is the system surface and $\partial\Omega$ is the system boundaries.

The active energy flow can be described by a similar expression:

$$I(M) = \int_{\Omega} \rho(S) H(S, M) dS + \int_{\partial\Omega} \sigma(P) \vec{u}_{SM} \vec{n}_P H(P, M) dP \quad (10)$$

where ρ corresponds to internal sources to the considered domain (here a two dimensional one) and σ corresponds to sources at the boundaries of the two dimensional space.

3. The inverse energy flow method IMES formulation

In what follows, a discretized format of the energy integral equations are employed. To that end, we define firstly T as the (N, N) geometric interaction matrix between N two dimensional edge elements. This operator is given below referring to figure 1.

$$T_{ij} = \int_{\partial\Omega} \int_{\partial\Omega} \frac{1}{2} \frac{1}{P_i P_j} e^{-\frac{\eta \omega}{c} P_i P_j} \vec{u}_{P_i P_j} \vec{n}_{P_i} dP_i dP_j \quad (11)$$

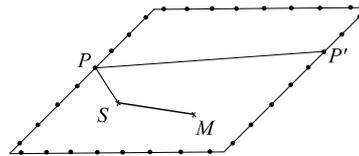


Figure 1. element/element interaction

Referring to the same figure 1, another needed (N, k) matrix is defined and named here Q . This operator is representing the geometric interaction between N boundary elements and k sources. It is expressed as follows:

$$Q_{ij} = \int_{\partial\Omega} \frac{1}{2} \frac{1}{S_j P_i} e^{-\frac{\eta\omega}{c} S_j P_i} \vec{u}_{S_j P_i} \vec{n}_{P_i} dP_i \quad (12)$$

Then, we define two matrices named Y^W and Y^I (having a dimension (k, m)) representing a geometric interaction between sources and measurement points. These matrices are determined respectively for the energy density W and the energy flow I as follows:

$$Y_{ij}^W = \int_{\partial\Omega} \frac{1}{S_j P_i} e^{-\frac{\eta\omega}{c} S_j P_i} dP_i \quad \text{and} \quad Y_{ij}^I = \frac{1}{c} \int_{\partial\Omega} \frac{1}{S_j P_i} e^{-\frac{\eta\omega}{c} S_j P_i} \vec{u}_{S_j P_i} dP_i \quad (13)$$

Ultimately, we define A and B as follows (Id being the identity matrix and ρ injected power vector):

$$A = Id - T \quad \text{and} \quad B = Q\rho \quad (14)$$

The power vector σ can thus be readily expressed as a function of injected power vector ρ as:

$$\sigma = A^{-1}B = A^{-1}Q\rho \quad (15)$$

Combining equations (9), (10) and (15), the energy density vector W and the energy flow vector I can be obtained from:

$$W = S^W \rho = R^W \rho + Y^W \sigma \quad \text{and} \quad I = S^I \rho = R^I \rho + Y^I \sigma \quad (16)$$

where R^W and R^I are defined as follows:

$$R_{ij}^W = \frac{1}{S_i M_j} e^{-\frac{\eta\omega}{c} S_i M_j} \quad \text{and} \quad R_{ij}^I = \frac{1}{S_i M_j} e^{-\frac{\eta\omega}{c} S_i M_j} \vec{u}_{S_i M_j} \quad (17)$$

S^W and S^I given by:

$$S^W = R^W + Y^W A^{-1}Q \quad \text{and} \quad S^I = R^I + Y^I A^{-1}Q \quad (18)$$

Finally, both W and \vec{I} fields can be expressed thanks to a linear operator denoted \mathcal{F} :

$$(W, \vec{I}) = \mathcal{F}(\rho) \quad (19)$$

The IMES formulation aims at inverting operator \mathcal{F} proposed in equation (19). Sources have to be detected thanks to measurements made on the plate. As the number of measurements is not equal to the number of sources, the problem is not directly invertible. Thus, we look for the \mathcal{G} operator that gives σ and ρ as a function of W and \vec{I} .

$$\rho = \mathcal{G}(W, \vec{I}) \quad (20)$$

Let now introduce the quantity $R(\mathcal{X})$ defined as follows:

$$R(\mathcal{X}) = \|(W, \vec{I}) - \mathcal{F} \circ \mathcal{X}(W, \vec{I})\| \quad (21)$$

We now have to minimize the quantity $R(\mathcal{X})$ among operator \mathcal{X} . In the following, the IMES will be applied to a 2D plate. Thus, Ω corresponds to the surface of the plate, whereas $\partial\Omega$ corresponds to the boundaries. In this case, only surface sources are treated. In what follows, The access

to structural intensities experimentally being a hard task to achieve, simulations have been made considering only energy densities W measurements in some locations of the structure. Thus, we will replace S^W by S for the sake of clarity. MES equation (9) is discretized to provide the following matrix formulation:

$$\begin{Bmatrix} W_1 \\ W_2 \\ \vdots \\ W_m \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} & \dots & S_{1k} \\ S_{21} & S_{22} & \dots & S_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ S_{m1} & S_{m2} & \dots & S_{mk} \end{bmatrix} \begin{Bmatrix} \rho_1 \\ \rho_2 \\ \vdots \\ \rho_k \end{Bmatrix} \quad (22)$$

where m is the number of the measurements and k is the number of the sources. To invert the formulation, as explained in equation (21), we have to find a matrix X that minimize the quantity $R(X)$:

$$R(X) = \|(I_m - SX)\| \left\| \begin{Bmatrix} W_1 \\ W_2 \\ \vdots \\ W_m \end{Bmatrix} \right\| \quad (23)$$

In fact, we choose matrix X as a pseudo-inverse of matrix S :

$$X = (S^T S)^{-1} S^T \quad (24)$$

The inverse formulation can then be performed. From a set of input energy parameters, the inversion can be achieved in order to characterize input sources.

4. Numerical simulations and Results

In what follows, a number of numerical tests are performed in order to validate the formulation. The first considered test case is very simple. It consists in achieving the inverse energy flow approach when a set of energy prediction using the direct energy flow method are used. This is an MES/IMES simulation. A more realistic test case is then considered. The input variables are in this situation computed from a full finite element model.

4.1 MES/IMES simulation

A first simulation named MES/IMES simulation is made. IMES calculations are processed using energy densities W coming from the direct MES formulation. The model studied here is a thin plate whose characteristics are mentioned in table (1).

Table 1. plate characteristics

<i>parameter</i>	<i>unit</i>	<i>value</i>
Length	<i>m</i>	1
Width	<i>m</i>	1
Height	<i>m</i>	$2e - 3$
Young's modulus of elasticity E	<i>Pa</i>	$2.1e11$
Poisson's ratio ν		0.3
Solid density ρ_s	<i>Kg/m³</i>	7800

This plate is excited by an input power of $1W/m^2$. Considering N to be the number of edge plate elements, IMES result found for $N = m = 64$ is presented in figure 2.

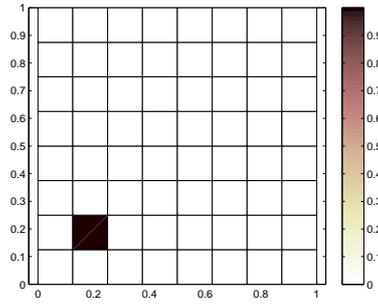


Figure 2. MES/IMES simulation for the plate

Table 2. Error on the input power

ξ	Error
0.06	30 %
0.14	25 %
0.5	18 %

It appears that the source are correctly identified in this simulation (localization and level). So, the inversion of $(S^T S)$ is validated. To illustrate the effect of the number of measurements m , we repeated the MES/IMES simulation while decreasing m . Let's then introduce the factor $\xi = \frac{m}{N}$ to quantify the efficiency of the method ($\xi = 1$ corresponds to a square matrix to be inverted).

It appears that by increasing the number of measurements m spread over the plate surface and corresponding to $\xi = 0.06$, $\xi = 0.14$ and $\xi = 0.5$ respectively, the source is well located and the error on the input power summarized in Table (2) decrease. Nevertheless, the input power of the source is equal to 0.7 W/m^2 for $\xi = 0.06$, 0.75 W/m^2 for $\xi = 0.14$ and 0.82 W/m^2 for $\xi = 0.5$ whereas it should be 1 W/m^2 . The right value of the input power is obtained for higher values of ξ . The IMES seems to correctly localize the input structural force but underestimates the injected power.

4.2 FEM/IMES simulation

The IMES/MES subsection allowed mainly the validation $(S^T S)$ inversion. In this subsection the validation of approach from a FEM simulated energies is considered. In fact, IMES calculations are processed using energy densities W coming from a finite element software (Comsol in the present study). An FEM model of the plate (characteristics provided below) are processed with 1 W/m^2 excitation and the same number of elements N . Given the input power ρ , the force F applied to a point of the plate is recalculated by:

$$\rho = \frac{1}{2} \frac{F^2}{8\sqrt{D\rho_s h}} \quad (25)$$

Where D is the plate stiffness defined by $D = \frac{Eh^3}{12(1-\nu^2)}$. FEM calculation is processed on the plate, and provides energy density W field used as an input for IMES algorithm. FEM calculations are first averaged first space and frequency averaged. Octave bands are considered and various center frequencies f_c are considered. The values of W recovered by FEM calculation and realized around f_c feed the IMES algorithm. Results found for $\xi = 0.06$, $\xi = 0.14$, $\xi = 0.5$ and $\xi = 1$ are presented in figure 3.

Moreover, the influence of f_c is also studied. Results found for $\xi = 1$ and for different values of f_c are given in figure 4. This figure shows a good localization of the source whatever the value of f_c belonging to the range of medium and high frequencies.

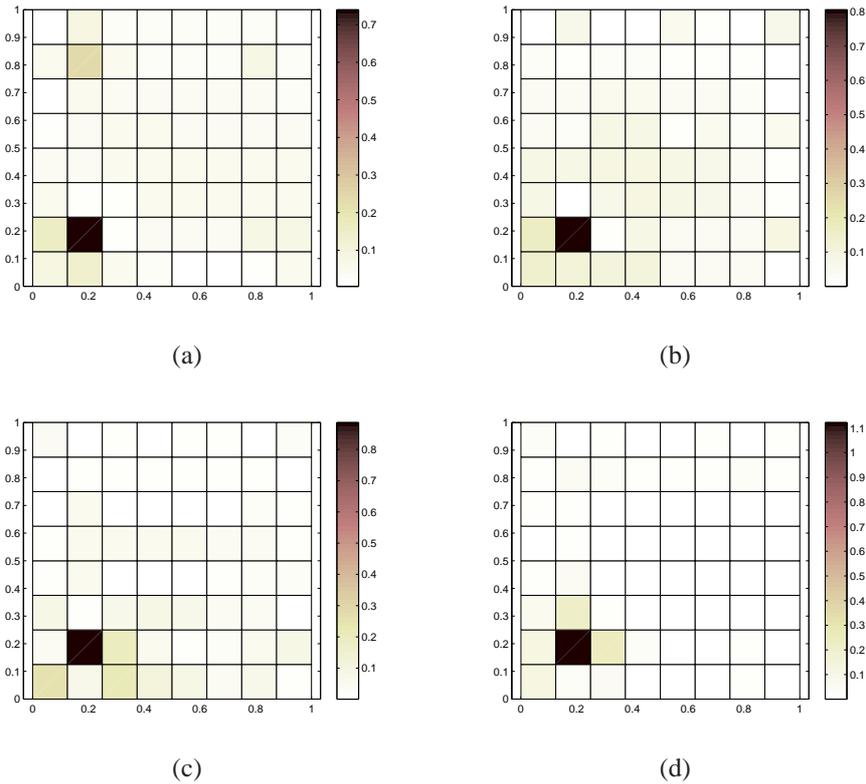


Figure 3. FEM/IMES simulations for $\xi = 0.06$ (a), $\xi = 0.14$ (b), $\xi = 0.5$ (c), $\xi = 1$ (d)

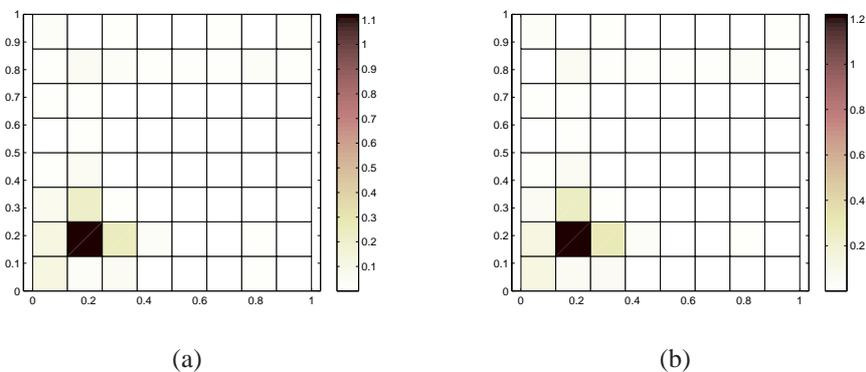


Figure 4. FEM/IMES simulations: $f_c = 950Hz$ (a) and $f_c = 1500Hz$ (b)

5. Conclusion

The aim of this paper is to analyze the effectiveness of the Simplified Energy Method (MES) to identify sources from a set of given measured energies in the plate. The MES formulation and assumptions were first detailed. Then, the Inverse formulation identifying sources through data energy recovered from the plate is given. After that, the MES is validated with FEM/IMES simulations taking numerical data from FEM software.

Validation of the strategy for 3D acoustic cavities is under investigation.

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