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## **NUMERICAL MODELING OF MATERIAL FLOW WITH AN INTERNAL LENGTH**

F. Dufour <sup>1</sup> and G. Pijaudier-Cabot <sup>2</sup>

### **ABSTRACT**

In this paper we propose a new numerical method to study material flow with an internal length. The finite element method with Lagrangian interpolation points (FEMLIP) is used for its capability to model large deformation processes for any type of materials. It is widely accepted in the literature that fresh concrete rheology is very well estimated by Bingham model in the classical continuum approach. Nevertheless in most cases the form size and the distance between reinforced bars are about few times greater than aggregate size itself. This means that the microstructure has a non negligible effect on the flow (aggregate rotation independent of material rotation + boundary layer effect) and that it must be taken into account to get a good estimation of the filling capability of the material. We propose to use a Cosserat continuum with a Bingham law to tackle this work.

**Keywords:** Cosserat model, Bingham rheology, fresh concrete flow.

### **INTRODUCTION**

The objectives of this work are to model heterogeneous two-phase fluid like fresh concrete flow during filling form processes with a particular concern about aggregate segregation and/or locking effect around reinforced bars. The aggregate mean size is only few times smaller than the distance between the bars or between the bars and the mould. Therefore, microstructure must be taken into account to get a good estimation of the concrete flow and boundary conditions must be addressed carefully.

We already have modelled (Dufour, 2002) concrete flow either with an homogeneous or an heterogeneous approach within the classical continuum framework. In the first approach, informations about material microstructure are clearly missing (e.g. material locking between bars cannot be modelled) whereas the second approach is very much demanding on computer resources and cannot be used for large scale applications.

Therefore, we propose in this work to combine the Bingham model with a Cosserat continuum in order to enhance microstructure effects into the model without having to use very fine and numerically costly meshes. We wish to avoid as much as possible heterogeneous modelling for large scale problems because of memory and computational time limits. Along these lines, heterogeneous modelling will be used only for validations purposes.

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## NUMERICAL METHOD

To tackle these objectives, we use a finite element method with Lagrangian interpolation points (FEMLIP) implemented in our *ellipsis* code (Moresi et al., 2002) where the problem domain is represented by an Eulerian mesh with an embedded set of Lagrangian integration points or particles. Unknowns (i.e. velocity field) are computed at the mesh nodes and the Lagrangian particles carry history variables during the large deformation process including material properties. This method is ideally suited to model fluid-like behavior of continuum solids like two-phase fluid. From the material properties point of view, unlike in a pure Eulerian method, material interfaces do not show convection during computations as fictitious lines representing interfaces are preserved during flow between two adjacent particles with different properties. From the computational point of view, interfaces are taken into account in one finite element for which rheological properties are averaged for integration. The consequence is that the modelling of heterogeneous material needs very fine meshes with the FEMLIP same as for any numerical method based on integration over volumes.

## RHEOLOGICAL LAWS

Concrete, in its fresh state, can be considered as a homogeneous fluid, provided that a certain degree of flow can be achieved and that the mix is quasi-homogeneous with regularly dispersed aggregate particles (in space and size). Modelling concrete rheology is a very difficult task as every concrete behaves differently depending on their composition and mixing method. Therefore it is impossible to write a relation between the stress and the strain rate based on every single parameter at the microscale, quantities and nature of each admixtures in particular, which are nowadays more and more numerous. Here, the concrete rheology is thought at a mesoscale and it is written as a relation between the shear stress  $\tau$  and the shear strain rate  $\dot{\gamma}$ . A complete review of rheological models for cement based materials has been done by Yahia and Khayat (2003). We discuss here only the three simplest ones, namely Newton's, Bingham's and Herschel-Bulkley's models.

The simplest model (one parameter) assumes that concrete is a Newtonian fluid:

$$\tau = \eta \dot{\gamma} \quad (1)$$

where  $\eta$  is the viscosity but Tattersall (1973) has shown that a single parameter is not enough to model concrete flow. At least two parameters are needed to model concrete with a slump lower than 30 cm. The first one needs to be related to the static behaviour (slump test) and the second one to the dynamic fluid response (rheometer test). Hence, a stress threshold has been added to the Newton's model to give the Bingham's model:

$$\begin{aligned} \tau &= \tau_0 + \eta \dot{\gamma} & \text{if } \tau \geq \tau_0 \\ \dot{\gamma} &= 0 & \text{if } \tau \leq \tau_0 \end{aligned} \quad (2)$$

where  $\tau_0$  is the yield stress and  $\eta$  is the plastic viscosity.

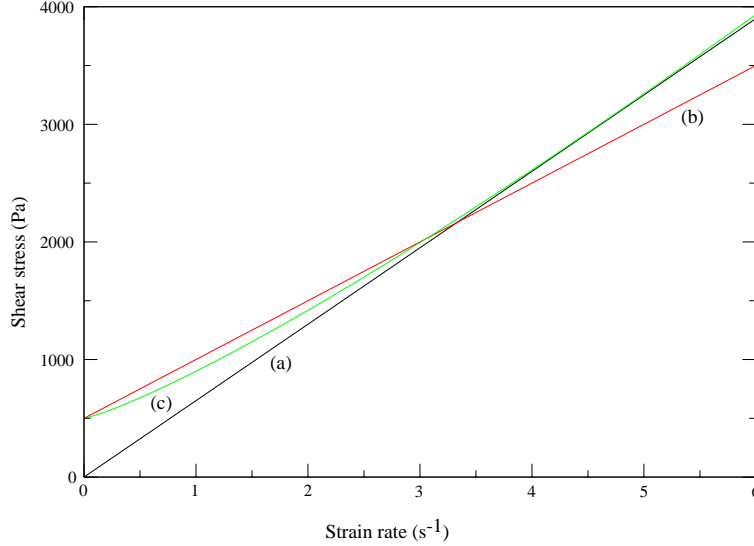
When experimental data are fitted for many different concretes and for self-compacting concrete especially, it often yields a negative value of the yield stress although very small. For this reason de Larrard et al. (1998) preferred to use a Herschel-Bulkley's model (3 parameters):

$$\tau = \tau_0' + a \dot{\gamma}^b \quad (3)$$

Ferraris and de Larrard (1998) have found a mean value for  $b$  of 1.53 for concrete without superplasticizer and 1.36 for others. However the Bingham's model (e.g.  $b = 1$ ) is most widely

used (Tsong et al., 1999) because it needs less experimental tests to be calibrated. For the same reason, we have chosen to use the Bingham's model in this study.

One example for each rheological model is plotted in Figure 1.



**FIG. 1. Different rheological models for fluid-like materials. (a) Newton ( $\eta = 650 \text{ Pa}\cdot\text{s}$ ), (b) Bingham ( $\tau_0 = 500 \text{ Pa}$ ,  $\eta = 500 \text{ Pa}\cdot\text{s}$ ) and (c) Herschel-Bulkley ( $\tau_0' = 400 \text{ Pa}$ ,  $a = 500 \text{ Pa}\cdot\text{s}^b$  and  $b = 1.2$ ).**

For an easier numerical implementation, the Bingham's model needs to be smoothed (Panastasiou, 1987) in order to avoid the corner at  $\tau = \tau_0$  on the material response and the use of two equations under and above the shear stress threshold. This yields:

$$\eta = \eta_0 + \frac{\tau_0}{\dot{\gamma}} (1 - e^{-m\dot{\gamma}}) \quad (4)$$

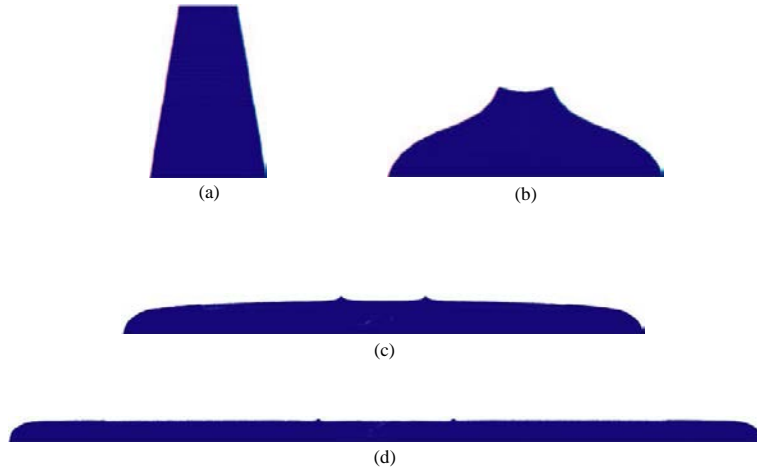
where  $m$  is the stress growth rate parameter. The higher  $m$ , the more accurate the approximation. We found out, for the range of strain rates reached during our numerical tests, that a value  $m=5000$  is high enough as we observed no differences with results obtained with a value of  $m=10000$ .

(Dufour and Pijaudier-Cabot, 2004) have already shown the ability of this rheological law together with the FEMLIP to model concrete flow during slump test (Fig.2) and Lbox filling (3).

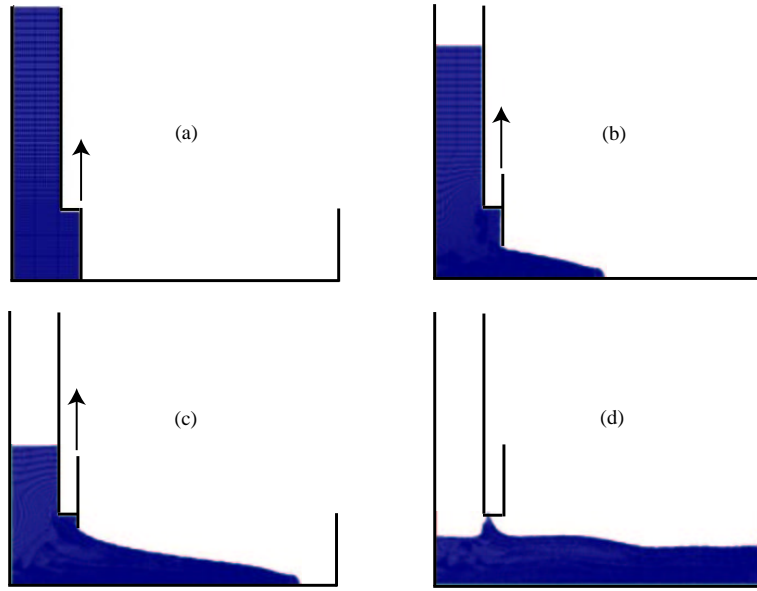
The use of Bingham's law in for a classical continuum needs two different types of experimental tests. The first one, the slump test, to identify the yield stress and the second one, a rheometer test, to identify the plastic viscosity. For the rheometer test we have chosen to measure the flow time of fresh concrete in a funnel. The reason is that it can be easily modelled by *ellipsis* as the flow is nearly 2D.

## ENHANCED MODEL

In a 2D conventional continuum the deviatoric part of an isotropic material is characterised by a shear viscosity  $\eta$ . For a Cosserat continuum we also have a Cosserat shear viscosity  $\eta^c$  and a bending viscosity  $M$ . The constitutive relation for a generalised Newtonian fluid can be



**FIG. 2.** Different snapshots of slump test on SCC at time 0 s. (a), 0.36 s. (b), 1.87 s. (c) and 7.24 s. (d).



**FIG. 3.** Different snapshots of the Lbox test on SCC at time 0 s. (a), 0.45 s. (b), 0.85 s. (c) and 7.63 s. (d).

written in the usual pseudo-vector form:

$$\boldsymbol{\tau} = \boldsymbol{\Lambda} \boldsymbol{D} \quad (5)$$

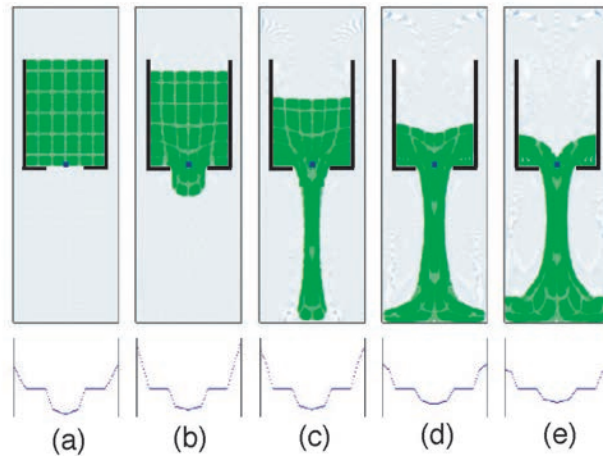
where  $\boldsymbol{\tau}$  is the deviatoric stress vector and the matrix  $\boldsymbol{\Lambda}$  is expressed as:

$$\begin{pmatrix} \eta & \eta & 0 & 0 & 0 & 0 \\ & \eta & 0 & 0 & 0 & 0 \\ & & \eta + \eta^c & \eta - \eta^c & 0 & 0 \\ & & & \eta + \eta^c & 0 & 0 \\ \text{symm.} & & & & M & 0 \\ & & & & & M \end{pmatrix} \quad (6)$$

It means that when the Bingham's law for a Cosserat continuum is implemented two additional parameters are required. Choi and Mühlhaus (1991) have proposed relations between bending viscosity and viscosity  $\eta$  and internal length  $d$  such as:

$$M = 2\eta d \quad (7)$$

If we assume that the internal length of such a material is the granular size, then only one more parameter needs to be identified. One example of behaviour of this Cosserat continuum is studied for Newtonian material. We consider the discharge of a Cosserat material from a model silo (Fig.4). In this case we are interested in the influence of the internal length parameter on the discharge velocity (Fig.5) of a Cosserat fluid as described by (5–6)

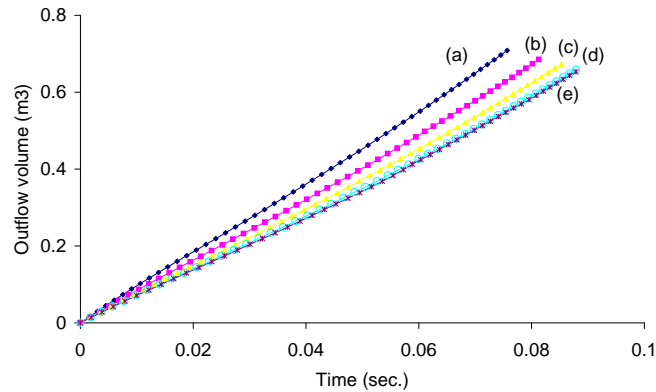


**FIG. 4. Snapshots of the model. (a) Initial conditions, (b)  $t = 4.75 \cdot 10^{-4}$  sec., (c)  $t = 1.42 \cdot 10^{-3}$  sec., (d)  $t = 2.85 \cdot 10^{-3}$  sec. and (e)  $t = 3.8 \cdot 10^{-3}$  sec.. The plot below each picture corresponds to the vertical velocity profile throughout the silo aperture.**

In Fig. 5 we plot the volume flowing out the reservoir versus time and for different values of the ratio  $\alpha = d/a$  where  $a$  is the silo aperture and  $d$  is the internal length of the material. The flow rate is almost identical for all values of  $\alpha$  (Fig. 5-d and 5-e) larger than the aperture (1.0) which corresponds, in the elastic case, to a situation in which no flow can occur. Note that in purely viscous materials static equilibrium states do not exist, ensuring that stable arches do not form. For  $0 \leq \alpha \leq 1.0$ , the smaller the internal length the faster the outflow. Indeed Dufour et al. (2001) have observed the same effects in the case of an infinite shearlayer. The first effect is a boundary layer effect along the boundaries. Here is a possibility to force the material rotation to zero for non spherical aggregates which can slide without rotating. The second effect is that for the same body force, a Cosserat fluid will flow slower than a classical one because energy is stored within the material due to bending stiffness. Frictional contacts between aggregates are modeled by Cosserat couple stresses.

### BINGHAM'S FLOW WITHIN A COSSERAT CONTINUUM

To get the microstructure description of a Cosserat model together with the Bingham rheology we propose to model Bingham's flow within a Cosserat continuum. In the Cosserat constitutive matrix the viscosity  $\eta$  can be identified to the Bingham's viscosity,  $\eta^c$  and  $M$



**FIG. 5. Outgoing volume for (a)  $\alpha=0$ , (b)  $\alpha=1/3$ , (c)  $\alpha=2/3$ , (d)  $\alpha=5/3$  and (e)  $\alpha=10/3$**

needs to be identified. The first term is a constant fraction of the Bingham's viscosity and the second one is calculated as a function of the internal length taken equal to the granular size  $D_{50}$ . The coupling of the Bingham's law with a Cosserat model allows the use of coarser mesh with a homogeneous fluid approach compared to the modelling of heterogeneous fluid. The microstructure effect is brought into equations instead of the geometry. This coupling will be used in the future whenever boundary layer effects are of importance in the concrete flow for instance in the Lbox problem or in order to study concrete flow in the casting of densely reinforced structural elements.

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