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Higher order X-FEM for curved cracks

K. Dréau¹, N. Chevaugeon¹, N. Moës¹

¹ Institut de Recherche en Génie Civil et Mécanique, UMR CNRS 6183, Ecole Centrale de Nantes, B.P. 92101, 44321 Nantes Cedex3, France
{kristell.dreau,nicolas.chevaugeon,nicolas.moes}@ec-nantes.fr

Simulation of crack growth is an important tool to assess the safety of mechanical parts for instance in the aerospace industry. Since about 10 years, the extended finite element method has been developed to model this type of problem. With this approach the elements do not need to fit exactly the crack location thus relieving a lot of constraint on the meshing tool. With specific singular enrichments on the front of the crack, another advantage of X-FEM is that the computation does not require extremely small elements on the front.

More precisely, the finite element approximation is enriched by additional functions through the partition of unity, that model the behaviour of cracks: A step function along the crack, and an asymptotic crack tip displacement solution are added to the finite element basis ([1]). These enrichment functions are defined thanks to level-sets values. Discontinuous functions can't be directly integrated on elements. Elements are split at the integration level along the iso-zero value, and regular integration is done separately on each side of the integration partition.

X-FEM gives good results using linear shape functions and linear level-sets in elements : optimal rate of convergence is achieved, that is to say energy error converges as $O(h)$, h being the element size. With shape functions of higher order, if the error is reduced, the optimal rate of convergence can't be achieved without improving the geometrical representation. ([3], [4])

The problem is of course related to the well known fact that, when increasing the order of the approximation field, the order of representation of the geometry must be increase accordingly to get optimal rate of convergence, in the classical finite element method. Iso-parametric elements have an energy error norm on regular problems with curved boundary that converges as $O(h^p)$, where h is the element size and p the polynomial basis order. In the case of straight cracks, convergence is quasi-optimal using higher order approximation shape functions ([4]).

To get optimal rate of convergence when curved discontinuities are represented with level-set, the obvious solution would be to increase the order of the level-set representation near the zero iso-values. Level-set is interpolated by quadratic shape functions in [3]. In this case, the integration problem can become quite difficult : within an element, the iso-zero surface can have complex shape and topology.

The aim is to improve the geometrical representation, while keeping the same computational mesh, and therefore without adding any degree of freedom. The alternative approach that we propose consists in representing the level-sets on finer grids, while keeping the level-set representation linear by element. The improvement of the representation of a crack surface is also presented in [6]. Level-sets are computed on a finer mesh, called "geometrical grid", than the computational mesh, and its elements are called "cells". Initially, the geometrical grid is a copy of the computational mesh. If the geometrical representation has to be improved, the geometrical grid has to be refined. Thus, each cell of this grid that is crossed by the iso-zero is split into cells of same nature: For example, a triangle in 2D is divided in 4 triangles. Level-set values are then computed on the new nodes. The procedure is done recursively up to a user

defined maximum depth, forming the final geometrical grid. Each element of the analysis is then linked to an quadtree-like partition where the level-set is represented and on which the integration procedure is applied for each sub-element. Refining the geometrical grid improves the representation of the interface, while keeping a level-set piecewise linear on elements. This method was proposed in [7] for problems including curved free surfaces and material interfaces in 2-D linear elasticity.

The algorithm is now implemented and tested on simple problems with curved cracks. Numerical experimentations were done up to order three polynomial approximation and we show that optimal convergence rate can be achieved, with an optimal depth of recursion. Numerical examples including curved cracks will be presented and compared to the results given in the literature. A curved crack in an infinite plate in 2-D, and a lens-shaped crack in a cube in 3-D will be considered.

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