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# Risk-Based-optimization of geo-positioning of sensors in case of spatial fields of deterioration/properties

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**ABSTRACT:** The localization of weak properties or bad behavior of a structure is still a very challenging task that concentrates the improvement of Non Destructive Testing (NDT) tools on more efficiency and higher structural coverage. In case of random loading or material properties, this challenge is arduous because of the limited number of measures and the quasi-infinite potential positions of local failures. The paper shows that the stationary property is useful to find the minimum quantity of NDT measurements and their position for a given quality assessment. A two stages procedure allows us (i) to quantify the properties of the ergodic, stationary field (ii) to assess the distribution of the characteristics. The paper focuses on optimization of sensor geo-positioning to reach (i): the criterion relies on the width of the confidence interval. The concept of critical upper bound spatial correlation is introduced.

## 1 INTRODUCTION

The localization of weak properties or bad behavior of a structure is still a challenge for Structural Health Monitoring (SHM) design (Okasha and Frangopol, 2012). In case of random loading or material properties, this challenge is arduous because of the limited number of sensors and the quasi-infinite potential positions of local failures. Deterministic algorithms and specific multi-sensors systems are developed to this aim. In that case, generally, the precision of the positioning of a defect goes with a lack of sizing. However, the stochastic field is rarely a pure white noise and has a stochastic structure and probabilistic properties. These properties should be used to provide rational aid tools for optimizing the number of sensor. The paper shows that the stationary property is sufficient to find the optimal quantity of sensors and their position in view to assess accurately the shape of the auto-correlation function (see Fig. 1) that guides the position of control through Non Destructive Techniques (NDT) during the service life. If we focus on embedded sensors, these stochastic properties are less sensitive to environmental conditions (sun, humidity, ...) at the surface of the structures where usual NDT tools are affected by. That will help the help the planning of NDT campaigns. The evolution of measurement with time is out of the scope of the present paper.

The paper starts with key concepts and illustration of spatial random field modeling.

In the next section, a method for optimal sensor positioning is suggested by focusing on the assessment of the correlation function. The quality is ex-

pressed in terms of confidence interval on the parameters of the auto-correlation function.

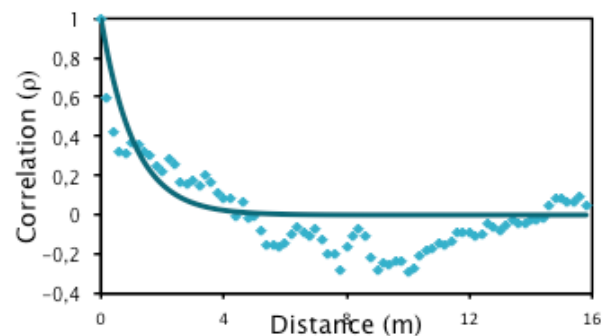


Figure 1. Auto-correlation function fitted with a parametric function.

## 2 SPATIAL RANDOM FIELD MODELLING

### 2.1 Usual approaches for spatial field modeling

The stochastic field could take several forms more or less complicated. The most simple when the degradation can be considered as homogeneous is the stationary stochastic field that can be used, for instance, to model chloride distribution (O'Connor & Kenshel 2013), concrete properties (Bazant, 1991; 2000a; 2000b) or soil properties. In some cases, when a stochastic process is influenced by several phenomena that vary with time (sea wave for instance) or with space (concreting of a structure in several steps with heterogeneity of these steps), it can be modeled as piecewise stationary. This more sophisticated stochastic field can represent, for ex-

ample, the variability of concreting materials by pieces or the corrosion of structures located in contiguous environments with different characteristics (Boéro et al 2009, 2012). That is the case for the corrosion of sheet-piles in marine environment where the corrosion process is governed by several phenomena that depend on the exposure zone  $E$ . For illustrating this case, Figure 2 presents the spatial variability of the mean and the 95% quantiles of steel thickness loss in different zones at 10, 25 and 50 years ( $t_{10}$ ,  $t_{25}$  and  $t_{50}$ ). It can be observed that both the mean and the 95% quantile are well defined for the considered zones –i.e.,  $E \in [E_A, E_T, E_L, E_I, E_M, E_S]$  limited by dashed lines: from the top to the bottom, Aerial (A), Tide (T), Low level of tide (L), Immersion (I), Mud (M) and Soil (S). In this case, a piecewise stationary stochastic field could be used to have a good representation of the loss of thickness.

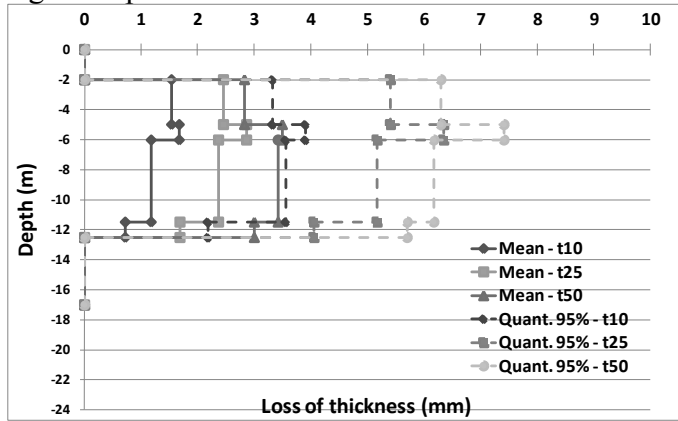


Figure 2. Mean value and 95% quantiles of steel thickness loss in each zone at time 10, 25 and 50 years.

## 2.2 Usual approaches for inspection modeling

During measurement, there are many factors that influence the quality of measurements –e.g., environmental conditions, error in the placement of the sensor, error due to sensor and error induced by the operator (Bonnet et al. unpubl., Breyse et al. 2009). These factors could lead, for a given measurement, to under or overestimations of the measured parameter. If the parameter is underestimated and the owner could decide, “do nothing” when repair is required. On the contrary, an overestimation generates a “wrong decision” where the early repair generates overcharges (Rouhan & Schoefs 2003, Sheils et al. 2012). In this paper we will consider inspection as perfect: that (i) there is no bias, (ii) error is negligible or redundancy allows us to assume it as negligible.

## 2.3 Main assumptions for the stochastic modeling

Starting from the previous sections and to limit the study, we consider five main assumptions about the sensor and the random field modeling.

- The stochastic field is statically homogeneous, and only few information on the marginal distribution is known: the type of the unique

probability density distribution. In the following applications we consider a Gaussian field;

- A limited number (from 30 to 50) of discrete sensors can be placed;
- Second order stationary stochastic field can piecewise or totally describe the spatial fields;
- Sensors are considered as perfect as defined by Schoefs et al. (2009).

Given a probability space  $(\Omega, \mathcal{F}, P)$ , a stochastic field or process with state space  $Z$  is a collection of  $Z$ -valued random variables indexed respectively by a set  $s$  “space” or  $t$  “time”. Let us denote  $Z(x, \theta)$  the one-dimensional stochastic field where  $\theta$  represents the randomness and  $x$  the spatial coordinate.

$Z(x, \theta_i)$  is called the  $i^{\text{th}}$  trajectory of this field and corresponds to a given realization  $\theta_i$  of the field for whatever location.  $Z(x_l, \theta)$  is a random variable that is generated by  $\theta$  at a given location  $x = x_l$ . We consider here homogeneous fields only. This means that the marginal distribution of  $Z(x_l, \theta)$  does not depend on the location.

A stochastic field is second order stationary if it follows three main properties:

- expectation  $E[Z(x, \theta)]$  does not depend on the location  $x$  –i.e.,  $E[Z(x, \theta)] = E[Z]$ ;
- variance  $V[Z(x, \theta)]$  does not depend on the location  $x$  –i.e.,  $V[Z(x, \theta)] = V[Z]$ ; and
- spatial covariance  $COV[Z(x, \theta), Z(x', \theta)]$  depends only on the distance  $(x-x')$ .

Thus, the second order stationary is a property restricted to the two first probabilistic moments. It can be shown that geometries of welds for ships or the spatial distribution of chloride concentration in reinforced concrete structures can be represented by stationary stochastic fields (Schoefs et al. 2011a). For instance, Figure 3 presents the correlation function of the chloride diffusion coefficient as function of the distance between two points (duratiNet project: <http://www.duratinet.org>, following O’Connor & Kenshel 2013). Note that if  $E[Z(x, \theta)]$  is not constant with space, the signal can be centered and the properties been proved for the field  $(Z(x, \theta) - E[Z(x, \theta)])$ .

Several approaches can be used to represent a stochastic field  $Z(x, \theta)$ : Karhunen-Loève expansion, approximation by Fourier series, and approximation EOLE (Li & Der Kiureghian 1993). In this paper, we select a Karhunen-Loève expansion to represent the stochastic field of resistance of a structure  $Z(x, \theta)$ . This expansion represents a random field as a combination of orthogonal functions on a bounded interval  $[-a, a]$ :

$$Z(x, \theta) = \mu_Z + \sigma_Z \cdot \sum_{i=1}^n \sqrt{\lambda_i} \cdot \xi_i(\theta) \cdot f_i(x) \quad (1)$$

where,  $\mu_Z$  is the mean of the field  $Z$ ,  $\sigma_Z$  is the standard deviation of the statistically homogeneous field  $Z$ ,  $n$  is number of terms in the expansion,  $\xi_i$  is a set of centered Gaussian random variables and  $\lambda_i$  and  $f_i$

are, respectively, the eigenvalues and eigen functions of the correlation function  $\rho(\Delta x)$ . It is possible to analytically determine the eigenvalues  $\lambda_i$  and eigen functions  $f_i$  for some correlation functions (Ghanem & Spanos 2003). For example, it can be determined if we assume that the field is second order stationary and we use an exponential correlation function (Vanmarcke, 1983):

$$\rho(\Delta x = x_1 - x_2) = \exp\left(-\frac{|\Delta x|}{b}\right); 0 < b \quad (2)$$

where  $b$  is the correlation length and  $\Delta x \in [-a, a]$ . Accounting for the stationarity of the process, the following transcendental equations can be obtained:

$$\begin{cases} \frac{1}{b} - \omega \tan(\omega a) = 0 \\ \omega - \frac{1}{b} \tan(\omega a) = 0 \end{cases} \quad (3)$$

where  $\omega$  is obtained by solving equation (3). If the solution of the second equation is noted  $\omega^*$ , the eigenfunctions are:

$$f_i(x) = \begin{cases} \frac{\cos(\omega_i x)}{\sqrt{a + \sin(2\omega_i a)/2\omega_i}} & \text{for even } i \\ \frac{\sin(\omega_i^* x)}{\sqrt{a - \sin(2\omega_i^* a)/2\omega_i^*}} & \text{for odd } i \end{cases} \quad (4)$$

and the corresponding eigenvalues are:

$$\lambda_i = \begin{cases} \frac{2}{b(1/b^2 + \omega_i^2)} & \text{for even } i \\ \frac{2}{b(1/b^2 + \omega_i^{*2})} & \text{for odd } i \end{cases} \quad (5)$$

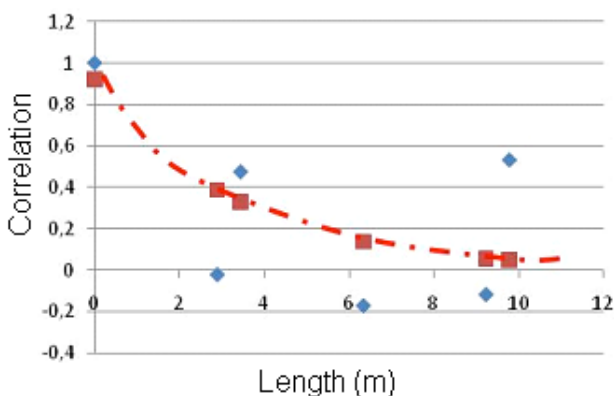


Figure 3. Spatial correlation of the chloride diffusion coefficient in concrete – data Trinity College of Dublin.

### 3 SENSOR POSITIONNING STRATEGY AND GOALS

#### 3.1 Key ideas for sensor geo-positioning

The paper focuses on the assessment of the auto-correlation function of a stationary field. An optimal geo-positioning of sensors along a trajectory (Fig. 4) should give an accurate value of the parameters (i.e.  $b$ ) with a limited number of sensors.

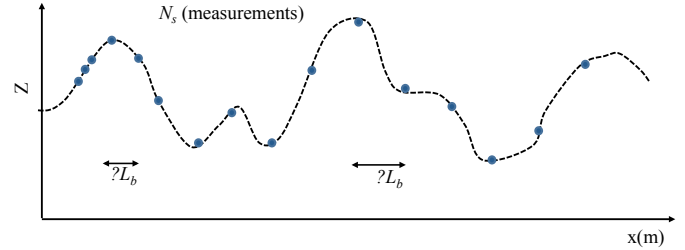


Figure 4. Non-regularly spaced sensors along a trajectory.

When looking to the usual shape of a correlation function (Fig.1) a regular spacing ' $L_b = a$ ' of  $N_s$  sensors appears not optimal: it provides a dense information for distances ' $a, 2a, \dots, (N_s-1)a$ ' but none between. We get a non-homogeneous quantity of data:  $N_s$  for distance 0,  $(N_s-i)$  for distance  $ia$  ( $1 < i < N_s$ ). The objective will be to get a spacing of sensor that leads to a more homogeneous quantity of data in the zone of high correlation as defined on Figure 5. Moreover, there is a limited feed-back on the auto-correlation length on site and the uncertainty on the value of ' $b$ ' (Eq. 2) when installing the sensors is significant.

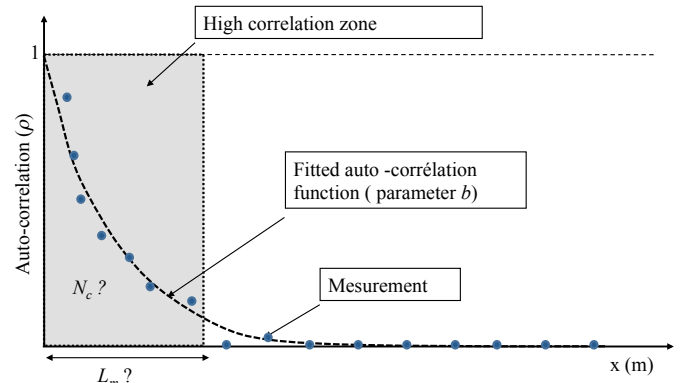


Figure 5. High correlation zone and homogeneous measurements long the trajectory.

These reasons lead us to suggest a non-regular spacing of the sensors.

#### 3.2 Definition of the Spatial Correlation Threshold

When the uncertainty of the stages inspection we suggest should provide both the parameters of the spatial correlation function and independent events that characterize the marginal distribution of  $Z$ . We consider in this paper a one-dimensional spatial field and we will apply the methodology on a set of trajectories: these trajec-

ries could be a set of 1D components (beams), or a very long 1D-component subdivided artificially or physically (expansion joint or construction joints) in a set of short components, or belong to a wall structure (steel sheet pile or concrete wall).

In view to limit the costs of the monitoring (number of sensor) we inspect a trajectory with a “sufficiently low distance  $L_b$ ” to assess the shape of the correlation function (2): for instance the shape parameter  $b$ . This “sufficiently low” can be seen as an Inspection Distance Threshold (IDT). Thus the non regular distances of sensor spacing  $L_{bi}$  should satisfy:  $L_{bi} \in ]0, IDT[$  where  $L$  is the length of the trajectory.

For illustrating the methodology, we consider in the following a set of 1D-components (beams) with infinite length  $L \sim \infty$ : we don't discuss the case of components with a limited size, especially those where  $L < L_b$  for which it is quite impossible to characterize the spatial variability. The IDT is defined by assuming that after a given distance, the events measured from an inspection can be assumed as statistically independent. A Spatial Correlation Threshold SCT of the spatial correlation gives this weak correlation. For instance in Figure 3 (red line) for  $SCT=0.4$ ,  $IDT=3$  meters. It is linked with IDT by the relationship:

$$IDT = b \cdot \ln(SCT) \quad (6)$$

### 3.3 Parameterization of the non-regular spacing

In view to reduce the set of potential solutions and simplify the design of the network of sensors, we suggest a parameterization. It is based on a division of the trajectory into  $n_p$  pieces of same size  $L_m$  (see Fig. 6) and then a subdivision of each piece into a decreasing number of pieces of same size following a series (7).

$$N_s^i \approx \left\lfloor \frac{N_s^{1i}}{j} \right\rfloor; \quad j \in \{1i \dots n_p\} \text{ with } \sum_j N_s^j = N_s \quad (7)$$

Where  $\lfloor \cdot \rfloor$  denotes the floor function,  $N_s^j$  is the number of sensors in the  $j^{\text{th}}$  piece and  $N_s^{1i}$  is the initial number of sensors in the 1<sup>st</sup> piece, the final number of inspection  $N_s^{1f}$  in the first section is computing following:

$$N_s^{1f} = N_s - \sum_{j>1} N_s^j \quad (8)$$

For instance, for two pieces and 6 sensors, 4 are placed in the first piece and 2 in the second (Fig. 6). Figure 7 presents the case 7 sensors and 3 pieces.

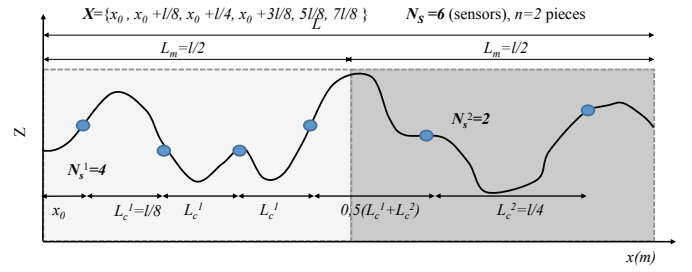


Figure 6. Distribution of sensors for  $N_s=6$  and  $n_p=2$ .

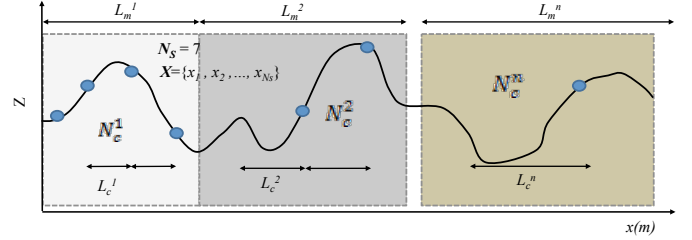


Figure 7. Distribution of sensors for  $N_s=7$  and  $n_p=3$ .

Our objective is now to optimize the number of inspections in view to reach a given quality of the result.

## 4 APPLICATION

### 4.1 Assumptions and inputs

We consider in this section a beam of length  $L=25$  m with a limited number of sensors  $N_s=30$ . The objective is to optimize their position in view to reach a good assessment of  $b$  (see Eq. 2). Only one trajectory is instrumented and the uncertainty due to the stochastic field is simulated by (1) with:  $\mu_z=100$  (MPa);  $\sigma_z=20$  (MPa);  $b=1$ m.

We analyze the error between the theoretical value and the assessed one.

### 4.2 Results and inputs

We vary the number of pieces from 1 (30 sensors) to 6 (12, 6, 4, 3, 3, 2 sensors on each piece). Figure 8 presents plots the difference  $(\hat{b}-b)$  between the estimated value and the theoretical one (here  $b=1$ ) for 1000 simulated trajectories. The regular spacing obtained for  $n_p=1$  shows the biggest uncertainty in the assessment of  $b$  with potential errors of 0.8 (near 100%). The cutting with 6 pieces shows a systematic underestimation of  $b$ .

Let us focus on  $\epsilon_b^*$ , the normalized mean error on  $b$  presented in Equation (9). Figure 9 gives the plot of this error as a function of  $n_p$ .

$$\epsilon_b^* = \frac{1}{1000} \sum_k \left( \frac{\hat{b}_k - b}{b} \right) \quad (9)$$

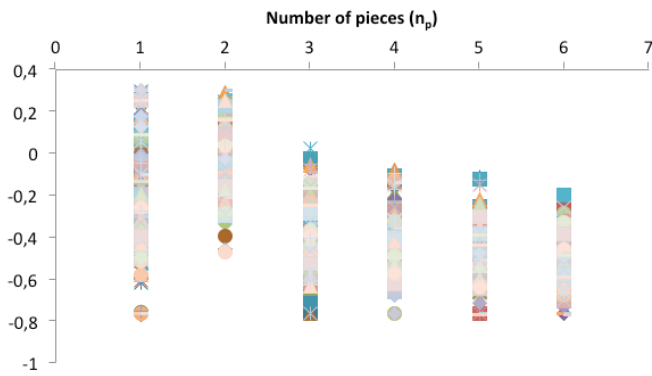


Figure 8. Distribution of sensors for  $N_s=7$  and  $n_p=3$ .

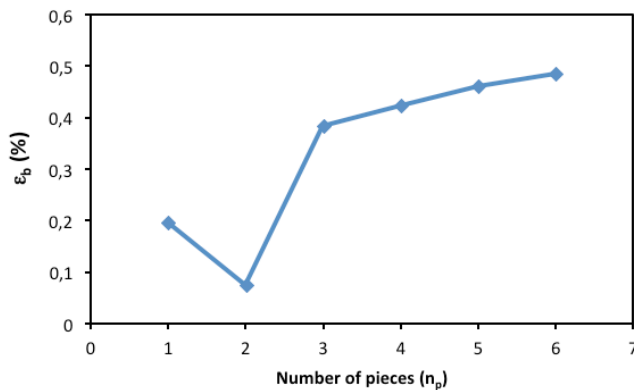


Figure 9. Normalized error on  $b$  as a function of  $n_p$ .

Following this criterion, it is shown that the spacing with  $n_p=2$  is optimal.

## 5 CONCLUSION

We propose in this paper an original method for the non-regular spacing of sensors devoted to the assessment of stationary fields from embedded sensors. The method is based on the uncertainty assessment of the parameter of the auto-correlation function. Numerical simulations of Gaussian stationary stochastic illustrate the potential of the method by showing that it provides a decision aid tool when a limited number of sensors are placed in a structure.

## 6 ACKNOWLEDGEMENTS

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