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Optimal design of corroded reinforced concrete structures by using time-variant reliability analysis

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ABSTRACT: The design optimization of RC structures should take into account the uncertainties related to all, material properties, geometry, loading and deterioration. The Time-Variant Reliability-Based Design Optimization (TV-RBDO) consists in finding the optimal design by satisfying appropriate safety level during the whole structure lifetime. This work proposes an efficient method for TV-RBDO, where the classical formulation is transformed into a sequence of equivalent deterministic design optimization sub-problems. This transformation is defined by the mean of optimally calibrated safety factors, linking the reliability requirement to the equivalent deterministic optimization. The proposed method is developed to account for deterioration (corrosion) and is applied to optimize a corroded RC bridge girder. The results show the effectiveness and the efficiency of the method.

1 INTRODUCTION

The design of reinforced concrete (RC) structures involves several types of uncertainties related to concrete and steel properties, structural dimensions, depth of the concrete cover, load fluctuations and accuracy of the analysis models. All these uncertainties contribute to make the performance of the RC structures different from the expected one. In addition, when a RC structure is located in a corrosive environment and subjected to chloride penetration, the corrosion can significantly influence the long-term performance. The deterioration of the RC structures (bridges, port, offshore platforms...) is a major problem for many companies and countries, where high costs are involved to maintain, repair or replace degraded structures. For example, CC Technologies Laboratories Inc. (2001) show that the annual direct cost of corrosion in USA highway bridges is estimated to be 8.3 billions of dollars.

Structural optimization is frequently applied for effective cost reduction of engineering systems. As the rational approach consists in finding the best compromise between cost reduction and safety assurance, the structural reliability theory provides an appropriate approach to take account for uncertainties (Ditlevsen & Madsen 1996). The Reliability-Based Design Optimization (RBDO) allows us to reach effectively balanced cost-safety configurations

(Frangopol & Moses 1994, Enevoldsen & Sørensen 1994, Aoues & Chateauneuf 2007). However, to design RC structures subjected to deterioration (e.g., corrosion, creep, fatigue, etc.), the RBDO approach should use time-variant instead the time-invariant reliability analysis, where load and environmental fluctuations are modeled by stochastic processes.

The classical formulation of the Time-Variant Reliability-Based Design Optimization (TV-RBDO) consists in minimizing an objective function defined by initial and expected failure costs under reliability constraints (Kuschel & Rackwitz 2000). This formulation is not suitable for real engineering structures because considerable time consumption is involved, and convergence can hardly be achieved. The major drawback lies in the time-variant reliability analysis, which requires considerable computational efforts. Moreover, in the classical approach of TV-RBDO this reliability analysis is performed in both the objective function and constraints.

Several works are focused on developing efficient and robust approaches for RBDO (Chateauneuf & Aoues 2008). However, few studies are interested by TV-RBDO. In this work, a new methodology of TV-RBDO of RC structures is developed on the basis of decoupling the time-variant reliability analysis and the optimization procedures. This concept has been already used in the RBDO, as in the SORA method developed by Du & Chen (2004).

The proposed approach is based on transforming the TV-RBDO problem into a sequence of equiva-

lent deterministic design optimization sub-problems. This transformation is defined by the mean of optimal safety factors, linking the reliability requirement to the equivalent deterministic optimization. At the end of each optimization sub-problem, the reliability constraint is verified by performing the time-variant reliability analysis based on the outcrossing procedures. The safety factors corresponding to the target reliability level at the initial time are calibrated by inverse probabilistic approach. Finally, these safety factors are provided to the following sub-problem of the equivalent deterministic optimization and so on, until convergence. The numerical example of optimizing a corroded RC bridge girder shows the efficiency and the good-standing of the proposed method, through the comparison with classical TV-RBDO approach.

2 TIME-VARIANT RELIABILITY ANALYSIS

The probabilistic models are usually used to describe the deterioration phenomena. When the time dependency affects only the degradation mechanism, where the material properties are decaying with time, the uncertainties are often modeled by random variables multiplied by deterministic functions of time describing the degradation kinetics. In this case, the problem can be treated by time-invariant reliability methods (Andrieu et al. 2002). However, there are many problems where the deterioration is influenced by phenomena that vary randomly with time, as the weather actions (i.e. temperature, relative humidity...) and loading (i.e. wind, traffic...). The stochastic processes are usually used to describe these phenomena. For these problems, the time-invariant reliability analysis is not valid.

The time-variant reliability aims at computing the probability of failure during the whole structure lifetime, when the time dependency lies in the loading and the degradation phenomena. Several approaches are addressed to assess the time-variant reliability, which can be done either by simulation techniques or by approximate approaches. Sampling methods such as the Monte Carlo simulation are computationally very expensive.

The approximate approaches are generally based on the outcrossing approach (Ditlevsen & Madsen 1996). Schall et al. (1991) have used the outcrossing approach and asymptotic integration to assess the probability of failure, which is related to the mean number of outcrossing of the random process through the limit state surface. Hagen & Tvedt (1991) suggested an original approach based on a parallel system reliability formulation for computing the outcrossing rate. This formulation is useful because it is based on time-invariant reliability tools. The PHI2 method developed by Andrieu et al. (2002) is based on this approach.

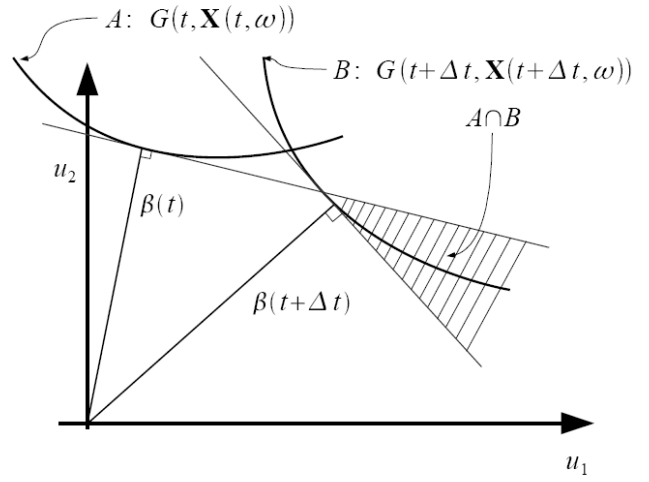


Figure 1. Computation of the outcrossing rate using PHI2.

2.1 The PHI2 method

Considering the following limit state function $G(t, \mathbf{X}(t, \omega))$, where $\mathbf{X}(t, \omega)$ denotes the set of random variables $X_j(\omega)$, $j = 1, \dots, p$ and the one-dimensional random process $X_k(t, \omega)$, $k = p+1, \dots, p+q$, t represents the time and ω describes the randomness in the mechanical model. The cumulative probability of failure $P_{f,c}(0, T_L)$ of the structure within the interval $[0, T_L]$ is given by:

$$P_{f,c}(0, T_L) = \Pr[\exists \tau \in [0, T_L] | G(t, \mathbf{X}(t, \omega)) \leq 0] \quad (1)$$

where T_L is the structural lifetime; $G(t, \mathbf{X}(t, \omega)) > 0$ indicates the safe domain and $G(t, \mathbf{X}(t, \omega)) \leq 0$ indicates the failure domain. The cumulative probability of failure $P_{f,c}(0, T_L)$ is bounded by (Hagen & Tvedt 1991):

$$\max_{0 \leq \tau \leq T_L} [P_{f,i}(\tau)] \leq P_{f,c}(0, T_L) \leq P_{f,i}(0) + \int_0^{T_L} \nu(t) dt \quad (2)$$

where $P_{f,i}(\tau)$ is the instantaneous probability of failure at a given time τ and $\nu(t)$ is the outcrossing rate function defined by:

$$\nu(t) = \lim_{\Delta t \rightarrow 0} \frac{\Pr[A \cap B]}{\Delta t} \quad (3)$$

where $A: G(t, \mathbf{X}(t, \omega)) > 0$ is the event where the structure is in the safe state at t and $B: G(t+\Delta t, \mathbf{X}(t+\Delta t, \omega)) \leq 0$ is the event that the structure is in the failure domain at $t+\Delta t$ (Figure 1). In the PHI2 method, the outcrossing rate is given by:

$$\nu(t) = \frac{\Phi_2(\beta(t), -\beta(t+\Delta t); \rho_G(t, t+\Delta t))}{\Delta t} \quad (4)$$

where $\Phi_2(\cdot)$ stands for the binormal cumulative distribution function; $\beta(t)$ and $\beta(t+\Delta t)$ are the reliability indexes corresponding to the two events A and B , respectively; and $\rho(t, t+\Delta t)$ is the correlation coefficient between the two events, which is given by the sensitivities of the random variables obtained in the two time-invariant reliability analyses.

$$\rho(t, t + \Delta t) = -\alpha(t) \cdot \alpha(t + \Delta t) \quad (5)$$

where $\alpha(t)$ and $\alpha(t + \Delta t)$ are the sensitivities of the two events A and B , respectively.

The advantage of the PHI2 approach consists in the use of the FORM approximation, where the time-variant reliability problem is converted to several time-invariant reliability analyses. Figure 1 shows the concept of the PHI2 approach. In this work, the PHI2 method is used, including the improvements proposed by Sudret (2008).

2.2 Discretization of random process

The discretization of the random process is required for time-variant reliability analysis especially when simulation methods are used. This discretization consists in representing the random process $X_k(t, \omega)$ by a finite set of deterministic functions weighted by a set of random variables. Several approaches can be used, as Fourier series expansion, Karhunen-Loève expansion (Ghanem, & Spanos 2003) and EOLE approach (Expansion Optimal Linear Estimation) (Li & Der Kiureghian 1993). The method retained in the present work is the Karhunen-Loève expansion:

$$X(t, \omega) = \mu_X + \sigma_X \sum_{i=1}^{n_{KL}} \sqrt{\lambda_i} f_i(t) \xi_i(\omega) \quad (6)$$

where μ_X and σ_X are respectively the mean and the standard deviation of the scalar Gaussian process $X(t, \omega)$; λ_i and f_i are respectively the eigenvalues and the eigenfunctions of the covariance function $\rho_{XX}(t_1, t_2)$; ξ_i is a set of independent normal variables and n_{KL} is the number of terms of the truncated discretization.

3 CLASSICAL FORMULATION OF TV-RBDO

Generally, the TV-RBDO is formulated like the RBDO problem, where the expected total cost is minimized under probabilistic constraints solved by nested optimization. The outer loop concerns the optimization and the inner loop concerns the reliability analysis leading to repeated evaluations of the performance function. This scheme leads to an expensive computation cost, where convergence and accuracy problems are observed.

$$\begin{aligned} \min_{\mathbf{d}} \quad & C_I(\mathbf{d}) + E[C_F(\mathbf{d}, \mathbf{X}, t)] \\ \text{s.t.} \quad & \begin{cases} P_{f,c}(\mathbf{d}, \mathbf{X}, t) \leq P_f^a; \forall t \in [0, T_L] \\ h_j(\mathbf{d}) \leq 0; j=1, \dots, m \end{cases} \end{aligned} \quad (7)$$

where \mathbf{d} is the vector of the design variables; \mathbf{X} is the vector of random variables and stochastic processes; $E[\cdot]$ is the expectation operator; C_I is the ini-

tial cost; C_F is the failure cost; P_f^a is the admissible probability of failure and h_j are deterministic functions, such as the lower and the upper bounds of the design variables. Kuschel & Rackwitz (2000) defined the expected cost of failure by :

$$E[C_F(\mathbf{d}, \mathbf{X}, t)] = \int_0^{T_L} f(\mathbf{d}, \mathbf{X}, t) C_f \delta(t) \quad (8)$$

where C_f is the annual failure cost; $f(\mathbf{d}, \mathbf{X}, t)$ is the time to failure and $\delta(t)$ is the capitalization or discount function. However, the PDF of time to failure $f(\mathbf{d}, \mathbf{X}, t)$ is not easy to compute. An approximation proposed by Sorensen & Tarp-Johansen (2005) for instantaneous probability of failure is extended in this work for cumulative probability of failure.

$$E[C_F] = \sum_{i=1}^{n+1} \frac{P_{f,c}(\mathbf{d}, \mathbf{X}, t_i) - P_{f,c}(\mathbf{d}, \mathbf{X}, t_{i-1})}{(1+r)^{t_i}} C_f \quad (9)$$

where $P_{f,c}(\mathbf{d}, \mathbf{X}, t_i)$ is the cumulative probability of failure at t_i ; r is the discount rate of the capitalization function $1/(1+r)^t$. This approximation fits the time-variant reliability analysis carried out by the PHI2 method, where the lifetime interval $[0, T_L]$ is discretized into $n+1$ sub-intervals.

4 PROPOSED APPROACH FOR TV-RBDO

The classical formulation of TV-RBDO converges slowly or even fails to converge. The proposed approach is based on the concept of decoupling the reliability analysis and the optimization procedures. This decoupled concept has been largely used in RBDO problems (Royset et al. 2001, Du & Chen 2004).

In this study, the TV-RBDO problem is decoupled and transformed into a sequence of sub-problems of equivalent deterministic design optimization followed by the time-variant reliability analysis. The proposed approach called SOTVRA (Sequential Optimization and Time-Variant Reliability Analysis) approximates the objective function at each deterministic optimization on the basis of the previous time-variant reliability analysis. At the end of each deterministic optimization and time-variant reliability analysis, the optimal safety factors are calibrated on the basis of the target reliability index at the initial time and provided to the next deterministic optimization sub-problem. In other words, the safety factors link the reliability requirement to the equivalent deterministic constraints. The Figure 2 shows the different steps of the SOTVRA approach, where the principal steps are described in the following subsections.

4.1 Approximation of the objective function

The objective function of the TV-RBDO problem formulated in Equation 7 requires a time-variant reliability analysis to evaluate the expected failure cost $E[C_F]$. In the SOTVRA approach, the objective function is approximated on the basis of the time-variant reliability analysis performed in the previous sub-problem. In this way, the reliability analysis is avoided in the optimization procedure. The objective function is approximated by a first-order Taylor expansion:

$$\tilde{C}_T(\mathbf{d}) = C_I + \sum_{i=1}^{n+1} \left[P_{f,c}^{(k)}(t_i) - P_{f,c}^{(k)}(t_{i-1}) + \left(\frac{\partial P_{f,c}^{(k)}(t_i)}{\partial \mathbf{d}} - \frac{\partial P_{f,c}^{(k)}(t_{i-1})}{\partial \mathbf{d}} \right)^T \cdot (\mathbf{d} - \mathbf{d}^{(k)}) \right] C_f \frac{1}{(1+r)^{t_i}} \quad (10)$$

where T indicates the transpose operator; $P_{f,c}^{(k)}(t_i)$ and $P_{f,c}^{(k)}(t_{i-1})$ are respectively the cumulative failure probabilities at times t_i and t_{i-1} calculated at the k^{th} sub-problem; $\mathbf{d}^{(k)}$ is the optimal design of the previous sub-problem; \mathbf{d} is the current design point; $\partial P_{f,c}^{(k)}(t_i)/\partial \mathbf{d}$ and $\partial P_{f,c}^{(k)}(t_{i-1})/\partial \mathbf{d}$ are the derivatives of the cumulative failure probabilities regarding the design variables, respectively. These derivatives are calculated by using the time-variant reliability results of the previous sub-problem. The cumulative probability is approximated by the upper bound given in Equation 2.

$$P_{f,c}^{(k)}(t_i) = P_{f,i}^{(k)}(t_{i-1}) + \int_{t_{i-1}}^{t_i} v(t) dt \quad (11)$$

we assume that the outcrossing rate $v(t)$ is constant in the interval $[t_{i-1}, t_i]$ because the step length $\Delta\tau = t_i - t_{i-1}$ is small, therefore the derivation is easily obtained.

$$\frac{\partial P_{f,c}^{(k)}(t_i)}{\partial \mathbf{d}} = \underbrace{\frac{\partial P_{f,i}^{(k)}(t_{i-1})}{\partial \beta(t_i)}}_{A_1} \cdot \underbrace{\frac{\partial \beta(t_{i-1})}{\partial \mathbf{d}}}_{A_2} + \underbrace{\frac{\partial \Phi_2(\beta(t_{i-1}), -\beta(t_i); \rho)}{\partial \mathbf{d}}}_{A_3} \quad (12)$$

with:

$$A_1 = -\varphi(-\beta(t_{i-1})) \quad (13)$$

The term A_2 depends on the nature of the design variable \mathbf{d} . When d_j is a deterministic design variable then A_2 becomes:

$$A_2 = -\frac{1}{\|\nabla_{\mathbf{u}} G(\mathbf{u}^*, t_{i-1})\|} \frac{\partial G(\mathbf{d}, \mathbf{X}, t_{i-1})}{\partial d_j} \quad (14)$$

where $\nabla_{\mathbf{u}} G(\mathbf{u}^*, t_{i-1})$ is the gradient of the limit state in the normal space at the most probable failure point \mathbf{u}^* and at the time t_{i-1} ; $\partial G/\partial d_j$ is the derivative in the physical space of the limit state regarding the design variable d_j . These sensitivities are already computed in the previous time-variant reliability analysis and optimization sub-problem.

However, when d_j is the mean of the random variable X_j then:

$$A_2 = \alpha_j(t_{i-1}) \times \left(\frac{\partial T(x_j)}{\partial d_j} \right) \quad (15)$$

where $\alpha_j(t_{i-1})$ is the direction cosine of the random variable X_j at the time t_{i-1} ($\alpha = \nabla_{\mathbf{u}} G / \|\nabla_{\mathbf{u}} G\|$) and T is the probability transformation, where $x=T(u)$ (e.g. if X_j is normal variable with mean d_j and standard deviation σ_j , then $\partial T(x_j)/\partial d_j = -1/\sigma_j$).

The term A_3 is given as:

$$A_3 = \frac{\partial \Phi_2(\beta(t_{i-1}), -\beta(t_i), \rho)}{\partial \beta(t_{i-1})} \cdot \frac{\partial \beta(t_{i-1})}{\partial \mathbf{d}} + \dots + \frac{\partial \Phi_2(\beta(t_{i-1}), -\beta(t_i), \rho)}{\partial \beta(t_i)} \cdot \frac{\partial \beta(t_i)}{\partial \mathbf{d}} \quad (16)$$

It is reasonable to assume that the correlation coefficient ρ stays constant in the interval $[t_{i-1}, t_i]$.

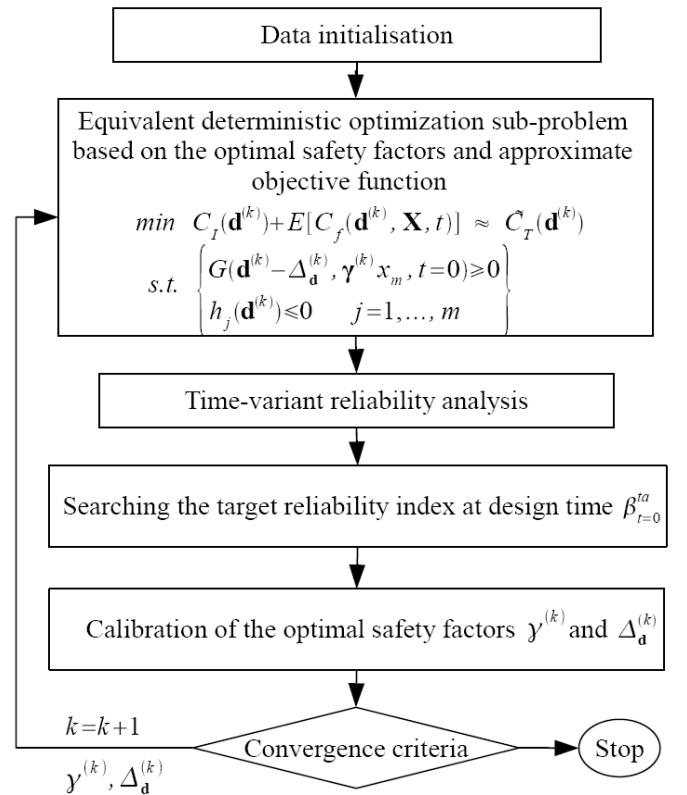


Figure 2. Flowchart of the proposed SOTVRA method.

4.2 Computing the target reliability index $\beta_{t=0}^{ta}$

In practice, the target index at the design time is unknown. The target index at the lifetime T_L is usually

linked to the admissible failure probability P_f^a by FORM approximation:

$$\beta_{T_L}^{ta} = \Phi^{-1}(P_f^a) \quad (17)$$

where Φ^{-1} is the inverse cumulative normal distribution. Generally, P_f^a is given by socio-economical considerations.

The design optimization is searched at the initial time $t = 0$, where the safety factors are calibrated on the basis of the target index at this time $\beta_{t=0}^{ta}$. Figure 3 shows that $\beta_{t=0}^{ta}$ can be deduced if the profile of the structural reliability index is known. This evolution is proposed as following:

$$\beta_{t=0}^{ta} = \beta_{T_L}^{ta} + \sum_{i=1}^{n+1} (\Phi^{-1}(P_{f,c}(t_{i-1})) - \Phi^{-1}(P_{f,c}(t_i))) \quad (18)$$

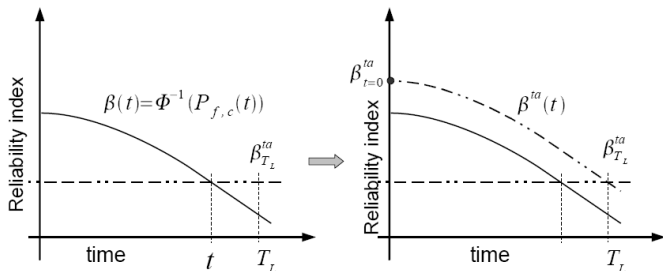


Figure 3. Computation of the target reliability index at design time $\beta_{t=0}^{ta}$.

4.3 Calibration of the safety factors

The safety factors link the reliability requirement to the deterministic optimization sub-problem. In this work, a probabilistic procedure is proposed to calibrate the safety factors on the basis of the target reliability index at the initial time $\beta_{t=0}^{ta}$. In this way, the design space of the next equivalent deterministic optimization sub-problem is adjusted, where the design variables are searched in order to satisfy the reliability requirement.

It is however possible to reach the same target reliability level by infinite ways of setting the safety factors. Nevertheless, only one combination of the safety factors verifies the target safety and minimizes the performance function. Besides, the minimum structural performance that satisfies the target reliability avoids over-designing of structures. In other words, the proposed procedure aims at searching for the safety factors corresponding to the target reliability level $\beta_{t=0}^{ta}$ by minimizing the structural performance function. This procedure is formulated as:

$$\min_{\boldsymbol{\gamma}^{(k)}} G(\mathbf{d}, \boldsymbol{\gamma}^{(k)} x_m, t = 0) \quad (19)$$

$$s.t. \quad \left\| \Phi^{-1}(F_{\mathbf{X}}(\boldsymbol{\gamma}^{(k)} x_m)) \right\| = \beta_{t=0}^{ta}$$

where F is the cumulative distribution of the random variables \mathbf{X} ; $\boldsymbol{\gamma}^{(k)}$ is the vector of safety factors at the k^{th} sub-problem and x_m is the vector of characteristic values of the random variables, which are taken as the median value. When the design variables are the

means of random variables, the shift factors $\Delta_{\mathbf{d}}$ are defined in order to take into account this situation. The vector $\Delta_{\mathbf{d}}$ is given by:

$$\Delta_{\mathbf{d}}^{(k)} = \mathbf{d}^{*(k)} (\boldsymbol{\gamma}_{\mathbf{X}_{\mathbf{d}}}^{(k)} - 1) \quad (20)$$

where $\mathbf{d}^{*(k)}$ is the optimal design point obtained in the k^{th} deterministic design optimization and $\boldsymbol{\gamma}_{\mathbf{X}_{\mathbf{d}}}^{(k)}$ are the optimal safety factors of the random variables $\mathbf{X}_{\mathbf{d}}$ which their means are the design variables \mathbf{d} . The updated safety factors $\boldsymbol{\gamma}^{(k)}$ and $\Delta_{\mathbf{d}}^{(k)}$ are provided to the next deterministic optimization sub-problem.

4.4 Implementation and Convergence criteria

The proposed approach SOTVRA and the classical TV-RBDO method are implemented in a computer program using the MATLAB environment, which contains a very useful optimization toolbox. The SQP (Sequential Quadratic Programming) method is used to solve the optimization problem. The convergence criteria are formulated by the absolute changes in design variables, the relative changes in the objective function and the constraint verification. The tolerance of these termination conditions is fixed to 10^{-3} and the maximum number of iterations is fixed to 100.

5 APPLICATION

Consider the RC girder in Figure 4 with rectangular cross-section, subjected to uniform live load. This application aims at searching for the optimal design for concrete cross-section defined by h and b , steel area A_s and the depth of the reinforcement cover c . The structure is designed by the classical TV-RBDO (Equation 7) and SOTVRA methods. The failure probability of the optimal design should be always less than the admissible probability $P_f^a = 7.23 \times 10^{-5}$ during the whole structure lifetime. This admissible probability corresponds to the target reliability index $\beta_{T_L}^{ta} = 3.8$ and the lifetime T_L of the structure is fixed to 50 years. The annual failure cost is supposed proportional to the initial cost, i.e. $C_f = 100 \times C_I$. Where C_I is the initial cost calculated from the concrete, steel and formwork costs.

$$C_I = b h L_b C_{con} + \rho_s C_{stl} L_s A_s + C_{fw} (2h + b) \quad (21)$$

where C_{con} , C_{stl} and C_{fw} are respectively the unit cost of concrete, steel and formwork taken to 150.5 €/m^3 , 1.46 €/kg and 47 €/m^2 ; $\rho_s = 7850 \text{ kg/m}^3$ is the unit weight of steel; L_b and L_s are respectively the total beam length and the length of steel bars.

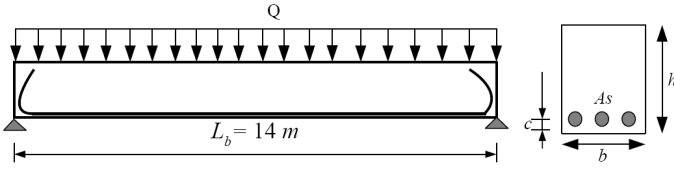


Figure 4. Rectangular RC bridge girder.

The operational life of the RC girder is affected by steel corrosion induced by chloride ingress. In saturated conditions, chloride ingress into concrete matrix can be simplified as a diffusion problem modeled by Frick's second law (Tuutti 1982). The time to corrosion initiation t_{ini} is the time at which a threshold concentration of chlorides that induces corrosion C_{th} reaches the cover depth:

$$t_{ini} = \frac{c^2}{4D_{cl}} \left[\text{erf}^{-1} \left(1 - \frac{C_{th}}{C_s} \right) \right]^2 \quad (22)$$

where D_{cl} is the chloride diffusion coefficient in concrete; $\text{erf}(\cdot)$ is the error function and C_s is the chloride surface concentration. After t_{ini} , the corrosion of reinforcement is controlled by a well-known electrochemical process. Corrosion can be quantified in terms of corrosion current intensity i_{corr} , also called corrosion rate. This parameter measures general or uniform loss of metal and relates corrosion current with section reduction. A corrosion rate of $i_{corr} = 1 \mu\text{A}/\text{cm}^2$, corresponds to a section loss of $11.6 \mu\text{m}/\text{yr}$ (Jones 1992). By assuming uniform corrosion (Figure 5), the diameter reduction $\phi_u(t)$ in mm at time t is computed as:

$$\phi_u(t) = \phi_0 - 0.0232 \int_{t_{ini}}^t i_{corr}(t) dt \quad (23)$$

where ϕ_0 , is the initial diameter of the bar in mm, and $i_{corr}(t)$ must be given in $\mu\text{A}/\text{cm}^2$. Equation 23 is only valid if uniform corrosion is assumed; however, chloride-induced corrosion is typically characterized by highly localized corrosion (i.e. pitting corrosion). According to González et al. (1995), the maximum penetration of pitting corrosion is about four to eight times that of the uniform corrosion. If the ratio between pitting and uniform corrosion depth is related by a factor α , the reduction in diameter at time τ , for pitting corrosion becomes (Figure 5):

$$\phi_p(t) = \phi_0 - 0.0116\alpha \int_{t_{ini}}^t i_{corr}(t) dt \quad (24)$$

Based on the work of González et al. (1995), Stewart (2004) found that for a 125 mm length of reinforcing bar, the parameter α follows a Gumbel distribution with mean of 5.65 and coefficient of variation of 0.22. By using Equations 23-24, it is possible to compute the effective reinforcement area of the bars i.e. $A_s(t) = \pi\phi(t)^2/4$, and subsequently, to evaluate the loss of capacity of a RC beam. Thus, the limit state function can be written:

$$G(\mathbf{d}, \mathbf{X}, t) = M_u(\mathbf{d}, \mathbf{X}, t) - M_{app} \quad (25)$$

where M_{app} is the applied moment and $M_u(\mathbf{d}, \mathbf{X}, t)$ the ultimate moment given by:

$$M_u(t) = A_s(t) f_y (h - c) \left(1 - \frac{0.59 A_s(t) f_y}{b(h - c) f_c} \right) \quad (26)$$

where f_c and f_y are respectively the concrete compressive strength and the steel yield stress, assumed as random variables. The statistical data of the random variables are detailed in Table 1. The applied moment is considered as Gaussian process with mean of 882 kN.m and coefficient of variation of 0.25. The autocorrelation function of this random process is assumed as:

$$\rho_{SS}(t_1, t_2) = \exp \left[- \left(\frac{t_1 - t_2}{L_c} \right)^2 \right] \quad (27)$$

where L_c is the length correlation fixed to 1 year. The time-variant reliability analysis is carried out by using a step length of 0.1 years. Figure 6 shows some trajectories of the random process M_{app} generated by using 10 terms of the the Karhunen-Loève expansion.

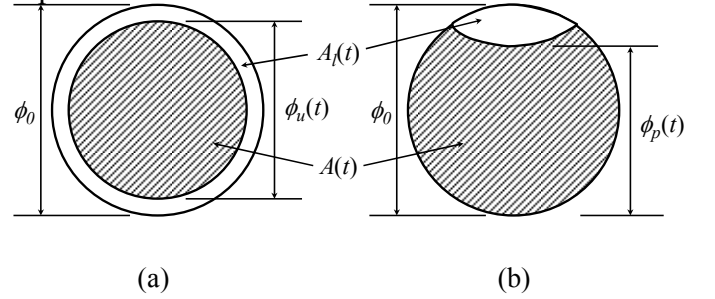


Figure 5. Diameter of bar as function of time (a) uniform corrosion (b) pitting corrosion.

Table 1. Statistical data of random variables (Bastidas et al. 2007).

Variables	Mean	C.V	Distribution
f_c (MPa)	30	0.15	Lognormal
f_y (MPa)	500	0.10	Lognormal
C_{th}	0.37	0.11	Lognormal
C_s	2	0.30	Lognormal
D_{cl} cm ² /s	1.62×10^{-8}	0.31	Lognormal
i_{corr} $\mu\text{A}/\text{cm}^2$	2	0.30	Lognormal
α	5.65	0.22	Gumbel

The design variables h , b , A_s and c are the means of normal random variables with coefficient of variation of 0.03. The initial design point, the lower and the upper bounds of design variables are detailed in Table 2.

Table 2. Initial design values, lower and upper bounds.

Variables	Initial value	Lower bound	Upper bound
h (cm)	80	20	120
b (cm)	60	20	120
A_s (cm ²)	50.27	12.57	251.1
C (cm)	4	3	5

Table 3 shows that both classical TV-RBDO and SOTVRA methods lead nearly to the same expected total cost. It can also be observed that the initial cost C_I and the expected failure cost C_F are almost the same for both TV-RBDO methods. These results show the validity of the approximation introduced in the SOTVRA to express the objective function. From the numerical point of view, the efficiency of SOTVRA is observed, as it requires only 21s CPU compared to 603s CPU for the classical TV-RBDO approach; i.e the computation time is divided by a factor of 28. In other words, the SOTVRA economises 96% of the computational effort. This reduction is extremely important in highly consuming finite element analyses of large-scale structures. The proposed approach requires less consumption time because it is based on the deterministic optimization procedure, already known to be numerically efficient and robust.

Table 3. Comparison of the optimal costs of the RC girder and numerical performances.

	TV-RBDO	SOTVRA
Expected total cost C_T (k€)	3.69	3.70
Initial cost C_I (k€)	3.67	3.68
Expected failure cost C_F (€)	16.57	16.82
CPU (s)	603	21

Table 4 indicates the optimal design obtained by both TV-RBDO approaches. The two design solutions are different. The classical TV-RBDO tends to reduce the concrete cross-section and increases the steel area. However, the SOTVRA gives other solution, where the steel area is reduced and the concrete cross-section is increased. The two solutions lead approximately towards the same expected total cost. Figure 7 shows that for both optimal solutions the reliability requirement is satisfied, where the reliability index is always higher than the target index of 3.8 for all the structural lifetime of 50 years.

Table 4. Comparison of the optimal design of the RC girder.

Optimal design var.	TV-RBDO	SOTVRA
h (cm)	66.1	68.4
b (cm)	49.5	51.3
A_s (cm ²)	91.4	85.7
c (cm)	3	3

Table 5 gives the optimal safety factors at the optimal solution. Figure 8 shows that the profile of the target index $\beta^{ta}(t)$ is deduced on the basis of the structural reliability index $\beta(t)$. The target index at the initial time $\beta^{ta}_{t=0}$ is improved at each sub-problem of SOTVRA, and after fourth cycles the reliability index $\beta(t)$ met the target index $\beta^{ta}(t)$ as shown in Figure 8. The optimal safety factors are calibrated on the basis of $\beta^{ta}_{t=0} = 4.4$.

It is important to notify that the random variables C_{th} , C_s , D_{cl} , i_{corr} and α play an important role in the time-variant reliability analysis only. The calibration

procedure does not consider these variables because the calibration is performed at the initial time $t = 0$. Furthermore, these random variables are not affected by the safety factors.

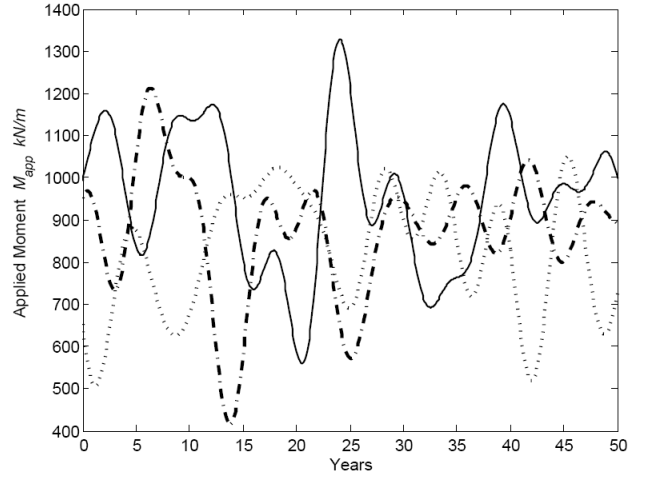


Figure 6. Random process trajectories.

Table 5. Optimal safety factors of SOTVRA

	M_{app}	f_c	f_y	Δh	Δb	ΔA_s	Δc
γ_X	1.91	1.19	1.23	2.38	0.35	1.62	-0.004

When the failure consequences are more important, the admissible failure probability becomes lower. The RC girder is designed when the annual failure cost is taken as $C_f = 10^4 \times C_I$ instead of $100 \times C_I$. Thus, the admissible failure probability is fixed to $P^a_f = 10^{-5}$ corresponding to the target reliability $\beta^{ta}_{T_L} = 4.27$. The optimal design obtained by SOTVRA for this situation is given in Table 6.

The new design of the RC girder is increased only from 4.3% with respect to the case when $C_f = 100 \times C_I$. The optimal safety factors for the situation of $C_f = 10^4 \times C_I$ are presented in Table 7. These safety factors are calibrated from the target index at the initial time $\beta^{ta}_{t=0} = 4.9$.

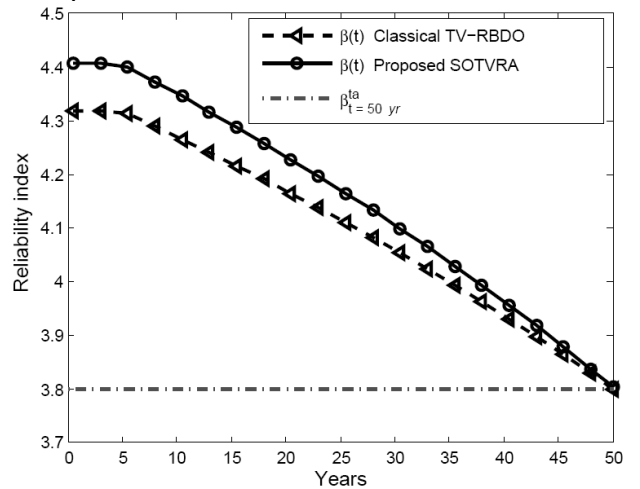


Figure 7. Time-variant reliability index for both optimal designs.

Table 6. Optimal design of the RC girder for $C_f=10^4 \times C_I$.

C_T (k€)	C_I (k€)	C_F (€)	h (cm)	b (cm)	A (cm ²)	c (cm)
4.08	3.83	248.34	70.2	52.7	89.92	3

Table 7. Optimal safety factors of SOTVRA for $C_F=10^4 \times C_I$.

	M_{app}	f_c	f_y	Δh	Δb	ΔA_s	Δc
γ_X	2.00	1.22	1.26	2.79	0.4	1.94	0.01

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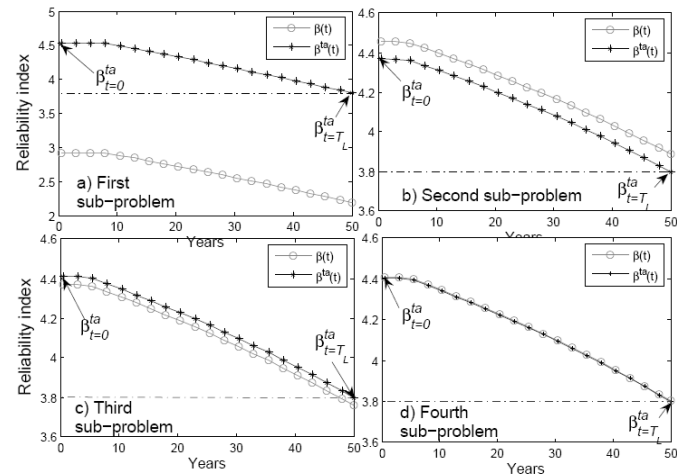


Figure 8. Time-variant reliability index and target index profiles at the first four sub-problems of SOTVRA.

6 CONCLUSION

This paper proposes a decoupled approach for TV-RBDO called SOTVRA. The proposed approach consists in transforming the TV-RBDO problem into a sequence of deterministic design optimization sub-problems based on the safety factor concept. The objective function of the TV-RBDO problem is approximated at the current design point by first-order Taylor series expansion and the reliability constraints are fulfilled by optimally calibrated safety factors. The deterministic optimization is then carried out by using the approximated objective function and reliability constraints including the safety factors. Thus, the proposed method does not involve any additional function evaluations. The time-variant reliability analysis and the optimization procedures are performed sequentially until convergence.

The accuracy and the efficiency of the proposed method are shown for RC girder affected by steel corrosion induced by chloride ingress. The results indicate that SOTVRA is very attractive for the following reasons: (1) its simple implementation in any general purpose optimization and finite element software; (2) the use of the safety factor concept is familiar for designers; and (3) the less consumption time allows to apply SOTVRA to large-scale structures. The robustness and the efficiency of SOTVRA lie in the fact that the time-variant reliability analysis is decoupled from the optimization process. Finally, the proposed method can be easily applied to any kind of structural durability design optimization without any loss of generality.