



*Centrale
Nantes*

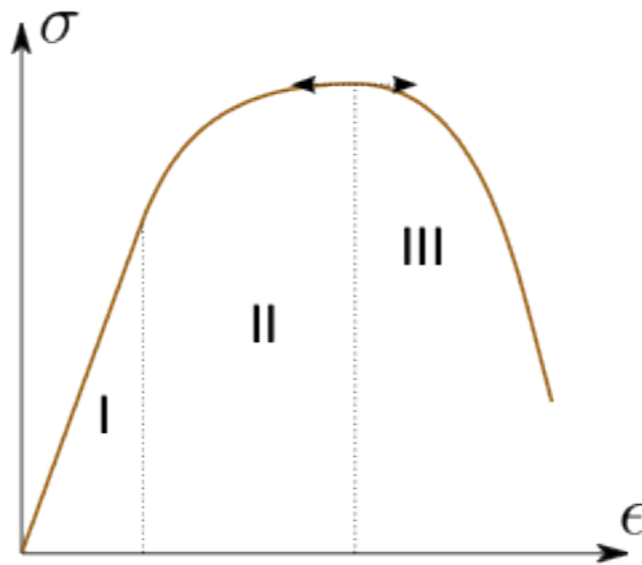
Damage growth modeling using Thick Level Set (TLS) approach

Nicolas Chevaugéon, Nicolas Moës,
Paul Emile Bernard.
Ecole Centrale de Nantes,
GEM Institute UMR CNRS 6183, France

ECCOMAS, Vienna, Austria,
September 2012.

Localization : A quick definition

- High strain zone in a structure leading to crack or shear band
- Takes place for material models exhibiting a limit point in the stress - strain curve (softening), this limit point may depend on the strain rate.



- As a consequence, for quasi-static analysis, there may exist no solution above certain loads: force or even displacement (snap-back)

Motivations for the work

- As localization takes place, the conditioning of the discretized problem is getting worse and worse and convergence is at stake. The cure is to place a crack and allow displacement jump.
- **But**, current damage model do not allow cracks to be easily placed
- The saying «a crack needs to be placed when d reaches 1» is an ill posed problem: extent and orientation of the crack ? loading on the crack ? what if branching ? Yet, effort are made mainly in 2D where it is already tedious.
- Goal I: reconciling damage mechanics and fracture mechanics through a new theoretical model in which cracks shows up automatically as a result of localization
- Goal II: Robust numerical implementation using X-FEM

Part I Theory

Fracture Mechanics

Energy

$$W(u, a)$$

State Law

$$G = -\frac{\partial W}{\partial a}$$

Dissipation

$$G\dot{a}$$

Evolution law

$$\dot{a} = \frac{\partial \psi^*(G)}{\partial G}$$

Damage Mechanics

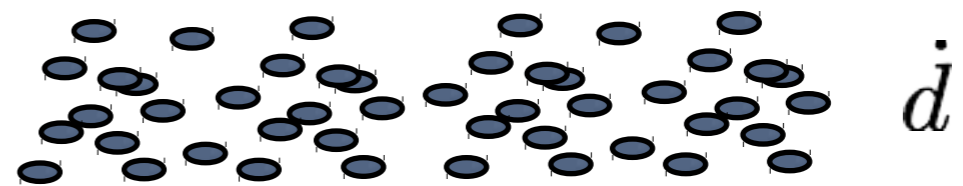
$$\int_{\Omega} w(u, d)$$

$$Y = -\frac{\partial w}{\partial d}$$

$$\int_{\Omega} Y \dot{d}$$

$$\dot{d} = \frac{\partial \psi^*(Y)}{\partial Y}$$

Initiation ?



Crack placement ?

The need of a material length

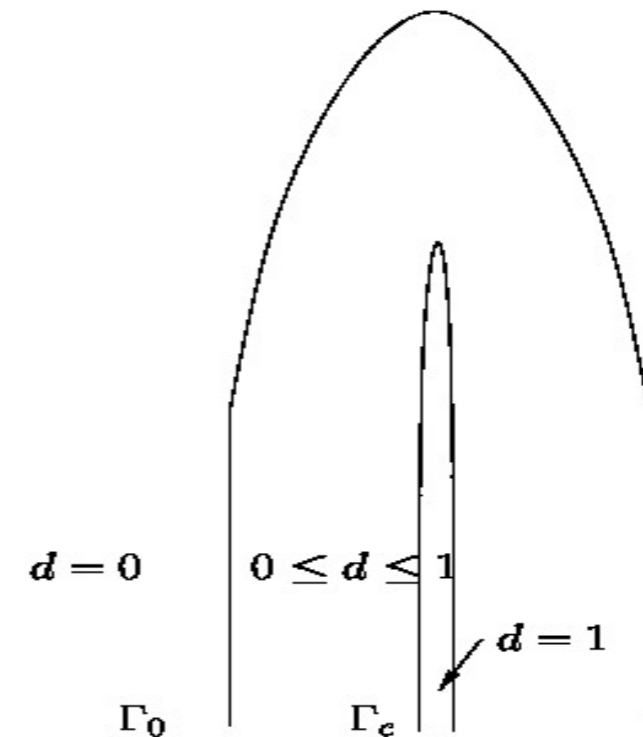
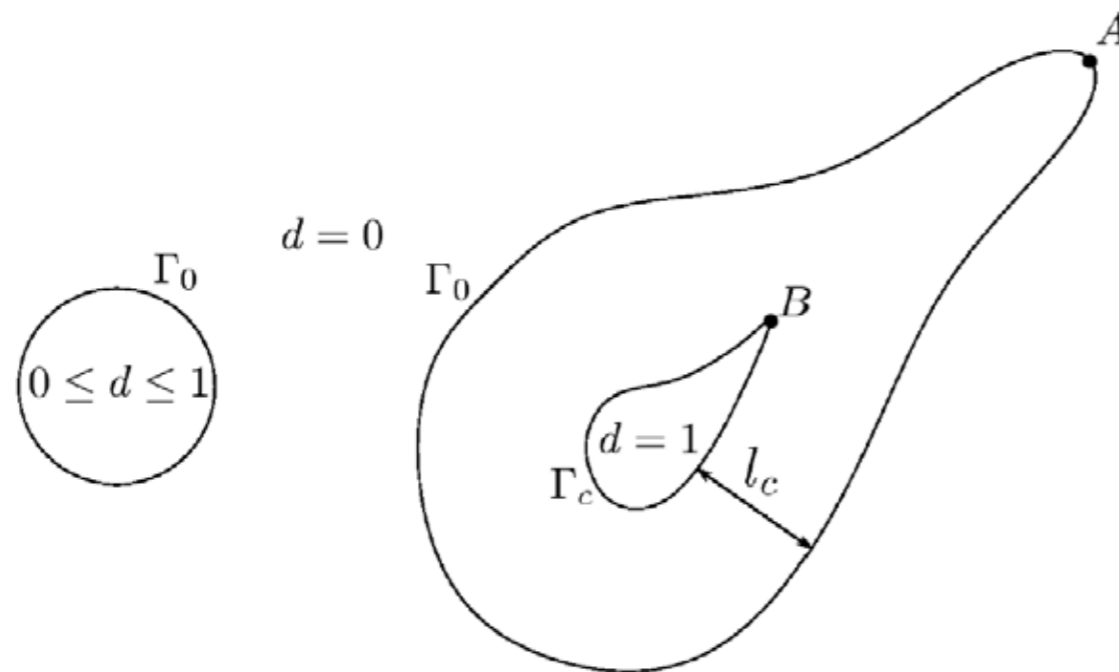
- Local damage models lead to spurious localization (zero dissipation). They need to be regularized by a length to become so-called non-local damage models.
- There exist several ways to introduce non-locality

Non-local Damage Models

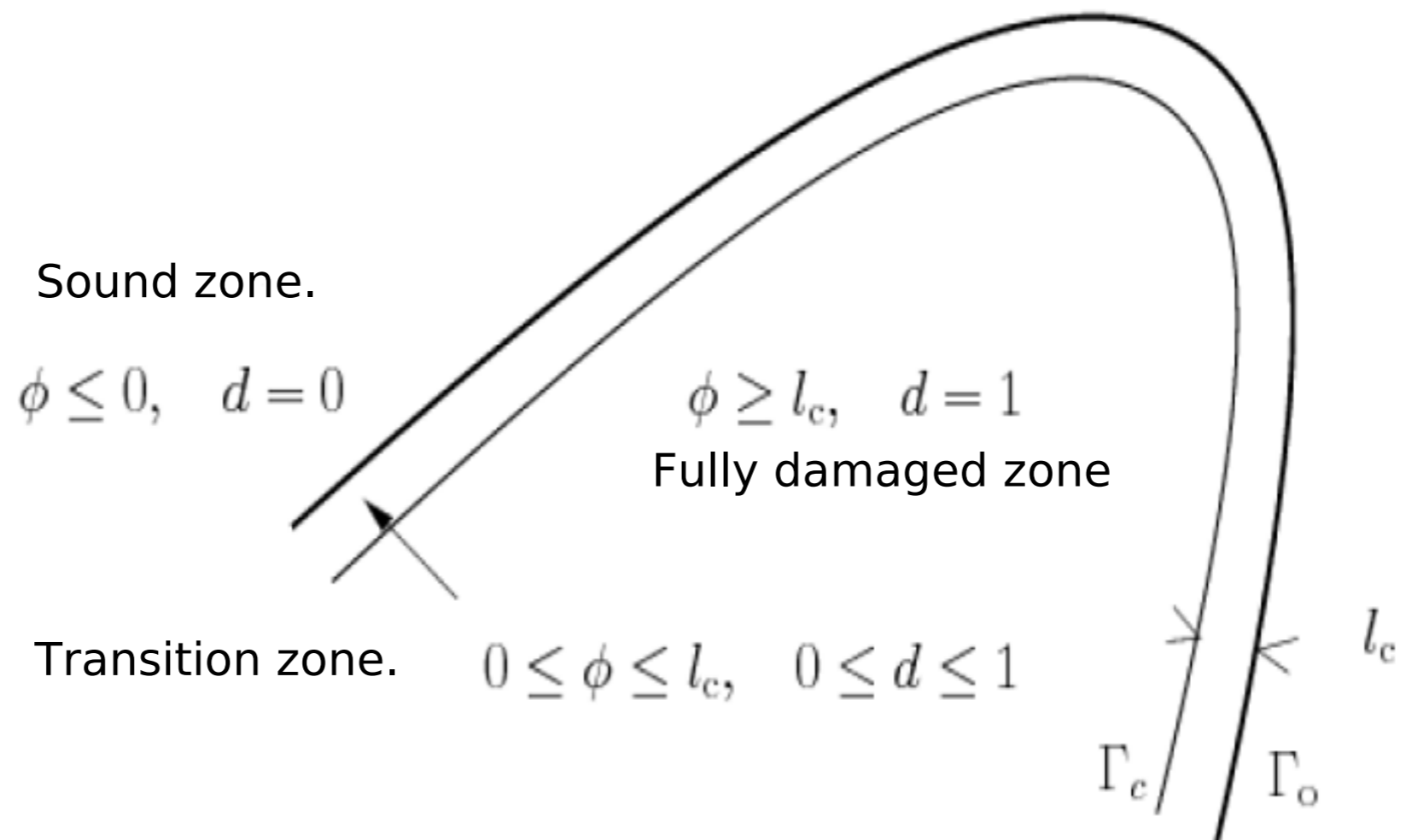
- **Integral approach:** the damage evolution is governed by a driving force which is non-local i.e. it is the average of the local driving force over some region: (Bazant, Belytschko, Chang 1984, Pijaudier-Cabot and Bazant 1987).
- **Higher order, kinematically based, gradient approach** involving higher order gradients of the deformation: (Aifantis 1984, Triantafillydis and Aifantis 1986, Schreyer and Chen 1986) or additional rotational degrees of freedom (Mühlhaus and Vardoulakis 1987).
- **Higher order, damage based, gradient models:** the gradient of the damage is a variable as well as the damage itself. This leads to a second order operator acting on the damage: (Fremond and Nedjar 1996, Pijaudier-Cabot and Burlion 1996, Nguyen and Andrieux 2005).
- **Variational approach of fracture:** (Francfort and Marigo 1998, Bourdin, Francfort and Marigo 2000, Bourdin, Francfort and Marigo 2008) with possible solve with shape optimization by level set (Allaire, Jouve and Van Goethem 2007).
- **Phase-field approach** emanating from the physics community: (Karma, Kessler and Levine 2001, Hakim and Karma 2005) and more recently revisited by (Miehe, Welschinger, Hofacker, 2010).

The Thick Level Set Model

- Damage evolves through the propagation of a front located by a level set
- The iso-zero level set separates damaged and undamaged zone.
- In the damaged zone, damage is a given function of the level set (distance to the front) rising from 0 to 1 as the level set rises from 0 to l_c
- Beyond the distance l_c , damage is set to 1.
- The crack is thus located by the contour of the iso- l_c

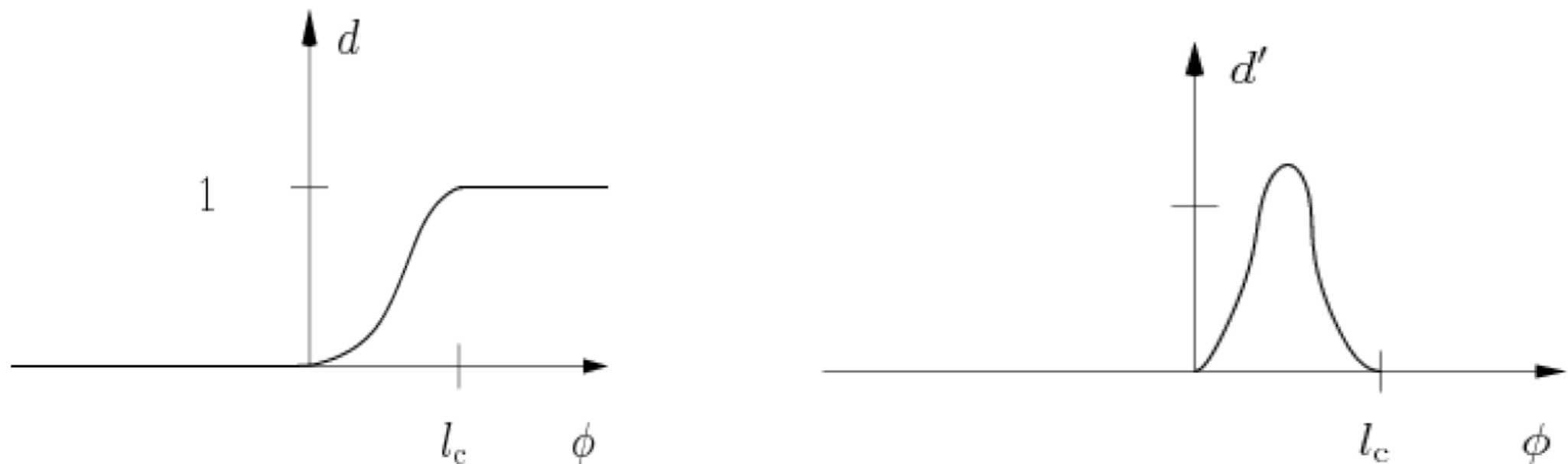


The Thick Level Set Model



The Thick Level Set Model

Example of evolution of \mathbf{d} as a function of the level set



- The TLS model is thus yet another type of non-local model but with the capability to automatically unveil cracks

Comparing models

Fracture	TLS Damage	Damage
$W(u, a)$	$\int_{\Omega} w(u, d(\phi))$	$\int_{\Omega} w(u, d)$
$G = -\frac{\partial W}{\partial a}$	$g = -\frac{\partial \int_{\Omega} w}{\partial \phi}$	$Y = -\frac{\partial w}{\partial d}$
$G\dot{a}$	$\int_{\Gamma_0} g\dot{\phi}$	$\int_{\Omega} Y\dot{d}$
$\dot{a} = \frac{\partial \psi^*(G)}{\partial G}$	$\dot{\phi} = \frac{\partial \psi^*(g)}{\partial g}$	$\dot{d} = \frac{\partial \psi^*(Y)}{\partial Y}$

Thermodynamics: The Free Energy

Free energy and local state laws

$$\varphi = \varphi(\boldsymbol{\epsilon}, d) = \frac{1}{2}(1 - d)\boldsymbol{\epsilon} : \boldsymbol{E} : \boldsymbol{\epsilon}$$

$$\begin{aligned}\boldsymbol{\sigma} &= \frac{\partial \varphi}{\partial \boldsymbol{\epsilon}} = (1 - d)\boldsymbol{E} : \boldsymbol{\epsilon} \\ Y &= -\frac{\partial \varphi}{\partial d} = \frac{1}{2}\boldsymbol{\epsilon} : \boldsymbol{E} : \boldsymbol{\epsilon}\end{aligned}$$

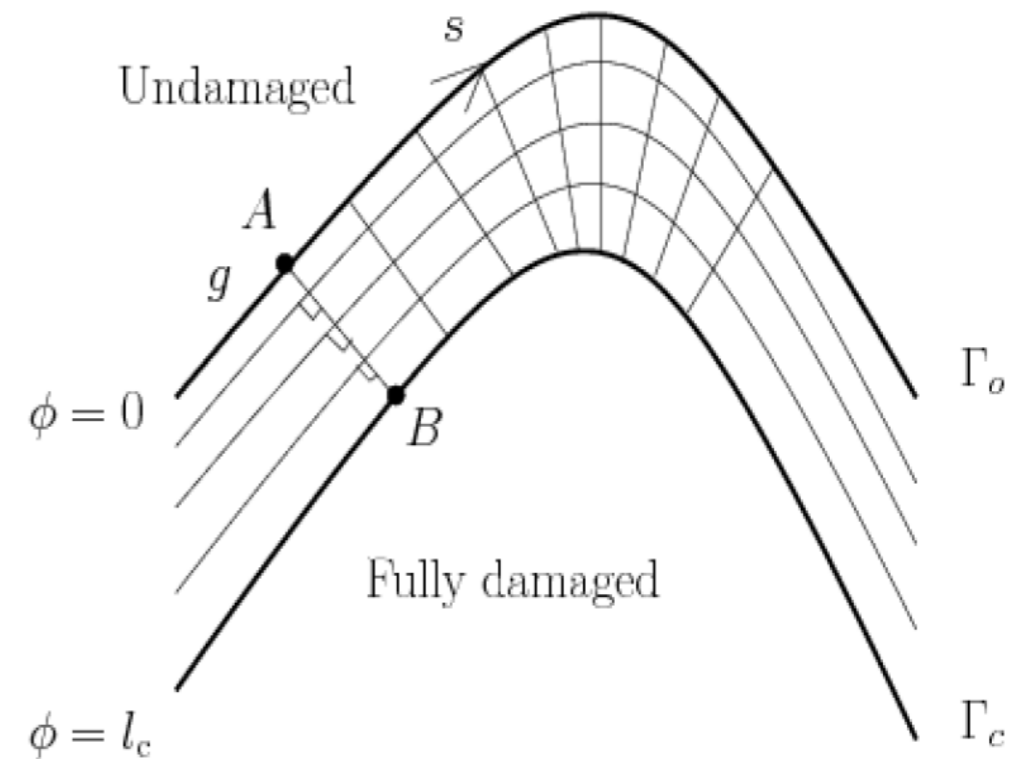
more complex free energy can be used to take into account dissymmetric behavior in tension and compression

Global potential energy

$$E(\boldsymbol{u}, \phi) = \int_{\Omega(\phi \leq l_c)} \varphi(\boldsymbol{\epsilon}(\boldsymbol{u}), d(\phi)) d\Omega - \int_{\Gamma_T} \boldsymbol{T} \cdot \boldsymbol{u} d\Gamma$$

Thermodynamics: The Free Energy

Global potential energy variation

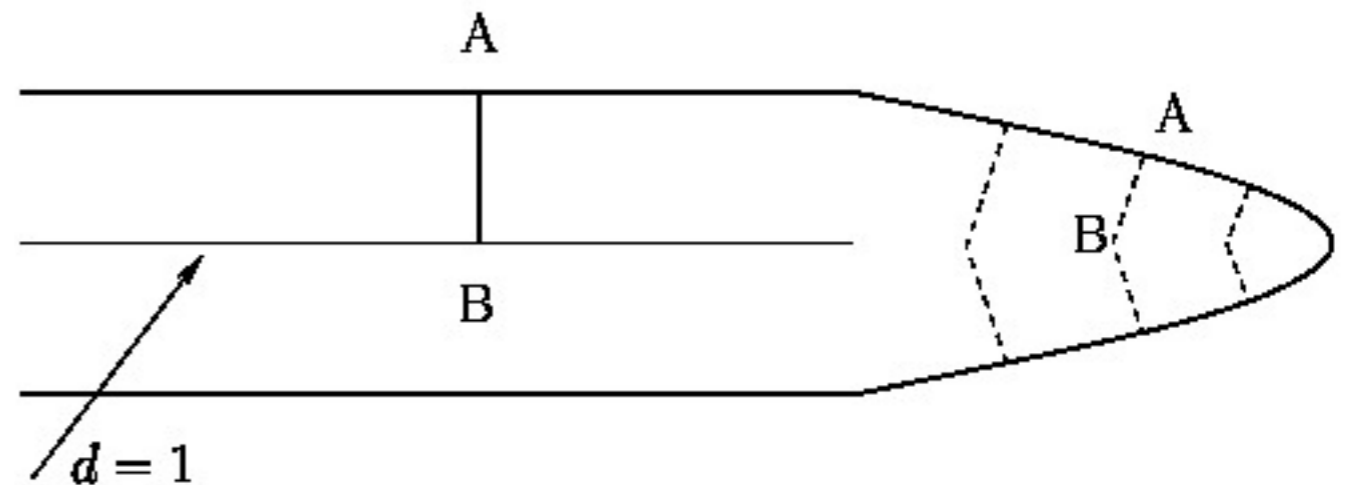
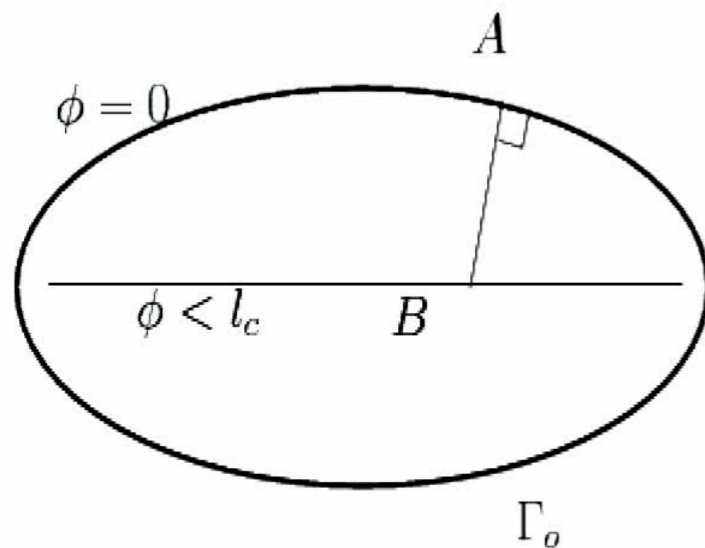


$$\begin{aligned}
 DE(\mathbf{u}, \phi)[(0, \delta\phi)] &= - \int_{\Omega_{oc}} Y d'(\phi) \delta\phi d\Omega \\
 &= - \int_{\Gamma_0} \int_0^l Y(\phi, s) d'(\phi) \delta\phi(s) (1 - \phi/\rho(s)) d\phi ds \\
 &= - \int_{\Gamma_o} g(s) \delta\phi(s) ds
 \end{aligned}$$

Thermodynamics: The Free Energy

- There exist a configurational force associated to the front movement

$$g(A) = \int_{AB} Y d'(\phi) \left(1 - \frac{\phi}{\rho_0}\right) d\phi$$

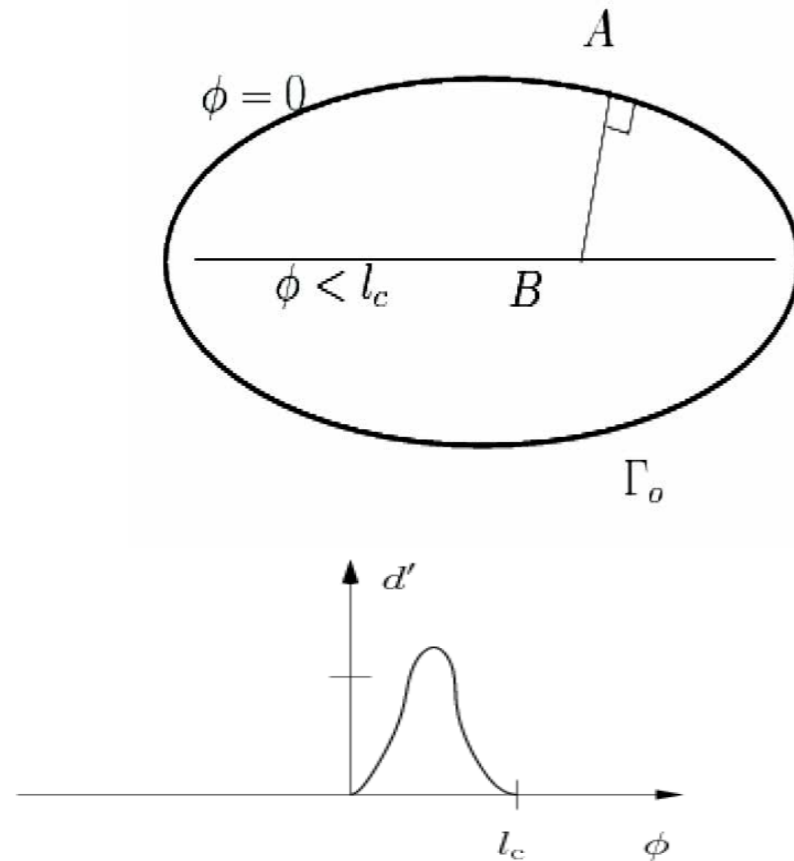


- We may also define a non-local energy release rate

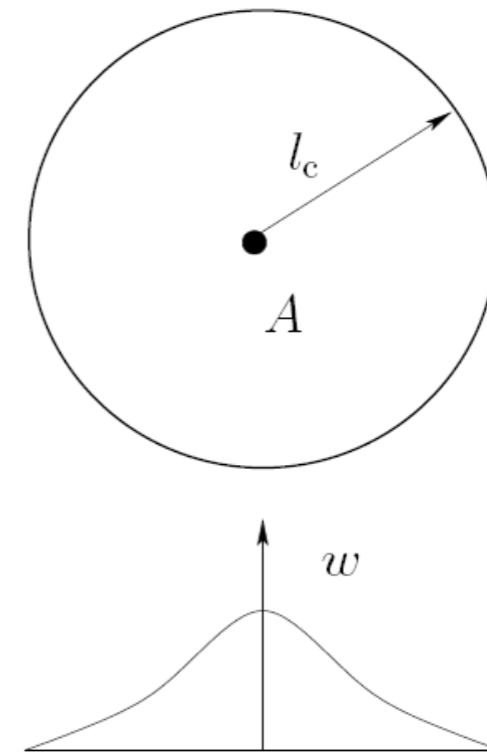
$$\bar{Y}(AB) = \frac{\int_{AB} Y d'(\phi) \left(1 - \frac{\phi}{\rho_0}\right) d\phi}{\int_{AB} d'(\phi) \left(1 - \frac{\phi}{\rho_0}\right) d\phi}$$

Similarity and difference with the non-local integral approach

$$\bar{Y}(AB) = \frac{\int_{AB} Y d'(\phi) \left(1 - \frac{\phi}{\rho_0}\right) d\phi}{\int_{AB} d'(\phi) \left(1 - \frac{\phi}{\rho_0}\right) d\phi}$$



$$\bar{Y}(A) = \frac{\int_{B_A} Y w d\Omega}{\int_{B_A} w d\Omega}$$



Pijaudier-Cabot, Bazant 1987

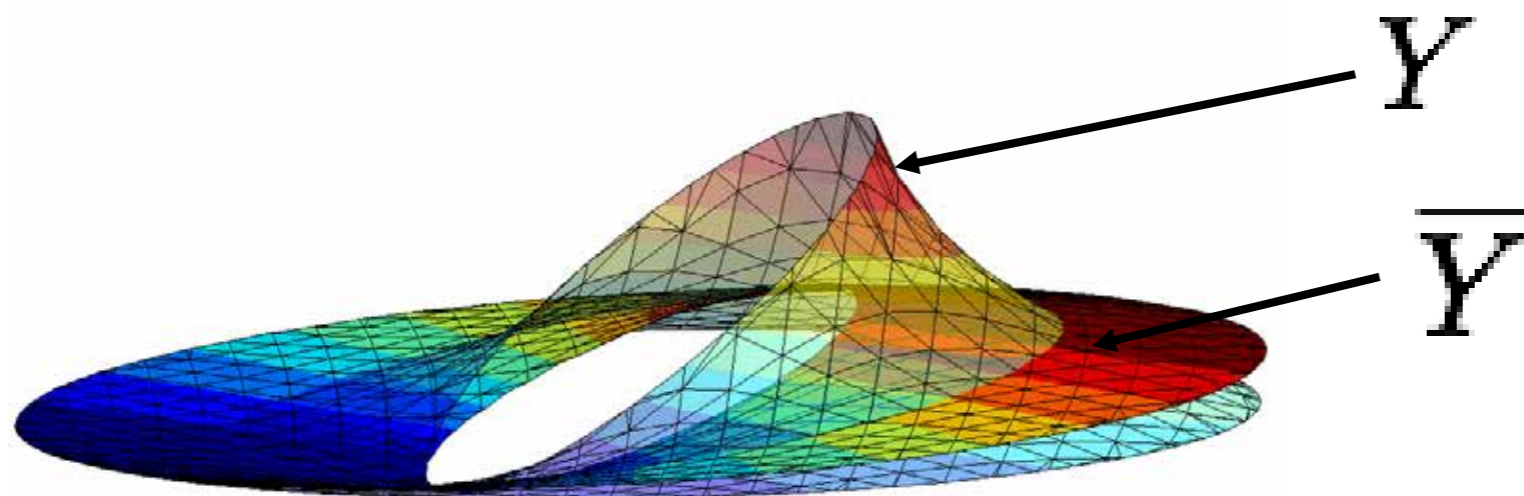
In the TLS model, the length over which averaging is performed is non-constant but evolving in time

A variational formulation to compute \bar{Y}

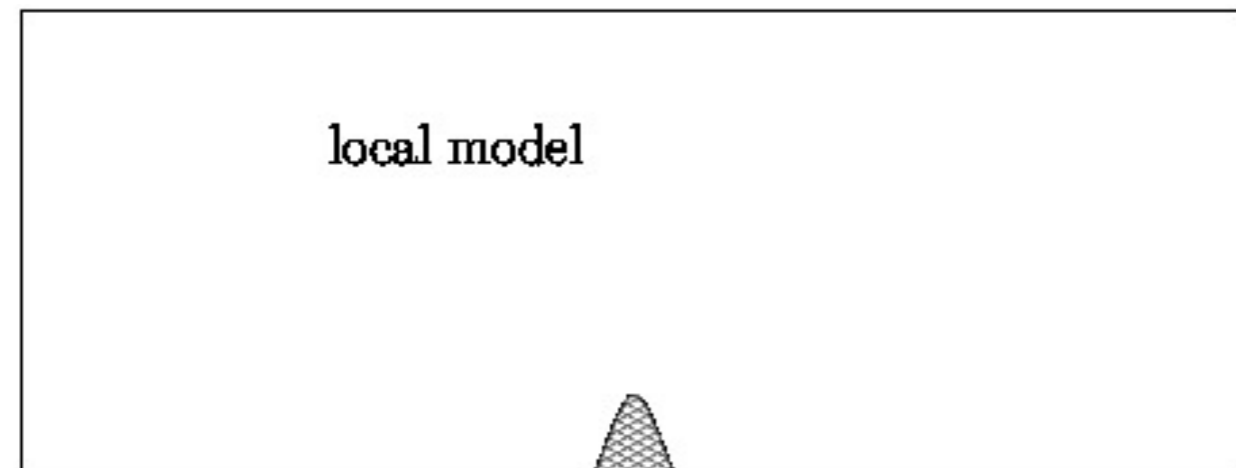
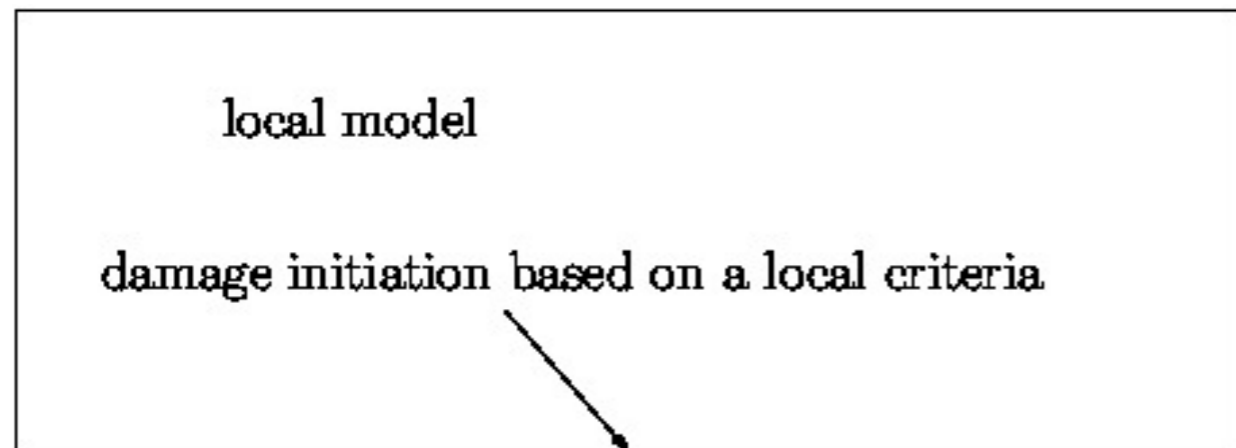
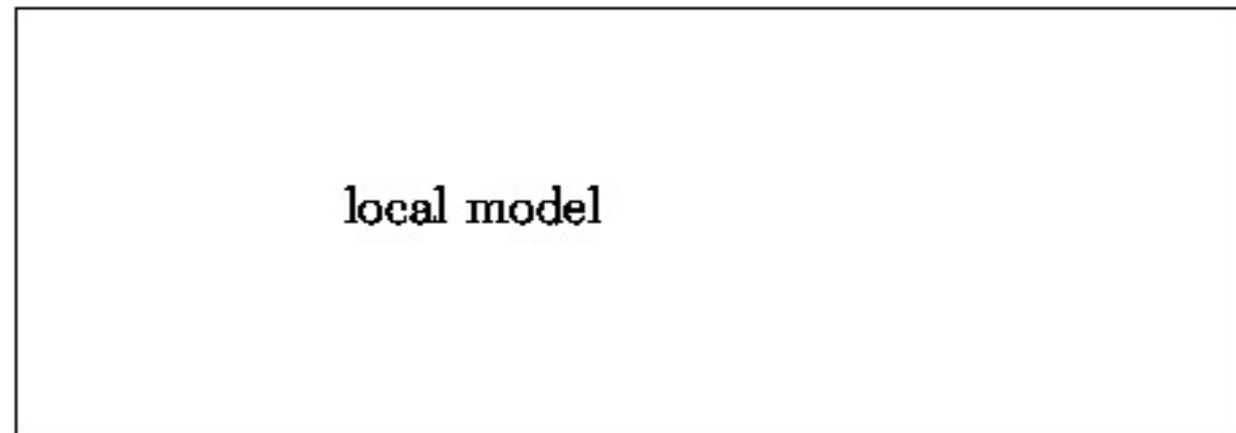
$$\int_{\Omega_{oc}} \frac{d'}{l_c} \bar{Y} Y^* + \partial_\phi \lambda \partial_\phi Y^* \, d\Omega = \int_{\Omega_{oc}} \frac{d'}{l_c} Y Y^* \, d\Omega \quad \forall Y^* \in \mathcal{Y}$$

$$\int_{\Omega_{oc}} \partial_\phi \bar{Y} \partial_\phi \lambda^* \, d\Omega + \int_{\Gamma_o} \mu \lambda^* \, d\Gamma = 0 \quad \forall \lambda^* \in \mathcal{L}$$

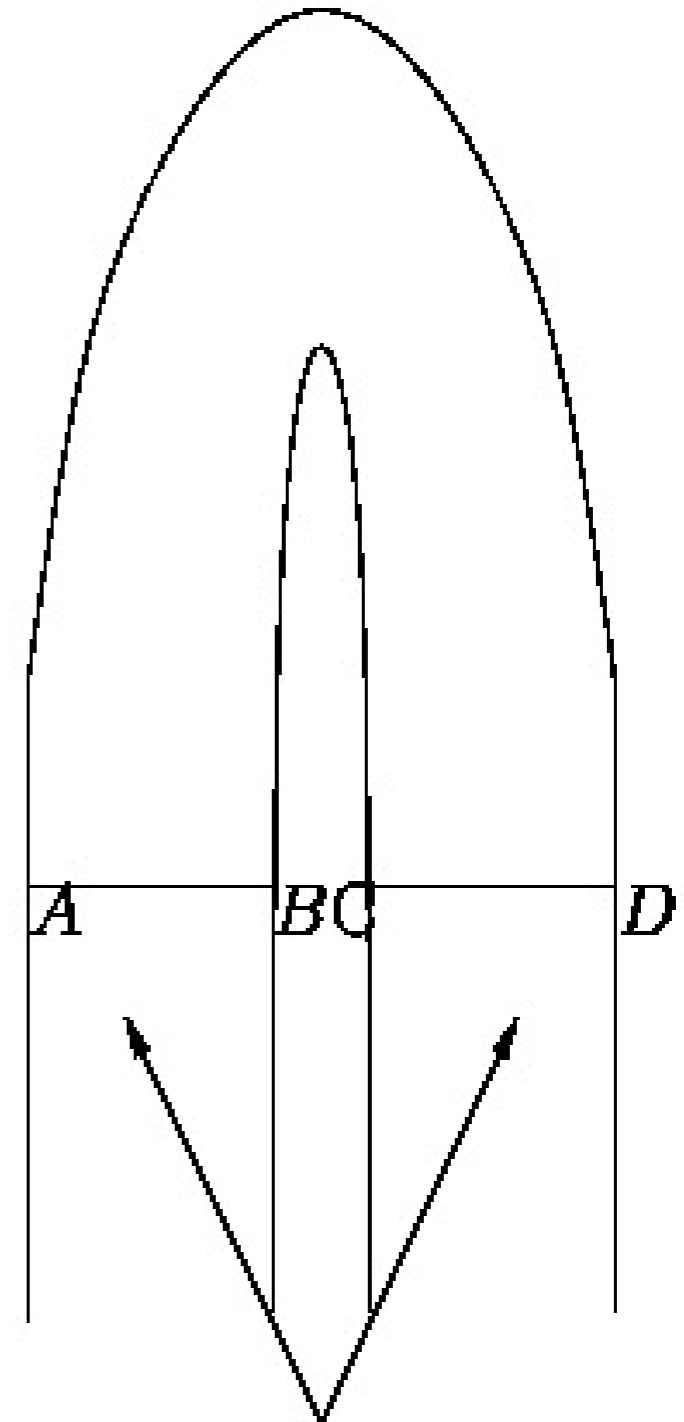
$$\int_{\Gamma_o} \lambda \mu^* \, d\Gamma = 0 \quad \forall \mu^* \in \mathcal{M}$$



Non-locality length is evolving from 0 to l_c



TLS non local model, averaging over an evolving length



Independent damage evolutions on each side of the crack

Thermodynamics: Dissipation

- **The dissipation is given by**

$$D = -\frac{dE(\mathbf{u}, \phi)}{dt} + \int_{\Gamma_T} \mathbf{T} \cdot \dot{\mathbf{u}} d\Gamma = \int_{\Omega_{oc}} Y \dot{d} d\Omega = \int_{\Gamma_o} g v_n d\Gamma$$

- We observe a **duality** between the **configurational force** g and the **front velocity** v_n and also a duality between the non-local \bar{Y} and the non-local $\bar{\dot{d}}$

$$g v_n = \int_0^{l(s)} Y \dot{d} \left(1 - \frac{\phi}{\rho_0(s)}\right) d\phi = \bar{Y} \bar{\dot{d}} \left(l - \frac{l^2}{2\rho_0}\right) \quad \bar{Y} \bar{\dot{d}} = \bar{Y} \bar{\dot{d}}$$

$$\bar{Y} \bar{\dot{d}} = \frac{\int_0^l Y \dot{d} \left(1 - \frac{\phi}{\rho_0}\right) d\phi}{\int_0^l \left(1 - \frac{\phi}{\rho_0}\right) d\phi}, \quad \bar{Y} = \frac{\int_0^l Y d'(\phi) \left(1 - \frac{\phi}{\rho_0}\right) d\phi}{\int_0^l d'(\phi) \left(1 - \frac{\phi}{\rho_0}\right) d\phi}, \quad \bar{\dot{d}} = \frac{\int_0^l \dot{d} \left(1 - \frac{\phi}{\rho_0}\right) d\phi}{\int_0^l \left(1 - \frac{\phi}{\rho_0}\right) d\phi}$$

Evolution laws

- The local evolution law is reused with the non-local quantities

$$\dot{d} = \frac{\partial \psi^*(Y)}{\partial Y} \quad \rightarrow \quad \bar{\dot{d}} = \frac{\partial \psi^*(\bar{Y})}{\partial \bar{Y}}$$

- The update of the front is based on the following sequence

$$Y \quad \rightarrow \quad \bar{Y} \quad \rightarrow \quad \bar{\dot{d}} \quad \rightarrow \quad v_n = \bar{\dot{d}} / \bar{d}'$$

Example : Brittle Time-Independent model with no hardening

Local

$$\dot{d} = \frac{\partial \psi^*(Y)}{\partial Y}, \quad \psi^*(Y) = I_-(Y - Y_c)$$

$$Y \leq Y_c, \quad \dot{d} \geq 0, \quad (Y - Y_c)\dot{d} = 0$$

Non-Local version

$$\bar{\dot{d}} = \frac{\partial \psi^*(\bar{Y})}{\partial \bar{Y}}$$

$$\bar{Y} \leq Y_c, \quad \bar{\dot{d}} \geq 0, \quad (\bar{Y} - Y_c)\bar{\dot{d}} = 0$$

Part II
Discretization
and
examples

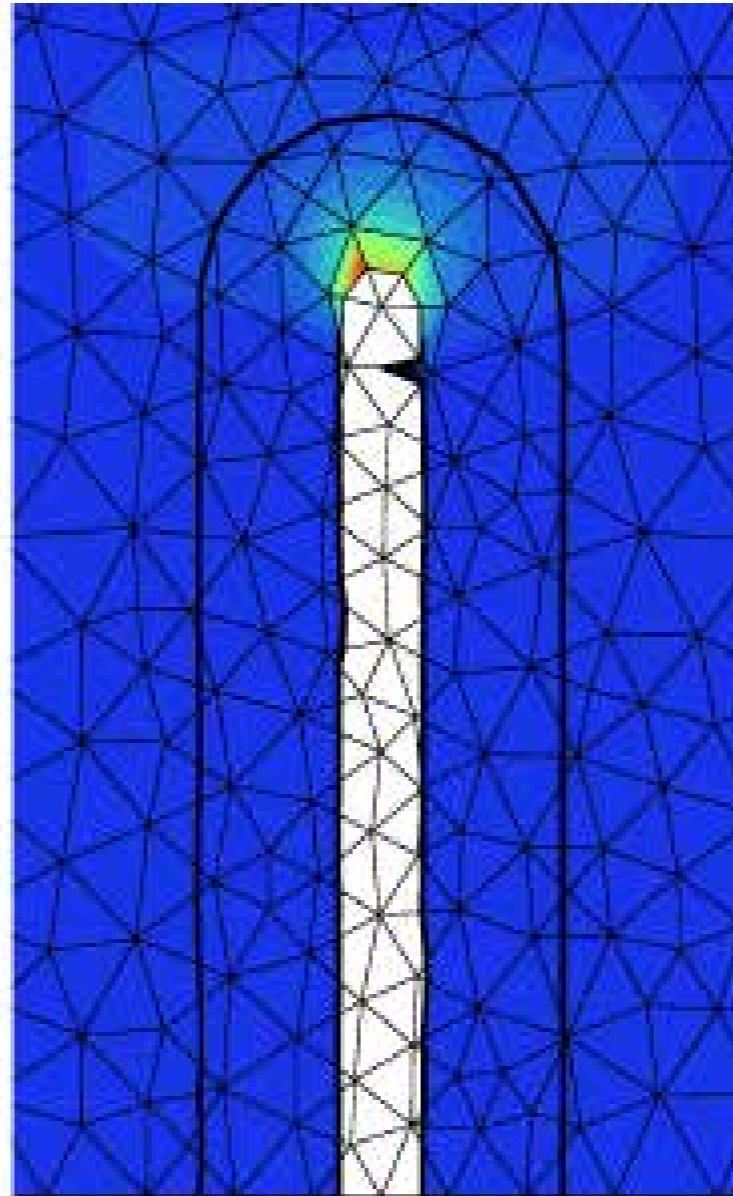
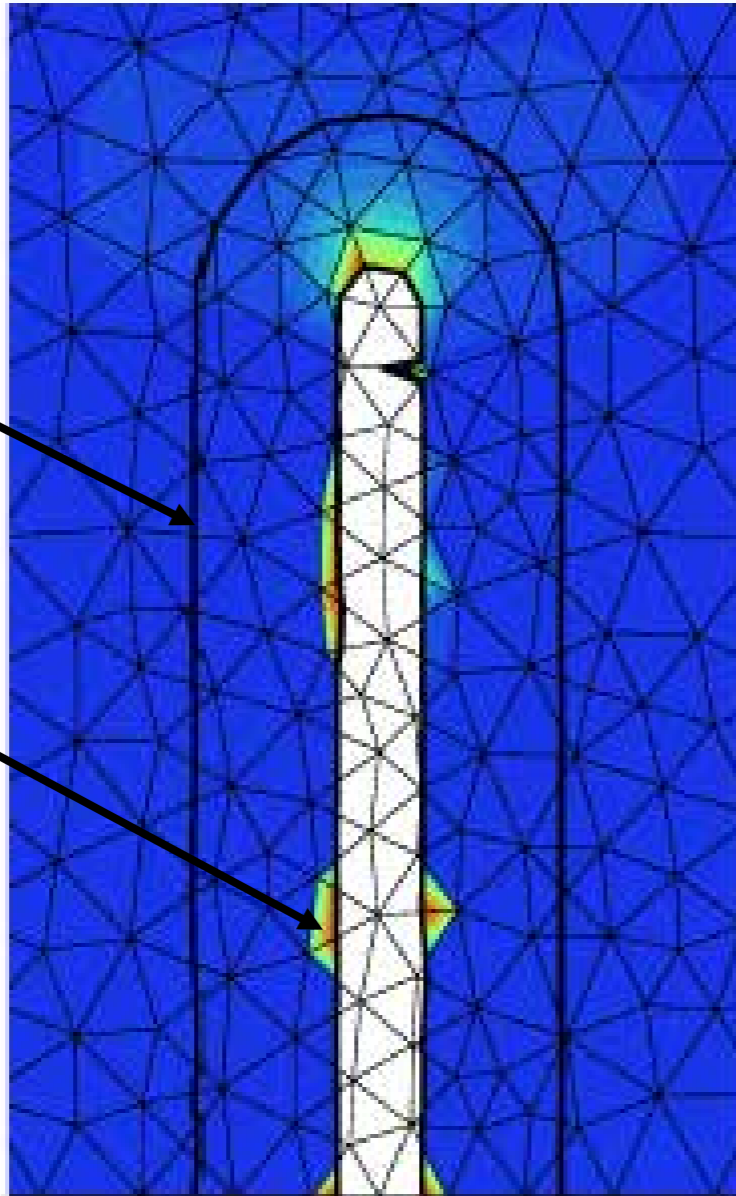
Implementation aspects

- Staggered explicit approach :
 - for a known damage front
 - the elastic field is computed (nonlinear problem in general), second order elements
 - the front velocity is computed as well as the load (dissipation control)
 - the front is updated and new front are inserted (if $Y > Y_c$)

Ramped Heaviside enrichment

Iso-0

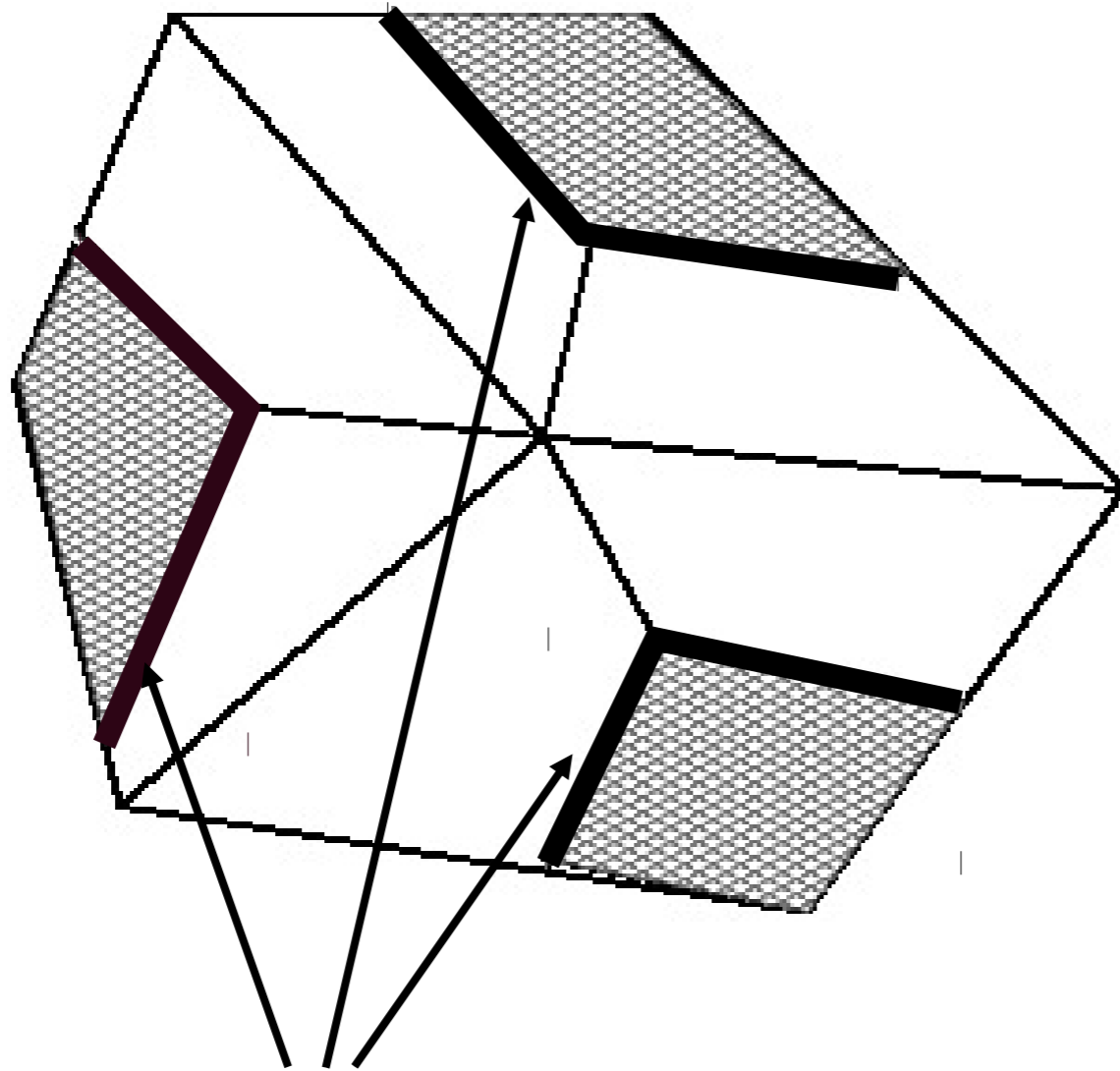
Iso-1c



No Enrichment

Enrichment

There may be more than one enrichment
needed on the support (two needed below)

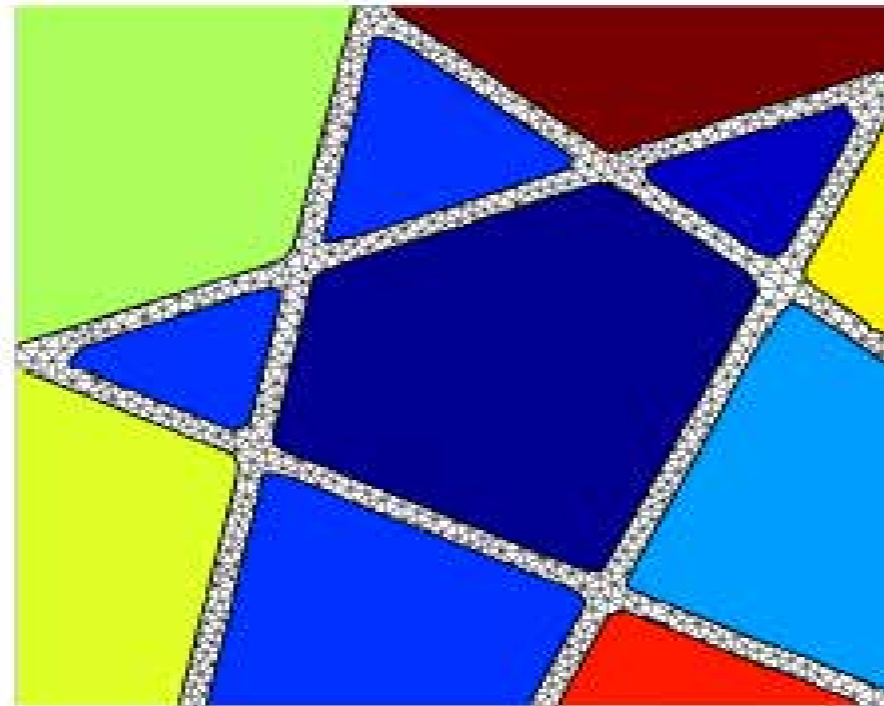


Three independent pieces
on the support

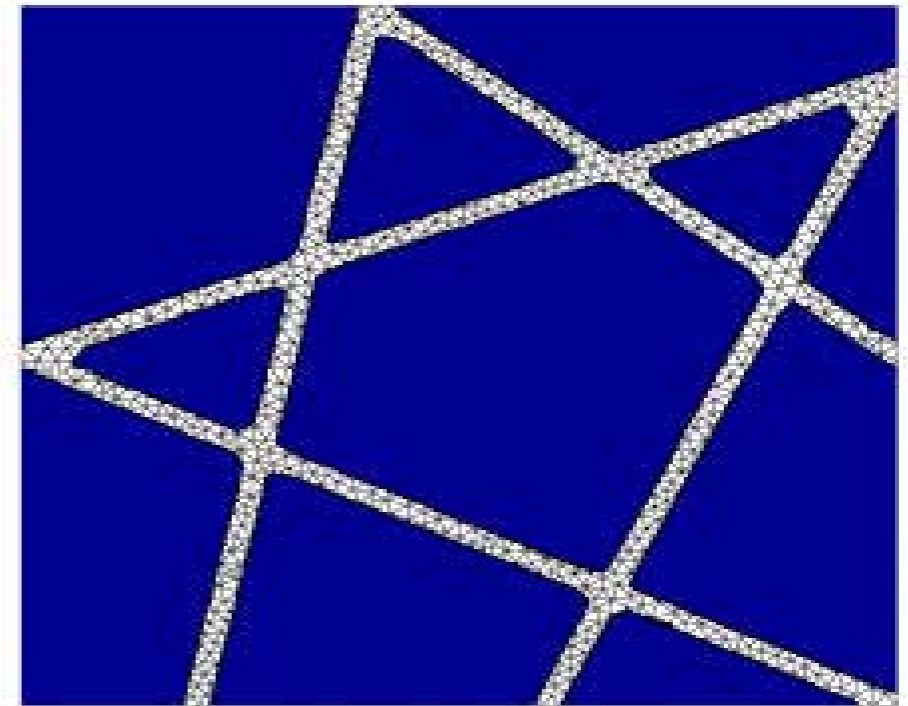
Iso-Ic

Benchmark to check the enrichment

Tension

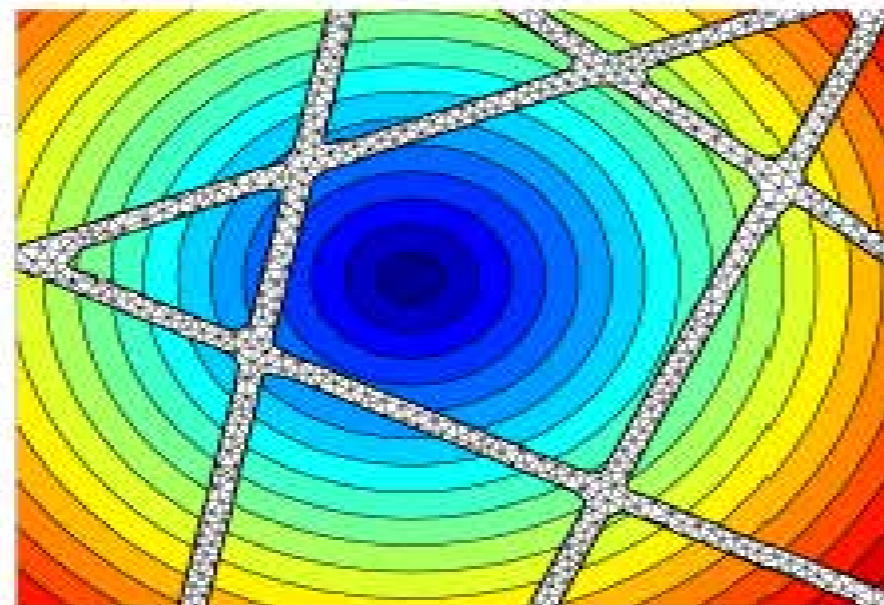


displacement
0 1.25e-05 2.51e-05

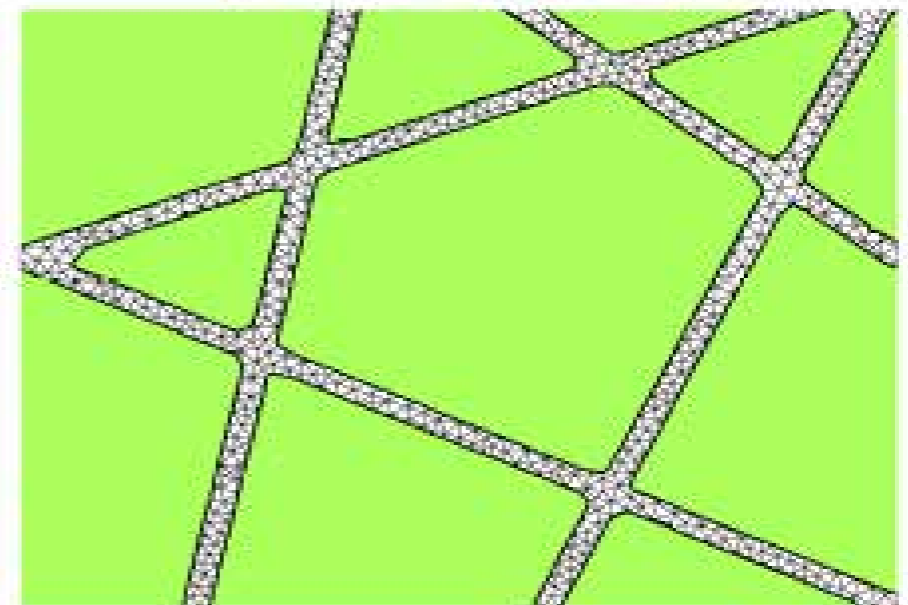


strain
2.03e-19 5.47e-11 1.09e-10

Compression

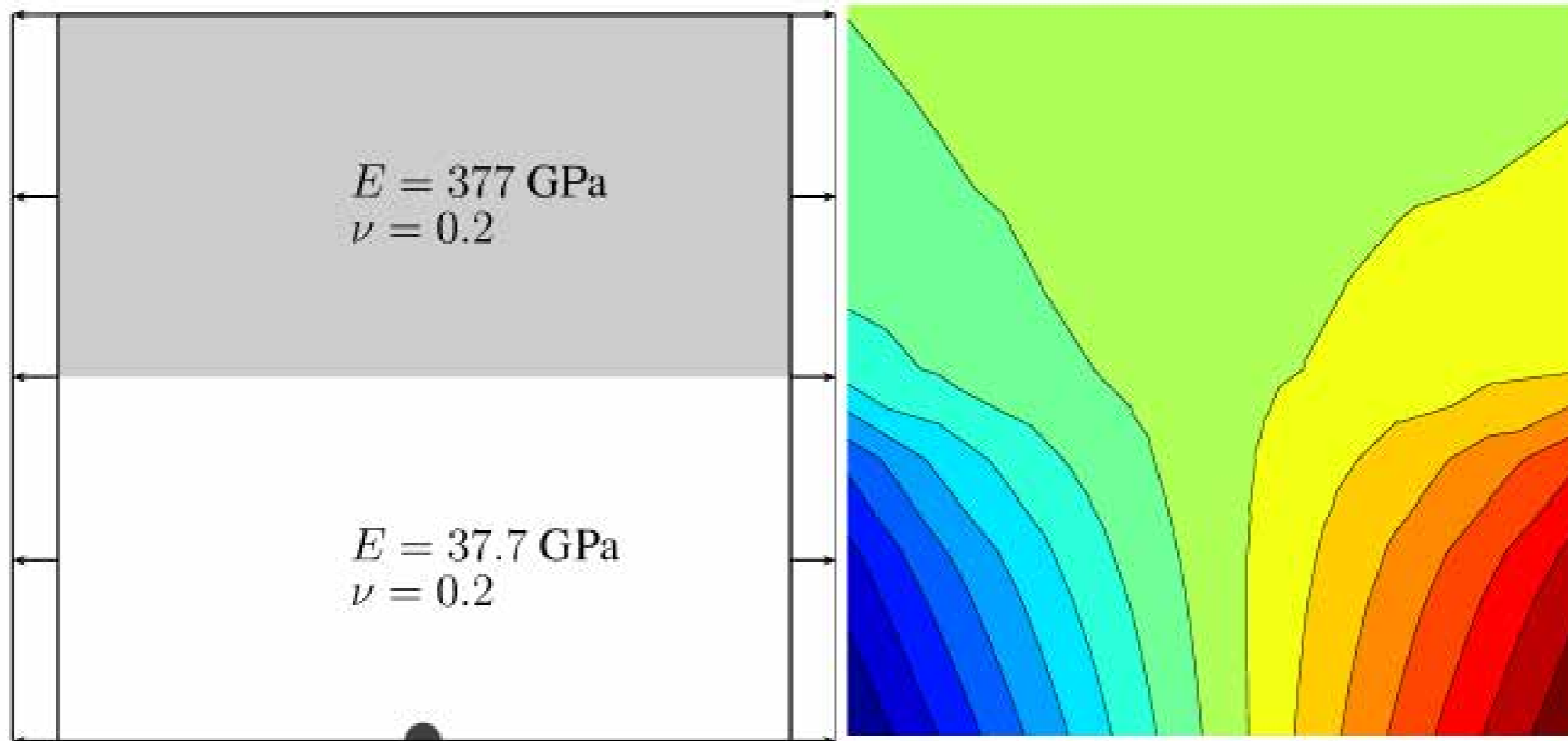


displacement
0 0.382 0.764

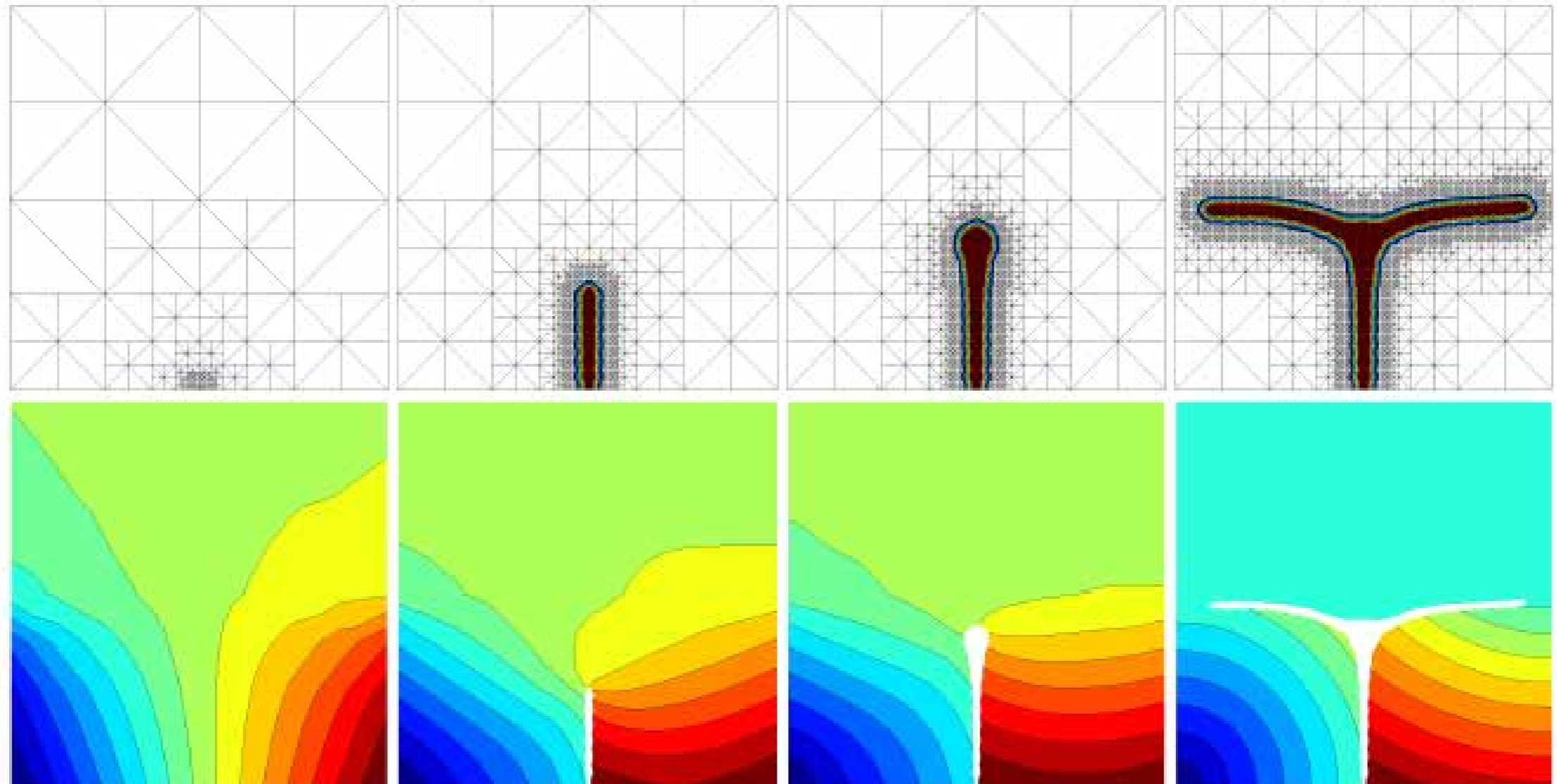


strain
1 1 1

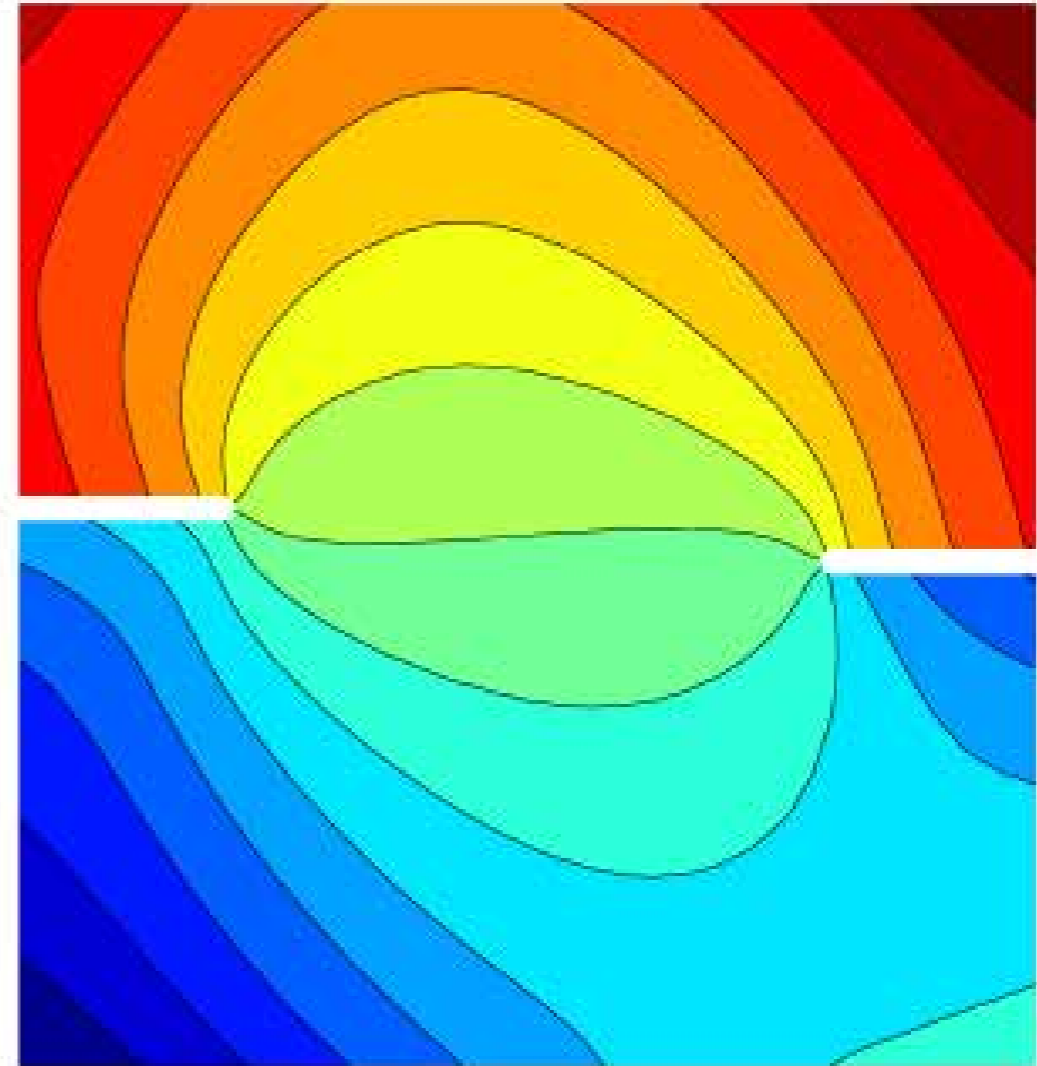
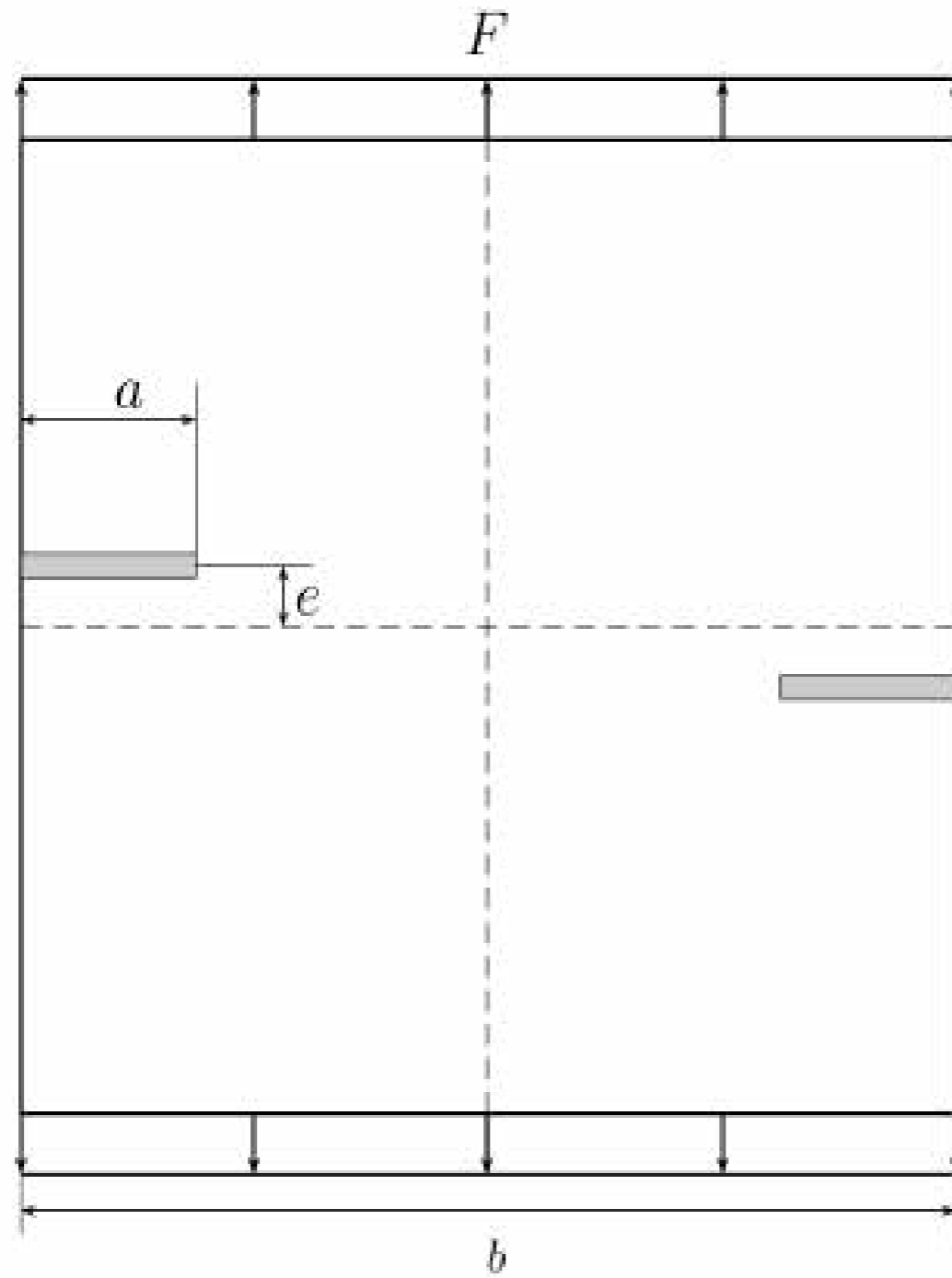
Crack Branching



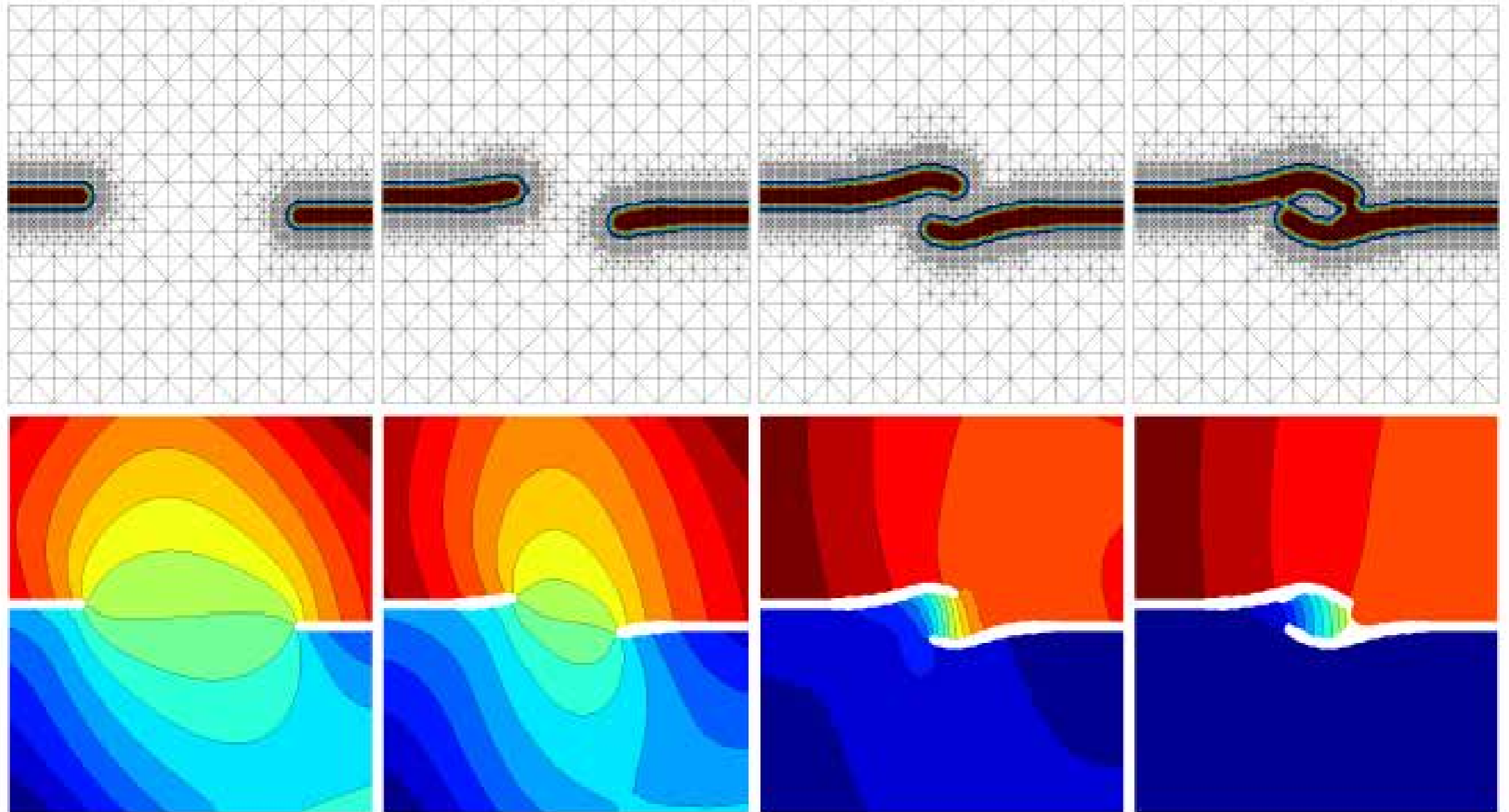
Crack Branching



Cracks coalescence



Cracks coalescence



Brazilian Test

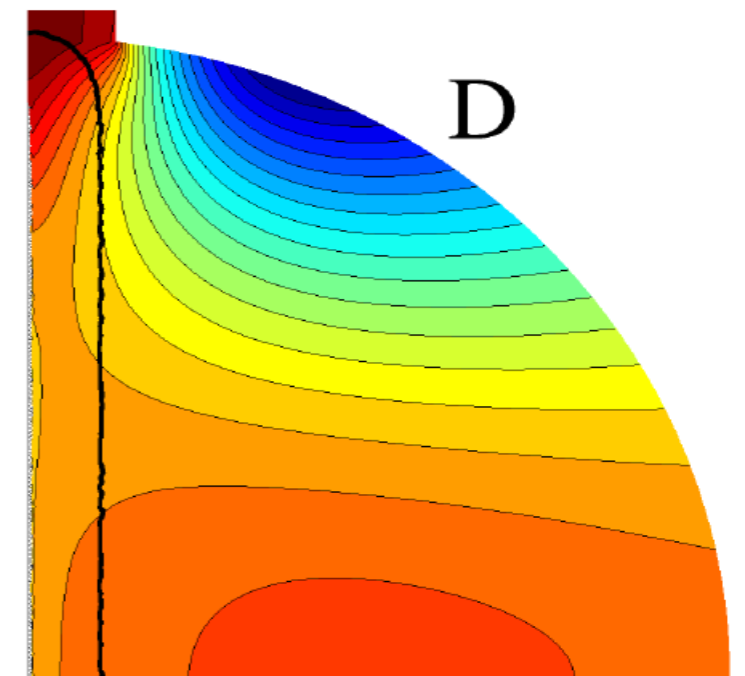
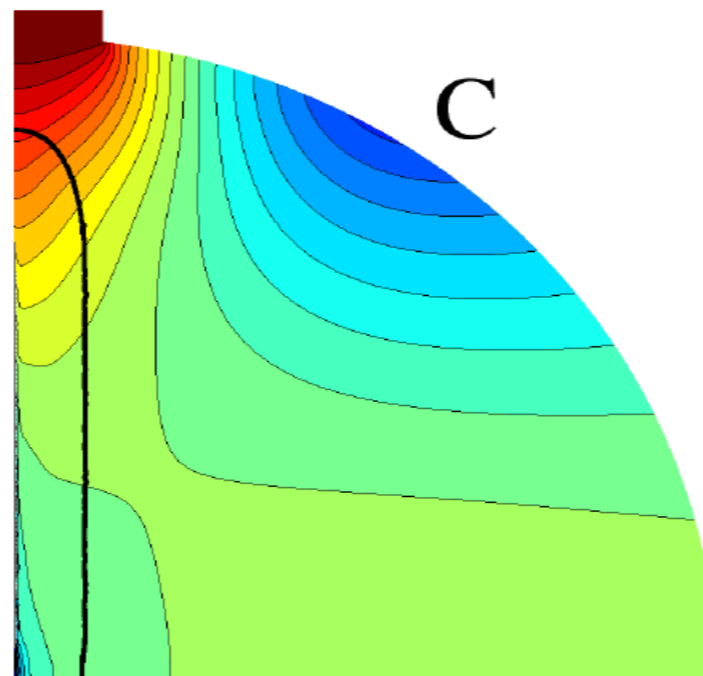
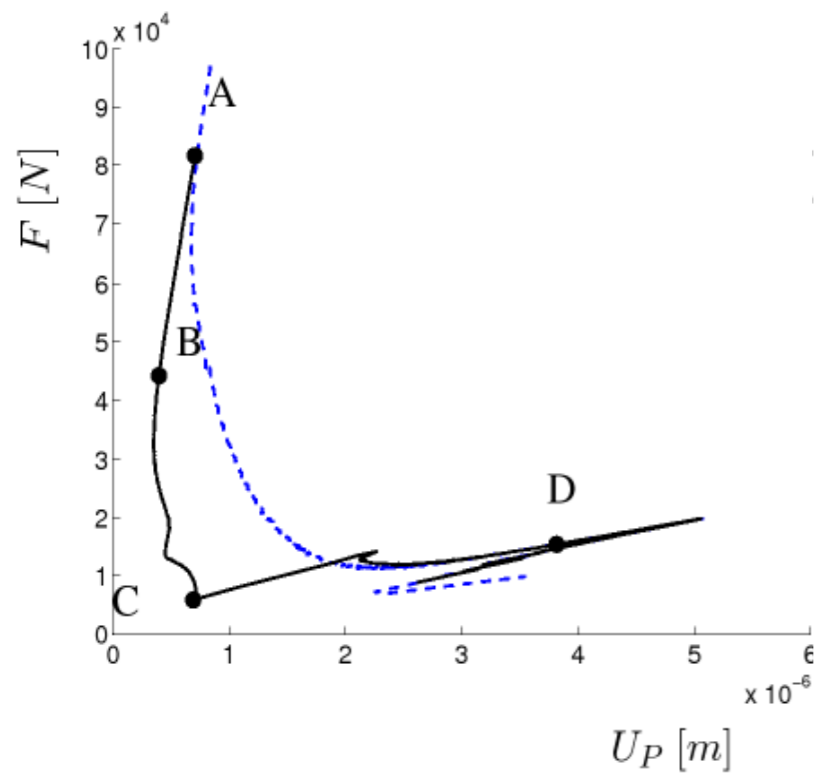
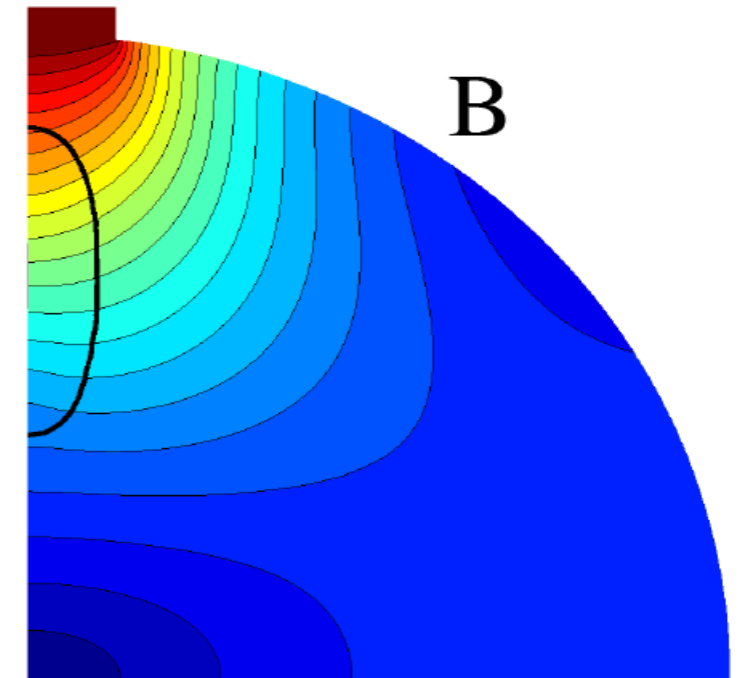
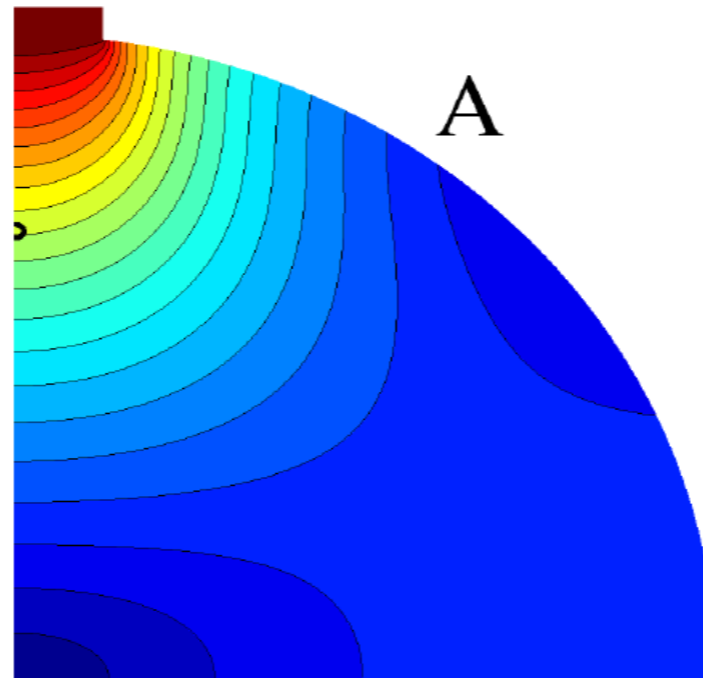
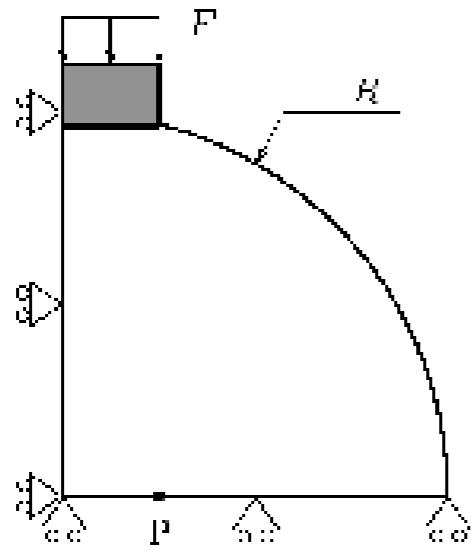
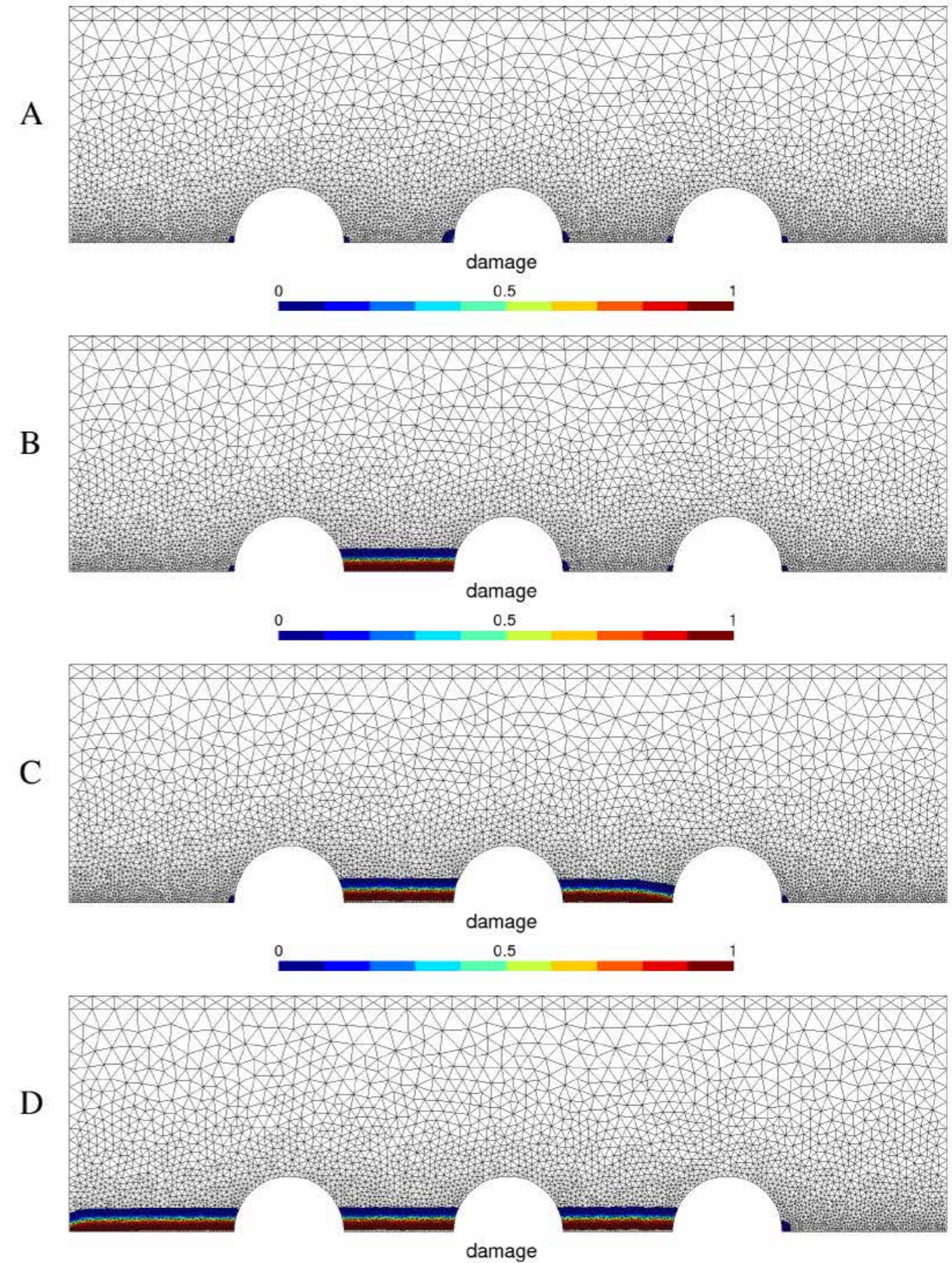
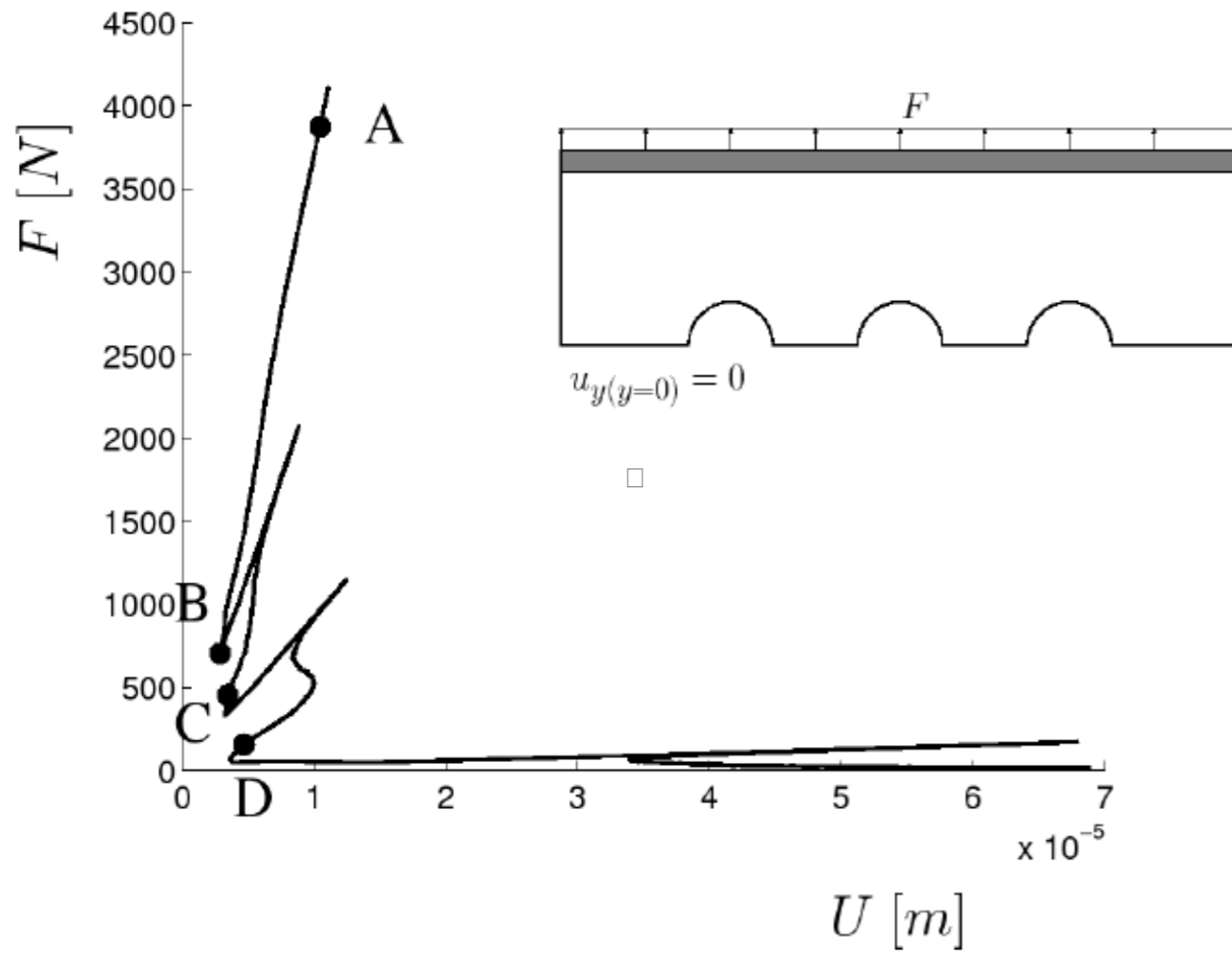
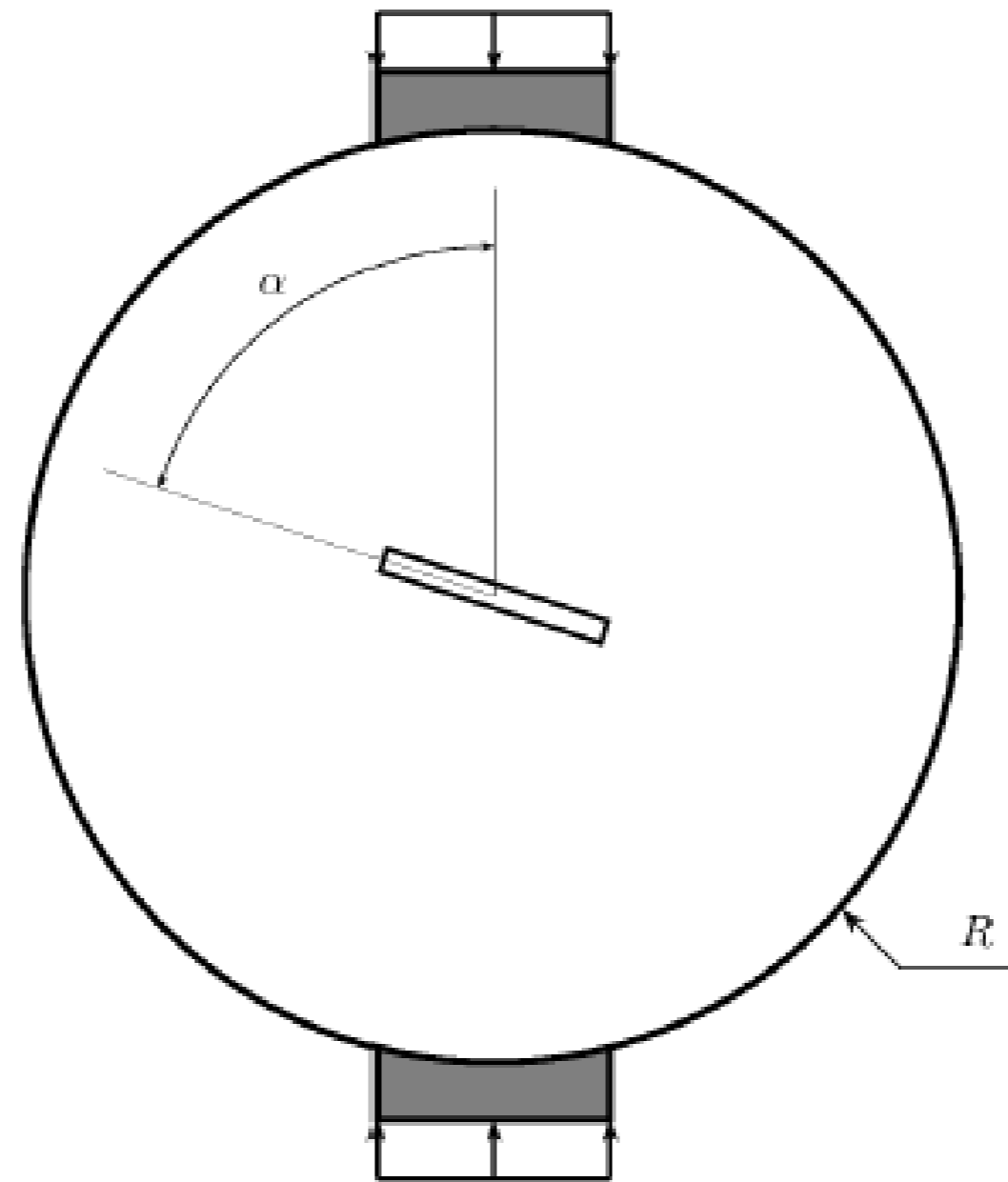


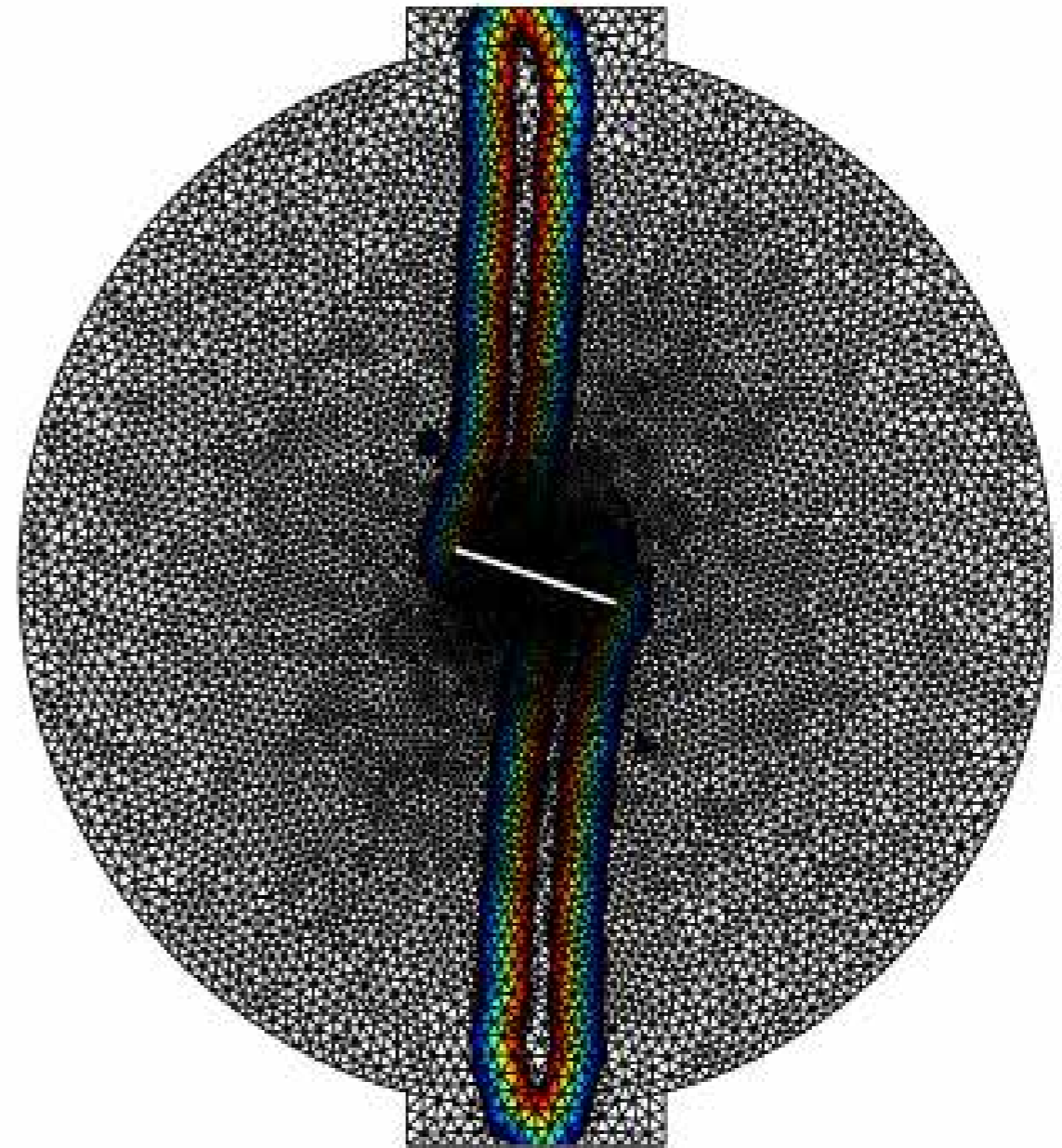
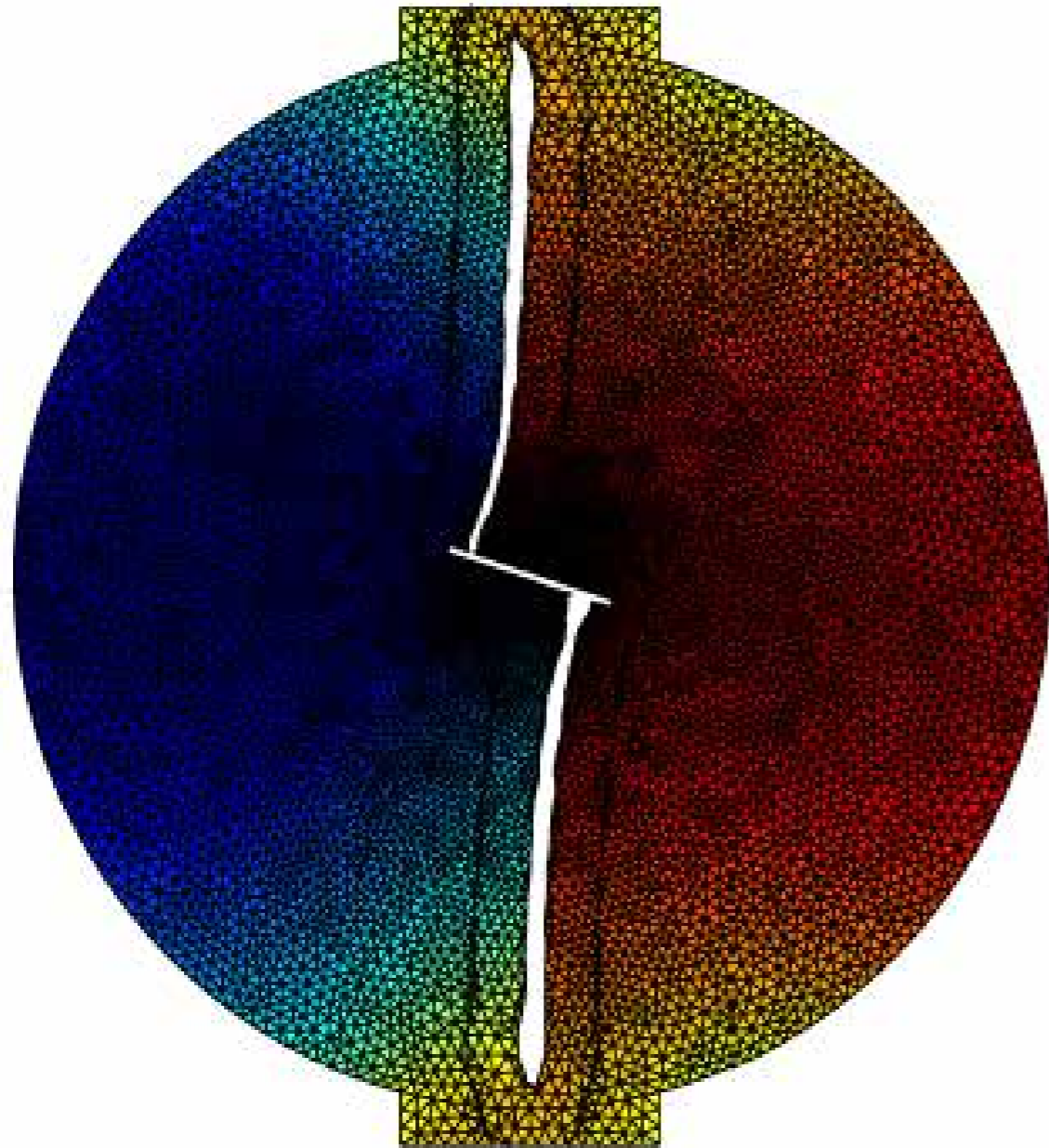
Plate with holes



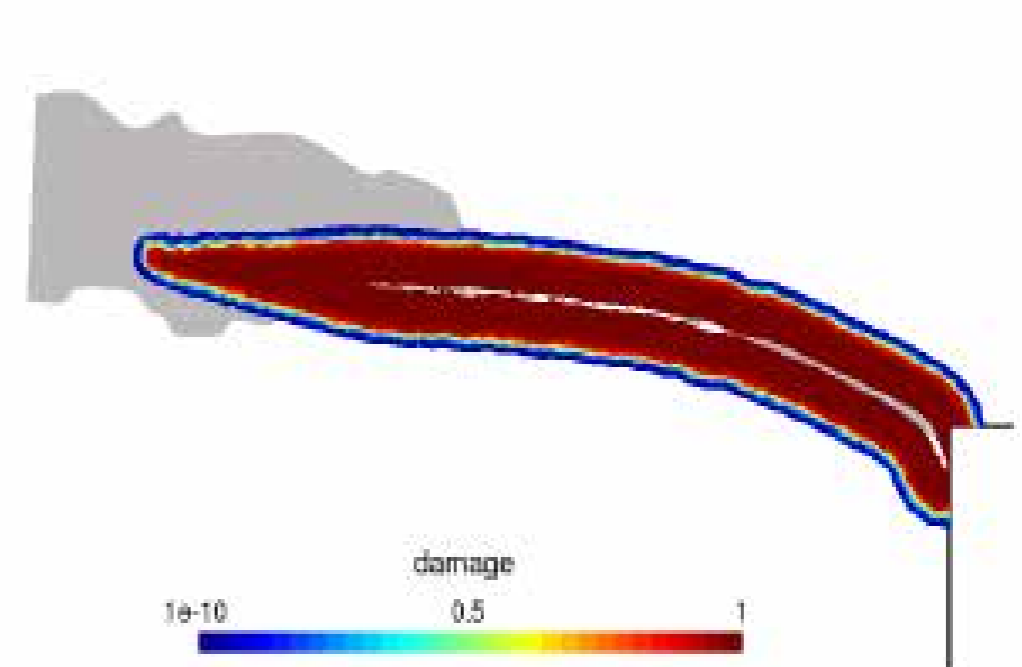
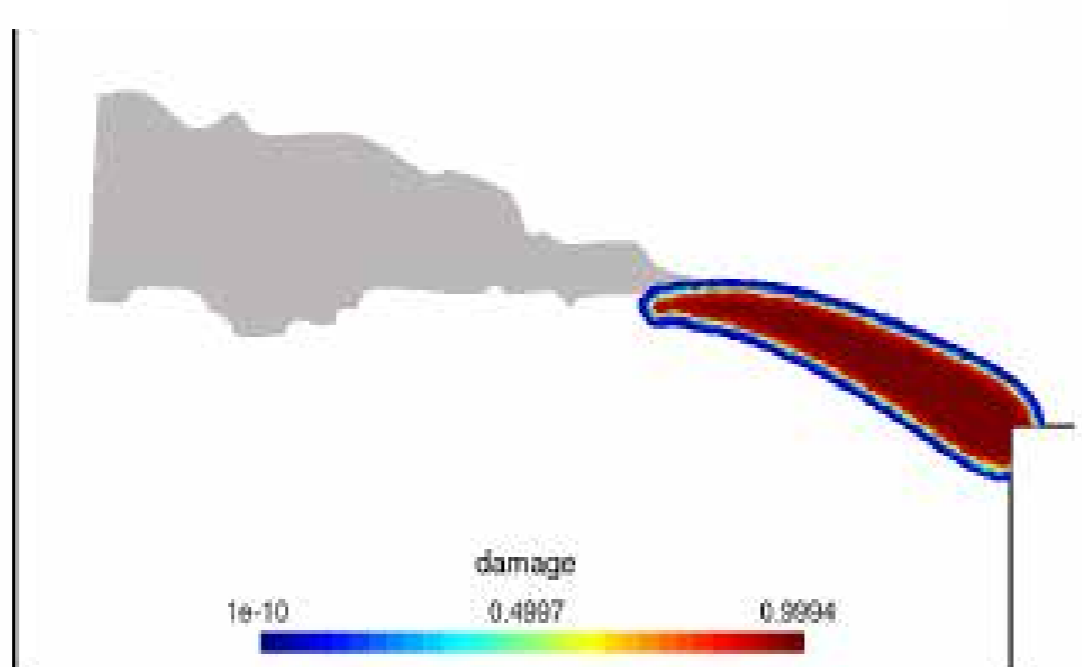
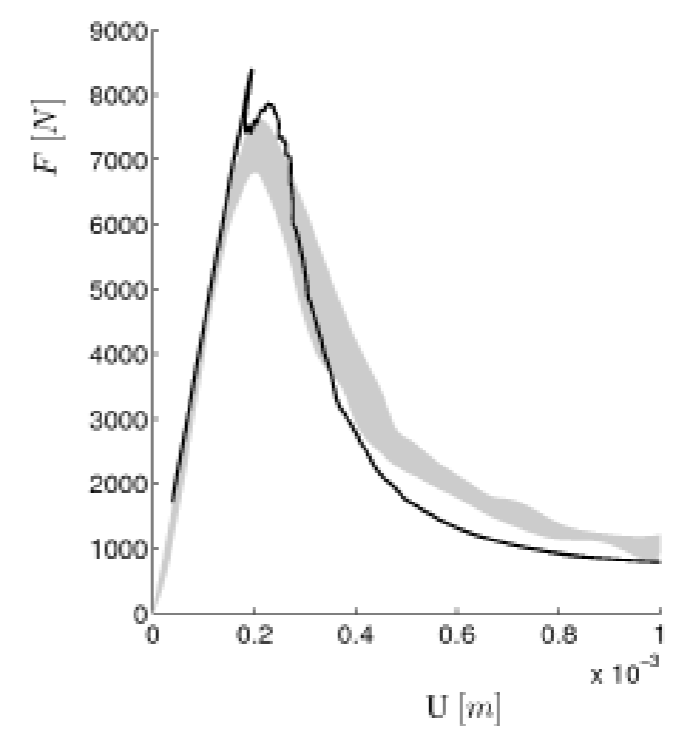
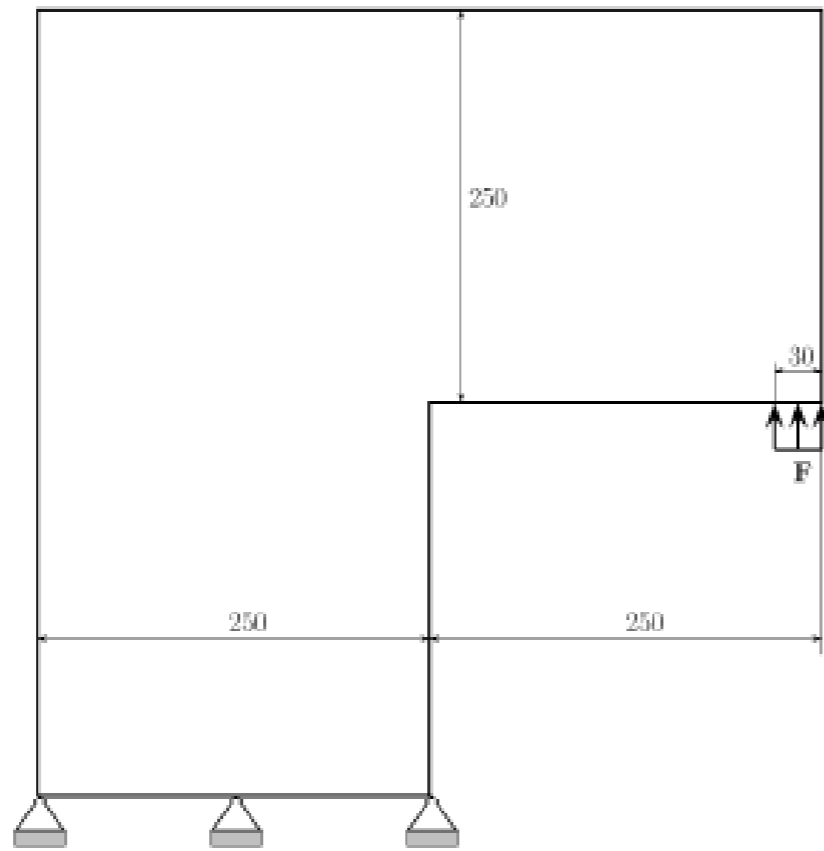
Notched brazilian test



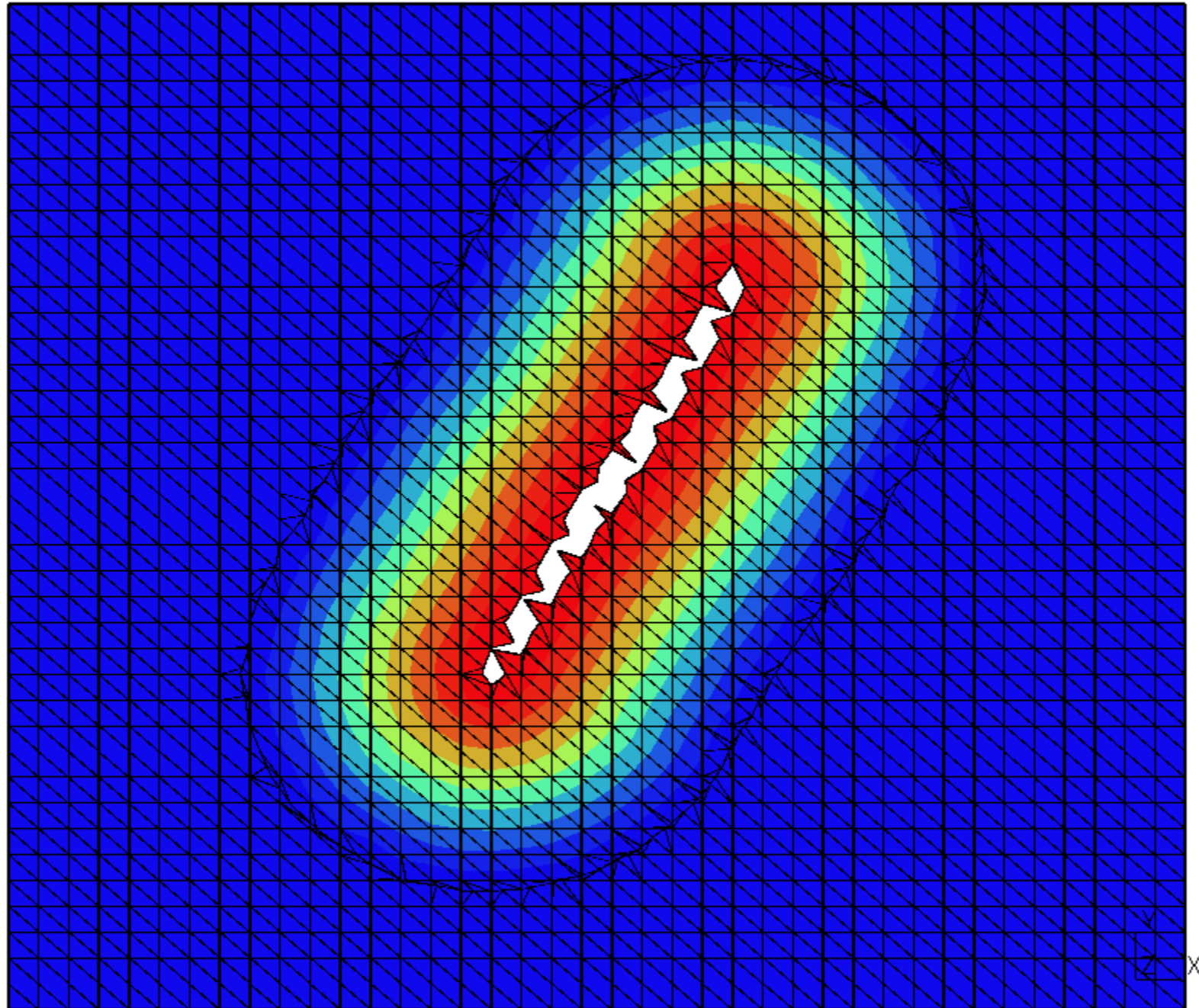
Notched brazilian test



L-shaped plate

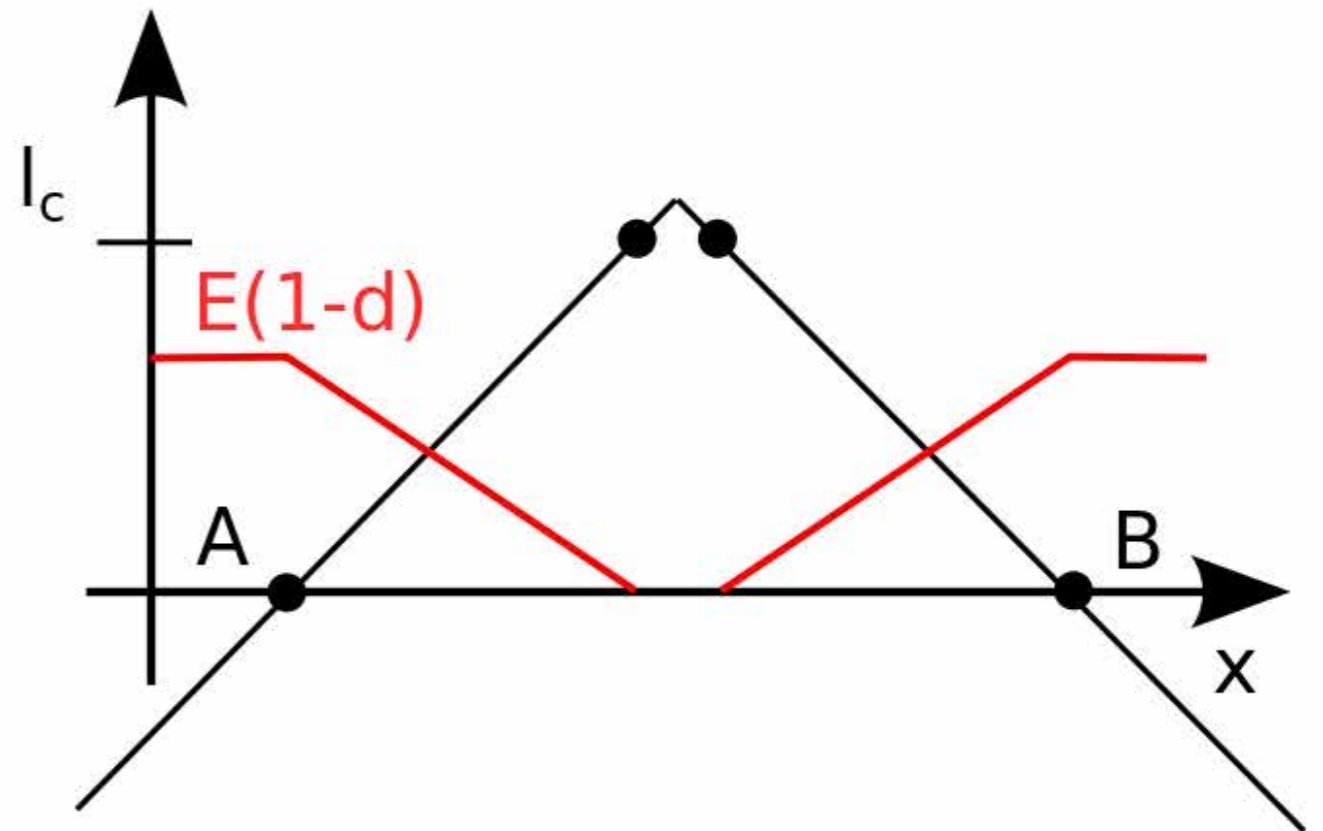
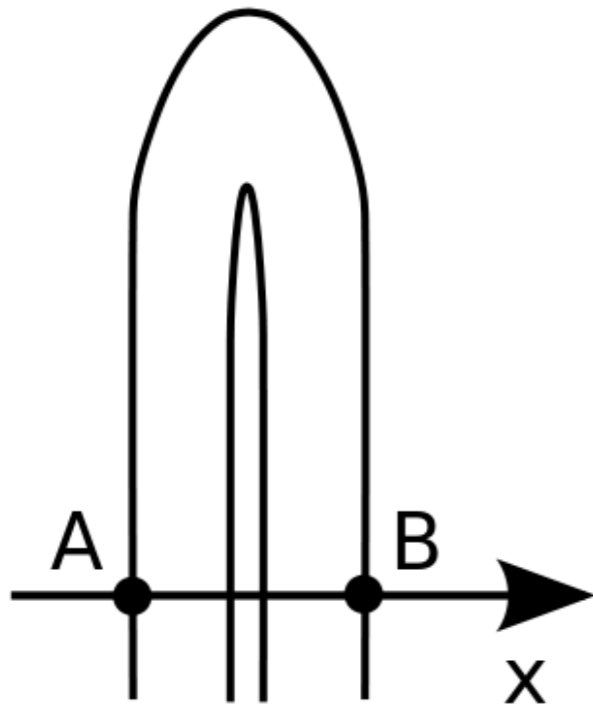


Mesh dependencies : Improving description of iso Ic

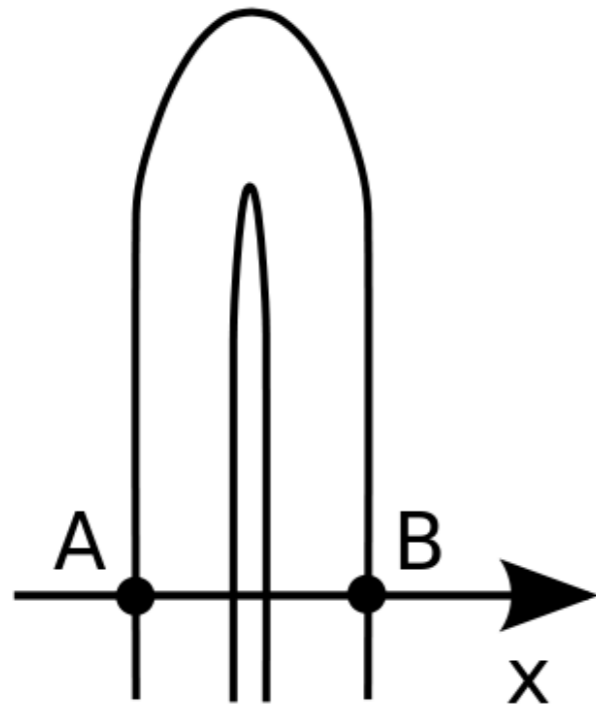


Source of the problem

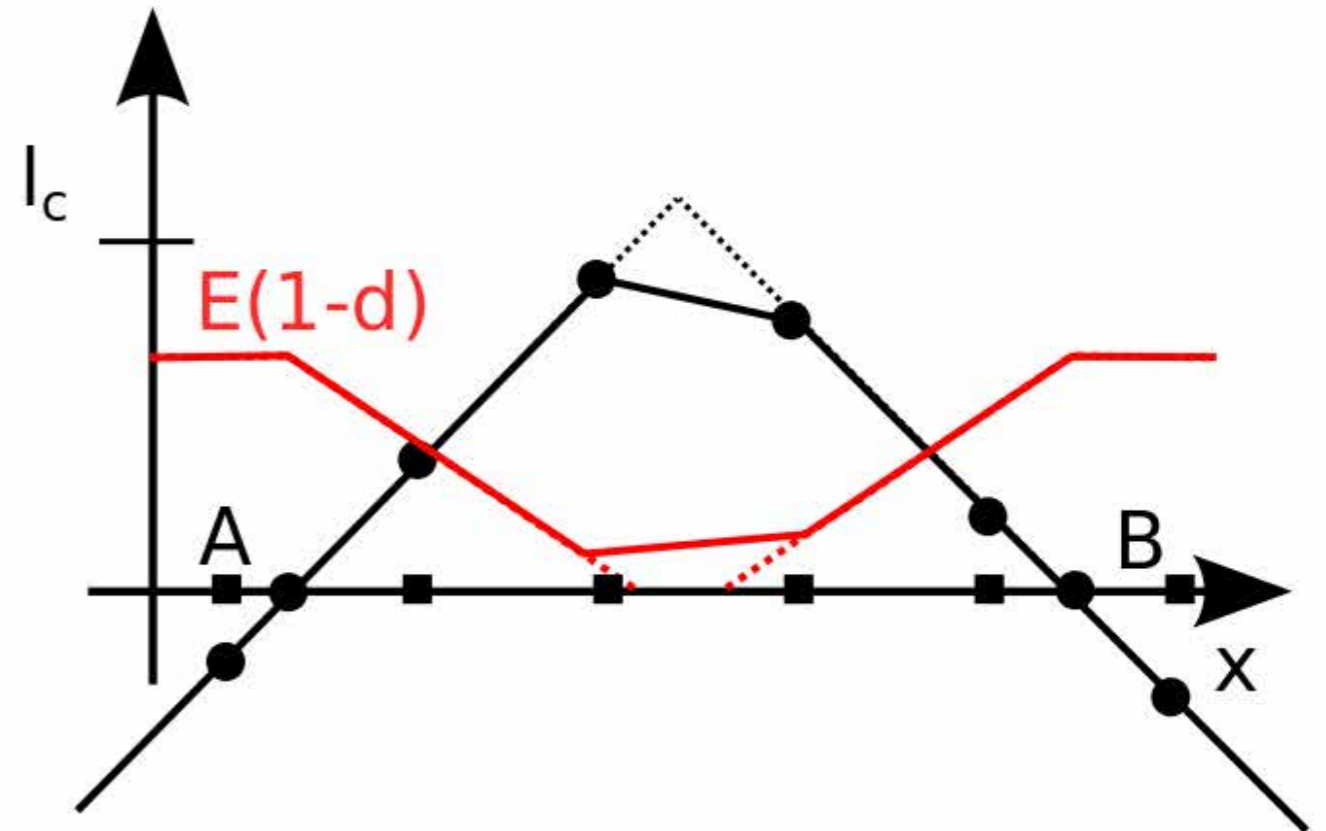
Exact case :



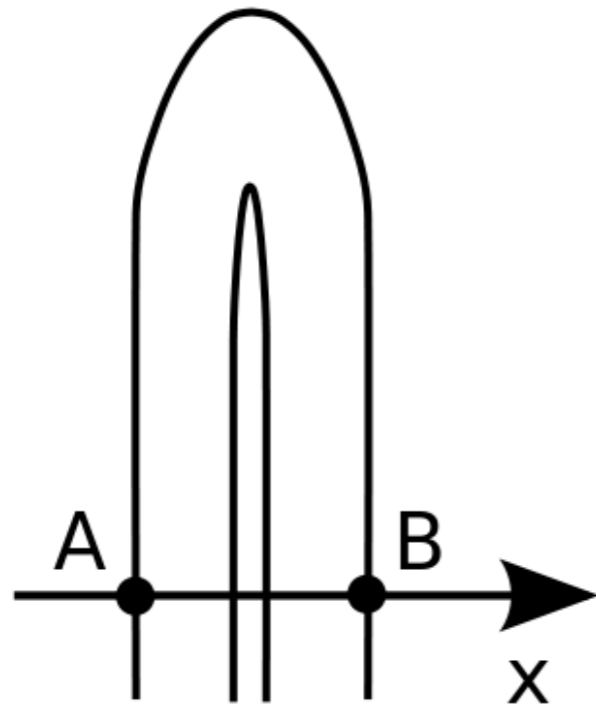
Source of the problem



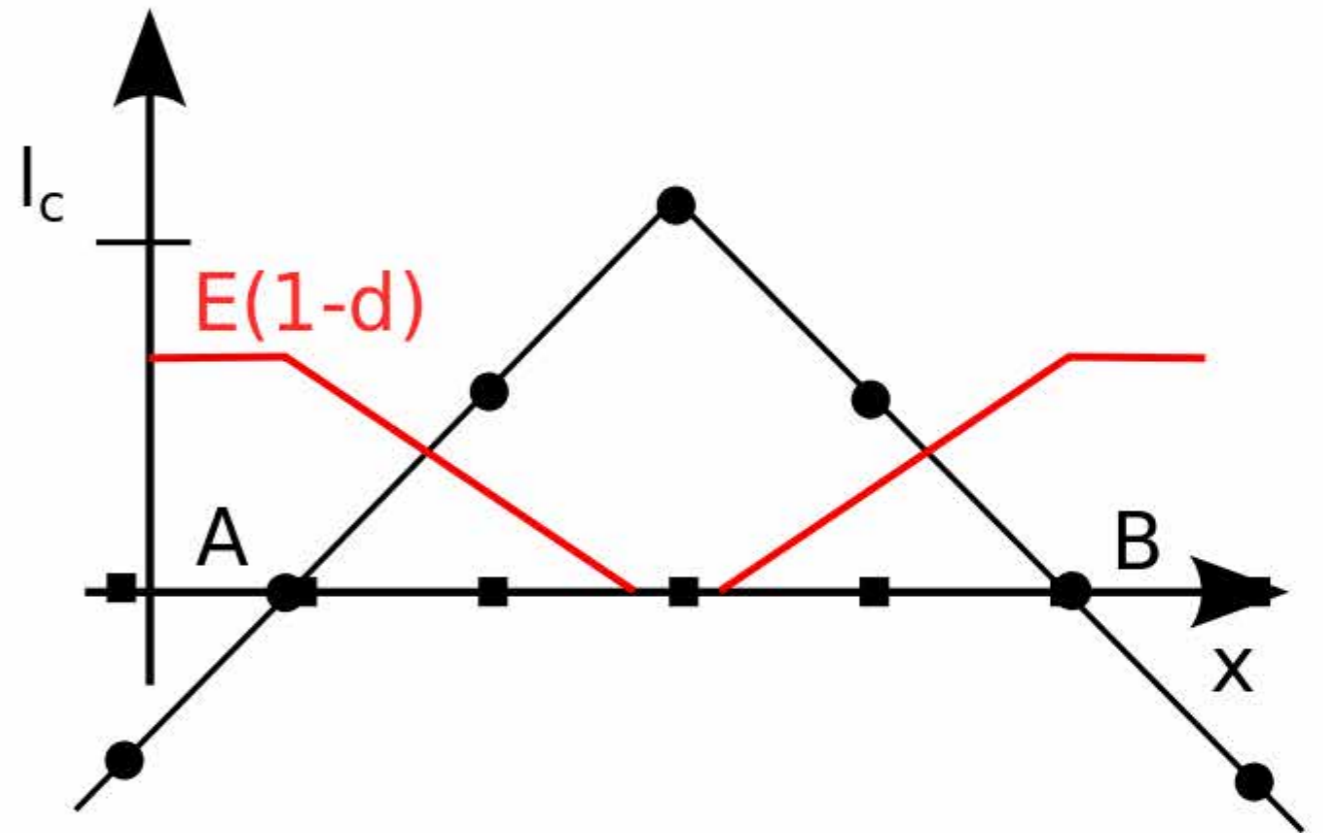
Discrete case :



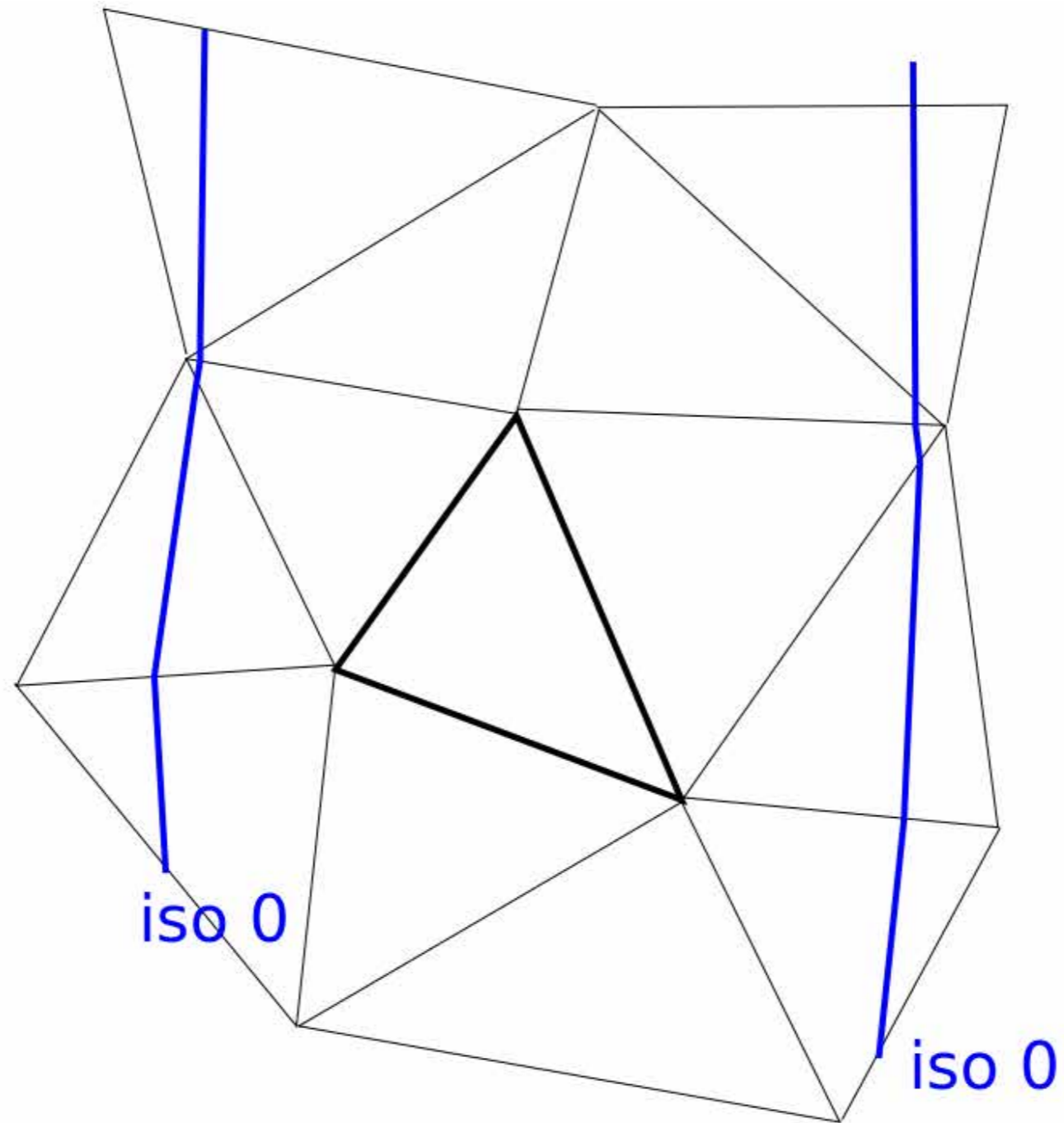
Source of the problem



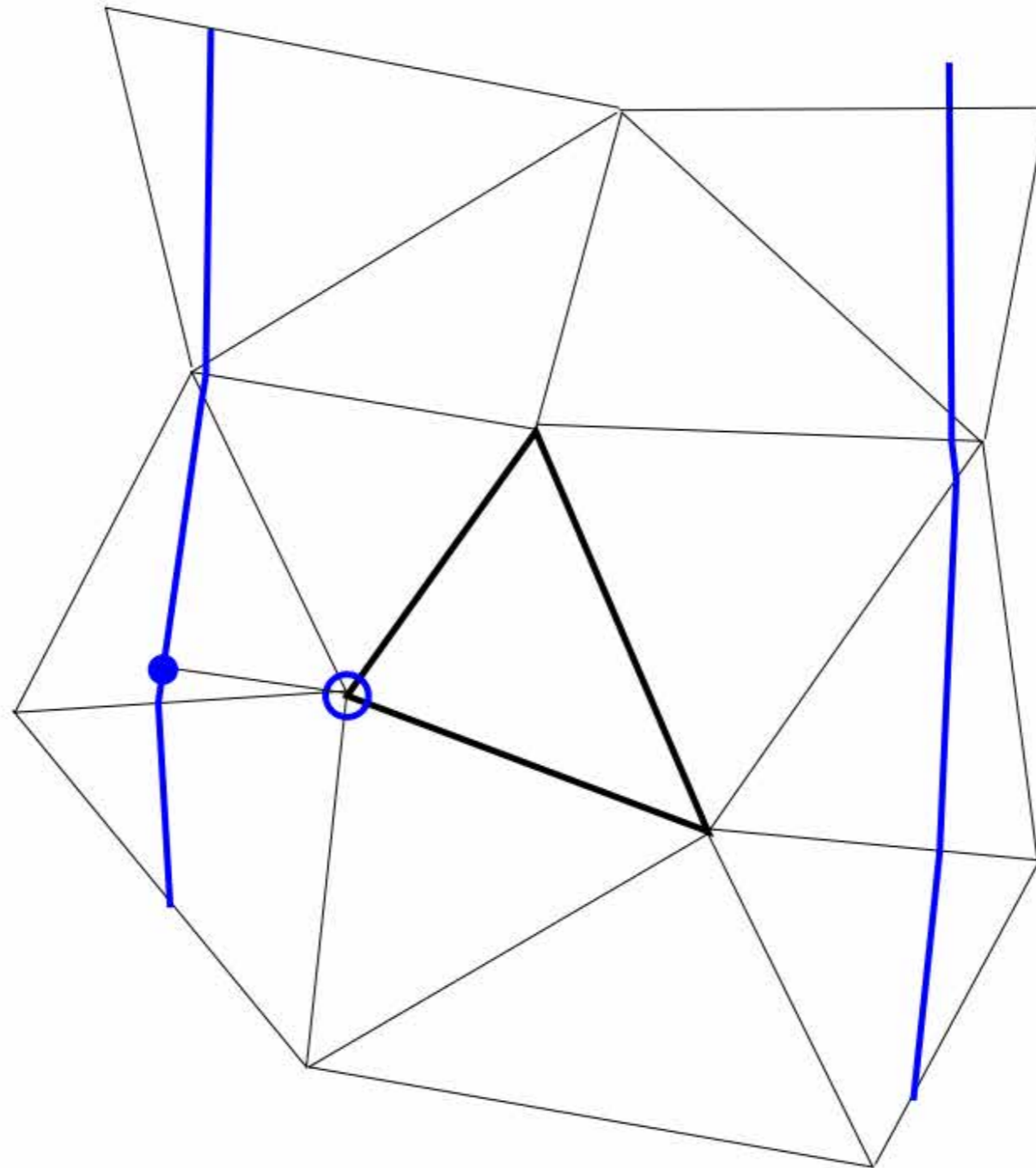
Discrete case :



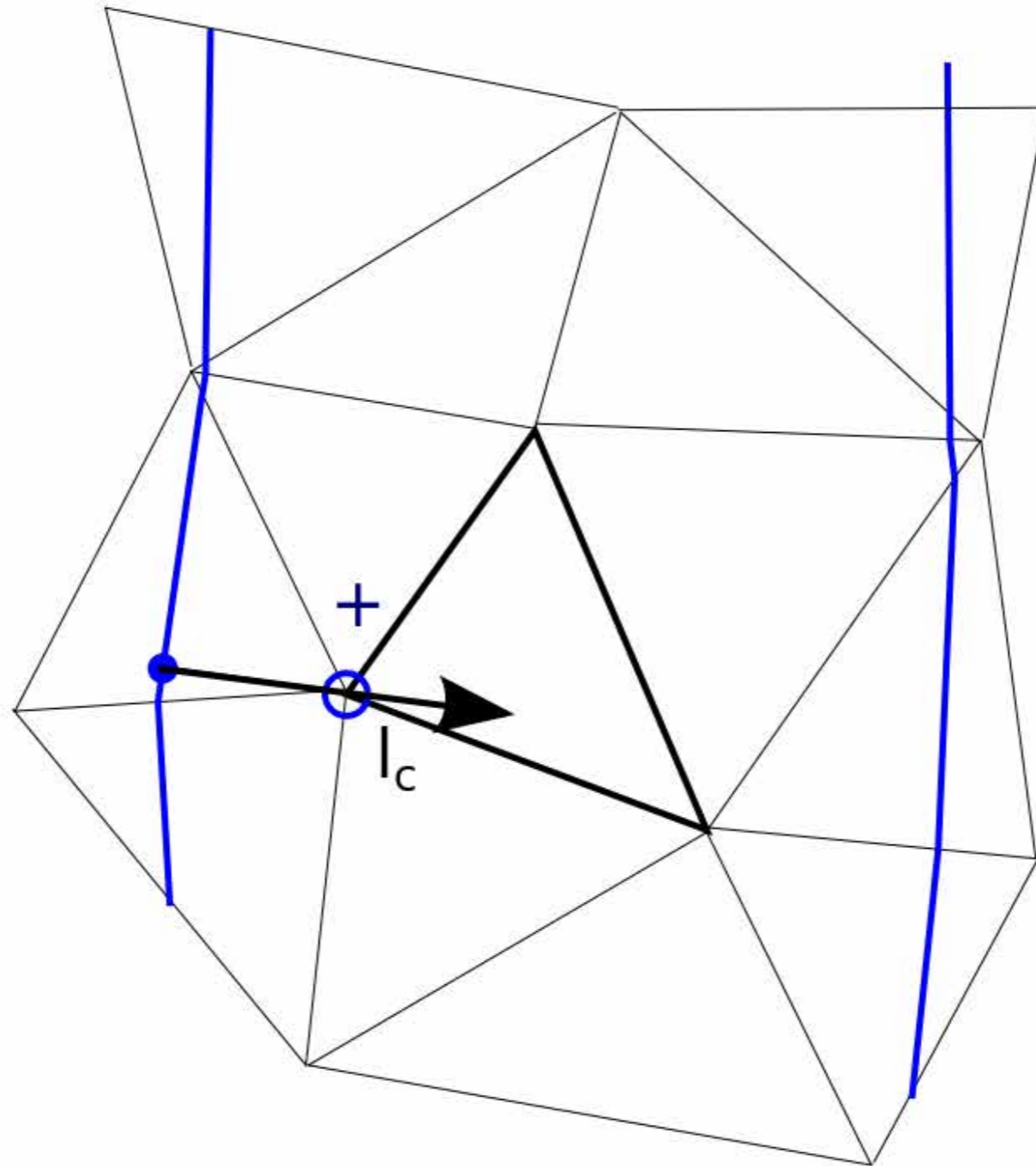
Double cut Algorithm



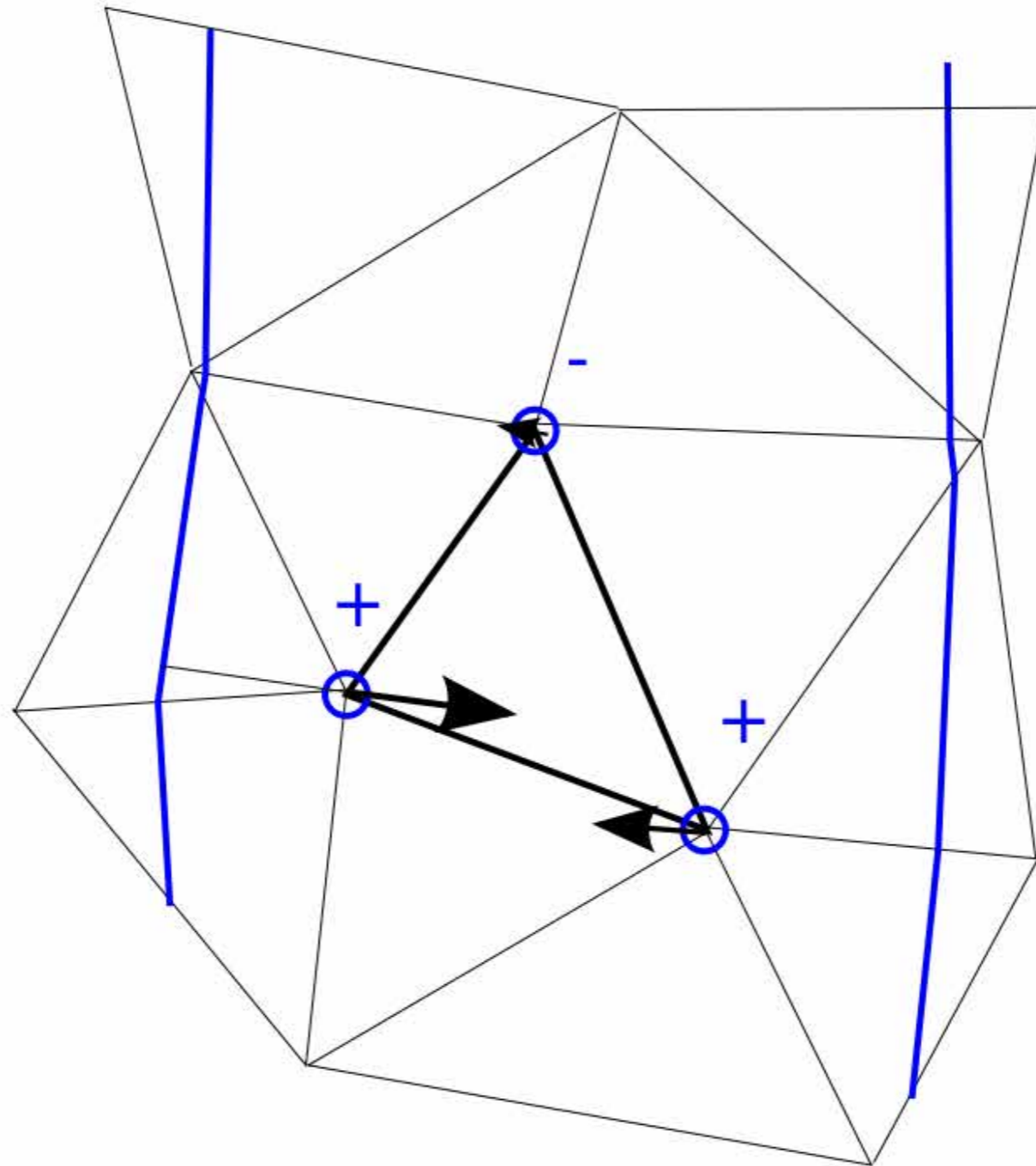
Double cut Algorithm



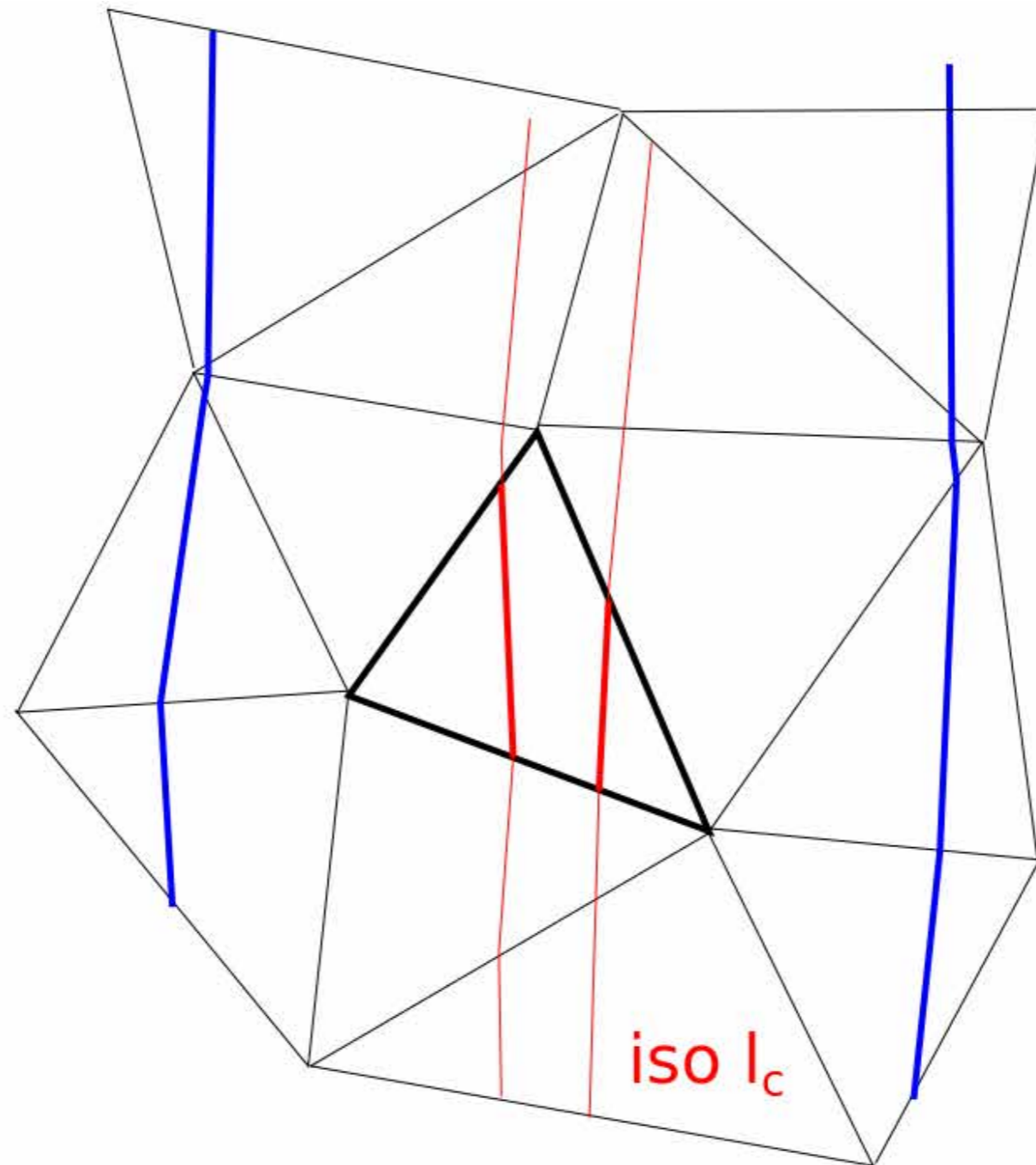
Double cut Algorithm



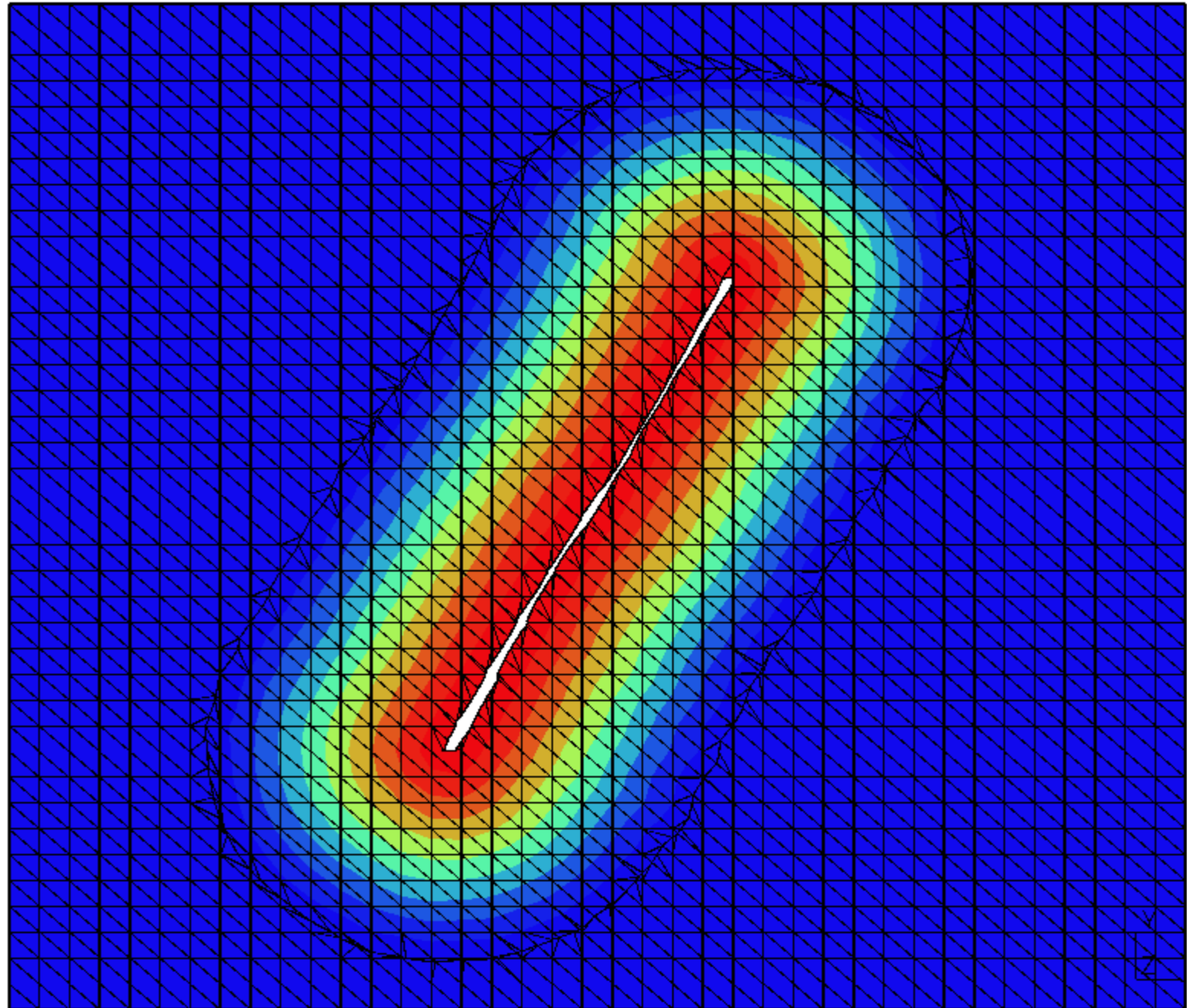
Double cut Algorithm



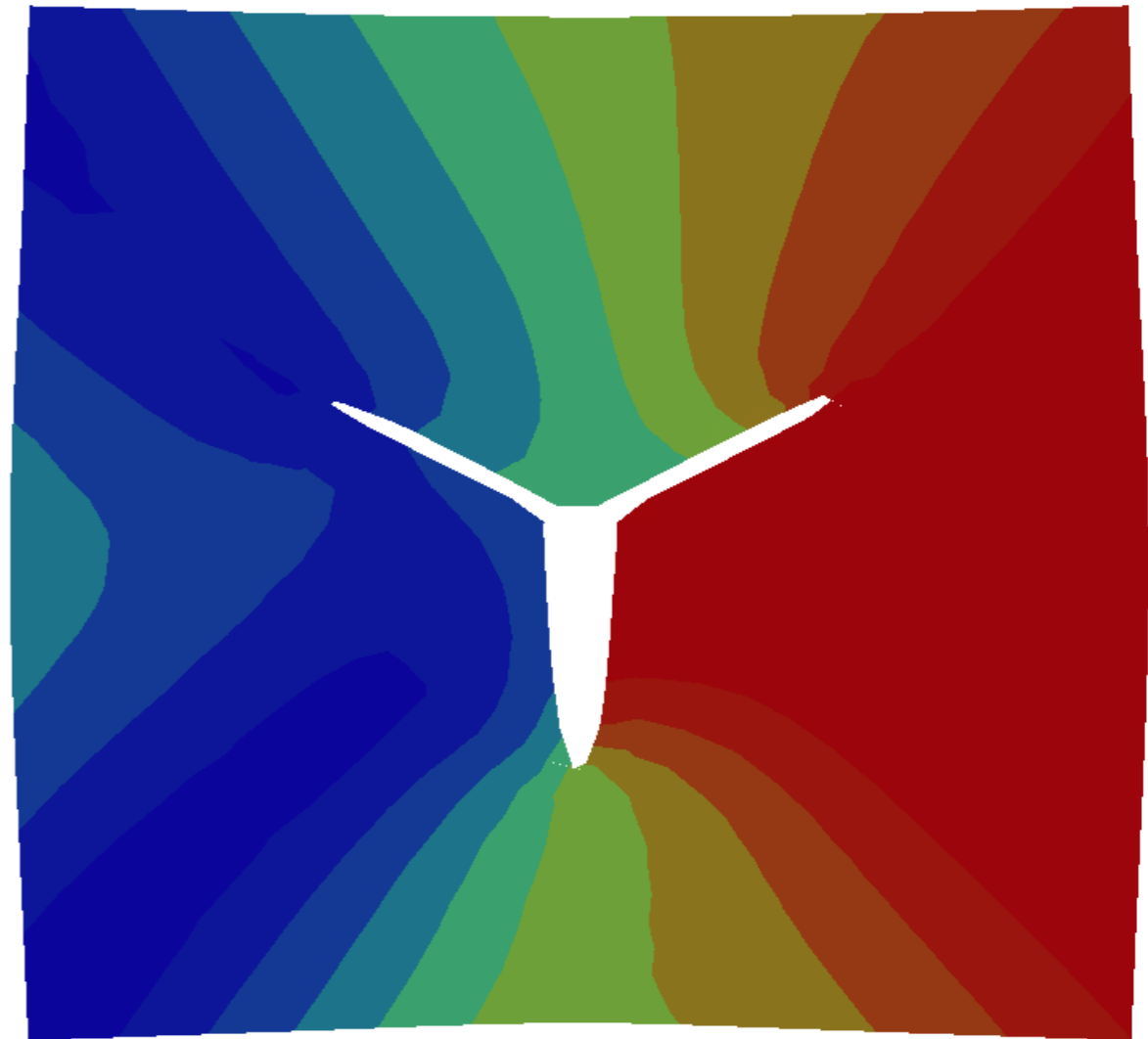
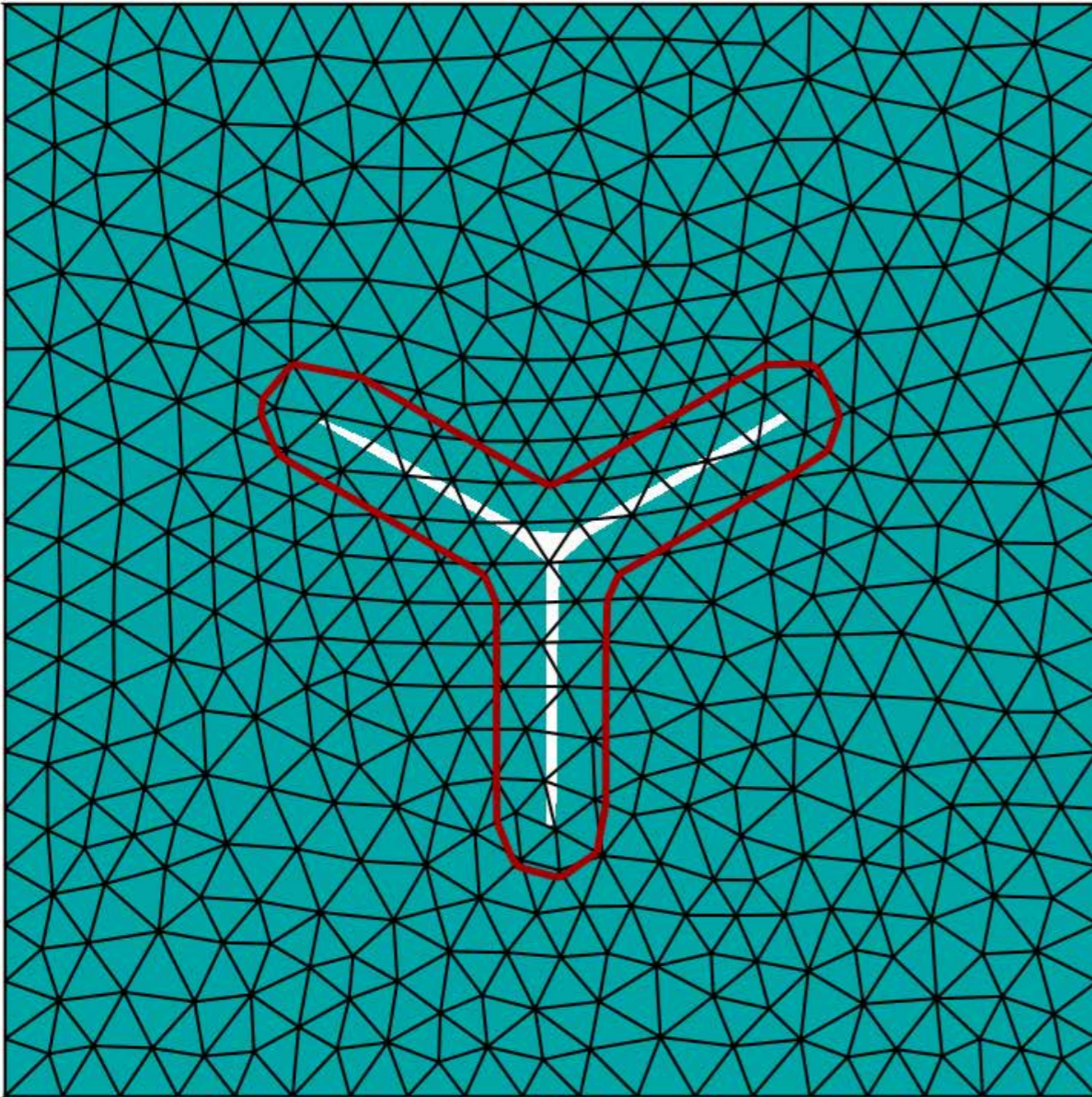
Double cut Algorithm



Double cut Algorithm

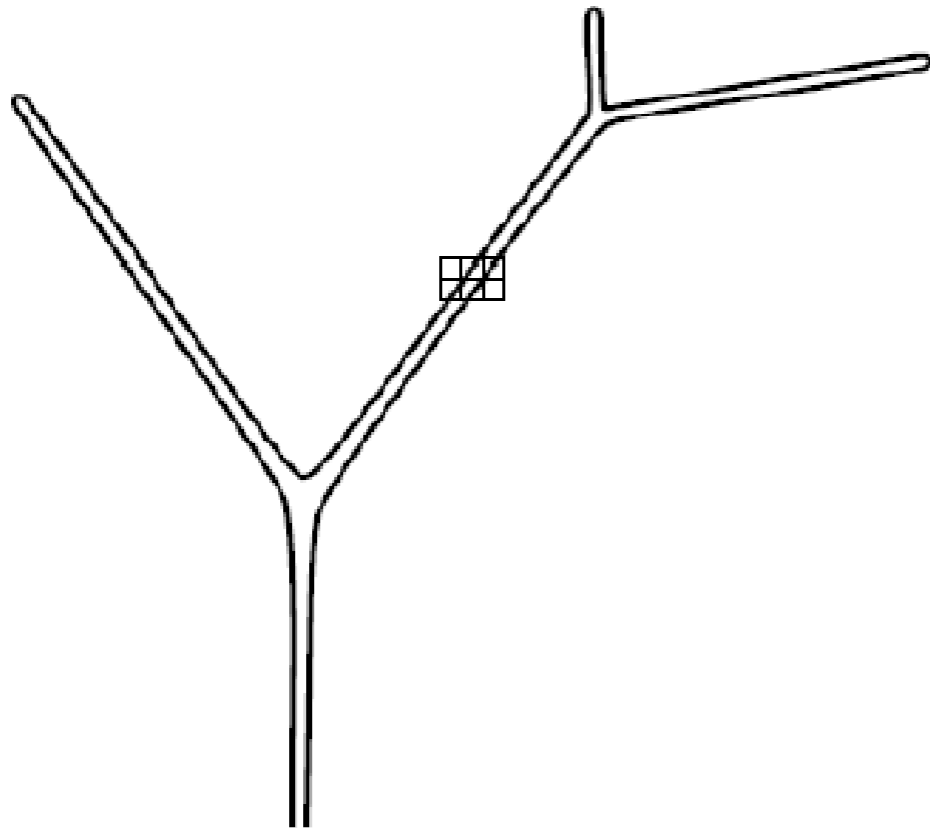


Double cut algorithm : Example

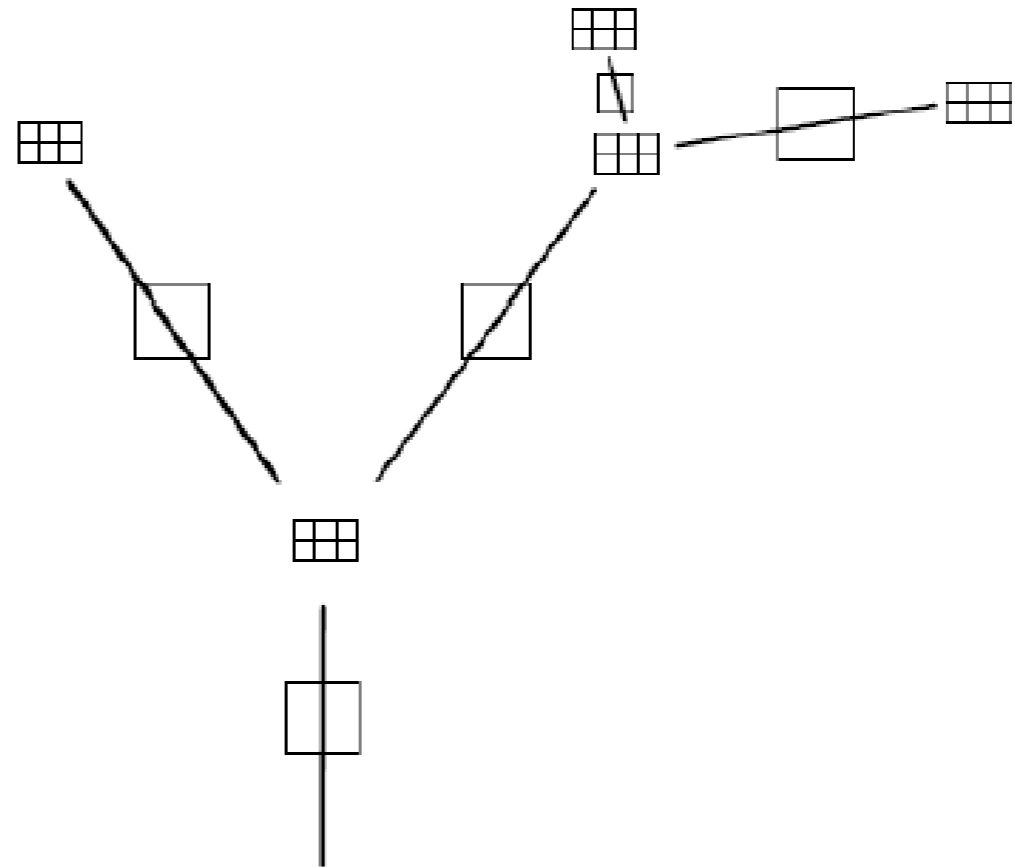


Futur work : Mesh Derefinement

Fine grid everywhere



Fine grid only where needed



Implementation on the way

Conclusions

- The TLS model fills the theoretical gap between damage and fracture
- TLS is a non-local model in which crack appears automatically
- It shares common feature with fracture mechanics: moving of a geometrical object and also shares feature with damage mechanics: usual damage models may be used
- Non-locality evolves gradually: l goes from 0 to l_c and no thickening of the damage band as it progresses.
- Preservation of duality in the non local quantity

Acknowledgements :

Part of this work has
been funded by the
ERC-XLS
and by the **FNRAE**



European Research Council

Established by the European Commission

References

IJNME, 2011, 36:358-
380

CMAME, 2012, 233-
236:11-27

IJF, 2012, 174:43-60

