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► **To cite this version:**

Francisco Chinesta, Adrien Leygue, Arnaud Poitou, Sylvain Chatel, Juan Antonio Garcia, et al.. Proper Generalized Decomposition of LCM Models. 10th International Conference on Flow Processes in Composite Materials, 2010, Ascona, Switzerland. <hal-01008708>

HAL Id: hal-01008708

<https://hal.science/hal-01008708v1>

Submitted on 5 May 2018

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Proper Generalized Decomposition of LCM models

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ABSTRACT: Simulation of RTM processes is usually performed in 2D. Thus, only a mesh of the middle plane is needed with the associated degrees of freedom savings with respect to fully 3D modeling. However such modeling needs the definition of an equivalent in-plane permeability representing the ignored dimension (the thickness). The definition of such permeability is not a trivial task because each ply in the thickness direction can be anisotropic, being the principal anisotropy direction different from one ply to the neighbor plies. In this work we propose a novel fully 3D modeling whose computational cost is equivalent to a 2D solution. It allows addressing properly the equivalent in-plane permeability issue.

KEYWORDS: RTM, Permeability, Separated representations, Model reduction

1 INTRODUCTION

In general the simulation of RTM processes assumes a 2D flow model. The most usual model results of combining the Darcy's law and the flow incompressibility:

$$\begin{cases} \mathbf{v} = -\mathbf{K} \cdot \nabla p \\ \nabla \cdot \mathbf{v} = 0 \end{cases}$$

that results in the second order BVP:

$$\nabla \cdot (\mathbf{K} \cdot \nabla p) = 0$$

The main issue in defining this model concerns the definition of the permeability tensor \mathbf{K} . Different techniques exist, but in that follows we are assuming that an averaged permeability has been determined for each type of reinforcement architecture.

2 PGD IN PLATE DOMAINS

In what follows we are illustrating the construction of the Proper Generalized Decomposition of a model defined in a plate domain $\Xi = \Omega \times I$ with $\Omega \subset \mathfrak{R}^2$ and $I = [0, H]$:

$$\nabla \cdot (\mathbf{K} \cdot \nabla p) = 0 \quad (1)$$

We consider that the laminate is composed of P different anisotropic plies each one characterized by a well defined permeability tensor $\mathbf{K}_i(x, y)$ -it is assumed constant in the ply thickness-. Moreover, without a loss of generality, we assume the same thickness for the

different layers of the laminate, that we denotes by h . Thus, we can define a characteristic function representing the position of each layer:

$$\chi_i(z) = \begin{cases} 1 & z_i \leq z \leq z_{i+1} \\ 0 & \text{otherwise} \end{cases}, \quad i = 1, \dots, P \quad (2)$$

where $z_i = (i-1) \times h$. Now, the laminated permeability can be given in the following separated form:

$$\mathbf{K}(x, y, z) = \sum_{i=1}^{i=P} \mathbf{K}_i(\mathbf{x}) \cdot \chi_i(z) \quad (3)$$

where $\mathbf{x} = (x, y) \in \Omega$.

The weak form of Eq. (1) writes:

$$\int_{\Xi} \nabla p^* \cdot (\mathbf{K} \cdot \nabla p) d\Xi = 0 \quad (4)$$

with the test function p^* in an appropriate functional space. The solution $p(x, y, z)$ is searched under the separated form:

$$p(\mathbf{x}, z) \approx \sum_{i=1}^{i=N} X_i(\mathbf{x}) \cdot Z_i(z) \quad (5)$$

In what follows we are illustrating the construction of one such decomposition. For this purpose we assume that at iteration $n < N$ the solution is already known:

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$$p^n(\mathbf{x}, z) = \sum_{i=1}^{i=n} X_i(\mathbf{x}) \cdot Z_i(z) \quad (6)$$

and that at the present iteration we look for the solution enrichment:

$$p^{n+1}(\mathbf{x}, z) = p^n(\mathbf{x}, z) + R(\mathbf{x}) \cdot S(z) \quad (7)$$

The test function involved in the weak form is searched under the form:

$$p^*(\mathbf{x}, z) = R^*(\mathbf{x}) \cdot S(z) + R(\mathbf{x}) \cdot S^*(z) \quad (8)$$

By introducing Eqs. (7) and (8) into (4) it results:

$$\begin{aligned} & \int_{\Xi} \left(\left(\frac{\tilde{\nabla} R^* \cdot S}{R^* \cdot \frac{dS}{dz}} \right) + \left(\frac{\tilde{\nabla} R \cdot S^*}{R \cdot \frac{dS^*}{dz}} \right) \right) \cdot \left(\mathbf{K} \cdot \left(\frac{\tilde{\nabla} R \cdot S}{R \cdot \frac{dS}{dz}} \right) \right) d\Xi = \\ & = - \int_{\Xi} \left(\left(\frac{\tilde{\nabla} R^* \cdot S}{R^* \cdot \frac{dS}{dz}} \right) + \left(\frac{\tilde{\nabla} R \cdot S^*}{R \cdot \frac{dS^*}{dz}} \right) \right) \cdot \mathbf{Q}^n d\Xi \end{aligned} \quad (9)$$

where $\tilde{\nabla}$ denotes the plane component of the gradient operator $\tilde{\nabla}^T \equiv (\partial/\partial x, \partial/\partial y)$ and \mathbf{Q}^n denotes the flux at iteration n :

$$\mathbf{Q}^n = \mathbf{K} \cdot \sum_{i=1}^{i=n} \begin{pmatrix} \tilde{\nabla} X_i(\mathbf{x}) \cdot Z_i(z) \\ X_i(\mathbf{x}) \cdot \frac{dZ_i(z)}{dz} \end{pmatrix} \quad (10)$$

Now, as the enrichment process is non-linear we propose to search the couple of functions $R(\mathbf{x})$ and $S(z)$ by applying an alternating direction fixed point algorithm. Thus, assuming $R(\mathbf{x})$ known, we compute $S(z)$, and then we update $R(\mathbf{x})$. The process continues until reaching convergence. The converged solutions allow defining the next term in the finite sums decomposition: $R(\mathbf{x}) \rightarrow X_{n+1}(\mathbf{x})$ and $S(z) \rightarrow Z_{n+1}(z)$.

We are illustrating each one of the just referred steps:

1. Computing $R(\mathbf{x})$ from $S(z)$:

When $S(z)$ is known the test function reduces to:

$$p^*(\mathbf{x}, z) = R^*(\mathbf{x}) \cdot S(z) \quad (11)$$

and the weak form (9) reduces to:

$$\begin{aligned} & \int_{\Xi} \left(\frac{\tilde{\nabla} R^* \cdot S}{R^* \cdot \frac{dS}{dz}} \right) \cdot \left(\mathbf{K} \cdot \left(\frac{\tilde{\nabla} R \cdot S}{R \cdot \frac{dS}{dz}} \right) \right) d\Xi = \\ & = - \int_{\Xi} \left(\frac{\tilde{\nabla} R^* \cdot S}{R^* \cdot \frac{dS}{dz}} \right) \cdot \mathbf{Q}^n d\Xi \end{aligned} \quad (12)$$

Now, as all the functions involving the coordinate z are known, they could be integrated in $I = [0, H]$. Thus, if we consider:

$$\mathbf{K} = \begin{pmatrix} \mathbb{k} & \mathbf{k} \\ \mathbf{k}^T & \kappa \end{pmatrix} \quad (13)$$

with $\mathbb{k} = \begin{pmatrix} k_{xx} & k_{xy} \\ k_{yx} & k_{yy} \end{pmatrix}$, $\mathbf{k} = \begin{pmatrix} k_{xz} \\ k_{yz} \end{pmatrix}$ and $\kappa = k_{zz}$, then we can define:

$${}^x \mathbf{K} = \begin{pmatrix} \int_{z=0}^{z=H} \mathbb{k} S^2 dz & \int_{z=0}^{z=H} \mathbf{k} \frac{dS}{dz} S dz \\ \int_{z=0}^{z=H} \mathbf{k}^T \frac{dS}{dz} S dz & \int_{z=0}^{z=H} \kappa \left(\frac{dS}{dz} \right)^2 dz \end{pmatrix} \quad (14)$$

and

$${}^x \mathbf{Q}^n = \sum_{i=1}^{i=N} \begin{pmatrix} \int_{z=0}^{z=H} \mathbb{k} S Z_i dz & \int_{z=0}^{z=H} \mathbf{k} S \frac{dZ_i}{dz} dz \\ \int_{z=0}^{z=H} \mathbf{k}^T \frac{dS}{dz} Z_i dz & \int_{z=0}^{z=H} \kappa \frac{dS}{dz} \frac{dZ_i}{dz} dz \end{pmatrix} \begin{pmatrix} \tilde{\nabla} X_i(\mathbf{x}) \\ X_i(\mathbf{x}) \end{pmatrix} \quad (15)$$

that allows writing equation (12) into the form

$$\begin{aligned} & \int_{\Omega} \left(\frac{\tilde{\nabla} R^*}{R^*} \right) \cdot \left({}^x \mathbf{K} \cdot \left(\frac{\tilde{\nabla} R}{R} \right) \right) d\Omega = \\ & = - \int_{\Omega} \left(\frac{\tilde{\nabla} R^*}{R^*} \right) \cdot {}^x \mathbf{Q}^n d\Omega \end{aligned} \quad (16)$$

that defines an elliptic 2D problem defined in the middle plane of the plate.

2. Computing $S(z)$ from $R(\mathbf{x})$:

When $R(\mathbf{x})$ is known the test function writes:

$$p^*(\mathbf{x}, z) = R(\mathbf{x}) \cdot S^*(z) \quad (17)$$

and the weak form (9) reduces to:

$$\begin{aligned}
& \int_{\Xi} \begin{pmatrix} \tilde{\nabla}R \cdot S^* \\ R \cdot \frac{dS^*}{dz} \end{pmatrix} \cdot \mathbf{K} \cdot \begin{pmatrix} \tilde{\nabla}R \cdot S \\ R \cdot \frac{dS}{dz} \end{pmatrix} d\Xi = \\
& = - \int_{\Xi} \begin{pmatrix} \tilde{\nabla}R \cdot S^* \\ R \cdot \frac{dS^*}{dz} \end{pmatrix} \cdot \mathbf{Q}^n d\Xi
\end{aligned} \tag{18}$$

Now, as all the functions involving the in-plane coordinates $\mathbf{x} = (x, y)$ are known, they could be integrated in Ω . Thus, using the previous notation, we can define:

$${}^z \mathbf{K} = \begin{pmatrix} \int_{\Omega} (\tilde{\nabla}R) \cdot (\mathbb{k} \cdot (\tilde{\nabla}R)) d\Omega & \int_{\Omega} (\tilde{\nabla}R) \cdot \mathbf{k} R d\Omega \\ \int_{\Omega} \mathbf{k} \cdot (\tilde{\nabla}R) R d\Omega & \int_{\Omega} \kappa (R)^2 d\Omega \end{pmatrix} \tag{19}$$

and

$${}^z \mathbf{Q}^n = \sum_{i=1}^{i=N} \begin{pmatrix} \int_{\Omega} \tilde{\nabla}R \cdot (\mathbb{k} \cdot \tilde{\nabla}X_i) d\Omega & \int_{\Omega} \tilde{\nabla}R \cdot \mathbf{k} X_i d\Omega \\ \int_{\Omega} R \mathbf{k} \cdot \tilde{\nabla}X_i d\Omega & \int_{\Omega} \kappa R X_i d\Omega \end{pmatrix} \begin{pmatrix} Z_i \\ \frac{dZ_i}{dz} \end{pmatrix} \tag{20}$$

that allows writing equation (18) into the form

$$\begin{aligned}
& \int_I \begin{pmatrix} S^* \\ \frac{dS^*}{dz} \end{pmatrix} \cdot \begin{pmatrix} S \\ \frac{dS}{dz} \end{pmatrix} \cdot {}^z \mathbf{K} \cdot \begin{pmatrix} S \\ \frac{dS}{dz} \end{pmatrix} dz = \\
& = - \int_I \begin{pmatrix} S^* \\ \frac{dS^*}{dz} \end{pmatrix} \cdot {}^z \mathbf{Q}^n dz
\end{aligned} \tag{21}$$

that defines a one-dimensional BVP.

3 CONCLUSIONS

This paper comes back to a recurrent issue in the numerical modelling of RTM flows, the one related to the pertinence of using 2D flow models making use of an averaged permeability of the different layers involved in the laminate. In complex situations significant deviations could be found, these deviations are being studied at present, and could justify the use of a fully 3D modelling. However, 3D simulations are reputed expensive from the point of view of the computational resources required for addressing complex scenarios. The use of separated representations as the ones involved in the proper generalized decompositions –PGD– could be an appealing alternative for addressing 3D models with a cost characteristic of 2D simulations. The

application of the PGD on the fully 3D simulation of flows encountered in RTM processes constitutes a work in progress.

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