



HAL
open science

Event-Triggered Observer-based Output-Feedback Stabilization of Linear System with Communication Delays in the Measurements

Sylvain Durand, Lizeth Torres, Fermi Guerrero Castellanos

► **To cite this version:**

Sylvain Durand, Lizeth Torres, Fermi Guerrero Castellanos. Event-Triggered Observer-based Output-Feedback Stabilization of Linear System with Communication Delays in the Measurements. ECC 2014 - 13th European Control Conference, Jun 2014, Strasbourg, France. 6 p. hal-01009025

HAL Id: hal-01009025

<https://hal.science/hal-01009025>

Submitted on 17 Jun 2014

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Event-Triggered Observer-based Output-Feedback Stabilization of Linear System with Communication Delays in the Measurements

Sylvain Durand*, Lizeth Torres†, Jose Fermi Guerrero-Castellanos‡

* Control Department of GIPSA-lab, CNRS, Univ. of Grenoble, Grenoble, France.

† Institute of Engineering, UNAM, Mexico City, Mexico.

‡ Faculty of Electronics, Autonomous University of Puebla (BUAP), Puebla, Mexico.

E-mail: sylvain@durandchamontin.fr

Abstract—In this paper, an original framework is proposed for the stabilization of a linear system with delays in the measurements: i) an observer estimates the full state information of the plant from a partial measurement, ii) an event-based control technique computes and updates the control signal only when a certain condition is satisfied and iii) an event-based corrector updates the model used to calculate the control law when it deviates from the estimated state. It is notably proved that such a proposal renders the closed-loop system stable for larger delays in the measurements than in the classical continuous-time control case. Simulation results are provided.

INTRODUCTION

With the development of embedded, miniaturized and interconnected systems, there is a growing interest in Networked Control Systems (NCSs) where the control loop is closed over a communication link. A network has several advantages, like flexibility in the configuration of the communication structure and the number of interconnected systems. However, it also has a considerable impact on the performance, notably because of communication delays and packet losses (not considered here) which avoid real-time control constraints to be met and can even cause the instability of the control loop. In this context, the event-based paradigm appears as a mean to reduce the communication bandwidth in the network since, contrary to the classical (periodic) scheme, an event-based control invokes a communication between the different nodes only when a certain condition is satisfied. Typical detection mechanisms are function of the state variation of the system, like in [5], [15], [14], [1], [7], [11], [6]. An alternative approach consists in taking events related to the variation of a Lyapunov function (and consequently to the state too) like in [19], or in taking events related to the time derivative of the Lyapunov function, like in [16], [12]. In the latter references, the updates ensure the strict decrease of the Lyapunov function, and so is asymptotically stable the closed-loop system.

The references above deal with state-feedback control. This means that the full state information is considered as measurable although, in practice, this assumption is often violated because only a small number of outputs (corresponding to the sensors that are available) is really measurable. For this reason, an output-feedback method, or an observer-based strategy where the whole state information is estimated, is more interesting. Such an approach was notably treated

in [10]. Furthermore, only few works consider event-based control under delay constraints. In [9] in particular, an event-based mechanism allows to control a plant with reduced communications for the measurements, but the control is still time triggered. In the present paper, both latter proposals are combined in order to propose an event-based output-feedback control of linear system with delays in the measurements. Note that *delays in the measurements* means the input signal of the controller is delayed.

The rest of the document is organized as follows. In section I, an overview of the context is provided and the problem is stated. The system architecture is introduced in section II. The proposal is then formalized in section III and the stability is analyzed. Simulation results are provided in section IV to highlight the capabilities of the proposed approach. Some discussions conclude the paper.

I. CONTEXT DESCRIPTION

A. Event-based state-feedback

Let consider the linear time-invariant dynamical system

$$\dot{x}(t) = Ax(t) + Bu(t) \quad (1)$$

$$y(t) = Cx(t) \quad (2)$$

$$\text{with } x(0) := x_0$$

with $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$ and $y \in \mathbb{R}^l$ are the state, input and output vectors. By **event-based state-feedback** we mean a set of two functions:

- i) an event function $\xi : \mathbb{R}^n \times \mathbb{R}^n \rightarrow \mathbb{R}$, that indicates if one needs (when $\xi \leq 0$) or not (when $\xi > 0$) to recompute the control law,
- ii) a state-feedback function $\mathbb{R}^n \rightarrow \mathbb{R}^m$ in the form $u(t) = -Kx(t)$, where the state-feedback matrix K is calculated to make the closed-loop system stable.

The solution of (1) with an event-based state-feedback starting in x_0 at $t = 0$ is then defined as the solution of the differential system

$$\dot{x}(t) = Ax(t) - BKx(t_i) \quad \forall t \in [t_i, t_{i+1}[\quad (3)$$

where the time instants t_i , with $i \in \mathbb{N}$ (determined when the event function ξ vanishes) are considered as *events* and $x(t_i)$ is the memory of the state value at the last event.

In [12], [17], it is proved that the linear system (1) can be asymptotically stabilized (as soon as (A, B) is a stabilizable pair) by means of a particular event-based state-feedback

$$u(t) = -Kx(t_i) \quad \forall t \in [t_i, t_{i+1}[\quad (4)$$

$$\text{with } K := 2R^{-1}B^T P \quad (5)$$

$$\xi(x(t), x(t_i)) = (\sigma - 1)x(t)^T Q_1 x(t) - 2x(t)^T Q_2 [\sigma x(t) - x(t_i)] \quad (6)$$

$$\text{with } Q_1 := PA + A^T P \quad \text{and} \quad Q_2 := PBR^{-1}B^T P$$

where P , Q and R are positive definite matrix solution of the Riccati equation $Q_1 - 2Q_2 = -Q$. Note that the tunable parameter $\sigma \in]0, 1[$ changes the frequency of events: the higher σ , faster is the convergence but more frequent are events in return.

It is also proved in [12] that the feedback (4)-(6) is **uniformly MSI** (Minimal inter-Sampling Interval). That means it is a piecewise constant control with non zero sampling intervals, which is useful to avoid *Zeno* phenomena.

B. Event-based output-feedback

Whereas the full state information x is considered as measurable in a state-feedback approach, in practice, only a small number of outputs y is really available. The idea behind an output-feedback approach is to directly use the output in the feedback law, i.e. $u(t) = -\bar{K}y(t)$, where \bar{K} is the output-feedback matrix for y as defined in (2), or to apply a state observer in order to have an estimation of all the state information (possible as soon as (A, C) is an observable pair), and then build a state-feedback control law using the estimated state. Typical Luenberger state observer for linear system (1) is

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + L[y(t) - C\hat{x}(t)] \quad (7)$$

$$\text{with } \hat{x}(0) := \hat{x}_0$$

where $\hat{x} \in \mathbb{R}^n$ is the estimated state vector and L is calculated to make stable the observation error defined by

$$\tilde{x}(t) := x(t) - \hat{x}(t) \quad (8)$$

Such an observer-based output-feedback method is considered here. By **event-based output-feedback** we mean a set of two functions:

- i) an event function $\xi : \mathbb{R}^n \times \mathbb{R}^n \rightarrow \mathbb{R}$ (defined as before),
- ii) an output-feedback function $\mathbb{R}^n \rightarrow \mathbb{R}^m$ in the form $u(t) = -K\hat{x}(t)$.

The solution of (1)-(2) with an event-based output-feedback based on the observer (7) and starting in x_0 at $t = 0$ is then defined as the solution of the differential system

$$\dot{\hat{x}}(t) = A\hat{x}(t) - BK\hat{x}(t_i) + L[y(t) - C\hat{x}(t)] \quad (9)$$

$$\dot{x}(t) = Ax(t) - BK\hat{x}(t_i) \quad \forall t \in [t_i, t_{i+1}[$$

The extension from event-based state-feedback (4)-(6) to an observer-based output-feedback version is easy. It only consists in applying \hat{x} instead of x in the event function and the control law, and so is asymptotically stable and **uniformly MSI** the closed-loop system for a given K and L .

C. Event-based control in NCSs with delays

Event-based control allows computing savings in embedded systems for the same performance as in a periodic scheme [1]. Similarly, it allows to reduce the communications in NCSs by sending signals over the communication link only when a given condition is satisfied. Note that only a communication link from the plant to the controller is considered here (and not from the controller to the plant).

Furthermore, delays in the measurements can be induced by the network and, as a consequence, a disturbance in the controlled systems can be detected too late by the (deported) controller, which can lead to an unstable behavior. The control mechanism has hence to be robust to such disturbances. For this reason, an *event-based corrector* is also applied. Such a technique was initially suggested in [9] for a classical (time-triggered) state-feedback control strategy and adapted in [3] for the particular event-based state-feedback (4)-(6) case. The principle is extended here to an output-feedback scheme using the observer (7). Output-feedback was treated in [10] in the case of a time-triggered control without delays, whereas event-based feedback and delays in the measurements are now considered. Finally, the idea is to make a copy of the undisturbed model (1) of the system to control, in both sides of the network, and correct them when they deviate too much from the estimated values, that is when a perturbation occurs if the observer is stable. The copy of the model in the control side is used to compute the control law and the measurement is sent over the communication link only when it has to be corrected.

Contributions of the paper

In this paper, a new setup is considered using *i) an event-triggered observer-based output-feedback controller* and *ii) an event-based corrector* with communication delays in the measurements. Note that, whereas the event-based control strategy is dedicated to previous works [12], [17], the proposal can be easily generalized to other (similar) strategies.

II. SYSTEM FRAMEWORK

The system architecture is presented in Fig. 1. Event-based mechanisms are used to minimize the computational cost and the sending of information over the communication link. They are in both the controller and the plant nodes: an event-based (output-feedback) technique updates the control signal based on a dynamical model of the plant, whereas an event-based (observer-based) corrector corrects this model when it deviates from the real measurements. The different events in both sides can occur at any time and independently, consequently, one needs to mark the time variable $t \in \mathbb{R}^+$ with respect to the source of event in order to formalize such a framework next. Also, the varying communication delays $\tau(t) \in \mathbb{R}^+$ are marked. Two indexes are used herein:

- Let t_i denote the time when an event is enforced for control, afterwards called *control's event*, with $i \in \mathbb{N}$. The delay from the controller to the plant is assumed to be null here.

- Let t_j denote the time when an event is enforced for correction, afterwards called *correction's event*, with $j \in \mathbb{N}$. Also, let $\tau_j := \tau(t_j)$ denote the delay in the measurements (since the state values of the plant will be sent over the communication link each time an event occurs for correction).

Remember that both indexes are independent in the sense there is no chronological relation between t_i and t_j .

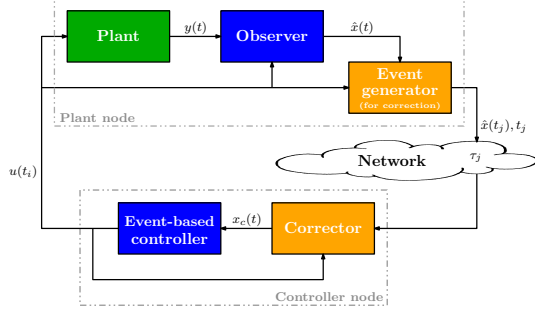


Fig. 1. System architecture.

A. Perturbed system to control

The plant is described by a linear perturbed model

$$\dot{x}(t) = Ax(t) + Bu(t) + Ed(t) \quad (10)$$

$$y(t) = Cx(t) \quad (11)$$

$$\text{with } x(0) := x_0$$

where $d \in \mathbb{R}^p$ is the disturbance. Several conditions are assumed in the sequel:

- i) the dynamics of the plant as well as the initial conditions are accurately known (A , B , C and x_0 are known), all model uncertainties are lumped into the disturbance d ;
- ii) matrices A and B are controllable;
- iii) matrices A and C are observable;
- iv) the output y and the time t are measurable;
- v) the disturbance is bounded by

$$\|d(t)\| \leq d_{max} \quad (12)$$

- vi) the communication delays are bounded and are smaller than the minimum inter-sampling interval between two consecutive events, that is

$$\tau_j \leq \bar{\tau}_j < t_{j+1} - t_j \quad \forall j \in \mathbb{N} \quad (13)$$

- vii) the plant and the controller nodes are synchronous in the sense they share a same clock, i.e.

$$t_i(i=0) = t_j(j=0) = 0$$

Then, considering the system (10) and applying the undelayed event-based state-feedback (4)-(6), the continuous-time closed-loop system becomes

$$\dot{\check{x}}(t) = A\check{x}(t) - BK\check{x}(t_i) + Ed(t) \quad \forall t \in [t_i, t_{i+1}[\quad (14)$$

$$\text{with } \check{x}(0) = x_0$$

where $\check{x} \in \mathbb{R}^n$ is the undelayed event-based controlled closed-loop state, K is defined in (5).

As already explained, the seminal works developed in [9], [10] are adapted here for the proposed framework. The main difference is that the control law is now piecewise constant whereas it was continuously updated before. In the initial setups, the closed-loop system when applying the classical (time-triggered) state-feedback control was

$$\dot{\check{x}}(t) = A_K\check{x}(t) + Ed(t) \quad (15)$$

$$\text{with } A_K := A - BK$$

where $\check{x} \in \mathbb{R}^n$ is the undelayed (continuous-time controlled) closed-loop state and A_K is the closed-loop matrix. The problem was highly simplified since one only needed to know the value of A_K to make a copy of the (undisturbed) closed-loop system. However, making such a copy means that the state-feedback control in the copies is computed from the undisturbed copy state. In other words, the control is not the same than for the real controlled system (computed from the disturbed real state), which makes the copies are more unprecised. Here, the same control input is used for the copies than for the plant. One hence needs to know the estimated state value at the last control's event, i.e. $\hat{x}(t_i)$, or at least the control signal value $u(t_i)$.

B. Observer

The observer determines an estimate $\hat{x}(t)$ of the plant state $x(t)$. This observer is not event triggered because one needs a continuous estimation in order to then detect the event time instants. However, its input is piecewise constant due to the event-based control setup. Reformulating the original form (7), the observer expression becomes

$$\dot{\hat{x}}(t) = A_L\hat{x}(t) + Bu(t_i) + Ly(t) \quad (16)$$

$$\text{with } A_L := A - LC$$

$$\text{and } \hat{x}(0) := \hat{x}_0$$

This model requires the control signal $u(t_i)$ which is applied to the real system (10) each time an event is enforced from the controller. This is discussed in the sequel.

C. Event-based corrector

1) *Event generator for correction*: This part runs a copy of the closed-loop system model (14) without disturbance

$$\dot{x}_e(t) = Ax_e(t) + Bu(t_i) \quad \forall t \in [t_i, t_{i+1}[\quad (17)$$

$$\text{with } x_e(0) = x_0$$

where $x_e \in \mathbb{R}^n$ is the state of the event generator. This model also requires the control signal $u(t_i)$, as discussed in the sequel.

An event is generated for correction when the difference between the estimated system state $\hat{x}(t)$ in (16) and the event generator state $x_e(t)$ in (17) reaches a given threshold \bar{e} , that is when

$$\|\hat{x}(t_j) - x_e(t_j^-)\| = \bar{e} \quad (18)$$

where t_j^- is the time just before the event, and so is corrected the value of the event generator state such that

$$x_e(t_j^+) = \hat{x}(t_j) \quad (19)$$

where t_j^+ is the time just after the event. This defines the correction's event instant t_j . The estimated system state $\hat{x}(t_j)$ and the event time t_j are then sent to the corrector (in order its model is also corrected) over the communication link.

2) *Corrector*: The corrector also runs a copy of the undisturbed closed-loop system model (14), that is

$$\begin{aligned} \dot{x}_c(t) &= Ax_c(t) + Bu(t_i) \quad \forall t \in [t_i, t_{i+1}[\\ \text{with } x_c(0) &= x_0 \end{aligned} \quad (20)$$

where $x_c \in \mathbb{R}^n$ is the state of the corrector. The control signal $u(t_i)$ is also required here, it is directly obtained from the controller (since it is in the same network side).

Actually, this model has also to be updated when the condition (18) is satisfied but, due to the network, the corrector receives the information $(\hat{x}(t_j), t_j)$ at the delayed time $t_j + \tau_j$. Fortunately, the communication delay τ_j can be easily deduced (knowing the time of sending t_j and the one when the data are received, since both nodes are synchronous). The update of the corrector state x_c can hence be easily determined. This is detailed in subsection III-A.

D. Event-based controller

In fact, the event-based feedback (4)-(6) is not directly computed for the system (10) to control, but for the copy of the model available in the controller node, that is the corrector model (20). The control's event instant t_i is hence determined by the vanishing of the event function (6) applied to x_c , that is when

$$\xi(x_c(t), x_c(t_i)) \leq 0 \quad (21)$$

Also, the control law (4) becomes

$$u(t) = -Kx_c(t_i) \quad \forall t \in [t_i, t_{i+1}[\quad (22)$$

where K is defined in (5). The control signal $u(t_i)$ is then sent to the plant (without communication delay) in order to be applied to the plant, the observer and the event generator for correction, and so it is available in (16) and (17). Also, note that the state x_c is updated using the estimated state \hat{x} (see subsection III-A) and, for this reason, the control strategy is an observer-based output-feedback law.

III. ANALYSIS AND MAIN RESULTS

A. Determination of the corrector state update $x_c(t_j^+ + \tau_j)$

The corrector state (20) is updated when data are received over the communication link from the event generator, that is at time $t_j + \tau_j$. Nevertheless, in order the system (20) at time $t_j + \tau_j$ holds like the one in (17) which was updated at time t_j , one needs to know how it behaved during the elapsed time. The analysis is divided into two steps:

1) *The control signal is not updated during the communication, i.e. $t_i \leq t_j < t_j + \tau_j < t_{i+1}$* : The system trajectory update of the model (20) is

$$x_c(t_j^+ + \tau_j) = e^{A\tau_j} \hat{x}(t_j) - \int_0^{\tau_j} e^{As} ds BK x_c(t_i) \quad (23)$$

based on an extension of [18], [4] and using (19).

2) *The control signal is updated during the communication, i.e. $t_j < t_i \leq t_j + \tau_j < t_{i+1}$* : The update of the corrector state depends on the state value of the model at the control's event time t_i , which depends on its own on the system state value at the correction time t_j . This gives

$$\begin{aligned} x_c(t_j^+ + \tau_j) &= e^{A(t_j + \tau_j - t_i)} \bar{x}_c(t_i) \\ &\quad - \int_0^{t_j + \tau_j - t_i} e^{As} ds BK \bar{x}_c(t_i) \end{aligned} \quad (24)$$

where $\bar{x}_c(t_i) = e^{A(t_i - t_j)} \hat{x}(t_j) - \int_0^{t_i - t_j} e^{As} ds BK x_c(t_{i-1})$ denotes the trajectory of the state x_c if the corrected value $\hat{x}(t_j)$ is applied at the correction's event time t_j .

Thus, $x_c(t) = x_e(t)$ holds for $t_j + \tau_j \leq t < t_{j+1}$. At the end, one only needs to know $\hat{x}(t_j)$ and t_j (which are sent over the communication link), τ_j (which is deduced from the transmitting time t_j and the receiving one) and t_i (which is provided by the controller) to update the corrector state (20). Also, one could note that assumption (13) has to be satisfied. This is discussed in next subsection.

B. Determination of the maximum error of observation

From (8), (10)-(11), (16) and (17), the error of observation becomes

$$\dot{\tilde{x}}(t) = A_L \tilde{x}(t) + Ed(t) \quad (25)$$

An upper bound of the error of observation \tilde{x} is obtained according to the relation (since the A_L matrix is assumed to be Hurwitz)

$$\begin{aligned} \|\tilde{x}(t)\| &\leq \tilde{x}_{max} \\ \text{with } \tilde{x}_{max} &:= \int_0^\infty \|e^{A_L s} E\| ds d_{max} \end{aligned} \quad (26)$$

where d_{max} is the disturbance bound defined in (12). The same results were obtained in [10] for the time-triggered control case without delays.

C. Determination of the maximum communication delay

A bound on the communication delays ensures that no event is enforced during the communication time intervals, as specified in (13). From (16), (17), (20), (22), one obtains

$$\begin{aligned} \dot{\hat{x}}_\Delta(t) &= A \hat{x}_\Delta(t) + LC \tilde{x}(t) \\ \text{with } \hat{x}_\Delta(t) &:= \hat{x}(t) - x_c(t) \end{aligned} \quad (27)$$

where \tilde{x} is defined in (8), and

$$\begin{aligned} \dot{x}_{e\Delta}(t) &= Ax_{e\Delta}(t) \\ \text{with } x_{e\Delta}(t) &:= x_e(t) - x_c(t) \end{aligned} \quad (28)$$

whose solutions on the time interval $t \in [t_j, t_j + \tau_j[$ are

$$\begin{aligned} \hat{x}_\Delta(t) &= e^{A(t-t_j)} \hat{x}_\Delta(t_j) + \int_{t_j}^t e^{A(t-s)} LC \tilde{x}(s) ds \\ x_{e\Delta}(t) &= e^{A(t-t_j)} x_{e\Delta}(t_j) = e^{A(t-t_j)} \hat{x}_\Delta(t_j) \end{aligned}$$

In the latter expression, $x_e(t_j)$ is replaced by $\hat{x}(t_j)$ thanks to (19). Then, as no correction's event should be enforced according to (18), the inequality

$$\|\hat{x}(t) - x_e(t)\| = \|\hat{x}_\Delta(t) - x_{e\Delta}(t)\| < \bar{\epsilon}$$

have to hold for all $t \in [t_j, t_j + \tau_j]$. This yields

$$\left\| \int_{t_j}^t e^{A(t-s)} LC \tilde{x}(s) ds \right\| < \bar{e}$$

Then, an upper bound of the communication delay τ_j for which this inequality is satisfied is easily determined by

$$\bar{\tau}_j = \arg \min_{\tau_j \geq 0} \left\{ \int_0^{\tau_j} \|e^{As} LC\| ds \tilde{x}_{max} = \bar{e} \right\} \quad (29)$$

where \tilde{x}_{max} is defined in (26) as an upper bound for the error of estimation $\tilde{x}(t)$, and so is satisfied the assumption (13) for all $\tau_j \leq \bar{\tau}_j$. This expression also defines the minimum inter-sampling interval of the event-based corrector (17)-(20), (23)-(24) which, as a consequence, is uniformly MSI. Furthermore, as already observed in [3] for the state-feedback case (without observer), the result in (29) is quite interesting since the achieved bound of the communication delay is larger than the one obtained in the original event-based corrector in [9], and its observer-based version in [10] (where there is no delay but the extension is intuitive using [9], [10], [3]). This is thanks to the piecewise constant control (21)-(22), because it allows that the plant runs without updating its input as often as in the original continuous-time case. Finally, as observed in [10], the delay bound (29) does not directly depend on the disturbance limit d_{max} but indirectly through the error of observation bound \tilde{x}_{max} .

D. Stability of the proposed framework

The stability property of the proposal comes from above results. Let first recall some definitions from [8].

Definition 3.1: The solution $x(t)$ of a continuous-time system is **Globally Uniformly Ultimately Bounded (GUUB)** if for every initial condition $x(0) \in \mathbb{R}^n$ there exists a positive constant μ and time ν such that $\|x(t)\| \leq \mu \forall t \geq \nu$.

Definition 3.2: The solution of the disturbed continuous-time state-feedback system (15) is GUUB if the feedback matrix K renders the undisturbed system (3) stable and the disturbance $d(t)$ is bounded.

Theorem 3.3 (Stability of the event-based framework): Consider the state observer (16). Consider the event-based corrector (17)-(20), (23)-(24) affected by communication delays (13) in the measurements, whose a bound is given in (29). Consider the event-based state-feedback (21)-(22). Then, the output-feedback control loop for the disturbed linear system (10)-(11), whose disturbance is bounded by (12), is uniformly MSI and GUUB.

Proof: The undelayed event-based state-feedback (4)-(6) renders the undisturbed linear system (3) asymptotically stable for a given feedback matrix K defined in (5). This was proved in [12], [17]. From Definition 3.2, the undelayed continuous-time state-feedback system (14) is hence GUUB for a bounded disturbance (12) and the stabilizing feedback (4)-(6).

From the observer theory, the undelayed event-based output-feedback – replacing the state x in (4)-(6) by its estimated value \hat{x} obtained thanks to the state observer (16) –

renders the undisturbed linear system (1) asymptotically stable for a given matrix of observation L if the corresponding state-feedback closed-loop system is stable [2].

On the other hand, let

$$e(t) := x(t) - \check{x}(t) \quad (30)$$

be the *approximation error*, i.e. the difference between *i*) the state $x(t)$ of the closed-loop system (10)-(12), (16), (17)-(20), (21)-(22), (23)-(24) with communication delay (13) and *ii*) the state $\check{x}(t)$ of the undelayed closed-loop system (14). The derivative of $e(t)$ gives

$$\begin{aligned} \dot{e}(t) &= Ae(t) - BK e(t_i) + BK x_\Delta(t_i) \\ \text{with } x_\Delta(t) &:= x(t) - x_c(t) \quad \text{and } e(0) = 0 \end{aligned} \quad (31)$$

which finally yields the upper bound of the approximation error as follows

$$\begin{aligned} \|e(t)\| &\leq x_{\Delta max} \int_0^\infty \|e^{As} BK\| ds \\ \text{with } x_{\Delta max} &:= \max_t \|x_\Delta(t)\| \end{aligned} \quad (32)$$

since the feedback matrix K defined in (5) renders the “undisturbed” approximation error dynamics (31) asymptotically stable (where x_Δ can be seen as the disturbance), and so becomes null the first right-hand term in (31). This result is quite close to the original work [9], [10], the closed-loop matrix A_K is only replaced by A in (32) due to the piecewise constant control of the present paper. Therefore, an upper bound of $\|x_\Delta(t)\|$ is obtained with the same method (adapted here for the piecewise constant observer-based output-feedback control case). The study is divided into two parts:

1) Firstly, $x_c(t) = x_e(t)$ holds for $t_j + \tau_j \leq t < t_{j+1}$ whatever the last control’s event time t_i thanks to the update mechanism (23)-(24). The bound is hence obtained from the inequality

$$\begin{aligned} \|x_\Delta(t)\| &= \|x(t) - x_c(t)\| \\ &\leq \|x(t) - \hat{x}(t)\| + \|\hat{x}(t) - x_e(t)\| + \|x_e(t) - x_c(t)\| \end{aligned}$$

whose first right-hand term is limited by the error of estimation bound \tilde{x}_{max} previously obtained in (26), second one is bounded by the correction event threshold \bar{e} because of (18), and the last right-hand side is zero. This gives

$$\|x_\Delta(t)\| \leq \tilde{x}_{max} + \bar{e} \quad \forall t \in [t_j + \tau_j, t_{j+1}[\quad (33)$$

2) Then, the analyze of $\hat{x}_\Delta(t)$ in (27) easily gives its solution on the second interval $t_{j+1} \leq t < t_{j+1} + \tau_{j+1}$

$$\hat{x}_\Delta(t) = e^{A(t-t_{j+1})} \hat{x}_\Delta(t_{j+1}) + \int_{t_{j+1}}^t e^{A(t-s)} LC \tilde{x}(s) ds$$

and since the error of estimation bound \tilde{x}_{max} in (26) also remains true, a bound of $x_\Delta = \tilde{x} - \hat{x}_\Delta$ is obtained, without any impact from the control’s events t_i

$$\begin{aligned} \|x_\Delta(t)\| &\leq \tilde{x}_{max} + \bar{c}\bar{e} + \bar{d}d_{max} \quad \forall t \in [t_{j+1}, t_{j+1} + \tau_{j+1}[\quad (34) \\ \text{with } \bar{c} &:= \max_{\tau_j \in [0, \bar{\tau}_j]} \|e^{A\tau_j}\| \quad \text{and } \bar{d} := \int_0^{\bar{\tau}_j} \|e^{As} E\| ds \end{aligned}$$

where $\bar{\tau}_j$ is the communication delay bound obtained in (29) and d_{max} is the disturbance bound defined in (12). At the end, since $\bar{c} \geq 1$ and $\bar{d} \geq 0$ by definition, the expression (34) can be generalized for all $t \in [t_j + \tau_j, t_{j+1} + \tau_{j+1}[$, and so is upper-bounded $\|e(t)\|$ in (32).

Since the undelayed continuous-time state-feedback system (14) is GUUB and $\|e(t)\|$ in (32)-(34) is upper-bounded, one can conclude the proposed event-triggered observer-based output-feedback control with event-based correction and communication delays is GUUB.

Also, the MSI property was demonstrated in [12] for the control case and in subsection III-C for the corrector one. This ends the proof. ■

IV. SIMULATION RESULTS

In this section, the proposal is tested in simulation, using the *Matlab/Simulink* environment. Firstly, the system is a double integrator, whose matrices in (10) are given by

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad C = [1 \quad 0] \quad \text{and} \quad E = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

The initial state and observer conditions are $x_0 = \hat{x}_0 = [1 \quad -3]^T$. The system is controllable and observable. The control parameters to calculate K in (5) are

$$Q = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}, \quad R = 1 \quad \text{and} \quad \sigma = 0.8$$

The poles of the observer are chosen to converge 2 times faster than the poles of the controller, and so is obtained the matrix of observation L . The corrector parameter is $\bar{e} = 0.05$. Also, the bound of the communication delays is $\bar{\tau}_j = 0.36$ s, calculated using (29) (whereas it is 0.16 s when applying the original setup [9]). A (randomly) varying disturbance whose maximum value is $d_{max} = 0.1$ and constant communication delays $\tau_j = 0.03$ s $\forall j \in \mathbb{N}$ are considered.

The simulation results of the whole proposal are compared in Fig. 2 with a state-feedback approach (see [3] for further details). The system is stabilized in both cases (with only 18 and 16 events respectively) even in the present case of disturbances and delays. Moreover, the number of samples and the final error are lower with the output-feedback strategy than in the state-feedback case. This is probably because the observer estimates the plant state but not its disturbance, and the control law is based on the resulting undisturbed model.

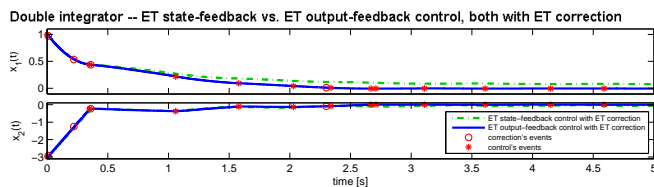


Fig. 2. Simulation results of the double integrator: event-based state-feedback vs. (observer-based) output-feedback control, both with event-based correction (and communication delays).

CONCLUSION AND FUTURE WORK

In this paper, it was proposed to combine i) a state observer, ii) an event-based control technique and iii) an event-based corrector for the stabilization of a linear system with delays in the measurements. It was proved this framework is stable and works with delays larger than in the original continuous-time control [9], [10]. Future work is to test the proposal on a real-time implementation, like a mini-helicopter with delays due to a vision-based measurement. Furthermore, next step is to consider packet losses and to add a disturbance observer, in order to use the estimation of the perturbations in the corrector to improve its accuracy.

REFERENCES

- [1] K. J. Åström and B. Bernhardsson. Comparison of Riemann and Lebesgue sampling for first order stochastic systems. In *Proceedings of the 41st IEEE Conference on Decision and Control*, 2002.
- [2] K. J. Åström and R. M. Murray. *Feedback Systems: An Introduction for Scientists and Engineers*. Princeton University Press, 2008.
- [3] S. Durand. Event-based stabilization of linear system with communication delays in the measurements. *Proceedings of the American Control Conference*, 2013.
- [4] S. Durand, J. F. Guerrero-Castellanos, and R. Lozano-Leal. Self-triggered control for the stabilization of linear systems. In *Proceedings of the 9th IEEE International Conference on Electrical Engineering, Computing Science and Automatic Control*, 2012.
- [5] S. Durand and N. Marchand. Further results on event-based PID control. In *Proceedings of the European Control Conference*, 2009.
- [6] A. Eqtami, D. V. Dimarogonas, and K. J. Kyriakopoulos. Event-triggered control for discrete-time systems. In *Proceedings of the IEEE American Control Conference*, 2010.
- [7] W. P. M. H. Heemels, J. H. Sandee, and P. P. J. van den Bosch. Analysis of event-driven controllers for linear systems. *International journal of control*, 81:571–590, 2009.
- [8] H. Khalil. *Nonlinear Systems, Third Edition*. Prentice Hall, 2002.
- [9] D. Lehmann and J. Lunze. Event-based control with communication delays. In *Proceedings of the 18th IFAC world congress*, 2011.
- [10] D. Lehmann and J. Lunze. Event-based output-feedback control. In *Proceedings of the 19th Mediterranean Conference on Control and Automation*, 2011.
- [11] J. Lunze and D. Lehmann. A state-feedback approach to event-based control. *Automatica*, 46:211–215, 2010.
- [12] N. Marchand, S. Durand, and J. F. Guerrero-Castellanos. A general formula for the stabilization of event-based controlled systems. In *Proceedings of the 50th IEEE Conference on Decision and Control and European Control Conference*, 2011.
- [13] M. Mazo Jr. and P. Tabuada. Input-to-state stability of self-triggered control systems. In *Proceedings of the Joint 48th IEEE Conference on Decision and Control and 28th Chinese Control Conference*, 2009.
- [14] J. Sánchez, M. Guarnes, and S. Dormido. On the application of different event-based sampling strategies to the control of a simple industrial process. *Sensors*, 9:6795–6818, 2009.
- [15] J. H. Sandee, W. P. M. H. Heemels, and P. P. J. van den Bosch. Event-driven control as an opportunity in the multidisciplinary development of embedded controllers. In *Proceedings of the American Control Conference*, pages 1776–1781, 2005.
- [16] P. Tabuada. Event-triggered real-time scheduling of stabilizing control tasks. *IEEE Transactions on Automatic Control*, 52:1680–1685, 2007.
- [17] J. J. Téllez-Guzmán, J. F. Guerrero-Castellanos, S. Durand, and N. Marchand. Event-based LQR control for attitude stabilization of a quadrotor. In *Proceedings of the 15th IFAC Latinamerican Control Conference*, 2012.
- [18] M. Velasco, P. Martí, and E. Bini. Control-driven tasks: Modeling and analysis. In *Proceedings of the IEEE Real-Time Systems Symposium*, 2008.
- [19] M. Velasco, P. Martí, and E. Bini. On Lyapunov sampling for event-driven controllers. In *Proceedings of the 48th IEEE Conference on Decision and Control*, 2009.
- [20] G. Walsh and H. Ye. Scheduling of networked control systems. *IEEE Control Systems Magazine*, 21:57 – 65, 2001.