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Topological Reconstruction of Complex 3D Buildings and Automatic Extraction of Levels of Detail

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Abstract

This paper describes a new method allowing to retrieve the indoor and outdoor topology of a detailed 3D building model from its geometry and to extract different levels of detail (LoD) from the resulting topological description. No prior information about the initial model, except its geometric information is needed as input, and using the combinatorial maps data structure, the method recovers the topological information of the identified parts of the building. The topology is needed for most of the applications using 3D building models after the architects design it. While classical models available are mainly furnished in a Boundary Representation (B-Rep) format, we discuss how to recover the components that allow to distinguish the several parts of the building (defined as volumes) then the spatial relationships linking them.

Categories and Subject Descriptors (according to ACM CCS): I.3.5 [Computer Graphics]: Computational Geometry and Object Modeling —Boundary representations

1. Introduction

Modelling of buildings is a widely investigated research area in the computer science community. While architects create models mainly for high detailed visualization matters, other fields will need the same model for engineering analysis (calculation, simulation, ...) or less detailed visualization (3D urban maps). Due to the differences between those disciplines, modelling of buildings is a victim of a lack of common model since each field will have particular needs, not always provided by the initial model.

A lot of work has been done in this sense and promising results come from Industry Foundation Classes (IFC) and City Geography Markup Language (CityGML) as standards offering complete description of a building model and its environment. Those formats are still in development and classical CAD files remain the most widely used.

Our main contribution consists in proposing a new approach allowing to recover the whole topological information of the indoor and outdoor space of a 3D building model, only from its geometrical information. We use the combinatorial maps data structure that have the advantage of dealing easier with the topology. It offers a simple and efficient formalism to describe a complex geometry by mean of its topo-

logical structure, providing a 3D cellular description with the incidence and adjacency relationships between the cells.

At the end of the topological reconstruction, we end up with a topological description composed of a set of consistent volumes, in the sense that they represent real components of the building model (wall, floor, roof, etc). Topological relations allow to retrieve different information such as *which wall has contact with another*. Thanks to its handling advantages compared to the initial model, our reconstruction model stands as a generic basis for many applications. Here, we illustrate it with the automatic extraction of different levels of details from the initial building model that is known as building generalization in the 3D Urban mapping area.

After a discussion about the previous work related to topology of building models, we will briefly describe the theory behind combinatorial maps. The methodology of the approach allowing to recover the topology from the geometry will be detailed in the second part, before exploring the application to building generalization in a third section. We will finally conclude by summarizing and proposing future improvement to enhance the work.

1.1. Previous Work

Two major research areas stand in the investigations regarding modelling of buildings: the Geographic Information Systems (GIS) and the Building Information Models (BIM). Many works have been done and each of them aims to deal with specific applications. Independently of the approach used, there are no pure topological, semantic or geometric models, but hybrid models using various levels of detail. A good overview of the subject (for building indoors) is given in [DGF12]. We will discuss here the contributions that are more closely related to our work and dealing with topology.

3D data acquisition of geographic information is still intensively studied (e.g. 3D reconstruction using LiDAR sensor). On the other hand, 2D images plans are still commonly available (from satellite, 2D sketches from architects, cadastre, etc). Therefore many methods use such 2D plans as input to propose a 3D reconstruction by extrusion [BZ03, HDMB07, HMDB09, CL09]. All of those previous work offered 3D topological data structures to represent spatial relationships between objects.

As for 2D problems, a good segmentation of the 3D building model into semantically meaningful parts is necessary to study properly the spatial relationships between components. Most of the methods take benefits from prior information like semantics (in IFC or CityGML files) [TRRF01, TR07], graph-based or grammar analysis [BHMT], to identify the components. Thiemman and Sester [TS04] derived the method in [RHG*01] to find features on 3D model before generalizing it based on defined significance criteria.

Another well investigated problem is the topological query on 3D models. Borrmann and Rank [BR08] proposed formal definitions of topological operators by means of an Octree-based representation of objects. The spatial relationships are described using the 9-intersection model [EF91]. The goal was to make topological predicates available in a 3D Spatial Query Language for BIM [BTR06]. Ellul and Haklay [EH09] proposed the Binary B-Rep structure and its modified version to improve binary relationship query performance in 3D GIS.

Combinatorial data structures have proven their efficiency to describe topological information and spatial relationships in building models [CL09, Wor11]. Similarly to our approach, Horna et. al [HDMB07, HMDB09] introduced a method to reconstruct geometry and topology of 3D buildings from 2D architectural plans based on the generalized maps data structure. While architects tend to produce more and more geometrically detailed 3D models, no method among the literature allows to retrieve the spatial decomposition of a 3D building and its complete topology without prior information in addition to its geometry. Our goal is to propose a complete framework to fully recover the indoor and outdoor topology of a 3D building model from its geometric information *only*, and to store it in a proper data structure.

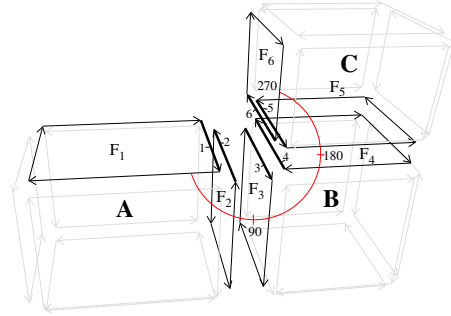


Figure 1: Combinatorial map representation of three volumes *A*, *B* and *C*. The six faces in dark lines share the same edge (described by the 6 darts drawn as bold arrows and numbered from 1 to 6). The figure illustrates the angular sorting method using dart 1 as reference. Face F_1 is linked by β_2 to face F_2 , face F_3 to F_4 , and face F_5 to F_6 .

1.2. Combinatorial Maps and Linear Cell Complex

A 3D building model can be seen as a set of volumes corresponding to specific parts (walls, roofs, floors, etc), linked to form rooms. We will describe these different parts and their interconnection by using *3D combinatorial maps* [Lie94, Dam13a]. A combinatorial map is an edge-centered data structure composed by a set of darts plus the relations between these darts. A dart can be seen as a part of an oriented edge, plus a part of incident vertex, face and volume. By linking these darts by β_1 , faces are obtained which are cycles of darts. Then, linking these faces by β_2 , volumes are obtained which are connected components of darts obtained using only β_1 and β_2 . The operation allowing to create the β_i links is called *i-sew*. Lastly volumes are linked by β_3 .

Thanks to the darts and the β relations, 3D combinatorial maps represent the complete topological information of 3D buildings. Indeed they describe the building subdivisions in cells: volumes (3-cells), faces (2-cells), edges (1-cells) and vertices (0-cells); plus all the incidence and adjacency relations between these cells (see an example of 3D combinatorial map in Figure 1). In order to describe also the shape of the buildings, 3D combinatorial maps are enriched with 3D points associated to the 0-cells. This corresponds to an embedding of a combinatorial map in a linear geometrical space which is called *3D linear cell complex (LCC)* [Dam13b].

The main interests of a 3D linear cell complex is to describe the full topological and geometrical information of 3D buildings with a data structure allowing several construction and modification operations, while guarantying the validity of objects thanks to a strong mathematical background [Lie94]. Furthermore, all the adjacency and incidence relationships information between the cells is stored in the data structure and available by simple query operations. Another benefit is the availability of an open C++

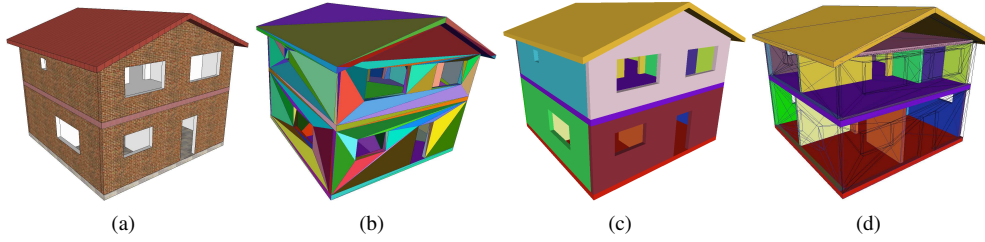


Figure 2: Topological reconstruction of a building model. (a) Original textured model. (b) Soup of isolated faces. (c) Result after the angular sorting. (d) Final result after linking volumes.

library named *Computational Geometry Algorithm Library* (CGAL) [CGA13] that offers all the tools needed to work with combinatorial maps and to produce the so-called LCC.

2. Topological Reconstruction

In order to fully describe the topological structure of a building model, the cells and adjacency relations of the building must be retrieved taking as input its geometrical description. This topological reconstruction is done in ascending dimension order, in basically three main steps (cf. Figure 2):

- Creation of isolated faces from the polygon's geometry;
- Link by β_2 the faces to create the volumes;
- Link by β_3 the volumes to put together all the volumes.

The first step is straightforward, since the 3D points and the sequence of vertices of each face are directly provided by the input file. They allow to directly create corresponding isolated faces in the linear cell complex (cf. Figure 2(b)).

2.1. Building Links Between Faces

The second step of our topological reconstruction consists in linking the faces by β_2 in order to create the different volumes of the LCC, each volume corresponding to a meaningful component of the building model (cf. Figure 2(c)). Contrary to the previous step (reconstruction of isolated faces), the information of which faces must be linked by β_2 is not directly given in the geometrical data but needs to be retrieved.

To reach this objective, we use the following three properties: (1) two faces can be linked by β_2 only if they have the same edge along their boundary; (2) when more than two faces share the same edge, a face is linked by β_2 with the closest face (angularly) around the edge; (3) two darts can be linked by β_2 only if they have opposite orientations. The two first properties are direct consequences of the fact that the volumes of the building form a partition of the 3D space, and the third property is a basic property of combinatorial maps.

Thanks to these three properties, we can reconstruct all the β_2 links between all the isolated faces by an angular-based sorting method consisting in three steps:

- Collect all faces sharing a common edge and pick one as a reference;
- Compute and sort angles between the reference face and the other faces around the common edge;
- 2 – sew each pair of consecutive darts in the angular ordering, having opposite orientation.

Figure 1 illustrates this principle. In this example, applying the angular sorting using F_1 as reference gives F_2 and F_3 as closest faces to F_1 (90°), and the only one having dart of opposite direction to dart 1 is F_2 . Thus darts 1 and 2 are linked by β_2 . Note that similar sorting approach is proposed in 2D in [HDMB07], where edges are sorted in order to retrieve links between edges.

2.2. Building Links Between Volumes

The third step of our topological reconstruction consists in linking the different unconnected volumes by β_3 (cf. Figure 2(d)). For this reconstruction, we have to detect all the surfaces of contact between all the volumes. This is achieved by searching all pairs of faces in the LCC which are coplanar and with non empty intersection. Each pair of coplanar faces characterizes an adjacency between two volumes. For each pair of faces, there are two possibilities: either the two faces have the same topology and the same geometry, or they have different shapes. In the first case, each pair of colinear darts of the two faces can be directly put in relation by β_3 (this is for example the case in Figure 1 for volumes A and B through the two faces F_2 and F_3).

In the second case, when the two faces have different shapes, we need to create the contact surface between the two incident volumes by cutting the two faces in order to obtain the common part in the two faces having both same topology and the same geometry. The process illustrated in Figure 3 is depicted by the following steps:

- Compute the *inner* and intersection points between the pair of faces and insert corresponding vertices in the LCC;
- Based on the new vertices added and the existing coplanar faces, build the new faces by inserting edges between the different vertices.

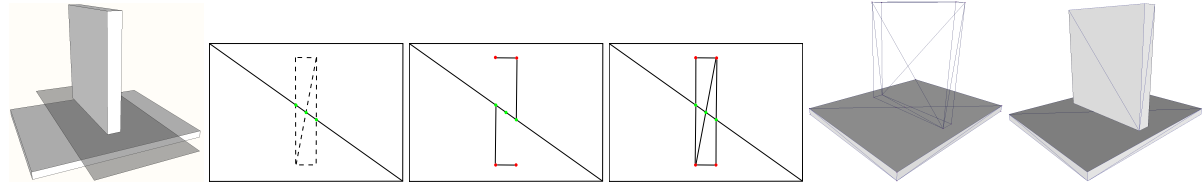


Figure 3: Steps of β_3 reconstruction for non similar faces. Starting from the left, the first image shows two blocks (a horizontal floor and a vertical wall) having contact in a common plane (separated for visual matters); the three following images illustrate a top view of the contact surface projected on the floor (the intersection points between both volumes are represented in green and the inner points in red). The two last images show the two volumes 3-sewn.

In the first step, we search the intersections between the boundaries of the coplanar faces. Some vertices will be missing after only intersection query, because they do not result from edge intersection, but they lie inside the surface areas of opposite faces (cf. Figure 3). Thus, those points that we call *inner points* are also queried and inserted.

In the second step, the new faces representing the contact surface between pairs of volumes are built by inserting edges between the proper vertices. The edges are created based on the darts of the faces they depict. All those steps are processed by means of classical geometrical algorithms like polygon normal vector calculation [SSS74], space projection and intersection computation, combined with the operations defined for the combinatorial maps.

2.3. Constraints on the Input Data and Rounding Issues

A few constraints on the input model are necessary to obtain good results. 3D models are expected to be quite well furnished in geometric information. From an architect we need to represent each component entirely with its whole geometry and to preserve it from changing while interfering with the other components. This is not a tricky task since recent modeling tools offer the possibility to handle each single or set of drawn geometries as a separated component. This way, we will also be able to remove the useless components which make the model unusable for fields like numerical simulation for example. Note that if the geometrical model does not satisfy these constraints, we could envisage some automatic correction tools. This is one of our perspectives.

On the other hand, it is well known that in 3D geometry the concept of contact between two entities is exposed to vagueness. The modelling tools also face such kind of problems. So, we must define a range in which a contact can be considered between two cells. The algorithm is fully implemented by using an *Epsilon Geometry* model [SSG89] to avoid as much inconsistencies as possible and to make it robust. Let ϵ be the margin error defined for a given process. For the collection of coplanar faces, assuming that the modelling tools can at least avoid the collision between two components supposed to have simple contact, we consider

that two faces in a distance of ϵ_1 and having small difference in their normal vector can be considered as coplanar. The ϵ used is usually very dependent of the model and its scale. For the intersection computation, the rounding issue may occur for the points and edges contact. To overcome this, the smallest distance between a point and an edge is compared to a margin error ϵ_2 to consider if there is contact or not. Then in the case when this distance is non zero, the closest point to the intersection point found, lying on the segment, is kept to build the new point on the edge.

3. Automatic Extraction of Levels of Detail

With the soaring of techniques allowing highly detailed 3D acquisition and the increasing popularity of 3D virtual city modelling these last years, the concept of 3D building generalization is being intensively investigated [FMJ09, Thi02, Ses07, Kad07]. It consists in simplifying complex 3D buildings to end up with lighter multi-scale models with less details, easier to handle. It is a crucial process for real-time visualization and navigation. Many recent buildings already have available 3D models produced by architects using CAD tools. Our goal here is to exploit those high detailed and complex models, to automatically extract their simplified versions, while preserving their visual shapes as much as possible. Our simplification framework is based on the four levels of detail (LoDs) defined by the OGC CityGML standard [Ope14] for 3D buildings. LoD1 is the building at its most simplified shape. It can be thought of as a footprint extrusion of the model or just its bounding box depending on its shape complexity. LoD2 is just a LoD1 characterized by a roof differentiation. LoD3 corresponds to the facade that is the only visible part from outside, and finally LoD4 include the indoor details of the building. The existing works on building generalization mainly focus on deriving LoD2 and LoD1 from LoD3 models. They are still the most used levels for visualization and navigation purposes, but other methods are being explored to overcome the limitations while interacting with huge scale models [DBCG*09]. Considering the initial model as of LoD4, we present our automatic extraction of LoD3 to 1.

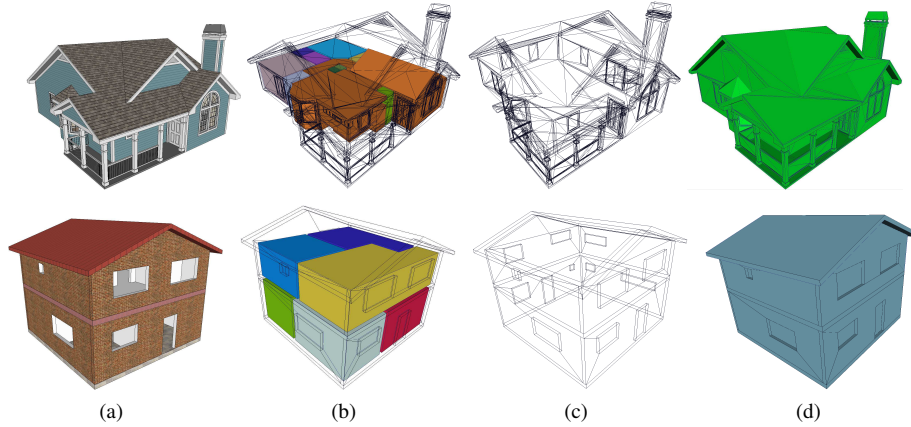


Figure 4: Indoor volumes and outer shell extraction. First row model: 23828 polygons, second row model: 560 polygons. (a) The original and textured model. (b) The indoor volumes. (c) The outer shell (visualized in wireframe). (d) The resulting exterior shell presenting detailed but lighter model (2510 polygons for the first row and 127 for the second).

3.1. Extraction of LoD3

Nowadays, most of the city buildings are reconstructed in LoD3 [FM12]. On the other hand, as discussed in [FMJ09], the exterior envelop of a 3D building can be seen as a good generalization in LoD3. Assuming that we are dealing with a closed building model (with windows and doors at every hole), we can extract the indoor volumes of the rooms and the exterior shell of the model from the LCC resulting from our topological reconstruction method. Indeed, by duplicating all the faces of the LCC which are not 3-sewn (i.e. having no other dart linked by β_3), and linking them by β_2 , both indoor and outdoor volumes are automatically generated.

Figure 5 illustrates the process in 2D and Figure 4 shows the results on a model available in the Trimble repository [Tri13]. The LCC allows to keep or remove volumes of interest. It is for example possible to remove all indoor volumes and components to keep only the outer shell. The size reduction of the model is significant: 89.5% for the first row model and 77.3% for the second row. There is no visual difference between the initial model and its shell. The later can be still generalized more and end up with even a lighter model. Thanks to that process, one can obtain a suitable model to insert into a 3D urban map from a detailed initial building model.

3.2. Extraction of LoD2 and LoD1

Thus to extract those two LoDs, we worked mainly on the roof of the building, similarly to the approach used in [FM12]. Despite the lack of semantical information, the roof can be automatically detected among the volumes. The largest 3-cell containing the highest vertices in addition to the sloping of its normal against the vertical direction (in the case of tilted roofs) is considered to be the roof. From that

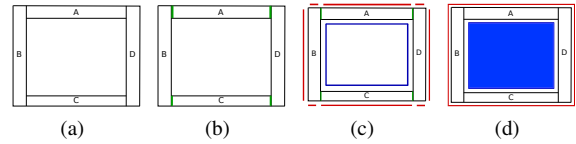


Figure 5: Face duplication process in 2D. (a) Four faces A, B, C and D. (b) Green edges symbolise common edges between two faces (2-sewn). (c) Unshared edges (not 2-sewn) are duplicated inside (blue edges) and outside (red edges). (d) Blue and red edges are 1-sewn to form the blue face (indoor volume in 3D) and the exterior envelop.

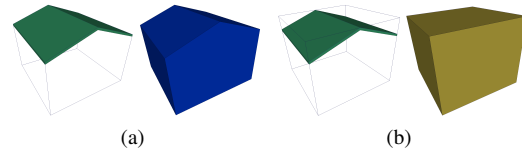


Figure 6: Roof-based extraction of LoD2 and LoD1. (a) LoD2 obtained by roof extrusion. (b) LoD1 that corresponds in this case to the bounding box of the LoD2.

volume an extrusion to the ground plan is processed to obtain the LoD2, and the LoD1 is finally obtained by flattening the roof (cf. Figure 6).

4. Conclusion and Outlooks

We presented a new method to recover the whole topology of a complex 3D building model from its geometry without any other kind of information. Thanks to the combinatorial map data structure and its formalism, we construct a cellular de-

composition of the model, and recover step by step the links between edges, faces and volumes. The space subdivision is done in such a way that the resulting volumes represent consistent parts of the building model, despite of the lack of semantical information. The output model rich of geometric and topological information can then be used by many applications. We showed how to extract from complex model lower LoDs for applications in GIS.

As future work, we plan to exploit the power of the LCC more by applying specific simulation processes (acoustics for example) with the indoor volumes extracted and their adjacency. Another interesting application, being recently more and more investigated, is the automatic semantization of the building components from a purely geometrical model. It will also be interesting to work with the IFC or CityGML standards, to contribute to the topological lacks and to take advantage of already furnished semantics for a better building cell decomposition. Indeed, only 3D models with available geometric, semantical and topological information can be of relevant help to the interoperability problem. Thus by providing tools able to recover or create such information where they are missing, we can considerably optimize the applications around modelling of buildings.

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