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The CR_4 index and the interval estimation of the Herfindahl-Hirschman Index: an empirical comparison

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Abstract. Concentration indices are employed to measure the level of competition within an industry. Among the several indices proposed in the literature, the Herfindahl-Hirschman Index (HHI) and the Four-firm concentration ratio (CR_4) are among the most established. However, the HHI requires the market shares of all market players to be known, while the CR_4 requires just the top four. In order to investigate whether we can always use the CR_4 in place of the HHI, we have compared the indices resulting from a selected group of datasets. This preliminary analysis shows that the relationship between the CR_4 and the HHI may not be monotonic, so that the CR_4 does not preserve the order relationship established through the HHI.

Keywords: Concentration indices; Competition; Herfindahl-Hirschman Index (HHI); Four-firm concentration ratio.

1 Introduction

Concentration indices are used in a number of contexts to measure how a quantity is distributed among a number of subjects (individuals or aggregates, such as households or companies). Several indices have been proposed in the literature and have been applied in many contexts, to see how income (or wealth) is distributed among the population, or analyze the market structure (e.g., in the analysis of merger and acquisition operations), or even to investigate the filing of patents (see, e.g., [11]). An extensive literature is surveyed for the banking sector in [3]. Among them, two have emerged as the most widespread: the Herfindahl-Hirschman Index (often indicated with its abbreviation HHI) and the CR_4 index. The former requires the knowledge of the market shares of all the companies participating in the market, while the latter concentrates on just the top four. The HHI has been compared to concentration indices established among statisticians (namely those due to Gini, Bonferroni, and Amato) and has been shown in [13] to provide the largest sensitivity in the case of the Zipf law

(see [15] for an example of application of the Zipf law also in contexts other than the distribution of firms). The CR_4 is instead periodically provided by the U.S. Census Bureau for a number of industries [21].

Given the large diffusion of those two indices, it is to be seen whether they lead to the same conclusions, so that either can be taken as a measure of concentration (in this case the choice would probably fall on the more parsimonious CR_4), or they provide complementary views of the phenomenon (with the HHI probably reflecting a more complete panorama).

However, as hinted above, we may not be in a condition to compute both indices, since the HHI requires the complete distribution of the market. However, it has been shown that, though we cannot compute the exact value of the HHI, we can obtain an interval estimate for it if we know just the largest market shares [14]. We can then compare the two indices on a level playing field, assuming that we know in both cases just the top 4 firms (which is what we need for the less demanding of the two, the CR_4).

In this paper, we report a preliminary analysis, using a limited number of datasets, with the aim of comparing the conclusions suggested by the two indices. We show in the cases examined that we cannot draw a monotonic relationship between the HHI and the CR_4, so that the latter cannot be considered as a full replacement of the more complete HHI.

After reporting the definitions of the two concentration indices in Section 2 and recalling the interval estimation of the HHI in Section 3, we describe the datasets in Section 4 and report the results in Section 5.

2 Concentration indices

Concentration indices are employed to measure the level of competition in an industry, often to examine whether concerns for dominant position creation exist in the case of mergers and acquisitions. Two major indices are the Herfindahl-Hirschman Index (often indicated with its abbreviation HHI) and the CR_4 index. In this section, we report their definitions and main properties.

2.1 The Herfindahl–Hirschman Index

The Herfindahl-Hirschman owes its name to the two economists who developed it, though independently. Albert O. Hirschman proposed the index in 1945 in [8], while Orris C. Herfindahl presented it in 1950 in his unpublished doctoral dissertation at Columbia University, "Concentration in the U.S. Steel Industry". Hirschman himself had to somewhat untangle the paternity dispute, since his index had been attributed to Gini [7]. Actually, though Herfindahl proposal appeared five years later than Hirschman's, the form of the index as used today is actually that proposed by Herfindahl (which leads to an index that is the square of the original proposal).

If we consider a market where n companies are operating, and the market share of the i-th company is s_i, the HHI is defined as the sum of the squares of

all the market shares

$$\text{HHI} = \sum_{i=1}^{n} s_i^2. \tag{1}$$

The HHI is obviously a positive figure. If the market shares are expressed as fractions of the whole market (i.e., $0 < s_i \leq 1$, $\forall i$), then we have $0 < HHI \leq 1$. Instead, if the market shares are expressed as percentages (i.e., $0 < s_i \leq 100$, $\forall i$), then we have $0 < HHI \leq 10000$. In the following we opt for the first choice, which gives us a naturally normalized value for the HHI.

Actually not the whole range of values may be taken by the HHI. If we have a monopoly, where a single company takes all the market, we have $n = 1$ and

$$s_1 = 1 \Longrightarrow \text{HHI} = 1, \tag{2}$$

but the opposite case, where the market is uniformly distributed between the company (perfect competition) does not lead to $HHI = 0$. In a market of n companies with perfect competition, we have

$$s_i = \frac{1}{n} \quad \forall i \Longrightarrow \text{HHI} = \sum_{i=1}^{n} \left(\frac{1}{n}\right)^2 = \frac{1}{n}, \tag{3}$$

which is then the minimum value attainable by the HHI.

The value of the HHI provides an indication of the level of concentration, with the maximum value corresponding to the case of the monopoly, and the minimum corresponding to perfect competition. Hence, the higher the value of the HHI, the higher the concentration of the market in the hands of a few companies. Though a precise correspondence cannot be drawn between the numerical value of the HHI and the qualitative indication of a level of concentration (or, equivalently, of competition), some guidelines have been provided for that purpose. The U.S. Department of Justice provided its guidelines for horizontal mergers first in 1985 and later revised them several times, till the latest version in 2010, proposing in Section 5.3 of its 2010 version a classification of markets into three types, reported in Table 1 [20]:

HHI	Competition level
<0.15	Unconcentrated Markets
0.15–0.25	Moderately Concentrated Markets
>0.25	Highly Concentrated Markets

Table 1. Levels of competition and the HHI

The European Commission, rather than stating just the levels of HHI for the same purpose of identifying horizontal competition concerns, has proposed

to define both the levels of post-merger HHI and the change ΔHHI incurred by the HHI because of the merger to identify the cases of no concern. Namely, it has stated that concerns are unlikely when any of the conditions listed in Table 2 apply [19]. All other cases are therefore likely to raise concerns about competition.

Post-merger HHI	ΔHHI
<0.1	Any
0.1–0.2	<0.025
>0.2	<0.015

Table 2. Alternative conditions for no competition concerns

The HHI has been used to measure concentration in a wide set of contexts. In a technical note of the Federal Reserve Bank, Rhoades showed with simple examples how it can be employed to analyse the competitive effects of bank mergers, by computing the HHI before and after the merger [18]. It has also been used in civil aviation [10], the book industry [4], the food processing industry [12], telecommunications, media and the Internet [16] [23] [17], and the newspaper industry [9]. Some criticism has come as well as to its capability to actually explain the role of market share inequalities in the banking system [1] [6].

2.2 The CR_4 index

The CR_4 index (the concentration ratio for the top 4 firms) has been the most relevant index to measure concentration before the advent of the HHI [24]. It is given by the sum of the market shares of the largest 4 firms in the market [2]

$$CR_4 = \sum_{i=1}^{4} s_i. \tag{4}$$

A foremost shortcoming of this index is that differences in the market structure may not show up: a market in which each of the four largest firms has 20% of the market has the same CR_4 value as another market in which the top four firms have shares of 55%, 20%, 4%, and 1%, respectively, i.e., 80% on the overall, though the competition level is quite different for four equal-size firms than for four firms of disparate size. A further aspect of this issue is that monopoly or oligopoly risks deriving from mergers may not be highlighted if mergers take place between the four top firms, and go completely ignored if the market is made just of four companies (which become three after the merger). In addition, the CR_4 does not consider the whole market, but just a limited number of companies.

While it is clear that a low value of the index represents a larger competition level, while a high value (close to 100) represents oligopoly situation, there is not a general consensus on the correspondence between the value of the index and intermediate concentrations. Typically, if $CR_4 < 40$, the industry is considered as very competitive. A complete classification table is proposed in [5] and reported here in Table 3.

CR_4	Competition level
0	Perfect Competition
0–40	Effective Competition or Monopolistic Competition
40–60	Loose Oligopoly or Monopolistic Competition
>60	Tight Oligopoly or Dominant Firm with a Competitive Fringe

Table 3. Levels of competition and the CR_4

The CR_4 has been employed in the construction industry [25], and the U.S. Census Bureau periodically updates it for a number of industries [21].

3 Interval estimation of the HHI

In Section 2, we have provided the definition of two concentration indices: the HHI and the CR_4. Though the HHI provides a more complete view of the market structure, in many cases we do not know the market shares of all the market players and are then unable to compute the HHI, while disposing just of the data about the top 4 firms allows us to compute the CR_4. However, in [14] it has been shown that we can obtain an interval estimate of the HHI if we know just the market shares of the top market players. This estimate is tight, in the sense that the bounds obtained are deterministic. Its goodness, as determined by the interval width, improves with the number of known market shares and the known fraction of the market. If we know just the market shares of the top four firms, we can therefore gain a better view of the market structure by estimating the HHI in addition to the exact computation of the CR_4. In this section, we report those bounds.

Let's consider a market made of n firms, with the market share of the i-th largest firm being represented by s_i. Let's assume that we know just the market shares of the top 4 firms. We indicate by $R = 1 - \sum_{i=1}^{4} s_i$ the residual market share, i.e. that in the hands of the remaining firms other than the top four. We also define the residual ratio $Q = \lfloor R/s_4 \rfloor$. The lower and upper bounds for the Hirschman-Herfindahl Index, as derived in [14] and reported here for the special case of the top four firms, are shown in Table 4. We do not report for sake of simplicity the derivation, which can however be found in [14].

Type	Bound
Lower (n known)	$\sum_{i=1}^{4} s_i^2 + \frac{\left(1-\sum_{i=1}^{4} s_i\right)^2}{n-4}$
Lower (n unknown)	$\sum_{i=1}^{4} s_i^2$
Upper ($R \leq s_4$)	$\sum_{i=1}^{4} s_i^2 + \left(1 - \sum_{i=1}^{4} s_i\right)^2$
Upper ($R > s_4$)	$\sum_{i=1}^{4} s_i^2 + s_4^2 Q + \left(1 - \sum_{i=1}^{4} s_i - s_4 Q\right)^2$

Table 4. Bounds on HHI

4 The datasets

In Section 2, we have defined both the concentration indices of interest: the HHI and the CR4. In Section 3, we have shown how the HHI can be estimated if we know either the market share of the top four firms in the market. In order to investigate the relationship between the CR4 and the HHI, i.e., to see if the CR4 can be effectively used to estimate the more complete concentration index represented by the HHI, we consider a set of test cases taken from the real world. In this section, we describe the datasets adopted in this paper for such an assessment.

Here we consider 4 datasets concerning the following industries:

- the U.S. consumer book industry;
- the civil aviation industry;
- the mobile phone industry.

For some of these cases we own the complete market data or, equivalently, the actual HHI. This gives us, or allows us to compute, the actual value of the HHI against which we can compare the estimate we obtain if we know the market share of the top four firms only. If we have just a partial knowledge of the market, i.e., we know just the market shares of the top k firms, we can gauge the goodness of the estimate by observing how wide the estimate interval is: the smaller the interval, the tighter the estimate.

We first deal with the publishing industry data, as reported in [4]. In Table 5, we report the market share of the top 20 firms in the U.S. consumer book industry in 1996. This is enough to compute the CR4 and to perform the interval estimate of the HHI. It wouldn't be enough to compute the HHI, but in [4] the true HHI value is also reported.

Our second dataset represents the production figures of smartphone worldwide. The data are reported in Table 6. In this case, we do not know the overall number of manufacturers. In addition, the number of known market shares is quite small (just the top 5 firms), which account for slightly more than 60% of the market. The data have been taken from [22].

The third and fourth datasets concern instead the civil aviation industry and reports the number of flights per carrier on two most important routes: Paris–

Publisher	Market share [%]
Random House	12.3
Simon and Schuster	9.2
Time Warner	8.6
Bantam Doubleday Dell	6.7
Reader's digest	5.7
Harper Collins	5.2
Penguin	4.0
Putnam Berkeley	2.9
Holzbrinck	2.7
Golden books	1.7
Rodale	2.2
Harlequin	2.3
Thomas Nelson	1.5
Hearst	1.6
Scholastic	1.6
Andrews and McMeel	1.0
Houghton Mifflin	0.8
Barnes and Noble	1.2
Disney	1.3
IDG books	0.9
Others	26.7

Table 5. Market shares in the U.S. consumer book industry

Manufacturer	Market share [%]
Samsung	30.3
Apple	19.1
Huawei	4.0
LG Electronics	3.8
Lenovo	3.2
Others	39.6

Table 6. Market shares in the worldwide smartphone industry

New York and London–New York [10]. Here we know the full sets of data, which are shown in Table 7.

Carrier	Paris-New York	London-New York
KLM	2	1
Air France	6	8
British Airways	5	12
Lufthansa	3	2
Delta	2	1
TWA	1	0
American Airlines	2	8
United	1	5
Continental	2	3
US Airlines	0	1
Canada Airlines	1	1
Sabena	2	2
Lingus	1	2
Virgin	0	5
Iceland Air	1	0
SAS	1	2
TAP	1	0
Kuwait Airlines	0	1
Iberia	1	0
India Airlines	0	1

Table 7. Number of flights per weekday

5 Results

In order to compare the two indices, we use the definition of the CR_4 and the interval estimation method presented in Section 3 for the HHI. We compare the results obtained for the two indicators in the datasets described in Section 4. In this section, we report the results.

In Table 8 we show the values of the two indicators for the four datasets.

At the same time, we can plot the interval estimate of the HHI versus the CR_4 to obtain an immediate idea of the relationship between the two. Though the interval estimate for the HHI does not allow us to predict the precise value of the HHI if we know the CR_4 (or, better, the top four market shares that we employ to compute the CR_4), we can examine if the CR_4 can be considered as equivalent to the HHI, in the sense that it preserves the order relationship: if one market situation exhibits a larger CR_4 than another, then it also exhibit a larger HHI. In that case, we would observe a monotonic relationship between

Industry	CR_4	HHI	
		min	max
Aviation (London-NY)	0.6	0.1115	0.1326
Aviation (Paris-NY)	0.5	0.093	0.1035
Publishing	0.368	0.0355	0.0767
Smartphones	0.572	0.1313	0.1473

Table 8. Concentration indices for some industries

CR_4 and HHI. We show in Fig. 1 that the monotonic relationship we may expect between the two is not confirmed. There is a growing trend for the datasets with the lowest values of the CR_4 , but with the dataset concerning the London-NY flights the HHI band falls.

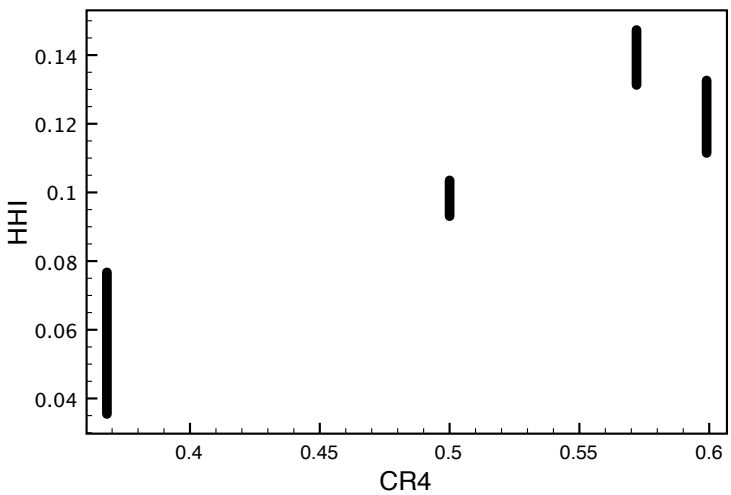

Fig. 1. Comparison of CR_4 and HHI

6 Conclusions

We have performed a preliminary analysis of the relationship between two major concentration indices: the Herfindahl-Hirschman Index (HHI) and the four-firm Concentration Ratio (CR_4). Our analysis aims at establishing first if the HHI can be safely estimated from the same information employed to compute the

CR_4 or, secondarily, if the CR_4 can be considered as equivalent to the HHI, in the sense that it preserves the order relationship.

We have employed 4 datasets, pertaining to the the U.S. consumer book industry, the civil aviation industry, and the mobile phone industry.

Our preliminary analysis shows that, if we sort results by the CR_4 values, we can observe a growing trend for the HHI (i.e., the order relationship appears to be preserved) for the first three datasets, but then the HHI for the dataset with the largest CR_4 falls, infringing the order relationship.

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