Matching permeability law from diffuse damage to discontinuous crack opening

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ABSTRACT: Experimental tests prove strong interaction between mechanical state and transport properties of concrete. In order to describe such interaction, relations either between damage (related to diffuse microcracking) and permeability or between macrocrack opening and permeability are applied. However none of these relations based on continuum and discontinuum approaches of concrete behaviour modelling, are able to represent by a unique relation the permeability evolution throughout the fracture process. Therefore, we propose to define a new relation matching the existing ones in the limit cases. At first, by means of a simple analytical approach we relate the crack opening with the state variable of the non local damage model. Then, using this variable substitution, the relation of permeability evolution with crack opening is reported to damage variable. Both relations of the evolution of permeability are thus reported to damage and can be weighted by the damage variable to form a single relation between permeability and damage presumed to be valid for diffuse damage state and discontinuous macrocracking. In order to emphasize influence of the matching law on structural response, we run finite element simulations of a Brazilian splitting test. In conclusion, numerical tendencies are similar to observed ones on experimental results.

1 INTRODUCTION

Transport properties of concrete, like permeability or diffusivity, are particularly important in case of structures with a tightness role, for instance containment vessels in French nuclear power plants. Evaluation of their gas tightness is critical during their service life, where concrete remains at most microcracked, but also during minor accidents, when macrocracks may appear locally. Results of experimental tests (Choinska *et al.*, 2007), performed on hollow concrete cylinders subjected to compressive loading, emphasize gas permeability increase with axial strain, ranging from 0 to 3 times the strain at the peak load. Furthermore, three regimes of permeability evolution, related to the mechanical response of concrete yielding different cracking regimes (microcracking or macrocracking), are observed.

The first regime, standing up to strain localization limit (just before the peak in simple compression test) and exhibiting relatively slight permeability increase, is probably due to the presence of microcracks homogenously spread out in material. Relationships between permeability and diffuse microcracking, described by damage variable, have already been derived theoretically (Dormieux & Kondo, 2004 and Chatzigeorgiou *et al.*, 2005) and investigated experimentally (Picandet *et al.*, 2001). The phenomenological models are most often exponential or power type with validity up to moderate diffuse damage. However, due to strain localization (transition between diffuse damage and crack opening), a second regime with rapid permeability increase is observed experimentally. Finally, the third regime, characterized by a slower rate of permeability evolution versus strain in comparison with the previous one, appears. Here, due to the formation of macrocracks, permeability becomes governed at the macrostructural level by Poiseuille's law and depends essentially on macrocrack(s) opening.

Finally, one may consider that damage variable is an appropriate parameter to model permeability evolution of microcracked material and crack opening is a relevant parameter to model permeability change of a macrocracked element. Existence of one and only one parameter governing permeability change from diffuse damage to discontinuous macrocrack opening is thus questionable. Therefore, in order to describe permeability evolution throughout the concrete fracture process by a unique relation, we propose to define a matching law between existing relations of permeability evolution with diffuse damage and with crack opening.

Analytical variables substitution, used in order to associate crack opening with state variable (regularized strain) and in turn with damage, is shown in

Section 2. In Section 3, concept and basic features of the proposed matching permeability law are presented. Details of numerical finite element simulations, run on a Brazilian splitting test using a nonlocal damage model and the proposed matching permeability law, are shown in Section 4. Finally, the concluding remarks are made in Section 5.

2 ANALYTICAL VARIABLES SUBSTITUTION

2.1 *Problem statement*

Fracture and damage mechanics are two correlated theories (Mazars & Pijaudier-Cabot 1996, Dufour at al. 2007). The bridge between fracture and damage mechanics can be considered to be the situation where damage is equal to one at a material point or in a small region defining the size of an initial flaw in the theories of fracture. Therefore, we may suppose equivalence at failure between continuous damaged domain and discrete macrocracked domain. This assumption is followed in order to substitute crack opening by damage field.

At first, nonlocal damage model is reviewed. Then, equivalence at failure between damage field and crack opening is analysed.

2.2 *Nonlocal damage model*

In our contribution, the scalar damage model developed by Mazars (1984), enriched by a nonlocal approach to strain softening has been chosen. In Mazars' model damage is assumed to be isotropic and it is held that it produces a degradation of the elastic stiffness of the material through a variation of the Young's modulus:

$$\sigma = (1 - D)C\varepsilon \tag{1}$$

where σ and ε are the Cauchy stress tensor and the strain tensor respectively. C is the fourth order tensor of elastic moduli. The damage variable D ranges from 0 for virgin material to 1 for completely damaged material with a zero stiffness and depends on state variable Y:

$$D = F(Y) \tag{2}$$

The state variable Y reaches a maximal value during loading history between the damage threshold Y_{D0} and the equivalent strain ε_{eq}:

$$Y = \max_{/t}(\varepsilon_{eq}, Y_{D0}) \tag{3}$$

The equivalent strain ε_{eq} is defined as follows (Mazars 1984):

$$\varepsilon_{eq} = \sqrt{\sum_{i=1}^{3}\left(\langle\varepsilon_i\rangle_+\right)^2} \tag{4}$$

Damage follows a damage evolution law which distinguishes tensile damage D_t and compressive damage D_c (Mazars 1984):

$$D = \alpha_t D_t + \alpha_c D_c \tag{5}$$

where α_t and α_c are the weights computed from the strain tensor. However, in the case of mode I loading (see Figure 1 (a)), α_t=1 and α_c=0. Therefore, the damage model used hereafter is based on the following evolution law of damage:

$$D = D_t = 1 - \frac{Y_{D0}(1 - A_t)}{Y} - \frac{A_t}{e^{[B_t(Y - Y_{D0})]}} \tag{6}$$

where A_t and B_t are the model parameters.

In this formulation damage depends only on the strain state at a point under consideration. Hence, this local formulation exhibits spurious strain localization (like any strain softening local formulation). Consequently, numerical simulations yield a pathological mesh dependency and physically unrealistic results are obtained (Bazant & Planas 1998). Nevertheless, different techniques exist to spread localized strain.

One possible remedy consists in reformulating the constitutive model in a nonlocal approach, with damage parameter at a material point depending on the strain not only at this point, but also in its neighbourhood determined by the interaction radius dependent on the material heterogeneity parameter. This nonlocal model, called integral, was developed by Pijaudier-Cabot & Bazant (1987). In this model, a nonlocal equivalent strain is the weighted average of the local strains over the representative volume Ω surrounding each point x in the material:

$$\bar{\varepsilon}_{eq}(x) = \frac{\int_{\Omega} \phi(x - s)\varepsilon_{eq}(s)d\Omega}{\int_{\Omega} \phi(x - s)d\Omega} \tag{7}$$

$\phi(x - s)$ is the classic weight function defined as:

$$\phi(x - s) = e^{-\left(\frac{2\|x - s\|}{l_c}\right)^2} \tag{8}$$

where l_c, the internal length of the material, is related to the material heterogeneity parameter quantifying the non local interactions (Bazant & Oh 1983).

Another possibility is the use of higher order gradient models in which strain derivatives are incorporated in the description of damage evolution. In gradient models, established by expansion of the integral relation into Taylor series which yields gradient terms (Bazant *et al.* 1984), a nonlocal equivalent strain may be determined as follows (de Borst *et al.* 1995):

$$\varepsilon_{eq} = \overline{\varepsilon}_{eq} - c\nabla^2 \overline{\varepsilon}_{eq} \quad (9)$$

where the parameter c is of the dimension of a square length, so that $\sqrt{c}$ can be regarded as the characteristic length of the model, related to the material heterogeneity parameter.

Finally, in nonlocal models damage becomes governed by a nonlocal state variable $\overline{Y}$ which follows Equation (3) in its nonlocal form:

$$\overline{Y} = \max_{/t}(\overline{\varepsilon}_{eq}, Y_{D0}) \quad (10)$$

$$D = F(\overline{Y}) \quad (11)$$

Integral and gradient models are strictly equivalent in the case of the infinite continuum and for a specific weight function (Peerlings 1999). Gradient model can thus be regarded as a particular case of integral model. Furthermore, numerical tests (Jason 2004) permit to establish a relation between the regularization parameters of an integral and a gradient Mazars' damage model:

$$\sqrt{c} \approx \frac{l_c}{4} \quad (12)$$

The proposed analytical approach to substitute variables is thus based on an integral model. The gradient model is used in further finite element simulations and permits to achieve mesh objective damage distributions.

2.3 *Equivalence at failure between crack opening and damage field*

In order to substitute crack opening by damage field, we suppose equivalence between two domains: a cracked one with a discontinuous crack opening $[u]$ (see Figure 1 (a)) and a damaged one with a damaged zone of width λl_c (see Figure 1 (b)).

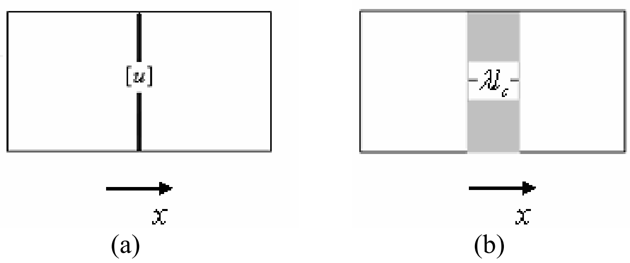


Figure 1. (a) Cracked domain (discrete case) (b) Damaged domain (continuous case).

Comparison of both domains is carried out at failure under the hypothesis of a uniform damage distribution in the damaged zone where $D = 1$.

Subsequently, we should calculate the crack opening in the continuous case. For this purpose, we suggest first to associate the crack opening with the state variable governing damage (see Equation (10)). Use of this variable, irreversible, permits to keep irreversible the relations between damage or crack opening and permeability. We propose thus to calculate the crack opening as:

$$[u] = \int_0^{\lambda l_c} (\overline{Y} - Y_{D0})dx \quad (13)$$

As the damage distribution in the damaged zone is uniform, one obtains the crack opening as:

$$[u] = (\overline{Y} - Y_{D0})\lambda l_c \quad (14)$$

To substitute this crack opening with a damage field, we relate the state variable with the damage through an inverse damage evolution law (see Equation (11)):

$$\overline{Y} = F^{-1}(D) \quad (15)$$

Finally, by substitution of Equation (15) with Equation (14), the crack opening may be represented as a function of the damage:

$$[u] = (F^{-1}(D) - Y_{D0})\lambda l_c \quad (16)$$

The hypotheses, on which computation of the crack opening is founded, are disputable. This approach may be improved by considering variation of the state variable in the damage band or by application of the method presented by Dufour *et al.* (2007). Nevertheless, this simplified approach is retained in further analysis.

3 MATCHING PERMEABILITY LAW

3.1 *Diffused damage – permeability interaction*

In order to represent interaction between diffused damage and permeability at material level, the phenomenological relation established by Picandet *et al.* (2001) for damage lower than 0.15 is retained in the further study:

$$k_D = k_0 \exp[(\alpha D)^\beta] \quad (17)$$

where k_D and k_0 are respectively the current and the initial material permeability. α and β are the parameters fitted by the author to 11.3 et 1.64 on experimental results of axial compressive damage on gas permeability of ordinary and high-performance concrete.

3.2 *Crack opening – permeability interaction*

At rupture, Poiseuille's law, applied to fluid flow between two plane and parallel plates distant of $[u]$, may be used to describe permeability evolution with macrocrack opening. This "crack permeability", called k_f, is therefore given by:

$$k_f = \frac{[u]^2}{12} \quad (18)$$

Crack opening represents amplitude of the discontinuity which appears within the material when it is completely degraded locally, therefore when its damage is close to 1. Consequently, for high damage value, we should find the permeability given by Poiseuille's law. Using Equation (16), this permeability is as follows:

$$k_f = \frac{(\lambda l_c)^2}{12}\left(F^{-1}(D) - Y_{D0}\right)^2 \quad (19)$$

3.3 *Matching law definition*

As two permeability evolution laws, k_D (Equation (17)) and k_f (Equation (19)), have already been formulated using the damage variable only, we propose now to associate them by means of a simple matching law which tends for a moderate damage (diffuse microcracking) to Picandet's permeability, and for a strong damage (macrocrack) to Poiseuille's permeability:

$$k = (1-D)k_D + Dk_f \quad (20)$$

An alternative for this matching law, based on logarithm of the permeability, may be used:

$$\log(k) = (1-D)\log(k_D) + D\log(k_f) \quad (21)$$

Besides, there is an obstacle concerning application of Picandet's exponential relation (Equation (17)) in the proposed matching laws (Equations (20) and (21)). Indeed, Picandet's relation, valid for diffuse and moderate damage ranging between 0 and 0.15, quickly tends towards infinite values when damage increases. To avoid it, we propose to introduce another function: simple, of power type, without any threshold, equivalent to Picandet's exponential relation for damage ranging between 0 and 0.15, and which, at the same time, does not quickly tend towards infinity. Thus, one may propose the following relation:

$$k_D{}^P = k_0\left[1 + (\alpha D)^\beta + \frac{(\alpha D)^{2\beta}}{2} + \frac{(\alpha D)^{3\beta}}{6}\right] \quad (22)$$

In the next paragraph, contribution of two alternatives matching laws, given by Equations (20) and (21), will be presented and commented, as well as influence of Picandet's exponential relation (Equation (17)) and the modified one (Equation (22)).

Finally, we propose to analyse four matching laws:

- Matching law 1 :

$$k = (1-D)k_D + Dk_f \quad (23)$$

- Matching law 2 :

$$\log(k) = (1-D)\log(k_D) + D\log(k_f) \quad (24)$$

- Matching law 3 :

$$k = (1-D)k_D{}^P + Dk_f \quad (25)$$

- Matching law 4 :

$$\log(k) = (1-D)\log(k_D{}^P) + D\log(k_f) \quad (26)$$

3.4 *Matching law properties*

To represent the proposed matching laws, let us consider a strongly damaged material with a behavior following Mazars' damage model. The model parameters are given in Table 1.

E GPa	ν	Y_{D0}	A_t	B_t
37.7	0.2	1.10^{-4}	1.0	15600

Table 1. Parameters of Mazars' damage model.

The internal length l_c is arbitrary chosen equal to 0.02m, while the parameter λ, which influences the width of a damaged band, is chosen equal to 2. Initial permeability k_0 considered in simulations is taken equal to $10^{-17}\,m^2$.

Considering the parameters of Mazars' damage model (see Table 1), the inverse damage evolution law, based on Equation (6), is used in order to represent the permeability k_f (see Equation (19)):

$$F^{-1}(D) = Y_{D0} - \frac{\ln(1-D)}{B_t} \quad (27)$$

Permeability evolutions, according to the proposed matching laws (Equations (23)-(26)), with damage and with state variable are shown in Figure 2 and Figure 3 respectively.

We observe that independently on the type of the matching law used (Equations (23) and (24)) application of Picandet's exponential relation (Equation (17)) leads systematically to an overestimation of the permeability (for damages higher than 0.15) in comparison with the permeability given by Poiseuille's law (Equation (19)).

However, concerning the modified Picandet's relation (Equation (22)), two types of the matching law (Equations (25) and (26)) give similar results in terms of orders of magnitude of permeability evolution. Nevertheless, a problem arises in the case of the matching law given by Equation (25), for which permeability evolution is not adequately reproduced for small damage lower than 0.3.

Notwithstanding, the matching law given by Equation (26), represents correctly the permeability for damage ranging between 0 and 0.15, as well as for strong damage where it tends towards the permeability given by Poiseuille's law. Therefore, we propose to retain this matching law for which permeability evolution is shown in Figures 4 and 5.

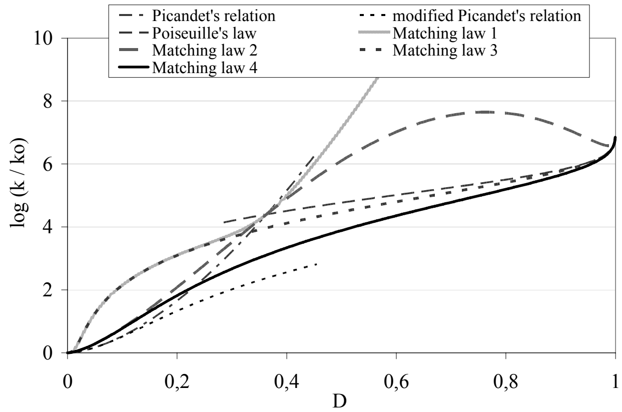


Figure 2. Logarithm of permeability evolution with damage.

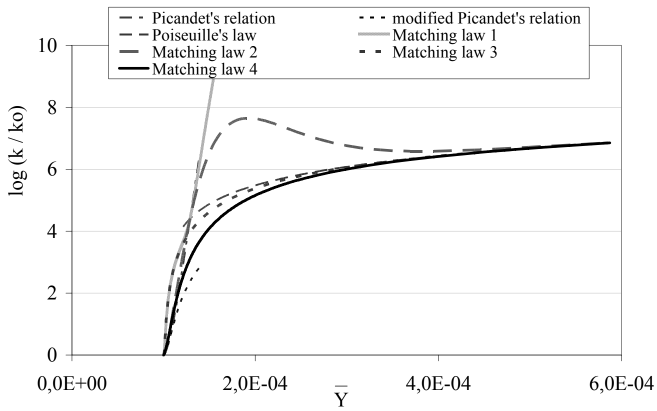


Figure 3. Logarithm of permeability evolution with state variable.

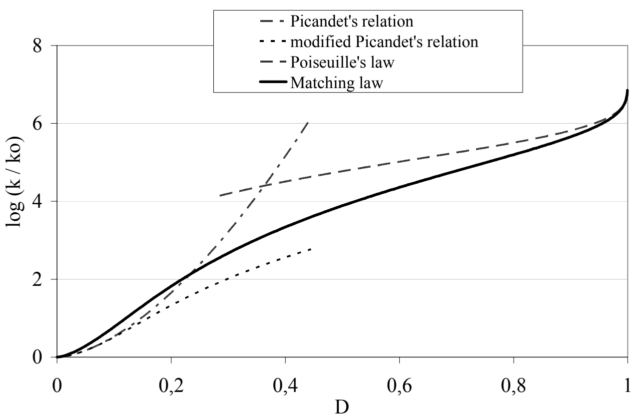


Figure 4. Logarithm of permeability evolution with damage for the retained matching permeability law.

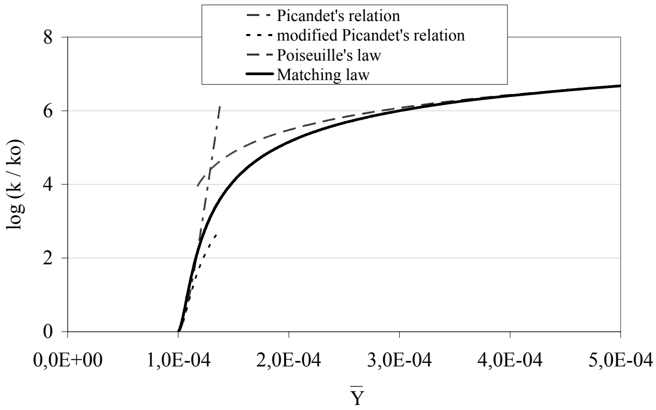


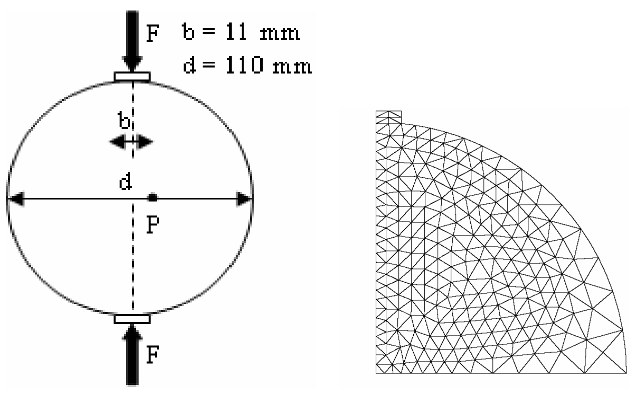
Figure 5. Logarithm of permeability evolution with state variable for the retained matching permeability law.

4 NUMERICAL SIMULATIONS

In order to emphasize influence of the retained local matching permeability law on structural response, we propose to run finite element simulations of a Brazilian splitting test.

4.1 *Mechanical problem*

The Brazilian splitting test is used as a standard measure of tensile strength of concrete, rocks and other geomaterials. The cylindrical specimen is loaded along a diametral plane by means of steel bearing plates, as shown in Figure 6 (a). As this study is not related to any experimental data, the steel bearing plates are arbitrary modelled with rigid plates, with high Young's modulus (E = 300 GPa) and Poisson's ratio ν of concrete in order to avoid a confinement effect of concrete. The set of parameters of Mazars' damage model (see Table 1) represents ordinary concrete behaviour.



(a) (b)

Figure 6: Brazilian splitting test (a) problem statement (b) FE mesh.

Numerical simulations are performed in the FE code *Code_Aster* with 6-node triangular elements. Due to double symmetry, a computation domain consists of one quarter of a specimen. A plane strain nonlocal gradient Mazars damage modelling (see

Equation (9)) is carried out in order to evaluate global mechanical response of structure as well as initiation, distribution and evolution of the damaged zone. As a parameter of the model, we use the characteristic material length √c, established according to Equation (12).

Solution's objectivity and mesh independence are provided on different mesh densities. The coarsest mesh, for which a large change in mesh density yields only a small change in a solution, is chosen for the further simulations (see Figure 6 (b)).
A nonlinear problem is solved incrementally by crack opening displacement (COD) control, i.e. the horizontal displacement of point P. F-COD plot is shown in Figure 7.

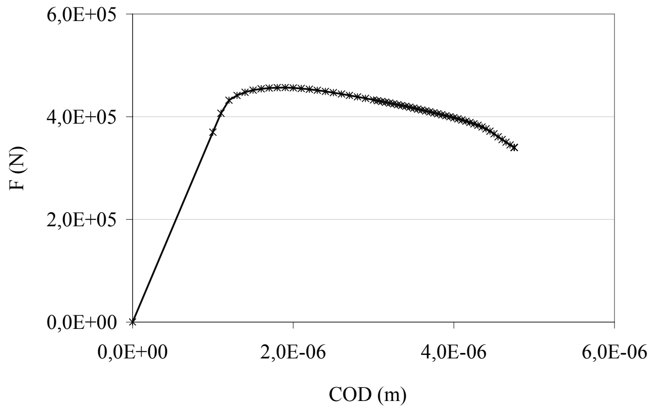
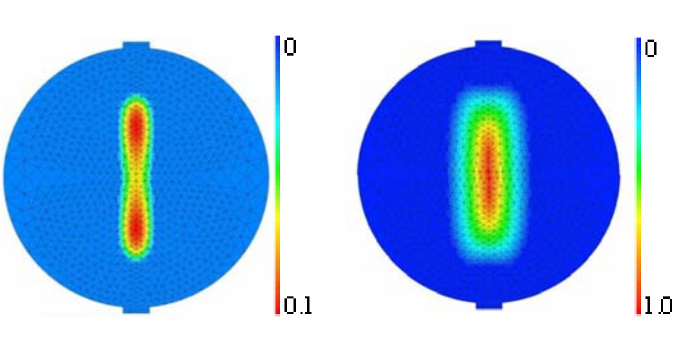


Figure 7: F-COD plot.

Damage distributions at damage initiation and at last loading step are depicted in Figures 8 (a) and (b). Maximal damage is initially located in some place along the vertical symmetry axis and then it translates downwards to the centre as the loading increases. Progression of damage in transversal section of the specimen (maximal values) is shown in Figure 9. One can observe that damage develops in a band of a limited width which is governed by model characteristic length. In addition, the height of the damage band at failure is the diameter.



(a) (b)

Figure 8: Damage distributions at (a) damage initiation and (b) at last loading step.

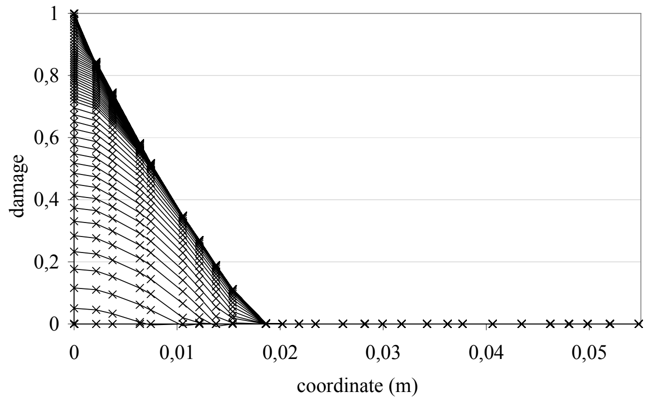


Figure 9: Damage progression in transversal section of the specimen (maximal values).

4.2 *Structural permeability evolution*

At each damage state, a local permeability is computed according to the matching law (Equation (26)) applied at each Gauss points of a discretized structure. Then, a structural permeability is determined by weighted-averaging of the local permeabilities.

Results plotted in Figure 10 show evolution of logarithm of the structural permeability (k str) with COD to peak COD ratio. One can observe that these numerical results are in qualitative agreement with experimental results of the tests performed recently in our laboratory on hollow concrete cylinders subjected to simple compressive test and radial gas flow under loading (Choinska *et al.* 2007) (see Figure 11). Finally, one may consider that, even if the type of test is not the same one, permeability varies according to the same type of the matching law.

5 CONCLUSIONS

In this contribution, a matching permeability law, derived from an analytical approach based on equivalence at failure between discontinuous crack opening and damage, has been proposed. By analytical variables substitution we have associated first crack opening with state variable (regularized strain) governing damage in nonlocal integral damage model and afterwards we have related state variable with damage through the evolution law using the hypothesis of a uniform damage distribution in the damaged zone. Consequently, we have conceived a matching permeability law from the relations of permeability evolution with diffuse damage and with crack opening both reported and weighted by the damage only. Using parameters representing mechanical and hydraulic behaviour of an ordinary concrete, several cases of a matching law have been tested. Finally, we have retained a matching law based on logarithms of permeability given by modified Picandet's relation and Poiseuille's law. In order to emphasize influence of this matching law on structural response, we have numerically simulated a Brazilian splitting test. Numerical tendencies are similar to observed ones on experimental results of the tests performed on cylinders in simple compression test. This observation permits to suppose that, even if the type of test is not the same one, permeability varies probably according to the same type of matching law.

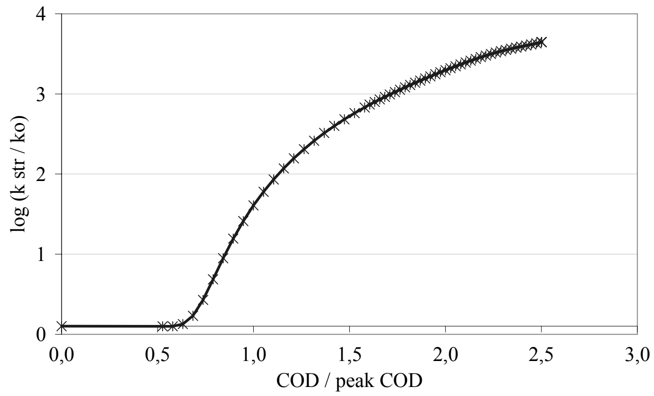


Figure 10: Evolution of logarithm of structural relative permeability with COD to peak COD ratio.

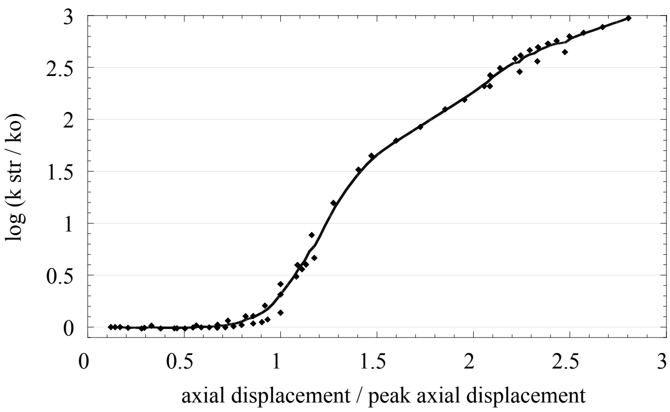


Figure 11: Evolution of logarithm of structural relative permeability with displacement to peak displacement ratio (experimental results, after (Choinska *et al.* 2007)).

Experimental validation of the proposed approach, emerging towards a continuous model capable to reproduce permeability variations of a concrete structure, constitutes a major perspective of this work. Experimental tests will be realised soon on concrete discs loaded in a Brazilian splitting test for which one single crack forms and boundary conditions are well identified, on contrary of a compression test. Measurement of the displacement field by image analysis and of the gas flow will be carried out during the tests and will be compared with numerical results in order to validate the proposed approach based on continuous non local damage modelling.

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