

# PGD Based Model Reduction: Towards Dynamic Data Driven Application Systems

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Process optimization and control problems can be addressed through a broad variety of approaches. The most common consists in considering the system as a black box relating a set of input variables to some outputs. Well established approaches of robust control and identification have been developed to handle, to some extent, sources of variability in the physical system, in the control system or in the data acquisition system. However, many systems need to be controlled and optimized at very fine levels that can only be simulated through computationally expensive models. By expensive, we understand that the time needed to solve the model is incompatible with the requirements of the control algorithm. For example in material processing the appropriate level of description usually is that of the fine microstructural scales involved in the physical and chemical transformations occurring during the process. The corresponding thermo-chemo-mechanical models are solved today at a pace that can be orders of magnitude too slow with respect to process control requirements. Moreover, as the relevant simulation scale is often connected to the different sources of variability through complex non-linear couplings, one simply cannot take advantage of system identification techniques or a posteriori model reduction such as the Proper Orthogonal Decomposition. These techniques would indeed yield too crude approximations of the real physical system.

In contrast to traditional, Simulation-based Engineering Sciences where one relies on the use of static values for model parameters, initial or boundary conditions that are not modified during the simulation, a new paradigm in the field of Applied Sciences and Engineering has emerged in the last decade: Dynamic Data-Driven Application Systems (DDDAS). By DDDAS we mean a set of techniques that allow the linkage of simulation tools with measurement devices and real-time control systems. As defined by the U.S. National Science Foundation, “DDDAS entails the ability to dynamically incorporate additional data into an executing application, and in reverse, the ability of an application to dynamically steer the measurement process”. A crucial aspect of DDDAS is that of real-time simulation. This means that the simulations must run at a pace comparable to that of data acquisition and more generally at least equal to the characteristic timescale of the simulated system.

Physical dynamical systems are usually modelled through a set of differential equations describing some quantity of interest as a function of space

and time. Additionally, the model usually contains some parameters that are traditionally fixed before the model is solved numerically. The dimensionality of the model is therefore at most 4: 3 space coordinates and time. However a higher dimensional model containing the some parameters as extra-coordinates, allows us to compute the quantity of interest as a function of space, time and the model parameters now considered as coordinates. There is no conceptual limitation to the nature of the parameters that can be considered as new coordinates of the model: material parameters, initial or boundary conditions, command of actuators used to steer the system, geometrical parameters The only limitation lies in the ability to solve the high dimensional model.

Having computed, in an off-line step, the solution of the high dimensional model, it is straightforward and numerically inexpensive to obtain, in an on-line step, the solution of the original model for any value of the parameters, as one only needs to particularize the solution with specific parameters values. Parameter identification or inverse problems are also efficiently solved: The solution of an inverse problem usually requires numerous solution of the so-called direct problem with different values of the parameters to identify. Here one can substitute the solutions of the direct problem with a simple evaluation of the multidimensional solution. From this general solution computed only once and off-line one can therefore perform on-line and in real time all the post-processing task involved in optimization, inverse analysis, analysis of sensibilities, stochastic analysis, required for the model based control of system that are computationally expensive to simulate or in DDDAS applications.

The price to pay is the solution of parametric models defined in high dimensional spaces that could involve hundreds of coordinates. Models that are defined in high dimensional spaces suffers the so-called curse of dimensionality. As computers at present can handle about  $10^{12}$  nodes, the mere storage of the solution of multi-dimensional models is a challenge. Computing this solution can be even more challenging with respect to computation time. Different techniques have been proposed to circumvent the curse of dimensionality, Monte Carlo simulations being the most widely used technique. Their main drawback is the statistical noise. To our knowledge there are few precedents of deterministic techniques able to circumvent efficiently the curse of dimensionality. We proposed recently a technique based on a separated representation of the solution that proceeds by expressing any generic multidimensional function in a separated form.

This approach of DDDAS based on the on-line/off-line use of multidimensional reduced models computed with the PGD will constitute a new paradigm in Simulation-based Engineering Sciences.