

PGD-BEM applied to the heat equation

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The Boundary Elements Method (BEM) allows efficient solution of partial differential equations whose kernel functions are known. The heat equation is one of these candidates when the thermal parameters are assumed constant (linear model). When the model involves large physical domains and time simulation intervals the amount of information that must be stored increases significantly. This drawback can be circumvented by using advanced strategies, as for example the multi-poles technique.

We propose an alternative radically different that leads to a separated solution of the space and time problems within a non-incremental integration strategy. The technique is based on the use of a space-time separated representation of the unknown field that introduced in the residual weighting formulation allows to define a separated solution of the resulting weak form. The spatial step can be then treated by invoking the standard BEM for solving the resulting steady state problem defined in the physical space. Then, the time problem that result in an ordinary first order differential equation is solved using any standard appropriate integration technique (e.g. backward finite differences).

In the case of the linear and transient heat equation here considered for the sake of simplicity, the PGD (proper generalized decomposition) leads to the solution of a series of steady state diffusion-reaction problems (accurately solved by using the BEM method) and a series of problems that consist of a simple time dependent ODE.

Separated representations were already applied for solving transient models in the context of finite element discretizations [1] [2] [3] [4] [6], but they never have been used in the BEM framework, and certainly in this context the main advantage is the possibility of defining non-incremental strategies as well as the possibility of avoiding the use of space-time kernels. In principle, this technique seems specially adapted for solving transient problems involving extremely small times steps.

The strategy proposed allows to define a non-incremental boundary element method strategy for solving linear parabolic partial differential equations. Obviously, by using an appropriate linearization the proper generalized decomposition based BEM could be extended for solving non-linear models.

References

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