

# Real Time Simulation of Surgery: POD vs. PGD

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**Workshop RB, POD/PGD**



1542

**Universidad  
Zaragoza**

Aragón Institute of Engineering Research

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1. Introduction
2. A POD approach
3. A PGD approach
4. Conclusions

# Contents

1. Introduction

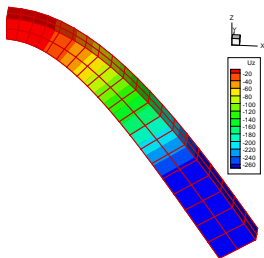
2. A POD approach

3. A PGD approach

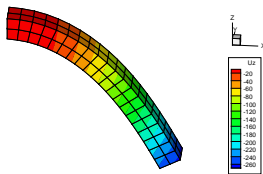
4. Conclusions

# Objective

- To develop the "engine" of a surgical simulator.
- Able to be used in medical environments.
- Many fields require **real-time** performance.
- Real-time response means 25 Hz for visual feedback and **500 Hz** for haptic feedback.
- Consensus: most of the surgical simulators do not work properly



(a) Linear elastic, small strains



(b) Linear elastic, finite strains

# State of the art



Adapted from Delinguette et al., Comm. ACM, 2005

First generation  
(Anatomy)

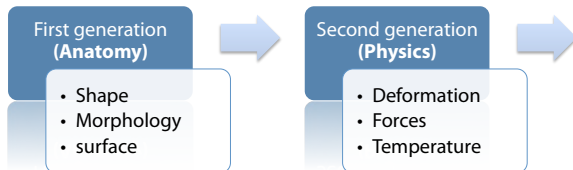


- Shape
- Morphology
- surface

# State of the art



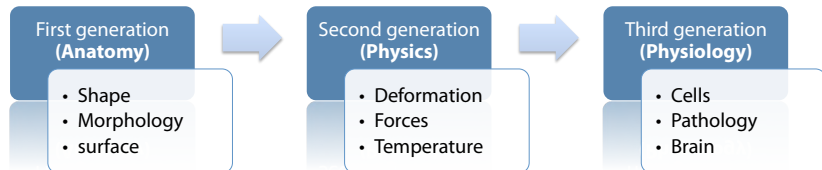
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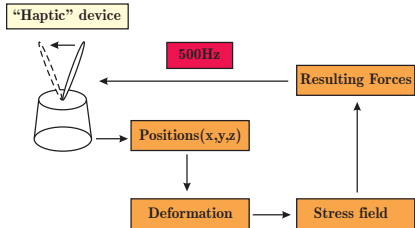
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Adapted from Delinguette et al., Comm. ACM, 2005

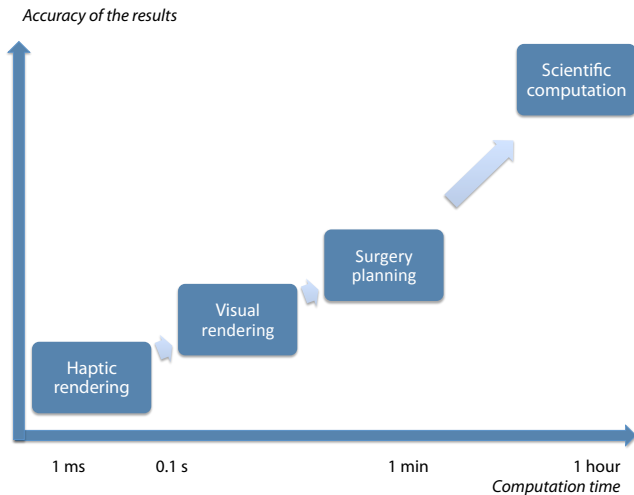


# Architecture of a surgical simulator

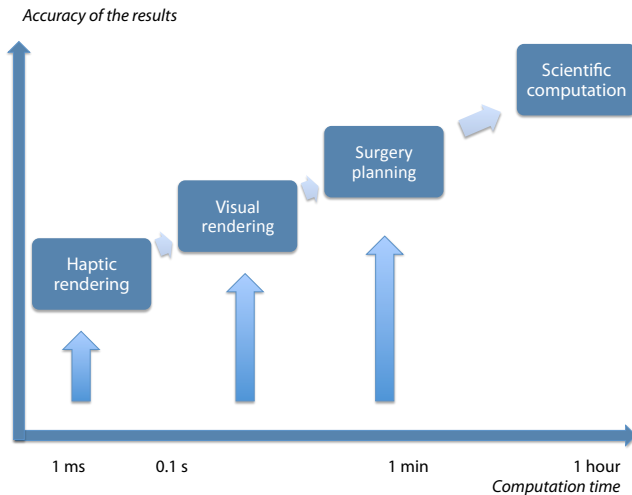




# Objective of this work



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# Contents

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2. A POD approach
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## 2. A POD approach.

### Main hypothesis

- Many movements in this field are low-rank.
- This low-rank subspace is determined by model reduction techniques (POD).
- We can perform as many *off-line* simulations as necessary.

## 2. A POD approach. *a posteriori* Model Reduction

- We perform some off-line simulations to determine  $\mathbf{u}_i^m$ ,  $\forall i \in [1, \dots, N]$  and  $\forall m \in [1, \dots, M]$ .
- Following these simulations, we extract the first  $n$  eigenvectors  $\phi_i$  related to the biggest  $n$  eigenvalues that contain the most relevant information of the model.
- We then interpolate among reduced models (see the work by Farhat).
- A classical approach:

$$\mathbf{u}^m = \sum_{i=1}^{i=n} \zeta_i^m \phi_i = B \zeta^m$$

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## 2. A POD approach.

### A hyperelastic model for the human cornea

- Internal collagen fiber reinforcement

$$\Psi(\mathbf{C}) = \Psi_{vol}(J) + \bar{\Psi}(\bar{\mathbf{C}}, \mathbf{m}_0 \otimes \mathbf{m}_0, \mathbf{n}_0 \otimes \mathbf{n}_0)$$

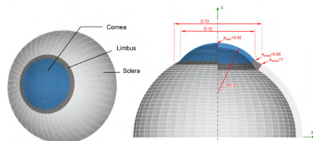


E. Lanchares et al., Journal of Biomechanics, 2008



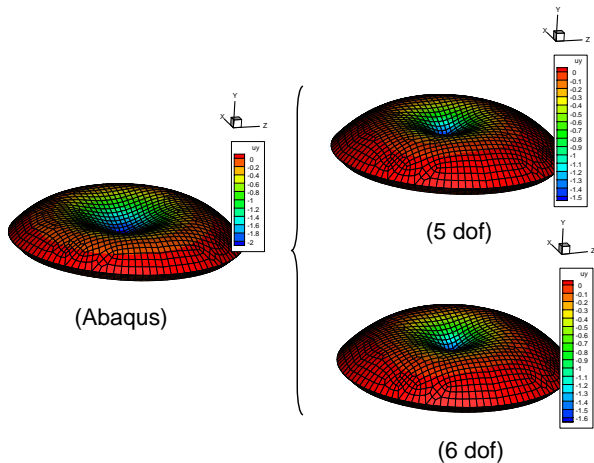
Alastrué et al., Biomechanical Engineering-Transactions ASME,

2006



## 2. A POD approach.

### Results



## 2. A POD approach. Non-linear POD-ANM approach.

### Difficulties

- It is not possible to update the tangent stiffness matrix
- Highly non-linear tissue behaviour
- Convergence is sometimes difficult

## 2. A POD approach. Non-linear POD-ANM approach.

- Formulation of a generic problem:

$$\nabla_{\mathbf{X}} \cdot \mathbf{P} + \mathbf{B} = \mathbf{0} \text{ in } \Omega_0 \text{ with } \mathbf{P} = \frac{\partial \Psi(\mathbf{F})}{\partial \mathbf{F}}$$

- Weak form:  
*find*  $\mathbf{u} \in W^{1,2}(\Omega_0)$  *such that*

$$\int_{\Omega_0} \mathbf{S} : \delta \mathbf{E} d\Omega = \lambda \int_{\partial\Omega_0} \bar{t} \delta \mathbf{u} d\Gamma \quad \forall \delta \mathbf{u} \in W_0^{1,2}(\Omega_0)$$

- We consider the Green-Lagrange strain tensor

$$\mathbf{E} = \frac{1}{2}(\mathbf{F}^T \mathbf{F} - \mathbf{1}) = \gamma_{nl}(\mathbf{u}, \mathbf{u}) + \gamma_l(\mathbf{u})$$



Yvonnet et al. CMAME, 2007

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Yvonnet et al. CMAME, 2007

## 2. A POD approach. Non-linear POD-ANM approach.

- Kirchhoff-Saint Venant constitutive model:

$$\Psi = \frac{\tilde{\lambda}}{2} (\text{tr}(\mathbf{E})^2) + \tilde{\mu} \mathbf{E} : \mathbf{E}$$

- We perform a series expansion at a time  $(\mathbf{u}^n, \mathbf{S}^n, \lambda^n)$  as a function of a load parameter  $a$ :

$$\begin{pmatrix} \mathbf{u}^{n+1}(a) \\ \mathbf{S}^{n+1}(a) \\ \lambda^{n+1}(a) \end{pmatrix} = \begin{pmatrix} \mathbf{u}^n(a) \\ \mathbf{S}^n(a) \\ \lambda^n(a) \end{pmatrix} + \sum_{p=1}^N a^p \begin{pmatrix} \mathbf{u}^p(a) \\ \mathbf{S}^p(a) \\ \lambda^p(a) \end{pmatrix}$$



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## 2. A POD approach. Non-linear POD-ANM approach.

- We obtain a series of problems for each order  $p$

$$\mathcal{L}(\mathbf{u}^p; \delta \mathbf{u}) = \lambda_p \Psi_{ext}(\delta \mathbf{u}) + \mathcal{F}_p^{nl}(\delta \mathbf{u})$$

with  $\Psi_{ext}(\delta \mathbf{u}) = \int_{\Omega_{0\sigma}} \bar{t} \delta \mathbf{u} d\Gamma$

- Giving rise to a sequence of problems
  - Order 1:

$$\mathbf{K}_t \mathbf{u}^1 = \lambda_1 \mathbf{f}$$

$$(\mathbf{u}^1)^T \mathbf{u}^1 = 1$$

- Order  $p$ :

$$\mathbf{K}_t \mathbf{u}^p = \lambda_p \mathbf{f} + \mathbf{f}_{nl}(\mathbf{u}^i) \quad (i < p)$$

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where  $\mathbf{f}_{nl}$  results from the discretisation of  $\mathcal{F}_{nl}$ .

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## 2. A POD approach. Non-linear POD-ANM approach.

- This sequence is solved sequentially:
  - Order 1:

$$\hat{\mathbf{u}} = \mathbf{K}_t^{-1} \mathbf{f}$$

$$\lambda_1 = \frac{\pm 1}{\sqrt{\hat{\mathbf{u}}^T \hat{\mathbf{u}}}}$$

$$\mathbf{u}^1 = \lambda_1 \hat{\mathbf{u}}$$

- Order  $p$ :

$$\mathbf{u}_{nl}^p = \mathbf{K}_t^{-1} \mathbf{f}_{nl}^p$$

$$\lambda_p = -\lambda_1 [\mathbf{u}_{nl}^p]^T \mathbf{u}^1$$

$$\mathbf{u}^p = \frac{\lambda_p}{\lambda_1} \mathbf{u}^1 + \mathbf{u}_{nl}^p$$

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## 2. A POD approach. A reduced-asymptotic numerical method.

- If we now apply the model reduction technique over the displacement field  $\mathbf{u}^p = \mathbf{B}\hat{\zeta}^p$
- Obtaining for the reduced model
  - order 1:

$$\hat{\zeta} = [\mathbf{B}^T \mathbf{K}_t \mathbf{B}]^{-1} \mathbf{B}^T \mathbf{f}$$

$$\lambda_1 = \frac{\pm 1}{\sqrt{\hat{\zeta}^T \hat{\zeta}}}$$

$$\zeta^1 = \lambda_1 \hat{\zeta}$$

$$\hat{\mathbf{u}} = \mathbf{B}\hat{\zeta}, \quad \mathbf{u}^1 = \lambda_1 \zeta^1$$

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$$\zeta_{nl}^p = [\mathbf{B}^T \mathbf{K}_t \mathbf{B}]^{-1} \mathbf{B}^T \mathbf{f}_{nl}^p$$

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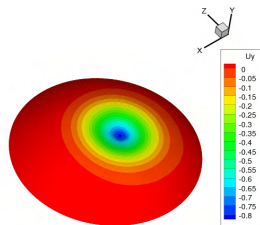
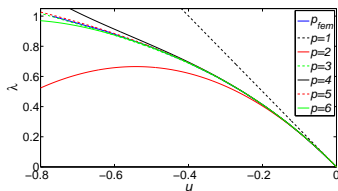
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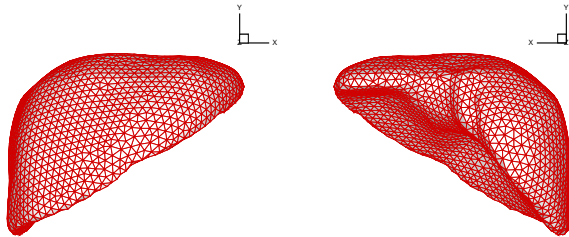


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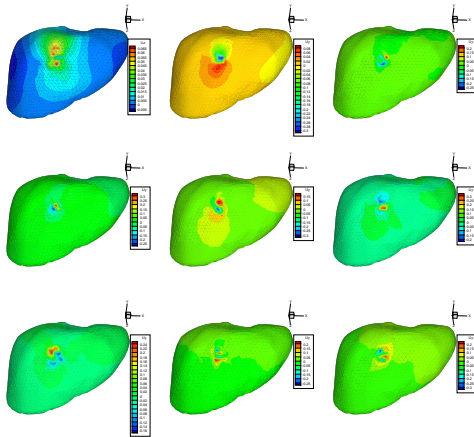
- Pinched cornea



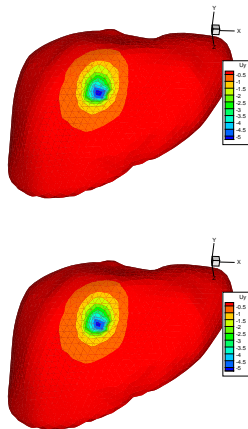
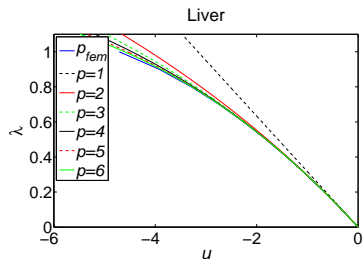
## 2. A POD approach. Palpation of a neo-hookean liver.



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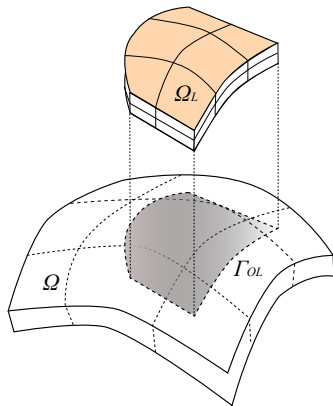
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## 2. A POD approach. Simulating surgical cutting.

- The use of Ritz-like basis functions very much complicates the simulation of surgical cutting.
- In order to avoid complex matrix updates, we have chosen X-FEM techniques coupled with POD.

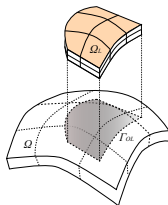
## 2. A POD approach. The $s$ -FEM.



$$\mathbf{u} = \mathbf{u}^0 + \mathbf{u}^L \text{ in } \Omega_L$$

$$\mathbf{u}^L = \mathbf{0} \text{ on } \Gamma_{OL}.$$

## 2. A POD approach. The $s$ -FEM.



- The reduced order model approximates the displacement everywhere,

$$\mathbf{u}^0(\mathbf{x}) = \sum_{i=1}^{i=r} \zeta_i \phi_i(\mathbf{x}),$$

- whereas in  $\Omega_L$  a traditional fem mesh, including discontinuous enrichment, is added:

$$\mathbf{u}^L(\mathbf{x}) = \sum_{j \in \Omega_L} \mathbf{d}_j N_j(\mathbf{x}) + \sum_{k \in J} \mathbf{b}_k N_k(\mathbf{x}) H(\mathbf{x}).$$

## 2. A POD approach. The $s$ -FEM.

- Weak form of the problem,  
find  $\mathbf{u} \in U = \{\mathbf{u} : \mathbf{u} \in \mathcal{H}^1, \mathbf{u} = \bar{\mathbf{u}} \text{ on } \Gamma_u\}$  such that

$$\int_{\Omega} \boldsymbol{\varepsilon}^* : \boldsymbol{\sigma} d\Omega = \int_{\Gamma_t} \mathbf{u}^* t d\Gamma + \int_{\Omega} \mathbf{u}^* \mathbf{b} d\Omega,$$
$$\forall \mathbf{u}^* \in U_0 = \{\mathbf{u}^* : \mathbf{u}^* \in \mathcal{H}_0^1, \mathbf{u} = \mathbf{0} \text{ on } \Gamma_u\},$$

- We arrive at a discrete system of equations of the type

$$\begin{pmatrix} \mathbf{K}^{\zeta\zeta} & \mathbf{K}^{\zeta d} & \mathbf{K}^{\zeta b} \\ \mathbf{K}^{d\zeta} & \mathbf{K}^{dd} & \mathbf{K}^{db} \\ \mathbf{K}^{b\zeta} & \mathbf{K}^{bd} & \mathbf{K}^{bb} \end{pmatrix} \begin{pmatrix} \zeta \\ d \\ b \end{pmatrix} = \begin{pmatrix} f^{\zeta} \\ f^d \\ f^b \end{pmatrix},$$

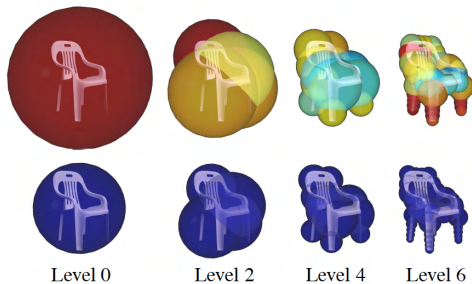
- Where the stiffness matrices  $\mathbf{K}^{\alpha\beta}$  are given by

$$\mathbf{K}_{IJ}^{\alpha\beta} = \int_{\Omega_i} (\mathbf{B}_I^{\alpha})^T \mathbf{D} \mathbf{B}_J^{\beta} d\Omega$$



## 2. A POD approach. Contact detection.

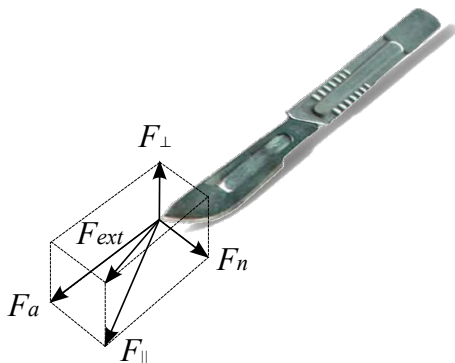
- Very efficient contact detection algorithms exist for reduced models



James and Pai, ACM Transactions on Graphics, 2004

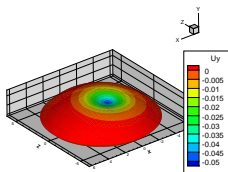
## 2. A POD approach. Simplified physics of cutting.

- Forces at scalpel tip:

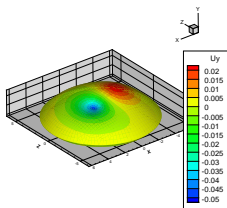


$$\mathbf{F}_{ext} = \mathbf{F}_{\perp} + \mathbf{F}_{\parallel} = \mathbf{F}_{\perp} + \mathbf{F}_{a} + \mathbf{F}_{n}$$

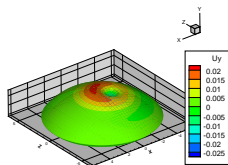
## 2. A POD approach. Modes.



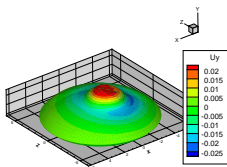
(c)



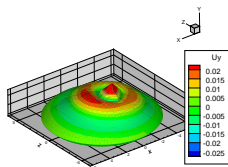
(d)



(e)

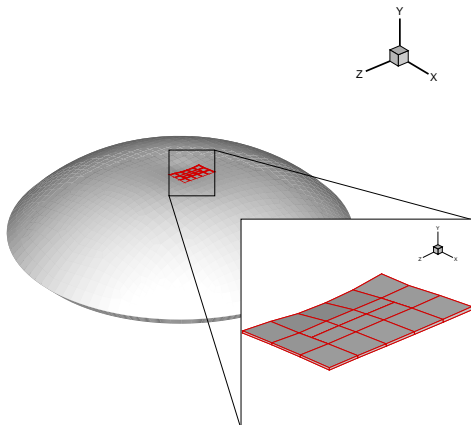


(f)

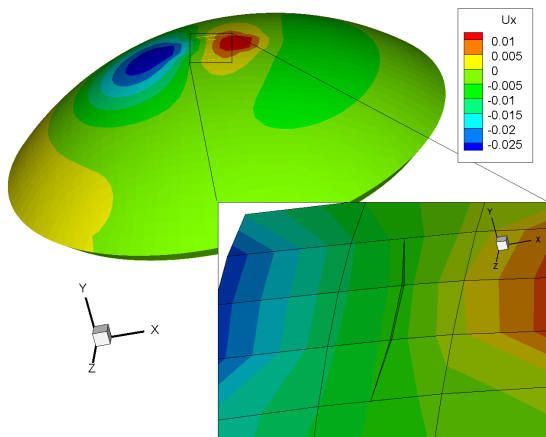


(g)

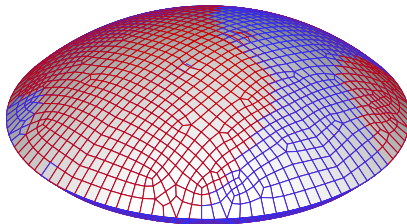
## 2. A POD approach. Simulating limbal relaxing incisions.



## 2. A POD approach. Simulating limbal relaxing incisions.



## 2. A POD approach. Simulating limbal relaxing incisions.



## 2. A POD approach. Conclusions.

- The examples shown here ran at 25 Hz on a MacBook Pro laptop running Matlab.
- While it is enough for visual feedback, it is not for haptic feedback.
- POD-ROM also helps with the time step in explicit dynamics (see Krysl *et al.* 2001).

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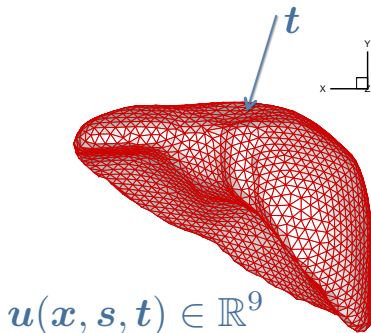


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### 3. A PGD approach. Motivation.

- PGD allows for **physics-based meta-modeling** without prior numerical experiments.
- We solve the problem (3D) for any load position (3D) and load value (3D), thus giving a 9D problem



### 3. A PGD approach. Problem setting.

- Weak form extended to all possible force vectors applied on the boundary:

$$\int_{\bar{\Gamma}} \int_{\Omega_0} \mathbf{S} : \mathbf{E}^* d\Omega d\bar{\Gamma} = \int_{\bar{\Gamma}} \int_{\Gamma_{t_2}} (\mathbf{u}^*)^T \mathbf{t} \delta(\mathbf{x} - \mathbf{s}) d\Gamma d\bar{\Gamma}$$

- We then regularize the load term:

$$t_j \approx \sum_{i=1}^n f_j^i(\mathbf{x}) g_j^i(\mathbf{s})$$

### 3. A PGD approach. Problem setting.

- We use a fixed-point algorithm:

$$u_j^n(\mathbf{x}, \mathbf{s}) = \sum_{k=1}^n X_j^k(\mathbf{x}) \cdot Y_j^k(\mathbf{s})$$

- We look for a new pair of functions:

$$u_j^{n+1}(\mathbf{x}, \mathbf{s}) = u_j^n(\mathbf{x}, \mathbf{s}) + R_j(\mathbf{x}) \cdot S_j(\mathbf{s})$$

- Admissible variations of the displacement will be given by

$$u_j^*(\mathbf{x}, \mathbf{s}) = R_j^*(\mathbf{x}) \cdot S_j(\mathbf{s}) + R_j(\mathbf{x}) \cdot S_j^*(\mathbf{s})$$

### 3. A PGD approach. PGD algorithm for a Kirchhoff-Saint Venant material.

- Explicit algorithm

$$\int_{\bar{\Gamma}} \int_{\Omega_0} \mathbf{S}^m : \mathbf{E}^* d\Omega d\bar{\Gamma} = \int_{\bar{\Gamma}} \int_{\Gamma_{t2}} (\mathbf{u}^*)^T \mathbf{t} \delta(\mathbf{x} - \mathbf{s}) d\Gamma d\bar{\Gamma}$$

- For a Kirchhoff-Saint Venant material, we have

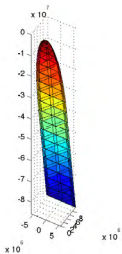
$$\Psi = \frac{\lambda}{2} (\text{tr} \mathbf{E}) + \mu \mathbf{E} : \mathbf{E}$$

- so that

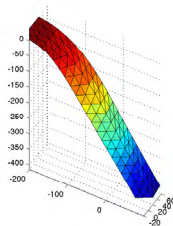
$$\mathbf{S}^m = \frac{\partial \Psi}{\partial \mathbf{E}} = \mathbf{C} : \mathbf{E}^n$$

### 3. A PGD approach. Some results.

- Results for a particular location of the load (beam end)



(h) Linear elastic, small strains

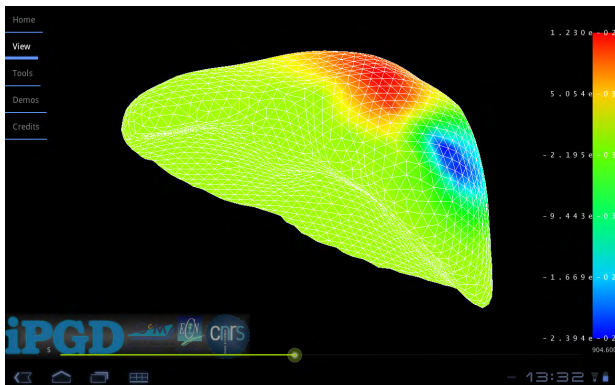


(i) Kirchhoff-Saint Venant

- These results are obtained at **kHz rates**.

### 3. A PGD approach. Some results.

- From PGD to iPGD...



### 3. A PGD approach. Some results.



### 3. A PGD approach. Some results.

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## 4. Conclusions.

- Model order reduction techniques provide a means for accurate solving of state of the art models of non-linear living tissues.
- POD-based techniques have demonstrated to provide feedback rates on the order of visual requirements (surgery planning, ...)
- PGD, on the contrary, seems to open new possibilities by constructing physics-based meta-models in a multi-dimensional setting.

# Real Time Simulation of Surgery: POD vs. PGD

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