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# A way to explore the possibility of new kinds of turbomachinery

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- **Abstract**

In this article we desire to explore the capability of conceiving new kinds of turbomachinery, and to estimate their performance and their application domain.

We need to extend the classical Euler equation for turbomachinery for non relative stationary flow and non permanent fluid boundaries.

To obtain reasonably tractable relations for exploratory purposes, we limit our study to incompressible non viscous flow.

The simplifying chosen boundaries conditions lead to hydraulic turbomachinery type and we focus on turbines.

It is possible to establish a chart of performance in the cases of limited effects of gravity on rotor flow and the domain of use is compared to classical hydraulic turbines.

In other cases the model can be applied and leads to turbomachines which appear more as water wheels but it offers another way to apprehend and design them.

- **Key Words**

Internal Flow, Turbomachinery, Unsteady Flow, Water Turbine, Water Wheel

- **Nomenclature**

$B$ : axial length	$R$ radius or rotor domain
$C$ constant	$S$ surface or area
$D$ diameter or fluid domain	$U$ frame velocity
$H$ head	$V$ absolute velocity
$L$ length with fluid in passage	$W$ relative velocity
$P$ power	
$\partial x$ limit of open domain $x$	$p$ static pressure
$e$ internal energy	$q$ volumetric flow rate
$f$ scalar quantity	$r$ radius
$g$ gravitational acceleration	$s$ curvilinear abscissa
$gH$ $g \times H$	$t$ time or unit vector tangent to a curve
$i$ incidence	$z$ altitude (from axis)
$l$ length	
$n$ rotational speed RPM	
$n_q$ specific speed	
$\alpha$ angle between meridional plane and absolute velocity	$\theta_0$ Initial injection azimuthal position
$\beta$ angle between meridional plane and relative velocity	$\sigma$ constraint
$\delta$ increment or derivative	$\tau$ time or tangential constraint
$\eta$ efficiency	$\omega$ rotor angular velocity
$\theta$ angular position counterclockwise from horizontal axes	$\rho$ fluid density

- Subscripts

$a$ ambient	0 time : beginning of injection
$c$ control surface	1 time : end of injection beginning of centripetal path
$ci$ canal inlet	2 time : end of centripetal path beginning of centrifugal path
$i$ inlet	3 time : end of centrifugal path beginning of ejection
$li$ limit of fluid (last entered)	4 time : end of ejection
$lo$ limit of fluid (first entered)	$M / F$ Exerted by the rotor on fluid
$m$ material surface	+ after inlet
$o$ outlet	- before inlet
$u$ peripheral velocity direction	
overbar is used for dimensionless quantities	tilde is used for characteristic units

## 1. Introduction

Dean [1] gives a necessary condition for turbomachinery to exchange energy with fluid but no way to obtain geometries apart of already well known.

To explore the possibility of conceiving turbomachinery different of the ones we currently use, we try to follow an intuitive approach based on a geometrical induction of their principle of action.

Axial, mixed, radial, Banki, Pelton, centripetal or centrifugal, open or closed, full or partial admission turbomachines base concept can be inferred by considering an absolute free path viewed in the relative frame then bending it to generate appropriate force on fluid to obtain desired energy transfer between fluid and rotor.

The most evident case is the one of an absolute linear free path with constant velocity parallel to the axis, the relative free path is an helix, no tangential force is applied to the fluid. We can modify the relative path with blades to obtain tangential force and so axial compressor or turbine.

There is multiple variants and arrangements of these turbomachines. Is it possible to imagine something notably different ? So arises the question: what all of these machines have in common ? A possible response is all of this machines are characterized by the fact that the outlet section is different of the inlet section (in the relative frame), so we will try to determine a type of turbomachine were the outlet section is the same as the inlet one (in the relative frame, preserving continuous flow at a fixed inlet).

## 2. One-dimensional approach

We use Euler equations for incompressible flow in relative frame.

$$\operatorname{div}(\vec{W}) = 0 \quad (1)$$

$$\left( \frac{\partial \vec{W}}{\partial t} \right)_r + \operatorname{rot} \vec{W} \times \vec{W} + 2 \vec{\omega} \times \vec{W} = -\operatorname{grad} \left( \frac{p}{\rho} + \frac{\vec{W}^2}{2} - \frac{U^2}{2} \right) + \vec{g} \quad (2)$$

$$\frac{W^2}{2} - \frac{U^2}{2} = \frac{V^2}{2} - U V_U \quad (3)$$

A more tractable form for discussing about the feasibility is the one - dimensional form along the flow passage.

$$\left( \frac{\partial W_{ii}}{\partial t} \right)_r \int_{li}^{lo} \frac{S_{ii}}{S} ds = \left( \frac{p}{\rho} + \frac{W^2}{2} - \frac{U^2}{2} \right)_{ii} - \left( \frac{p}{\rho} + \frac{W^2}{2} - \frac{U^2}{2} \right)_{io} + \int_{li}^{lo} \vec{g} \cdot \vec{i} ds \quad (4)$$

Four phases: Injection, deceleration, opposite acceleration , ejection, will be necessary in the passage .

Ignoring gravity effect for the moment, and if the pressure is considered as constant  $p_a = 0$  on fluid limits apart of injection, a simplification which is permitted for example if the exit is in an ambient fluid of density  $\rho_a$  such that  $\rho_a \ll \rho$ , it is necessary to have deceleration near velocity inversion than  $R_{lo} < R_{li}$ .

The simplest configuration model is when the entrance is the one of a centripetal machine and so the exit is of centrifugal type.

## 3. Moment of momentum equation derivation

We search for an equivalent to Euler formula for turbomachine which link moment of momentum to flow properties on inlet and outlet fixed control surfaces  $S_i$  and  $S_o$ . We can write:

$$M = \int_V \rho r \frac{\partial}{\partial t} (W_\theta) dv + \int_{S_i \cup S_o} \rho r V_\theta (\vec{W} \cdot \vec{n}) dS \quad (5)$$

Generally in turbomachinery field we rely on relative stationary flow to eliminate volume integration term and axisymmetric hub and shroud to restrict surface integration to inlet and outlet (at least as a first approximation), the steadiness hypothesis of the relative flow can be replaced by the one of relative time periodicity to eliminate the volume integration term.

In our case we admit flow periodicity in the absolute frame (induced by the rotor rotation) but a priori not in the relative frame. We examine the reasons why strategies to obtain classical Euler equation fail in our case.

In absolute frame the flow is periodic in time but not in space, the integration volume vary, and the surface integrand does not reduce to fixed  $S_i$  and  $S_o$  integrand.

In relative frame the flow is not periodic in time nor in space, the integration volume vary, and the surface integrand does not reduce to  $S_i$  and  $S_o$  integrand.

The same difficulties are valid for the mass conservation (in case of compressible flow) and energy equation derivation.

So willing to avoid prematurely recourse to infinite number of blade hypothesis we turn to another approach.

#### 4. Evolution of a scalar quantity in the rotor

##### 4.1. Mean value over a time period

One considers a rotor which the material free volume  $R$  is limited by  $\partial R$  composed of an immaterial axisymmetric control surface  $\partial R_c$  and of the material surface  $\partial R_m$ . The flow in the absolute frame has a periodicity  $\delta t$ .

The fluid volume  $D$  limited by  $\partial D$  flows in  $R$  during a time interval containing  $[0, \delta t]$ .

Given  $f$  a continuous scalar function with continuous derivatives in  $D \cup \partial D$  we are interested in:

$$\int_{D \cap R} f dv \text{ its convective derivative } \frac{d}{dt} \int_{D \cap R} f dv \text{ and its mean value over a period : } \frac{1}{\delta t} \int_0^{\delta t} \left( \frac{d}{dt} \int_{D \cap R} f dv \right) d\tau$$

These quantities are continuous with time.

$$\frac{d}{dt} \int_{D \cap R} f dv = \int_{D \cap R} \frac{\partial f}{\partial t} dv + \int_{\partial(D \cap R)} f \vec{V} \cdot \vec{n} dS \quad (6)$$

We will note  $\frac{\delta}{\delta t}$  the time derivative of  $\int_{D \cap R} f dv$  to distinguish from convective derivative.

$$\frac{\delta}{\delta t} \int_{D \cap R} f dv = \int_{D \cap R} \frac{\partial f}{\partial t} dv + \int_{\partial(D \cap R)} f \vec{X} \cdot \vec{n} dS \quad (7)$$

Where  $\vec{X}$  is the velocity of the surface  $\partial(D \cap R)$ .

One have  $\vec{X} \cdot \vec{n} = \vec{V} \cdot \vec{n}$  except on  $\partial R_c$  where  $\vec{X} \cdot \vec{n} = 0$  this leads to :

$$\frac{d}{dt} \int_{D \cap R} f dv = \frac{\delta}{\delta t} \int_{D \cap R} f dv + \int_{\partial R_c \cap D} f \vec{V} \cdot \vec{n} dS \quad (8)$$

$\frac{\delta}{\delta t} \int_{D \cap R} f dv$  can be discontinuous with time but as  $\int_{D \cap R} f dv$  is continuous we can integrate it and :

$$\frac{1}{\delta t} \int_0^{\delta t} \left( \frac{d}{dt} \int_{D \cap R} f dv \right) d\tau = \frac{1}{\delta t} \left( \int_0^{\delta t} \left( \frac{\delta}{\delta t} \int_{D(\tau) \cap R(\tau)} f dv \right) d\tau + \int_0^{\delta t} \left( \int_{\partial R_c \cap D(\tau)} f \vec{V} \cdot \vec{n} dS \right) d\tau \right) \quad (9)$$

$$\frac{1}{\delta t} \int_0^{\delta t} \left( \frac{d}{dt} \int_{D \cap R} f dv \right) d\tau = \frac{1}{\delta t} \left( \int_{D(\delta t) \cap R(\delta t)} f dv - \int_{D(0) \cap R(0)} f dv + \int_0^{\delta t} \left( \int_{\partial R_c \cap D(\tau)} f \vec{V} \cdot \vec{n} dS \right) d\tau \right) \quad (10)$$

From periodicity :

$$\frac{1}{\delta t} \int_0^{\delta t} \left( \frac{d}{dt} \int_{D \cap R} f dv \right) d\tau = \frac{1}{\delta t} \left( \int_0^{\delta t} \left( \int_{\partial R_c \cap D(\tau)} f \vec{V} \cdot \vec{n} dS \right) d\tau \right) \quad (11)$$

#### 4.2. Application to Mass flow conservation equation

Making  $f = \rho$  leads to mass flow conservation over a time period (even in case of compressible flow).

$$\int_0^{\delta t} \left( \int_{\partial R_c \cap D(\tau)} \rho \vec{V} \cdot \vec{n} \, dS \right) d\tau = 0 \quad (12)$$

#### 4.3. Application to moment of momentum equation

The mean value of the torque applied to the fluid will be given with  $f = \rho r V_\theta$ .

$$M_{M/F} = \frac{1}{\delta t} \left( \int_0^{\delta t} \left( \int_{\partial R_c \cap D(\tau)} \rho r V_\theta \vec{V} \cdot \vec{n} \, dS \right) d\tau \right) \quad (13)$$

#### 4.4. Application to energy equation

$$\int_{D \cap R} \rho \vec{g} \cdot \vec{V} \, dV = - \int_{D \cap R} \rho g \frac{dz}{dt} \, dV = - \frac{d}{dt} \int_{D \cap R} \rho g z \, dV \quad (14)$$

$$\partial(D \cap R) = (\partial R \cap D) \cup (\partial D \cap R) \cup (\partial R \cap \partial D) \quad (15)$$

$$\int_{\partial(D \cap R)} \vec{\sigma} \cdot \vec{V} \, dS = \int_{\partial R_m \cap \partial D} \vec{\sigma} \cdot \vec{V} \, dS + \int_{\partial R_c \cap D} \vec{\sigma} \cdot \vec{V} \, dS + \int_{\partial D \cap R} \vec{\sigma} \cdot \vec{V} \, dS \quad (16)$$

$$\vec{\sigma} = -p \vec{n} + \vec{\tau} \quad (17)$$

Following classical assumption we neglect viscous forces on  $\partial R_c$ ,

$$\vec{\tau} = \vec{0} \quad \text{on } \partial R_c \cap D \quad (18)$$

As  $\rho_a \ll \rho$  :

$$\vec{\sigma} = \vec{0} \quad \text{on } \partial D \cap R \quad (19)$$

And:

$$\int_{\partial R_m \cap \partial D} \vec{\sigma} \cdot \vec{V} \, dS = P_{M/F} \quad (20)$$

$$\int_{\partial(D \cap R)} \vec{\sigma} \cdot \vec{V} \, dS = \int_{\partial R_c \cap D} (-p \vec{n}) \cdot \vec{V} \, dS + P_{M/F} \quad (21)$$

Making  $f = e + \frac{V^2}{2}$  first principle can be written:

$$\frac{1}{\delta t} \left( \int_0^{\delta t} \left( \int_{\partial R_c \cap D(\tau)} \rho \left( \frac{p}{\rho} + \frac{V^2}{2} + g z \right) \vec{V} \cdot \vec{n} \, dS \right) d\tau \right) = \frac{1}{\delta t} \left( \int_0^{\delta t} P_{M/F}(\tau) \, d\tau \right) \quad (22)$$

The disappearance of instationnary term in absolute frame but only for mean value over a time period is consistent with the conclusions of Dean[1].

We now turn to more practical aspect of conception and we will use the previous relations later.

## 5. Velocity triangles

To continue simply we focus on pure 2D radial flow. We define velocity triangles examples supposing inlet and exit radial velocities are about the same magnitude and ignoring inlet incidence and exit slip. The triangles are drawn on figures 1 and 2, blue color is for absolute velocity, red for frame velocity and green for relative velocity.

### 5.1. Pump case

To realize suitable energy exchange we have to add kinetic energy to the fluid. Consequently at inlet the peripheral velocity will be chosen such that to subtract to the relative tangential velocity and add to it at exit.

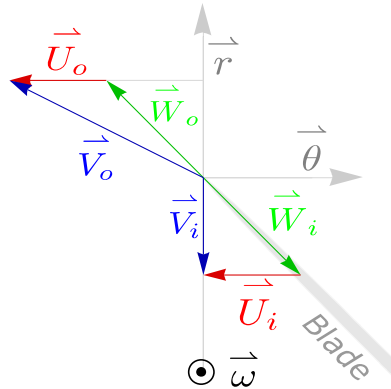


Figure 1: Velocity triangles examples for pump case

### 5.2. Turbine case

To realize suitable energy exchange we have to subtract kinetic energy to the fluid. Consequently at inlet the peripheral velocity will be chosen such that to add to the relative tangential velocity and subtract to it at exit.

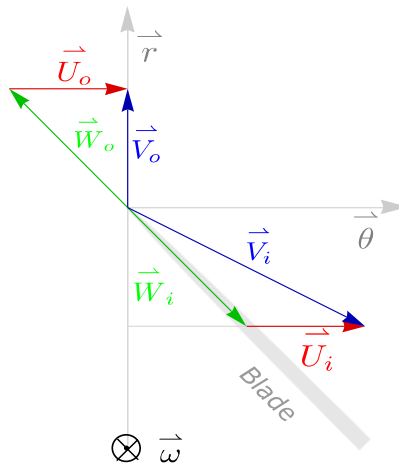


Figure 2: Velocity triangles examples for turbine case

We focus now on turbine case.

## 6. Illustration of turbine case flow path

We illustrate now the path of fluid in a 2D rotor. After entering in a flow passage it travels to the axes, reverses and then exits. Colored regions red, green, blue and magenta correspond to injection, centripetal path, centrifugal path, and ejection respectively.

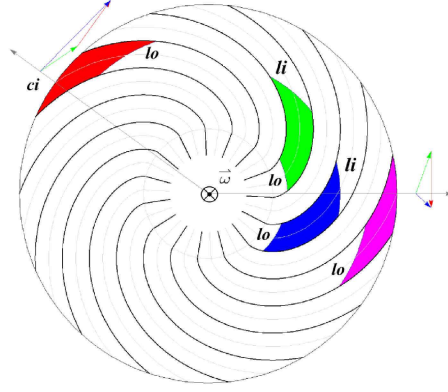


Figure 3: Illustration of fluid motion for turbine case.

## 7. Flow calculation

### 7.1. Inlet conditions

We suppose the inlet conditions are such that the fluid entrance takes place on a short distance and that the flow direction is such that the inlet losses are small so that we can write:

$$\left( \frac{p}{\rho} + \frac{W^2}{2} - \frac{U^2}{2} \right)_+ = \left( \frac{p}{\rho} + \frac{W^2}{2} - \frac{U^2}{2} \right)_- \quad (23)$$

Which can be written:

$$\left( \frac{p}{\rho} + \frac{W^2}{2} \right)_+ = \left( \frac{p}{\rho} + \frac{V^2}{2} \right)_- + \frac{U_i^2}{2} - U_i W^+ \cos \beta^+ \operatorname{tg} \alpha^- \quad (24)$$

$\left( \frac{p}{\rho} + \frac{V^2}{2} \right)_-$  is supposed constant at inlet, and we adjust  $\alpha^-$  such that  $W^+ \operatorname{tg} \alpha^-$  be constant at inlet.

$$\cos \beta^+ W^+ \operatorname{tg} \alpha^- = C_t \quad (25)$$

$\left( \frac{p}{\rho} + \frac{W^2}{2} \right)_+$  is then a constant, we will verify that our hypothesis does not lead to too high incidence at leading edge and we define:

$$\frac{W_{ci0}^2}{2} = \left( \frac{p}{\rho} + \frac{w^2}{2} \right)_+ \quad (26)$$

$$\left( \frac{p}{\rho} + \frac{V^2}{2} \right)_- = \frac{W_{ci0}^2}{2} - \frac{U_i^2}{2} + U_i C_t \quad (27)$$

### 7.2. Injection

In a flow passage in the case of a high number of infinitely thin blades the area for a unit axial length is given by:

$$S = r \cos \beta \delta \theta \quad (28)$$

$$r \cos \beta = C \quad (29)$$

If C is a constant W is constant and so the length of passage with fluid after injection and before ejection. In that case we have:

$$R_{li}^2 - R_{lo}^2 = 2 C L \quad (30)$$

For analytically obtained solutions we restrict to cases where the  $\beta=0$  radius is not crossed.

$$\left( \frac{\partial W_{li}}{\partial t} \right)_r L = \left( \frac{p}{\rho} + \frac{W^2}{2} - \frac{U^2}{2} \right)_{li} - \left( \frac{W^2}{2} - \frac{U^2}{2} \right)_{lo} \quad (31)$$

As W is constant along the passage we can write the first order ordinary differential system:

$$\begin{cases} \frac{dW}{dt} L = \frac{W_{ci0}^2}{2} - \frac{W^2}{2} - \omega^2 C L \\ \frac{dL}{dt} = W \end{cases} \quad (32)$$

As  $L(t=0) = 0$  :

$$W(t=0) = W_{ci0} \quad (33)$$

Deriving the system we get:

$$\frac{d^2 W}{dt^2} L = - \left( 2 \frac{dW}{dt} + \omega^2 C \right) W \quad (34)$$

Which is verified in particular if:

$$\frac{d^2 W}{dt^2} = 0 \quad 2 \frac{dW}{dt} + \omega^2 C = 0 \quad \forall t \quad (35)$$

One verifies that a solution is:

$$\begin{cases} W = W_{ci0} - \frac{\omega^2 C}{2} t \\ L = W_{ci0} t - \frac{\omega^2 C}{4} t^2 \end{cases} \quad (36)$$

To ensure inward velocity we suppose injection stops at time  $t_1$  such that :

$$W(t_1) \geq 0 \quad \Rightarrow \quad t_1 \leq \frac{2 W_{ci0}}{\omega^2 C} \quad (37)$$

### 7.3. Centripetal and centrifugal path

Centripetal part begins at time  $t_1$ , ends at time  $t_2$  when relative velocity is zero, whereas centrifugal part begins at time  $t_2$  and ends at time  $t_3$  when external radius is reached.

$$\frac{dW}{dt} = -\omega^2 C \quad (38)$$

$$W = W_1 - \omega^2 C (t - t_1) \quad (39)$$

$$t_2 = \frac{W_{ci0}}{\omega^2 C} + \frac{t_1}{2} \quad (40)$$

$t_3$  is defined by :

$$\int_{t_1}^{t_3} W dt = 0 \quad (41)$$

$$t_3 = \frac{2 W_{ci0}}{\omega^2 C} \quad (42)$$

$$W_3 = -W_1 \quad (43)$$

### 7.4. Ejection

$$\begin{cases} \frac{dW}{dt} = -\omega^2 C \\ \frac{dL}{dt} = W \end{cases} \quad (44)$$

$$W = W_3 - \omega^2 C (t - t_3) \quad (45)$$

Ejection ends at time  $t_4$  such that :

$$\int_{t_3}^{t_4} W dt = -L_1 \quad (46)$$

### 7.5. Pressure

The mean pressure can be calculated when velocity is known.

Up to now for clarity we have used dimensional variables. However this leads to relations with inutile complexity for exploratory purposes. So we use now nondimensional variables to involve fewer ones.



## 8. Nondimensional form of Equations

As characteristic units we take:

$$\tilde{L} = R_i \quad (47)$$

$$\tilde{t} = \frac{1}{\omega} \quad (48)$$

$$\tilde{M} = \rho \tilde{L}^3 \quad (49)$$

These has the advantage of equalling time and angular wheel rotation, the lengths are divided by  $R_i$  and the velocities are divided by  $U_i$ .

Overbars will be used for nondimensional variables.

$$\bar{t}_1 \leq \frac{2 \bar{W}_{ci0}}{\bar{C}} \quad (50)$$

$$\bar{t}_2 = \frac{\bar{t}_1}{2} + \frac{\bar{W}_{ci0}}{\bar{C}} \quad (51)$$

$$\bar{t}_3 = \frac{2 \bar{W}_{ci0}}{\bar{C}} \quad (52)$$

$$\bar{t}_4 = \frac{1}{2 \bar{C}} \left( \bar{C} \bar{t}_1 + 2 \bar{W}_{ci0} + \sqrt{4 \bar{W}_{ci0}^2 - \bar{C}^2 \bar{t}_1^2 + 4 \bar{C} \bar{t}_1 \bar{W}_{ci0}} \right) \quad (53)$$

So if  $\beta_i$ ,  $\bar{W}_{ci0}$  and  $\bar{t}_1$  are chosen the entire flow is determined,  $\theta_0$  has no effect.

The maximum for the s abscissa is obtained at time  $\bar{t}_2$  and its value is:

$$\bar{s}_2 = \frac{1}{8} \left( -\bar{C} \bar{t}_1^2 + 4 \bar{t}_1 \bar{W}_{ci0} + \frac{4 \bar{W}_{ci0}^2}{\bar{C}} \right) \quad (54)$$

The angle  $\beta$  is zero for the  $\bar{s}_M$  abscissa and we have:

$$\bar{s}_M = \frac{1}{2 \bar{C}} (1 - \bar{C}^2) \quad (55)$$

To have  $\bar{s}_2 < \bar{s}_M$  it is necessary to have  $\bar{W}_{ci0}^2 < 1 - \bar{C}^2$  and if  $\bar{W}_{ci0}^2 > \frac{1 - \bar{C}^2}{2}$  the duration of injection must be such that:

$$\bar{t}_1 < 2 \frac{\bar{W}_{ci0} - \sqrt{\bar{C}^2 + 2 \bar{W}_{ci0}^2 - 1}}{\bar{C}} \quad (56)$$

In practice for rotors with blades such that  $\bar{C}$  is constant,  $\bar{s}_2 < \bar{s}_M$  and  $\bar{g} = 0$  we use analytical solution for finding the relative velocity and numerical integration for finding the absolute path, in all others cases we use numerical integration.

## 9. Examples

All the shown examples will be for  $\bar{C}$  constant, that is  $\bar{C} = \cos \beta_i$ . On figure 4 flow path and velocity triangles are drawn for  $\beta_i = 70^\circ$   $\bar{W}_{ci0} = 0.6$   $\bar{t}_1 = 90^\circ$  and for  $\beta_i = 65^\circ$   $\bar{W}_{ci0} = 0.8$   $\bar{t}_1 = 45^\circ$  on figure 5. Colored regions belong to  $D \cap R$ . Colors are the same as for figures 1, 2 and 3.

For illustration the rotors are drawn with 20 blades. Inner circle figures  $\beta = 0$  radius.

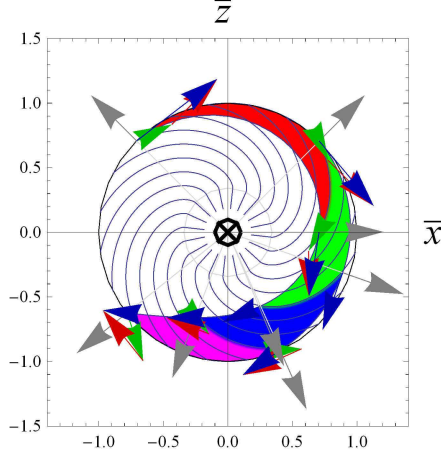


Figure 4: Absolute flow path

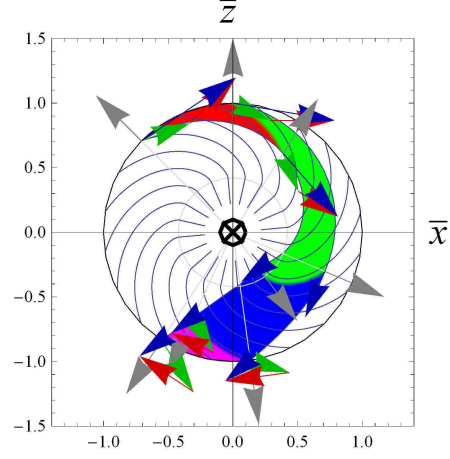


Figure 5: Absolute flow path

## 10. Global Characteristics

We calculate now volume flow and power in the case of infinite number of blades.

### 10.1. Volume Flow

$$q_v = R_i B_i \int_0^{\theta_1} W \cos \beta_i d\theta \quad \text{or} \quad q_v = -R_i B_i \int_{\theta_3}^{\theta_4} W \cos \beta_i d\theta \quad (57)$$

If one defines:

$$\bar{q}_v = \frac{q_v}{U_i B_i R_i} \quad (58)$$

$$\bar{q}_v = \bar{C} \int_0^{\bar{t}_1} \bar{W} d\bar{t} \quad (59)$$

$$\bar{q}_v = \bar{C} \bar{t}_1 \left( \bar{W}_{ci0} - \frac{\bar{C}}{4} \bar{t}_1 \right) \quad (60)$$

### 10.2. Power

If we apply moment of momentum equation to the calculated fluid flow :

$$P_{M/F} = \omega M_{M/F} \quad (61)$$

$$P_{F/M} = \omega \rho R_i^2 B_i \left( \int_0^{\theta_1} W \cos \beta_i V_U d\theta + \int_{\theta_3}^{\theta_4} W \cos \beta_i V_U d\theta \right) \quad (62)$$

If one defines :

$$\bar{P} = \frac{P_{F/M}}{\rho B_i R_i U_i^3} \quad (63)$$

$$\bar{P} = -\frac{\sin \beta_i}{24} \left( 8 \bar{W}_{ci0}^3 - 3 \bar{C} \bar{t}_1 (\bar{C}^2 \bar{t}_1^2 - 6 (\bar{C} \bar{t}_1 - 2 \bar{W}_{ci0}) \bar{W}_{ci0}) - (-\bar{C}^2 \bar{t}_1^2 + 4 \bar{C} \bar{t}_1 \bar{W}_{ci0} + 4 \bar{W}_{ci0}^2)^{3/2} \right) \quad (64)$$

It is clear as our incompressibility hypothesis and so  $\rho_a \ll \rho$  to obtained simplified model for exploratory purpose has led to hydraulic turbomachinery type so we will calculate head efficiency and specific speed as it is usual for that type of turbomachine.

### 10.3. Head

We define  $H_0$  such that:

$$g H_0 = \left( \frac{p}{\rho} + \frac{V^2}{2} \right) \quad (65)$$

$$g H_0 = \frac{W_{ci0}^2}{2} - \frac{U_i^2}{2} + U_i C_t \quad (66)$$

$$\overline{gH_0} = \frac{\overline{W}_{ci0}^2}{2} - \frac{1}{2} + \overline{C}_t \quad (67)$$

The conservation of moment of momentum across  $\partial R_c$  at inlet determines  $\overline{C}_t$ .

$$\overline{C}_t = \frac{\int_0^{\bar{t}_1} \overline{V}_R \overline{V}_U d\bar{t}}{\overline{q}_v} \quad (68)$$

### 10.4. Efficiency

If one defines efficiency as the ratio of the turbine power to the input.

$$\eta = \frac{P_{F/M}}{\rho q_v g H_0} \quad (69)$$

$$\eta = \frac{\overline{P}}{\overline{q}_v \overline{gH_0}} \quad (70)$$

### 10.5. Specific Speed

$$n_q = n \frac{q_v^{1/2}}{H_0^{3/4}} \quad (71)$$

$$n_q = 30 \frac{g^{3/4}}{\pi} \left( \frac{B_i}{R_i} \right)^{1/2} \frac{\overline{q}_v^{1/2}}{\overline{gH_0}^{3/4}} \quad (72)$$

### 10.6. Chart

As we are able to calculate efficiency and specific speed given inlet relative angle, inlet relative velocity at  $\bar{t}=0$  and  $\bar{t}_1$  we can draw the following chart.

Maximum value of  $\bar{t}_1$  such that  $\alpha < 85^\circ$   $\bar{t}_4 < 315^\circ$  and minimum radius is not crossed is chosen. For all the domain we found  $30^\circ < \bar{t}_1 < 120^\circ$  and extreme values for  $i$  is about  $\pm 10^\circ$ .

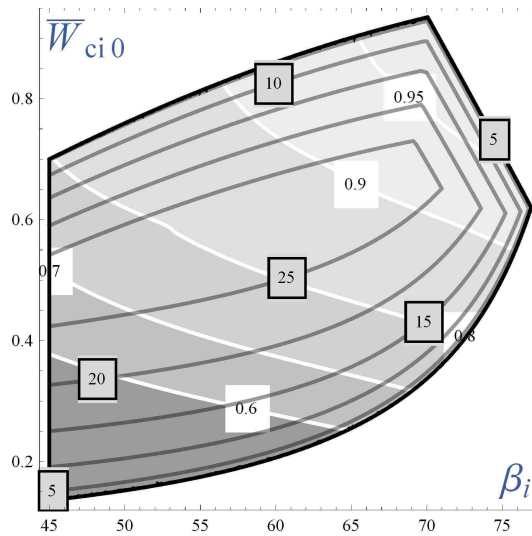


Figure 6: Specific speed and efficiency as function of  $\beta_i$  and  $\overline{W}_{ci0}$  ( $\frac{B_i}{R_i} = 1$   $\overline{g} = 0$ )

## 10.7. Operation Domain

With the same constraints we can estimate the utilisation domain of the actual turbine compared to others type.

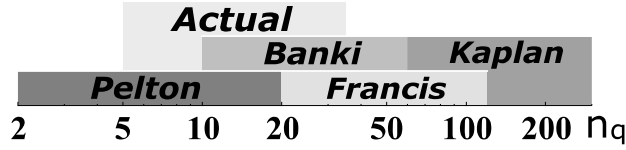


Figure 7: Specific speed domain ( $\frac{B_i}{R_i} = 1$   $\bar{g} = 0$ )

This utilisation domain can be extended by modifying  $\frac{B_i}{R_i}$ . We have:

$$\bar{g} = \overline{gH_0} \frac{R_i}{H_0} \quad (73)$$

As we neglect gravity effect in the rotor the previous results can only be used if  $\frac{R_i}{H_0}$  is small. We now try to extend the validity to other cases.

## 11. Evaluation of gravity effects

If the gravity effect is not negligible in the rotor we will take it into account.

### 11.1. Inlet

$$\left( \frac{p}{\rho} + \frac{V^2}{2} + g z \right)_- = g H_0 \quad (74)$$

$$\text{tg } \alpha^- = \frac{\bar{C}_t - \bar{g} \bar{z}}{\bar{W}^+ \cos \beta^+} \quad (75)$$

The conservation of moment of momentum across  $\partial R_c$  at inlet determines  $\bar{C}_t$ , in that case.

$$\bar{C}_t = \frac{\int_0^{\bar{r}_1} \bar{V}_R \bar{V}_U d\bar{r}}{\bar{q}_v} + \bar{g} \frac{\int_0^{\bar{r}_1} \bar{V}_R \bar{z} d\bar{r}}{\bar{q}_v} \quad (76)$$

$$\overline{gH_0} = \frac{\bar{W}_{ci0}^2}{2} - \frac{1}{2} + \bar{C}_t \quad (77)$$

### 11.2. Outlet

$$\overline{gH_o} = \frac{\int_{\bar{r}_3}^{\bar{r}_4} \bar{V}_R \frac{\bar{V}^2}{2} d\bar{r}}{\bar{q}_v} + \bar{g} \frac{\int_{\bar{r}_3}^{\bar{r}_4} \bar{V}_R \bar{z} d\bar{r}}{\bar{q}_v} \quad (78)$$

$$\bar{g} \bar{z}_o = \bar{g} \frac{\int_{\bar{r}_3}^{\bar{r}_4} \bar{V}_R \bar{z} d\bar{r}}{\bar{q}_v} \quad (79)$$

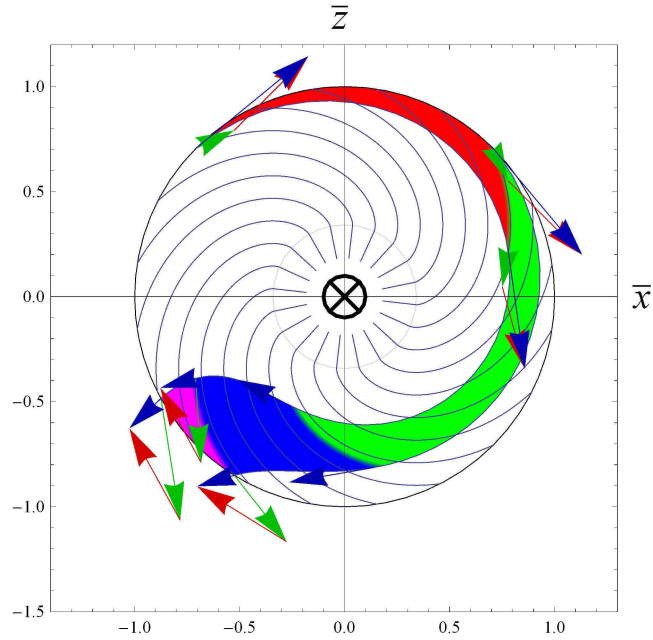
### 11.3. Efficiency

$$\eta = \frac{\overline{gH_0} - \overline{gH_o}}{\overline{gH_0} - \bar{g} \bar{z}_o} \quad (80)$$

$$\bar{H}_0 = \frac{\overline{gH_0}}{\bar{g}} \quad (81)$$

## 12. Examples with gravity effects inside the rotor

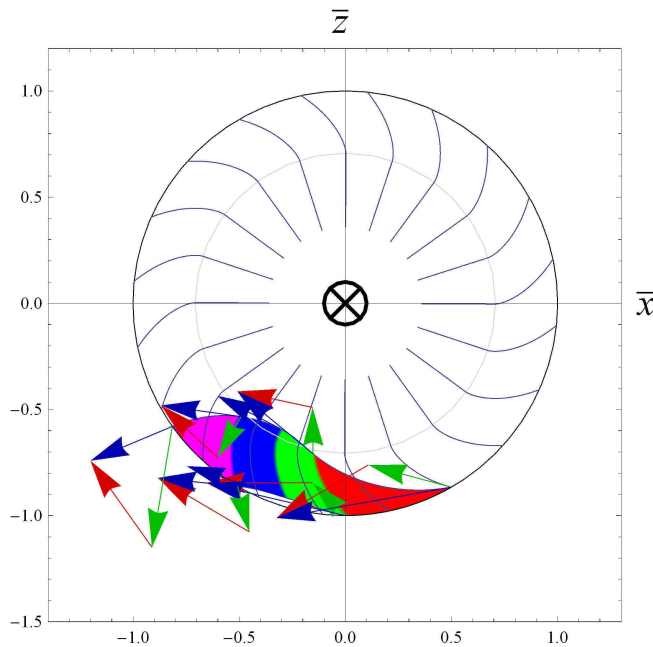
The first injection point position  $\theta_0$  play now an important role. We recall that in these cases we used numerical integration for relative and absolute flow path, also for global characteristics evaluation.



$$\beta_i = 70^\circ \quad \bar{g} = 0.4 \quad \bar{W}_{ci0} = 0.4 \quad \theta_0 = 135^\circ \quad \bar{t}_1 = 90^\circ \quad \eta = 0.96 \quad n_{\alpha} = 16 \quad \bar{H}_0 = 3.1$$

Figure 8: Absolute flow path with gravity effects

Due to gravity acceleration it was necessary to diminish  $\bar{W}_{ci0}$ , here  $\frac{H_0}{R} = 3.1$ , we tend to overshoot water wheel. The difference with habitual overshoot water wheel is that the fluid is always in motion in relative frame, there is no bucket but blades even if the flow reverses.



$$\beta_i = 45^\circ \quad \bar{g} = 1 \quad \bar{W}_{ci0} = 0.81 \quad \theta_0 = -60^\circ \quad \bar{t}_1 = 30^\circ \quad \eta = 0.73 \quad n_{\alpha} = 25 \quad \bar{H}_0 = 0.345$$

Figure 9: Absolute flow path with gravity effects.

In that case we increase  $\bar{W}_{ci0}$ , here  $\frac{H_0}{R} = 0.345$ , we tend to breast or undershot water wheel.

### 13. Discussion

It is clear that familiar criteria in turbomachinery field (velocity ratio, blade loading, etc.) should be used with caution in our case, as they were developed mostly for stationary relative flow.

About Hypothesis:

A one-dimensional form of the unsteady energy equation has been used by Fukutomi [2] to calculate fluid forces in a Banki turbine but with measured flow velocities as input, calculated and measured fluid forces agree well.

The particular choice of blade form (which leads to constant onedimensional velocity along blade at a given time) and the infinitely thin blades hypothesis can be easily changed especially if numerical integration is used.

The absolute angle distribution at inlet could be replaced by a coupling between flow calculation in rotor and upstream flow with the help of numerical resolution.

Deviation angle or slip factor should be taken into account to remove no slip hypothesis at exit.

The blades velocity distribution could be obtained from simplified method such as given by Stanitz [3] to estimate finite blade number effect, but with the additional complexity of relative unstationary flow.

Effect of flow viscosity can be estimated by losses models but this will be difficult because of the unsteadiness and differences with classical machines. Experimental or CFD investigation is possible but with considerable less commodity for exploratory purposes.

About possible others use :

Pure radial flow is not strictly necessary as such as a radius difference impart centrifugal force.

The  $\rho_a \ll \rho$  and open exit hypothesis which lead to  $\vec{\sigma} = \vec{0}$  on  $\partial D \cap R$ , or incompressibility could be removed for others use.

### 14. Conclusions

A possibility of completing the existing variety of turbomachinery types is proposed.

Classical relationship for turbomachinery are still valid even in the case of absolute and relative unsteadiness and non permanent fluid volume but in the sense of mean value over an absolute time period if this period exists.

A simplified evaluation of the performances for hydraulic turbine application in case of negligible effects of gravity in the rotor flow is given.

In case of noticeable gravity effects on the rotor flow, injection position must and can be taken into account, but it is not possible to establish a simple 2D chart of the performances.

The simple 2D blade geometry can be attractive.

Others possibilities although not explored are possible.

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