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Wind Farm Portfolio Optimization under Network Capacity Constraints

Hélène Le Cadre, Anthony Papavasiliou and Yves Smeers

Abstract—In this article, we provide a new methodology for optimizing a portfolio of wind farms within a market environment, for two Market Designs (exogenous prices and endogenous prices). Our model is built on an agent based representation of a certain number of interacting geographic demand markets, each facing a bilevel program to optimize its production level and bilateral trades with other markets, while anticipating the grid congestion. The Nash Equilibria resulting from this Signaling Game are characterized using Algorithmic Game Theory. The Markowitz Frontier, containing the set of efficient wind farm portfolios, is derived theoretically as a function of the number of wind farms and of their concentration. Finally, using a case study of France, Germany and Belgium, we simulate the Markowitz Frontier contour in the expected cost-conditional variance plane.

Index Terms—Algorithmic Game Theory, Market Design, Renewable Capacity Investment, Markowitz Frontier.

I. INTRODUCTION

THE most advanced Market Design in the restructuring of the European (EU) electricity market is Market Coupling. As several other electricity markets, Market Coupling is organized as a two tiered system with a day-ahead market followed by a real-time market or EU type balancing system. The day-ahead market is run by a Power Exchange totally independently of balancing that is run by Transmission System Operators (TSOs). European power exchanges operate as zonal markets that ignore Kirchoff’s laws and assume no congestion within zones. Zonal models can lead to dispatch that violates transmission constraints. Counter-trading is required in order to redispatch the system such that transmission constraints are not violated. In their models, Smeers et al. aggregated nodes into zones and ignored Kirchoff’s laws [27]. They determined the flows over interconnections and the total amount of power production (and consumption). The clearing of their zonal market followed by counter-trading might be sub-optimal because there is no congestion anticipation in the day-ahead market and no representation of the uncertainty associated with the integration of renewable supply.

Market Coupling relies on a separation of the energy market (the power exchanges) and the transmission system organized

by the TSO [27]. Moreover, Market Coupling is progressively moving from a decentralized to a more centralized organization, whereby national power exchanges are cleared simultaneously with an implicit auctioning of transmission capacity in order to capture the efficiencies of pooling diverse resources from different regions. This trend may become more and more relevant with increasing wind power penetration, which is a result of both European and national policies [15], [23].

Oggioni et al. compared the effect of two wind policies (“priority dispatch” under which the TSO must accommodate all wind energy produced and the “no priority dispatch” under which the TSO can decide not to inject all potential wind power in the grid in order to limit congestion problems) in a context of Market Coupling [23]. The authors showed, using stochastic programming models depending on the different wind penetration levels, that “no priority dispatch” removes most of the problems resulting from Market Coupling. However, the relevance of this conclusion relies on the strong assumption that the power exchanges and the TSOs are perfectly coordinated among zones. While Oggioni et al. focused on the day ahead modeling, Nair et al. explicitly characterized the impact of growing wind power penetration, assuming that conventional energy may be procured in three stages (i.e., day ahead, intra day and real time) to balance supply and demand [21]. Our model extends the approach of Nair et al. by taking into account the European Market Coupling and characterizing the optimal investment policy in renewable energy supply capacity, which is becoming a major consideration for utilities that are subject to aggressive renewable portfolio standards. Furthermore, we model TSOs interactions thereby capturing the competition among national energy markets which has not been considered so far.

Accurate short-term forecasts of wind farms power output over the next few hours to days are important factors for secure and low cost operations of power systems with high wind power penetration [20], [24]. According to Girard et al. [8], it is difficult to quantify the economic benefit of improved forecasting. The recent literature dealing with the placement of wind turbines concludes that the aggregation of wind farms can produce significant effects in terms of variability and cost reduction [8] since forecast errors might compensate each other. Furthermore, a portfolio of wind farms is likely to give better results in terms of the trade-off between cost and profit and its variability than relying on a single wind farm [9]. Considering both problems of wind farm expansion and optimal wind farm portfolio generation, Girard et al. checked, using real data from Western Denmark, that power producer

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H. Le Cadre is with MINES ParisTech, PSL Research University, CMA - Center for Applied Mathematics, CS 10207 rue Claude Daunesse, 06904 Sophia Antipolis Cedex, France e-mail: helene.le_cadre@mines-paristech.fr

A. Papavasiliou and Y. Smeers are with CORE, Center for Operations Research and Econometrics, Catholic University of Louvain, Belgium e-mails:{anthony.papavasiliou,yves.smeers}@uclouvain.be.

revenue is linear with respect to the wind farm capacity factor and that the accuracy of short-term wind power forecasts has only a very small impact on revenue. However, as the authors explained, their results do not quantify the benefit of predictability from a global system point of view. Adopting a more systemic approach, Green showed, using 18 years of hourly wind speed data coming from 120 sites around Great Britain, that careful market analysis is needed if investors are to build optimal portfolios of wind stations [9]. Baringo and Conejo already made the link, dealing with the optimization of a strategic wind power investor which sells wind power in a two tiered market (including a day-ahead and a balancing market) [1].

The main contribution of our work is an analytical treatment of the strategic positioning of suppliers in two tiered electricity markets, where real-time conditions are dictated by the uncertainty of renewable energy supply. A by-product of this analysis is the characterization of the optimal investment policy in renewable energy supply capacity, for suppliers acting as investors in wind power. The resulting installations are driven largely by aggressive renewable portfolio standards that are imposed on utilities; as is happening in the US.

According to the literature mentioned above, a careful (simplified) modeling of Market Coupling is the most crucial modeling aspect. We consider a certain number of geographic demand markets, described in Section II. Since we want to characterize the agents' general behaviors, we do not consider explicitly Kirchoff's laws and aggregate the supply and the demand at the market level. The originality of our approach relies on its capability to cope with competition among suppliers that are operating in spatially separated electricity markets, which was ignored in the previously cited models. After having defined the agents' roles in Subsection II-A, we assume that a bilevel optimization problem occurs over each geographic market in the day ahead. Its timing is described in Subsection II-B. It is based on the anticipation of what will happen in the real time market. The game is solved by backward induction. The link between day-ahead and real-time markets is guaranteed by the existence of a forward position in the day-ahead energy market that compensates for the uncertainty of supply in real time. The bilevel optimization problem is solved for two Market Designs: two tiered with exogenous prices in Section III and two tiered with endogenous prices in Section IV. The way this bilevel optimization problem is solved depends on the received signals i.e., which information is shared among the agents. These signals can come from price (real-time prices, in Subsection IV-C) or from quantity (inter-zonal trades, in Subsection IV-D)¹. Efficient wind farm portfolios are then characterized on the basis of the Markowitz Frontier definition. Its exploration is detailed in Section V. Contrary to traditional approaches that ignore the occurrence of rare events when performing risk minimization, the Markowitz Frontier is defined in a context of rare events, guaranteeing the robustness of the wind farm distribution. Illustrations based on real wind speed and energy consump-

tion data for France, Germany and Belgium are provided in Section VI.

SPECIFIC NOTATION

Parameters:

t_f	Day-ahead market time of occurrence
t_0	Real-time market time of occurrence
N	Number of geographic demand markets
$\kappa_{i,j}$	Equivalent capacity between markets i and j
\mathcal{C}_i	Market i cluster set

Probability density functions:

$\Gamma(\alpha)$	Gamma function evaluated in α
$\Gamma_{inc}(\alpha, \chi)$	Incomplete Gamma function evaluated in α, χ
$\mathcal{N}(0; \sigma^2)$	Gaussian density function centered in 0 and of standard deviation σ

Market i setting:

p^f	Day-ahead price
p_i^0	Real-time price
U_i	Supplier expected cost
Π_i	Conventional generator expected profit
s_i^f	Supply of conventional in the day ahead
s_i^0	Supply of conventional in real time
c_i^f	marginal cost of conventional at t_f
c_i^0	marginal cost of conventional at t_0
α_i^f, b_i^f	marginal cost parameters at t_f
α_i^0, b_i^0	marginal cost parameters at t_0
q_i^f	market i demand of conventional at t_f
q_i^0	market i demand of conventional at t_0
r_i	Day-ahead position
α_i	Average wind production of a single wind farm
γ_i	Number of wind farms
θ_i	Concentration of the wind farms
$\lambda_{i \rightarrow j}^f$	Bilateral trade between market i and j at t_f
$\lambda_{i \rightarrow j}^0$	Bilateral trade between market i and j at t_0
S_i	Sum of market i trades in real time
S_i^+	Sum of market i exports in real time
d_i	Market i total demand in real time
\hat{d}_i	Day-ahead forecast of market i total demand
v_i	Forecast error made on the demand prediction
σ_i^v	Demand forecast error standard deviation
w_i	Wind power produced in real time
\hat{w}_i	Day-ahead forecast of wind power production
ϵ_i	Forecast error on wind power prediction
Δ_i	Difference between wind power and demand forecast errors
σ_i^e	Wind power forecast error standard deviation
σ_{Δ_i}	Forecast error difference standard deviation
$\tilde{\epsilon}_i$	Forecast error on a single wind farm
$\tilde{\sigma}_i^e$	Single farm forecast error standard deviation
\bar{F}_{Δ_i}	Complementary cumulative distribution function associated with Δ_i

Strategic wind capacity investment problem:

T	Investment problem horizon
$J(i, T)$	Market i supplier long-term cost
$RV\text{ar}(i, T)$	Market i supplier conditional variance

II. THE MARKET MODEL

We consider suppliers (distributors or utility companies) with long term contracts for renewable energy. Given such

¹It is classical to categorize non-cooperative games, on the basis of strategic variables, among games based on quantity (Cournot) and price (Bertrand).

a long term contract, the suppliers participate in a two tiered market for conventional energy production. It consists of a day-ahead market occurring at $t_f > 0$ and of a real-time market, occurring at $t_0 > t_f$; meaning that t_0 occurs after t_f . It is worth mentioning that we do not address bilateral forward contracting but adopt a standard assumption in the literature focusing on two-settlement systems [1], [19].

In the European Union (EU), the real-time markets introduced in this article can be assimilated to the EU balancing system [8], [14] where positive and negative deviations are treated asymmetrically. Positive deviations receive no real-time payments, whereas negative deviations pay the real-time price. This reflects an uninstructed deviation penalty for discouraging an unscheduled over-supply of renewable power in real time, and is consistent with existing market practices. For example, in ERCOT a wind resource generating more than 10% above its expected base point is charged for the deviation based on real-time prices. In our model, this charge is in fact equal to the real-time price itself². Asymmetric settlements of real-time deviations have also been modeled in the literature [19].

Market design cannot be separated from the physical transmission constraints resulting from the country interconnection [9]. The super grid is the network backbone enabling the power flow exchanges between the $N \in \mathbb{N}^*$ interconnected markets. The interconnection among the markets is performed through $L \in \mathbb{N}^*$ links. Each link $l = 1, \dots, L$ is defined by its Available Transmission Capacity (ATC) which can be reduced due to losses occurring through the transmission lines or due to reliability considerations. Losses are all the more important as renewable sources of production are often located at the extremities of the power network [13], as in the case of offshore wind power installations.

Inside the economic system formed by the power markets, bilateral trades occur among the markets. We let $\lambda_{i \rightarrow j}^f$ (resp. $\lambda_{i \rightarrow j}^0$) be the bilaterally traded flow of energy between market i and market j in the day-ahead market (resp. real-time market). A negative sign indicates an import from j to i , whereas a positive sign indicates an export from i to j . Throughout the article, we will use the following conventions: $(\lambda_{i \rightarrow j}^0)_+ \triangleq \max\{0; \lambda_{i \rightarrow j}^0\}$ and $(\lambda_{i \rightarrow j}^f)_+ \triangleq \max\{0; \lambda_{i \rightarrow j}^f\}$ and the simplifying notations: $S_i \triangleq \sum_{j \neq i} \lambda_{i \rightarrow j}^0$ and $S_i^+ \triangleq \sum_{j \neq i} (\lambda_{i \rightarrow j}^0)_+$. The power transfer distribution factor (PTDF) matrix, which depends non linearly on the impedances of the transmission lines, enables the linear scaling of the bilateral trades among the markets to physical flows along lines [10], [22].

Market Coupling clears energy and transmission in the day ahead with an "implicit auction" of transmission capacity [10], [22], and is followed by intra-day trading and real-time balancing. Commonly, markets having smaller prices export to markets having higher prices, until a common price is reached or congestion occurs due to limited ATCs at the interconnections [6], [13], [27]. If congestion occurs between market i and market j , they are split; otherwise, they are coupled through a unique integrated market. Congestion management remains

a controversial issue in the restructured European electricity sector. Congestion occurs when the infrastructure constrains transactions, thereby influencing the long term evolution of generation and consumption [6].

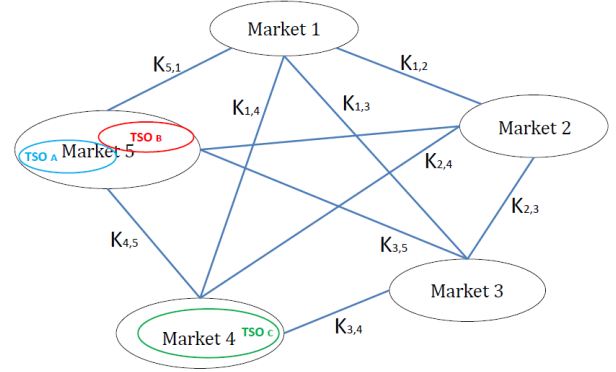


Fig. 1. Equivalent interconnection capacity between the split markets ($N = 5$).

Our network model is based on a virtual representation of the European area where each node coincides with a geographic³ demand market containing one or more TSOs and is fully interconnected with other geographic demand markets. We give an illustration of this representation in Figure 1 for five demand markets ($N = 5$). This representation of the European area is based on two abstractions: firstly, we assume that inside each demand market, the balancing markets (managed by the TSOs) are coupled whereas they are split with the $N - 1$ other demand markets; secondly, the equivalent interconnection capacity between any couple of demand markets corresponds to the aggregation of the capacities of the paths linking supply nodes in one market to demand nodes in the other market. This representation is consistent with the French Energy Transport Operator (RTE) ambition to simplify European grid modeling by partitioning the grid in zones (equivalent, here, to geographic demand markets) and by defining an equivalent network between these zones [4]. As in the classical ATCs/PTDFs model, the interconnections among the virtual geographic areas are limited by line capacity. We introduce the equivalent interconnection capacity between market i and market j : $\kappa_{i,j} \in \mathbb{R}$ such that $\kappa_{i,j} < +\infty$, $i \neq j$. The equivalent interconnection capacities are currently being evaluated by RTE using tests performed throughout the super grid [4]. According to Hutcheon and Bialek's representation of the copper plate [11], for any $i, j = 1, \dots, N$, $i \neq j$, $\kappa_{i,j}$ can be either positive or negative; furthermore, we have the relation: $\kappa_{i,j} = -\kappa_{j,i}$. Following this virtual representation of the European area in N split demand markets, we note that all the equivalent interconnection capacities are congested because, otherwise, the markets which can still export would export until being coupled, leading to less than N split markets. As a by-product, for any couple of demand markets, real-time trade equals equivalent interconnection capacity minus day-ahead trade i.e., $\lambda_{i \rightarrow j}^0 = \kappa_{i,j} - \lambda_{i \rightarrow j}^f$, $\forall i, j = 1, \dots, N$, $i \neq j$.

²Reference source: <http://www.pjm.com/~media/committees-groups/task-forces/irtf/20121022/20121022-item-05-cost-allocation-of-wind-forecasting-tool-and-operating-reserves.ashx>

³The term "geographic" will be understood in the rest of the article.

A. The agents

There is a certain number of geographic demand markets, each characterized by a price insensitive⁴ and random demand. This demand is unknown in the day ahead but revealed in real time. Similarly, wind generation in each geographic market is price insensitive and random: it is unknown in the day ahead and revealed in real time. We now describe the different categories of agents interacting over each demand market.

- Suppliers (distributors or utility companies) deliver energy to consumers characterized by their aggregated demand. They are price takers in the first Market Design (MD 1) detailed in Section III. In contrast with standard assumptions, they are not price takers in the second Market Design (MD 2), described in Section IV. In this latter Market Design, they are aware that their decisions modify prices and take that knowledge into account to minimize their procurement cost. Since consumers are exposed to a retail price that is unrelated to wholesale market conditions, we assume an inelastic consumer demand whose real-time level is uncertain in the day-ahead time frame.
- Conventional energy producers are characterized by their supply function. There is conventional generation in each market. Marginal costs are higher in real time than in the day ahead for a given output level. We will assume that generators do not exercise market power, and suppliers buy electricity at marginal cost.

Finally, we assume that the suppliers are driven by renewable portfolio standards to invest in renewable capacity. Suppliers have to decide how to compose an optimal wind farm portfolio for participating in the electricity market.

B. Timing

We make the assumption that a clearing price is reached at t_f ; the analysis of the implications of relaxing this assumption is an interesting topic of future research. Because the transfers are limited by the equivalent interconnection capacity, it will be harder to align the market prices at t_0 . By anticipation of what may happen in real time, a certain number of scenarios can be envisaged:

Congestion anticipation: Congestion occurs and the markets are then split in $N \in \mathbb{N}^*$ geographic areas. Note that, as already mentioned, two or more TSOs can be grouped in the same geographic area.

Coupling anticipation: There is no congestion and the markets are coupled. The mathematical analysis of this scenario will not be detailed in this article; all analytical and numerical details can be found in [15].

We now describe the bilevel program faced by each market. Over each market i , at time instant t_f :

⁴We do not consider demand side management in the present article. Models dealing with decentralized demand response integration through distributed learning approaches can be found in [14]. In [16], the end user's demand is price responsive and storage is possible either at the end users' level, for instance through the battery of their electric vehicles, or at the microgrid aggregator level.

(i) Anticipating what will happen in the real time market i.e., at time instant t_0 , the conventional producers optimize independently and simultaneously the bilateral trades with the other markets so as to maximize their expected profit under equivalent capacity constraints.

(ii) The suppliers optimize independently and simultaneously their purchase of conventional energy so as to minimize their expected cost while ensuring that the total purchased quantity satisfies the residual demand.

Under congestion anticipation of all the equivalent interconnection capacities, the trades at t_f are linked to the optimal trades at t_0 according to the relation: $\lambda_{i \rightarrow j}^f = \kappa_{i,j} - \lambda_{i \rightarrow j}^0, \forall i, j, i \neq j$. The usual way to solve the bilevel problem described above is to proceed by backward induction. Proceeding backwards, the optimization of conventional energy purchases depends on the expectations of the market prices which themselves depend on the bilateral trades concluded by the producers on the power markets. We will explain in more details in Section IV why such games are called Signaling Games (SGs) [28]. Here the signal (the shared information) is based on price but it might also be possible to consider that the game signal is based on quantity. Under this latter assumption, it will be more appropriate to invert Steps (i) and (ii) in the bilevel problem described above. The SG based on quantity will also be solved by backward induction. We prove formally in Subsections IV-C and IV-D that the SG based on solving backwards for quantity coincides with the SG based on solving forwards for price.

Justifying which form of SG occurs in practice depends on which information is shared among the agents i.e., is the information shared based on market prices or on quantities? In the extreme case where no information is shared, the bilevel game becomes a simultaneous game i.e., Steps (i) and (ii) occur simultaneously. In this case, the conventional energy purchase is optimized through the method described in Subsection IV-C and the bilateral trades are optimized using the results established in Subsection IV-D.

C. Description of the markets

Market i is defined by:

- d_i , the end users' total demand of energy at time t_0 . It satisfies the relation: $d_i = \hat{d}_i - v_i$ where \hat{d}_i is the forecast made at t_f of the end users' total demand of energy at t_0 . v_i is a random variable, representing the forecast error made on the demand prediction, and distributed according to a Gaussian density function centered in 0 and of standard deviation σ_i^v : $v_i \sim \mathcal{N}(0; (\sigma_i^v)^2)$.
- w_i , the energy produced at time t_0 by the market renewable energy producers. It satisfies the relation: $w_i = \hat{w}_i - \epsilon_i$ where \hat{w}_i is the forecast made at t_f of the quantity of renewable energy that market i producer will produce at t_0 . ϵ_i is a random variable, representing the forecast error made on the prediction of the renewable production, distributed according to a Gaussian density function centered in 0 and of standard deviation σ_i^ϵ : $\epsilon_i \sim \mathcal{N}(0; (\sigma_i^\epsilon)^2)$. The forecast error on

the production of a *single* wind farm will be denoted $\tilde{\epsilon}_i$. Being consistent with the assumption made on ϵ_i generation, it is distributed according to a Gaussian density function⁵ centered in 0 and of standard deviation $\tilde{\sigma}_i^\epsilon$: $\tilde{\epsilon}_i \sim \mathcal{N}(0; (\tilde{\sigma}_i^\epsilon)^2)$. The relation between ϵ_i and $\tilde{\epsilon}_i$ will be discussed explicitly in Subsection II-E.

- The forecast error vector for wind production and demand: $(\tilde{\epsilon}_i \ \nu_i)^\top$ ⁶ is also supposed to be a Gaussian random vector. According to Sinden [26], wind power output in the United Kingdom (UK) has a weak positive correlation to current electricity demand patterns i.e., $\frac{\mathbb{E}[\tilde{\epsilon}_i \nu_i]}{\tilde{\sigma}_i^\epsilon \sigma_i^\nu} > 0$ ⁷. This implies that $\Delta_i \triangleq \epsilon_i - \nu_i$, which is the difference between renewable production and demand forecast errors, is distributed according to a Gaussian distribution function centered in 0 and of variance $\sigma_{\Delta_i}^2 = (\sigma_i^\epsilon)^2 - 2\mathbb{E}[\epsilon_i \nu_i] + (\sigma_i^\nu)^2$. In the rest of the article, we will let: $\Delta_i \sim f_{\Delta_i} \equiv \mathcal{N}(0; \sigma_{\Delta_i}^2)$; \bar{F}_{Δ_i} will represent the associated complementary cumulative distribution function. Δ_i is supposed to be independent of any $\Delta_j, \forall j = 1, \dots, N, j \neq i$ i.e., the prediction errors made on one geographic market are independent of the ones made on the other geographic markets.
- s_i^f (resp. s_i^0) market i supply of conventional energy in day-ahead (resp. real time) markets.
- $c_i^f(s_i^f) = a_i^f + b_i^f s_i^f$ (resp. $c_i^0(s_i^0) = a_i^0 + b_i^0 s_i^0$) the marginal cost function of conventional energy produced by market i and purchased at t_f (resp. t_0), with $a_i^0 > a_i^f > 0$ and $b_i^0 > b_i^f > 0$ guaranteeing that the marginal cost on the real-time market remains larger than in the day-ahead market for a given output level. Indeed, balancing is often supplied from machines that are reserved on a long-term basis, and these are the older machines. The two supply curves are then different because they come from different machines. If so, the current formulation that separates day-ahead and real-time machines, the latter having a higher marginal cost that the former, may be seen as a reasonable approximation. Our modeling approach is an alternative that facilitates the development of our analysis and approximates non-convexities associated with real-time operations such as the wear and tear of machinery in emergency operation (emergency startup costs, wear due to emergency ramping, etc.).
- q_i^f (resp. q_i^0) market i demand of conventional energy in day-ahead (resp. real-time) markets.

The amounts of energy purchased by market i at t_f and at t_0 are defined as follows: $q_i^f = (\hat{d}_i - \hat{w}_i + r_i)_+$ and $q_i^0 = (d_i - w_i - q_i^f)_+$ where r_i is a reserve of energy purchased in day-ahead (lower cost) market because of uncertainty of supply at t_0 . Reserve r_i is determined by the energy supplier in market i for the consumers' demand d_i to be satisfied at t_0

⁵Other density functions might be considered without adding any changes in the derived theoretical results except in the numerical illustrations where the Gaussian assumption greatly simplified the computations.

⁶ \top stands for the transpose of the vector.

⁷Data analysis ran on 66 onshore weather recording sites for the period 1970 – 2003 in the UK showed a correlation of 0.28 [26]. This is the value that we will use in the simulations.

at the lowest possible cost. Market i knows \hat{d}_i and \hat{w}_i . Hence it is equivalent for the supplier to determine q_i^f or r_i . The hypothesis that $q_i^f > 0$ holds as long as the demand exceeds the average wind capacity. In the rest of the article, we will assume that: $q_i^f \triangleq \hat{d}_i - \hat{w}_i + r_i$.

D. Suppliers' expected cost and producers' expected profits

We define U_i , as the expected cost at t_f , that the supplier has to pay for its end user energy consumption:

$$U_i = q_i^f p^f + \mathbb{E}[q_i^0 p_i^0] \quad (1)$$

We let Π_i be the expected profit at t_f of market i conventional energy producer. It is defined as the difference between the price paid by all the markets for the purchase of conventional energy and the cost of the energy⁸. We assume that all the supply is sold at each time. Then:

$$\begin{aligned} \Pi_i &= \sum_{j \neq i} (\lambda_{i \rightarrow j}^f)_+ p^f + [s_i^f - \sum_{j \neq i} (\lambda_{i \rightarrow j}^f)_+] p^f \\ &\quad - \int_0^{s_i^f} c_i^f(s) ds + \mathbb{E} \left[\sum_{j \neq i} (\lambda_{i \rightarrow j}^0)_+ p_j^0 \right] \\ &\quad + \mathbb{E} \left[(s_i^0 - \sum_{j \neq i} (\lambda_{i \rightarrow j}^0)_+) p_i^0 \right] - \mathbb{E} \left[\int_0^{s_i^0} c_i^0(s) ds \right] \\ &= s_i^f p^f - \int_0^{s_i^f} c_i^f(s) ds + \mathbb{E} \left[\sum_{j \neq i} (\lambda_{i \rightarrow j}^0)_+ p_j^0 \right] \\ &\quad + \mathbb{E} \left[(s_i^0 - \sum_{j \neq i} (\lambda_{i \rightarrow j}^0)_+) p_i^0 \right] - \mathbb{E} \left[\int_0^{s_i^0} c_i^0(s) ds \right] \quad (2) \end{aligned}$$

E. Renewable energy modeling

The renewable wind energy production of market i is a function of the number of wind farms and of their concentration which is characterized by their spatial distribution over market i geographic area. To determine the renewable energy procurement for market i , we use the model of Nair et al. [21]. For market i , we introduce:

- α_i , the average wind production of a single wind farm over the geographic area of market i .
- γ_i , the number of wind farms over market i geographic area.
- $\theta_i \in [\frac{1}{2}; 1]$ (resp. $1 - \theta_i \in [0; \frac{1}{2}]$), a constant capturing the concentration (resp. the scattering) of the wind farm locations over market i geographic area. The more (resp. the less) concentration, the more (resp. the less) correlation there is between the wind farm productions.

We suppose that, at t_f , α_i is the best forecast of wind energy procurement of a wind farm [21]. Then: $\hat{w}_i(\gamma_i) = \alpha_i \gamma_i$. The forecast error will depend on the wind penetration too, and we choose the coefficient θ_i so that $\epsilon_i(\gamma_i) = \gamma_i^{\theta_i} \tilde{\epsilon}_i$ where $\tilde{\epsilon}_i$, as already introduced, represents the forecast error for the production of a single wind farm. We propose the following

⁸The cost of transport of power in the bilateral trades will not be considered since we focus on a market scale.

interpretation for the scaling of θ_i : If the wind farms are co-located they will all produce the same quantity of energy at the same time i.e., their productions are strongly correlated. This is the case when $\theta_i = 1$. This implies in turn that: $\epsilon_i = \gamma_i \tilde{\epsilon}_i$ and that: $\hat{w}_i = w_i + \gamma_i \tilde{\epsilon}_i$. On the contrary, if they are spatially distributed so that their productions are independent from one another i.e., uncorrelated, and under the assumption that the forecast errors are distributed according to Gaussian distribution functions, the Central Limit Theorem tells us that: $\sigma_i^\epsilon = \sqrt{\gamma_i} \tilde{\sigma}_i^\epsilon$ [21]. Therefore, the wind farm productions are independent from one another if, and only if, $\theta_i = \frac{1}{2}$. Note that in case of more general forecast error distribution functions, it can be interpreted as an approximation for γ_i large enough. Finally, in case where $\theta_i \in]\frac{1}{2}; 1[$, the wind farms are randomly located over the market geographic area and their spatial distribution is intermediate between perfect independence and co-location. With these notations, we obtain:

$$\begin{aligned} w_i(\gamma_i) &= \hat{w}_i(\gamma_i) - \epsilon_i(\gamma_i) = \alpha_i \gamma_i - \gamma_i^{\theta_i} \tilde{\epsilon}_i \\ \sigma_i^\epsilon(\gamma_i) &= \gamma_i^{\theta_i} \tilde{\sigma}_i^\epsilon \\ \epsilon_i(\gamma_i) &= \gamma_i^{\theta_i} \tilde{\epsilon}_i \end{aligned}$$

In the rest of the article, for the sake of simplicity, the dependence of w_i , σ_i^ϵ and ϵ_i on γ_i will be omitted.

In the following sections, we derive the suppliers' optimal reserves and real time prices, for two Market Designs.

III. MD 1: TWO TIERED WITH EXOGENOUS PRICES

In this first Market Design, the prices are supposed exogenous and such that:

$$0 < p^f < p_i^0, \forall i = 1, \dots, N$$

Market i supplier's expected cost takes the form:

$$\begin{aligned} U_i &= q_i^f p^f + \mathbb{E}[q_i^0 p_i^0] \\ &= (\hat{d}_i - \hat{w}_i + r_i) p^f + p_i^0 \mathbb{E}[(\Delta_i - r_i)_+] \end{aligned}$$

We solve the bilevel Program described in Subsection II-B by backward induction. In Step (ii), each market i supplier determines independently and simultaneously the quantity of energy to purchase, q_i^f , or, equivalently, its reserve, r_i , so as to minimize its expected procurement cost⁹:

$$\min_{r_i \geq 0} U_i \quad (3)$$

We made the assumption that $r_i \geq 0$ since otherwise this means that a supplier could be short in the day ahead, something that one may find unrealistic given that conventional plants are more expensive in real time.

Derivating market i expected cost with respect to r_i , we obtain: $\frac{\partial U_i}{\partial r_i} = p^f + p_i^0 \frac{\partial}{\partial r_i} \mathbb{E}[\Delta_i - r_i | \Delta_i \geq r_i] = p^f - p_i^0 \bar{F}_{\Delta_i}(r_i)$. Then:

$$\frac{\partial U_i}{\partial r_i} \Big|_{r_i=r_i^*} = 0 \Leftrightarrow r_i^* = \bar{F}_{\Delta_i}^{-1}\left(\frac{p^f}{p_i^0}\right) \quad (4)$$

⁹The reserve is used by the suppliers to compensate for their forecast errors in the real time. This can be compared with a forward position in the day-ahead energy market.

Since $\frac{\partial^2 U_i}{\partial r_i^2} = p_i^0 f_{\Delta_i}(r_i) > 0$, it coincides with a minimum for U_i . Furthermore, market i 's optimal reserve being independent of the other markets' optimal reserves, Equation (4) leads to a unique Nash Equilibrium for the market reserves.

Note that inverting the bilateral game steps leads exactly to the same optimal reserve since it depends exclusively on exogenous market prices (p_i^0, p^f) and on the forecast error difference standard deviation σ_{Δ_i} . The supplier long-term cost, which will be detailed in Section V and used to optimize its wind farm portfolio, depends only on the market prices and on the reserves. Market prices and the optimal reserves being independent of the bilateral trades, we do not give here the detail of their computation.

IV. MD 2: TWO TIERED WITH ENDOGENOUS PRICES

In the following subsections, we derive analytically the endogenous prices in the day-ahead and in the real-time markets.

The global day-ahead market is characterized by the equilibrium between the supply and the demand: $q_{tot}^f(N) = \sum_{i=1, \dots, N} q_i^f = \sum_{i=1, \dots, N} s_i^f$ which is the global quantity of conventional energy exchanged on day-ahead markets. Furthermore, for any market i , we suppose, at t_0 , that the difference between the supply and the demand for conventional energy over market i coincides with the sum of bilateral trades with the other markets:

$$\sum_{j=1, \dots, N, j \neq i} \lambda_{i \rightarrow j}^0 = s_i^0 - q_i^0 \Leftrightarrow S_i = s_i^0 - q_i^0 \quad (5)$$

We make the assumption that the prices p_i^f and p_i^0 paid by market i suppliers for the energy purchased at t_f and t_0 respectively equal the marginal costs. This assumption is justified in Subsection II-A by the decoupling of the cost functions at the day-ahead and real-time markets. In two-stage market models, represented by stochastic programming, such an assumption does not hold [25]. There are two major reasons why we do not work with a two stage stochastic program at the generation side. One is the analytical character of our work. Our model assumes suppliers that obey a two-stage optimization paradigm with market power. Adding generators that also solve a two-stage optimization problem and finding the equilibrium between the suppliers and these generators is not amenable to an analytical solution. But, we also believe that this additional technical complexity would bring little in terms of realism in the EU context where the day-ahead market and the balancing are quite different systems that can in no way be modeled or approximated by two-stage stochastic program. The 2013 Survey on Ancillary Services Procurement and Electricity Balancing Market Design by ENTSO-E [30] shows that there is no relation in EU mind between what takes place in the energy market and in balancing. The maps contained in [30] also show that quite different systems can prevail in neighboring countries that are part of the Market Coupling, with the consequence that it is simply unrealistic to formulate the coupling of the day ahead and real time as a stochastic program. This separation is in fact our justification for our modeling of the system as two separate markets. It

implies that: $p_i^f = c_i^f(s_i^f)$ and $p_i^0 = c_i^0(s_i^0)$. Furthermore, we assume that a clearing price is reached at t_f i.e., $p_i^f = p_j^f \triangleq p^f, \forall i, j = 1, \dots, N, i \neq j$ meaning that all the markets are integrated in a single one at that time. Because the transfers are limited by the available transmission capacities, it will be harder to align the market prices at t_0 : if the markets clear then $p_i^0 = p_j^0 \triangleq p^0, \forall i, j = 1, \dots, N, i \neq j$; otherwise there exists at least one market $i \in \{1, \dots, N\}$ in which the supplier pays $p_i^0 \neq p_j^0$ for $j \in \{1, \dots, N\}$ and $j \neq i$ ¹⁰.

A. Derivation of the coupling price

We set: $A^f \triangleq \sum_{i=1, \dots, N} \frac{a_i^f}{b_i^f}$ and $B^f \triangleq \sum_{i=1, \dots, N} \frac{1}{b_i^f} > 0$.

Lemma 1. *The coupling price for the day-ahead market is:*

$$p^f = \frac{\sum_{i=1, \dots, N} q_i^f + A^f}{B^f}.$$

Proof of Lemma 1. Using the assumption of the supply and demand equilibrium guaranteed by the day-ahead market rules, we have:

$$\begin{aligned} q_{\text{tot}}^f(N) &= \sum_{i=1, \dots, N} q_i^f = \sum_{i=1, \dots, N} s_i^f \\ &= \sum_{i=1, \dots, N} \frac{p_i^f - a_i^f}{b_i^f} \\ &\quad \text{under the assumption that } p_i^f = c_i^f \\ &= \sum_{i=1, \dots, N} \frac{p^f - a_i^f}{b_i^f} \\ &\quad \text{since the } N \text{ markets are coupled at } t_f \\ &= p^f \left(\sum_{i=1, \dots, N} \frac{1}{b_i^f} \right) - \sum_{i=1, \dots, N} \frac{a_i^f}{b_i^f} \end{aligned}$$

We infer from the following equations the day-ahead price on the coupling zone: $p^f = \frac{\sum_{i=1, \dots, N} q_i^f + A^f}{B^f}$. \square

We set: $A^0 \triangleq \sum_{i=1, \dots, N} \frac{a_i^0}{b_i^0}$ and $B^0 \triangleq \sum_{i=1, \dots, N} \frac{1}{b_i^0} > 0$.

We proved in [15] that the N markets being coupled at time t_0 , the coupling price for the real-time market is: $p^0 = \frac{\sum_{i=1, \dots, N} q_i^0 + A^0}{B^0}$.

B. Derivation of the split market prices

We set: $A_i^0 \triangleq \frac{a_i^0}{b_i^0}$ and $B_i^0 \triangleq \frac{1}{b_i^0} > 0$. As in the proof of Lemma 1, we infer the real-time price on the i -th split market:

Lemma 2. *The N markets being split in N geographic areas, at time t_0 , market i price for the real-time market is:*

$$p_i^0 = \frac{q_i^0 + A_i^0 + \left[\sum_{j=1, \dots, N, j \neq i} \lambda_{i \rightarrow j}^0 \right]}{B_i^0}.$$

¹⁰In case where $p_i^0 \neq p_j^0$, a congestion rent $CR_{i,j} = (p_i^0 - p_j^0) \lambda_{i \rightarrow j}^0$ is paid to the Transmission Operator. $CR_{i,j}$ is: positive if the lower price market is exporting energy to the higher price market; null if the interconnection lines, binding market i to market j , are not congested and $p_i^0 = p_j^0 = p^0$; negative if the lower price market is importing energy from the higher price market.

	Day-ahead price	Real-time price
Coupling anticipation	$\frac{1}{B^f} \left[\sum_{i=1, \dots, N} q_i^f + A^f \right]$	$\frac{1}{B^0} \left[\sum_{i=1, \dots, N} q_i^0 + A^0 \right]$
Congestion anticipation	$\frac{1}{B^f} \left[\sum_{i=1, \dots, N} q_i^f + A^f \right]$	$\frac{1}{B_i^0} \left[q_i^0 + A_i^0 + \sum_{j=1, \dots, N, j \neq i} \lambda_{i \rightarrow j}^0 \right]$

TABLE I
ENDOGENOUS DAY-AHEAD PRICE (p^f) AND REAL-TIME PRICE (p_i^0).

Proof of Lemma 2. Using the real-time market rules defined through Equation (5), we have: $q_i^0 = s_i^0 - \sum_{j=1, \dots, N, j \neq i} \lambda_{i \rightarrow j}^0$. This implies that: $q_i^0 + \sum_{j=1, \dots, N, j \neq i} \lambda_{i \rightarrow j}^0 = p_i^0 \frac{1}{b_i^0} - \frac{a_i^0}{b_i^0}$ using the fact that $c_i^0 = p_i^0$. Then: $p_i^0 = \frac{q_i^0 + \frac{a_i^0}{b_i^0} + \left[\sum_{j=1, \dots, N, j \neq i} \lambda_{i \rightarrow j}^0 \right]}{\frac{1}{b_i^0}} = \frac{q_i^0 + A_i^0 + \left[\sum_{j=1, \dots, N, j \neq i} \lambda_{i \rightarrow j}^0 \right]}{B_i^0}$

using the simplifying notations. \square

In Table I, we summarize the analytical expressions of market i endogenous prices in the day ahead and real time, depending on whether we are in coupling or in congestion.

Day-ahead and real-time prices being now determined analytically, we let \mathbf{r}_{-i} be a $N-1$ dimensional vector containing the reserves of all the suppliers except market i supplier. As mentioned at the end of Subsection II-B, depending on what the supplier, which decides r_i given \mathbf{r}_{-i} , solves for, the game can be classified as a price or quantity SG.

C. Signaling Game based on price

The price SG assumes that the supplier substitutes the real-time price with a linear function of r_i and \mathbf{r}_{-i} (this will be proved in Lemma 3), and optimizes with respect to r_i , given \mathbf{r}_{-i} . In this sense, the real-time price acts as a signal i.e., a mapping of real-time price to r_i and \mathbf{r}_{-i} . All of the equivalent interconnection capacities being congested in real time, we solve the bilevel game described in Subsection II-B proceeding by backward induction. Reserves and bilateral trades cannot be derived analytically since, as we will see, it is intractable to inverse the complementary cumulative distribution function \bar{F}_{Δ_i} in the fixed point equation defining the Nash Equilibrium in the reserves¹¹. However, we provide conditions guaranteeing the existence and uniqueness of a Nash Equilibrium for the reserves and detail algorithmically how reserves and bilateral trades at the optimum should be computed.

1) *Minimization of the suppliers' expected cost:* We determined the analytical expressions of the endogenous coupling price for the integrated day-ahead market in Subsection IV-A and of the endogenous prices for the split markets in real time in Subsection IV-B. Substituting these values in the suppliers' expected costs, each market i supplier determines independently and simultaneously the quantity of energy to

¹¹Except for simple distribution functions such as the Uniform density function.

purchase, q_i^f , or, equivalently, its reserve, r_i , so as to minimize its expected procurement cost, as described in optimization Program 3.

Market i supplier determines the best answer, $r_i^{BA}(\mathbf{r}_{-i})$, which minimizes its expected procurement cost. The decentralized program output is a Nash Equilibrium, $(r_i^*)_{i=1,\dots,N}$, defined by: $r_i^* = r_i^{BA}(\mathbf{r}_{-i}^*)$, $\forall i = 1, \dots, N$.

Lemma 3. *The sum of the bilateral trades of market i in the real-time market, S_i , can be expressed as a linear function in r_i and \mathbf{r}_{-i} .*

Proof of Lemma 3. By definition: $q_i^f = \sum_{j \neq i} \lambda_{i \rightarrow j}^f + s_i^f$ and $p^f = \alpha_i^f + b_i^f s_i^f$. This implies, in turn, that: $\sum_{j \neq i} \lambda_{i \rightarrow j}^f = q_i^f - s_i^f$ and $s_i^f = \frac{p^f - \alpha_i^f}{b_i^f}$. This implies that: $\sum_{j \neq i} \lambda_{i \rightarrow j}^f = q_i^f - \frac{p^f - \alpha_i^f}{b_i^f} = q_i^f - \frac{p^f - \alpha_i^f}{\sum_j q_j^f + A^f} + \frac{\alpha_i^f}{b_i^f}$ using the definition of p^f under congestion anticipation. Then: $\sum_{j \neq i} \lambda_{i \rightarrow j}^f = \sum_{j \neq i} \kappa_{i,j} - \sum_{j \neq i} \lambda_{i \rightarrow j}^f = \sum_{j \neq i} \kappa_{i,j} - (q_i^f - \frac{\sum_j q_j^f + A^f}{B^f b_i^f} + \frac{\alpha_i^f}{b_i^f}) = (\frac{1}{B^f b_i^f} - 1)(\hat{\Delta}_i - \hat{\omega}_i + r_i) + \frac{1}{B^f b_i^f} \sum_{j \neq i} (\hat{\Delta}_j - \hat{\omega}_j + r_j) + \sum_{j \neq i} \kappa_{i,j} - \frac{1}{b_i^f} (\alpha_i^f - \frac{A^f}{B^f})$. \square

This means that there exists a linear function $\varphi_i : \mathbb{R}_+^N \rightarrow \mathbb{R}$ such that $S_i = \varphi_i(r_i, \mathbf{r}_{-i})$.

As a corollary of Lemma 3, we obtain: $\frac{\partial S_i}{\partial r_i} = \frac{\partial}{\partial r_i} (\sum_{j \neq i} \lambda_{i \rightarrow j}^f) = \frac{1}{B^f b_i^f} - 1$.

Proposition 4. *If $\sum_{j \neq i} \lambda_{i \rightarrow j}^0 > -A_i^0$ (i.e., low quantity of imports), there exists a positive Nash Equilibrium solution of Program 3. Otherwise, the result still holds provided the standard deviation of Δ_i is smaller than $-\frac{1}{\sqrt{2\pi}} [\frac{A_i^0 + \sum_{j \neq i} \lambda_{i \rightarrow j}^0}{2B_i^0} + \frac{1}{2}(3 - \frac{1}{B^f b_i^f})]$ (i.e., small variance for the difference between the error associated to renewable production forecast and the error related to demand forecast).*

Proof of Proposition 4. It is provided in Appendix A.

Using the methodology described in Proposition 4 proof, market i determines the best answer: $r_i^* = r_i^{BA}(\mathbf{r}_{-i}^*)$ which minimizes its expected cost. Going a step further in the computations detailed in the proof of Proposition 4, we prove that this best answer is obtained as the solution of a fixed point equation: $\bar{F}_{\Delta_i}(r_i^*) = B_i^0 \{A_i^0 + (\frac{1}{B^f b_i^f} - 1)(\hat{\Delta}_i - \hat{\omega}_i + r_i^*) + \frac{1}{B^f b_i^f} \sum_{j \neq i} (\hat{\Delta}_j - \hat{\omega}_j + r_j^*) + \sum_{j \neq i} \kappa_{i,j} - \frac{1}{b_i^f} (\alpha_i^f - \frac{A^f}{B^f}) - 2r_i^*\}^{-1} \left[\frac{\sum_j (\hat{\Delta}_j - \hat{\omega}_j) + \sum_{j \neq i} r_j^* + r_i^* + A^f}{B^f} + \frac{\hat{\Delta}_i - \hat{\omega}_i + r_i^*}{B^f b_i^f} + (\frac{1}{B^f b_i^f} + 1) \frac{\sigma_{\Delta_i}}{B_i^0 \sqrt{2\pi}} \exp(-\frac{(r_i^*)^2}{2\sigma_{\Delta_i}^2}) \right]$. This fixed point equation is solved simultaneously by all of the markets. Nash Equilibria are obtained at the intersections of the best answers.

To show uniqueness of the resulting Nash Equilibrium, we apply the contraction mapping approach. As shown in Bertsekas [2], it is sufficient to show that the Hessian of the expected cost functions fulfills the diagonal dominance condition i.e., $\sum_{j \neq i} |\frac{\partial^2 U_i}{\partial r_i \partial r_j}| < |\frac{\partial^2 U_i}{\partial r_i^2}|$, $\forall i = 1, \dots, N$. We show, below, that the uniqueness of the Nash Equilibrium heavily relies on the number of interacting markets:

Proposition 5. *Providing there is a low quantity of imports (i.e., $S_i > -A_i^0$, $\forall i = 1, \dots, N$ according to Proposition 4), if $\frac{1}{B^f} (3 - N) + b_i^0 (3 - N) \bar{F}_{\Delta_i}(r_i) + \frac{A_i^0 + S_i}{B_i^0} f_{\Delta_i}(r_i) > 0$, $\forall i = 1, \dots, N$ then there exists a unique Nash Equilibrium solution of Program 3.*

Proof of Proposition 5. Derivating first U_i with respect to r_i and then, a second time, with respect to r_j , $j \neq i$, we obtain: $\frac{\partial^2 U_i}{\partial r_i \partial r_j} = \frac{1}{B^f} + \frac{b_i^0}{b_i^f B^f} \bar{F}_{\Delta_i}(r_i) > 0$. We remind that, by definition, $B^f b_i^f > 1$, $\forall i = 1, \dots, N$. The assumption that $S_i > -A_i^0$ implies, according to Proposition 4: $\frac{\partial^2 U_i}{\partial r_i^2} - \sum_{j \neq i} \frac{\partial^2 U_i}{\partial r_i \partial r_j} = \frac{1}{B^f} (3 - N) + b_i^0 (3 - \frac{N}{b_i^f B^f}) \bar{F}_{\Delta_i}(r_i) + \frac{A_i^0 + S_i}{B_i^0} f_{\Delta_i}(r_i) > \frac{1}{B^f} (3 - N) + b_i^0 (3 - N) \bar{F}_{\Delta_i}(r_i) + \underbrace{\frac{A_i^0 + S_i}{B_i^0} f_{\Delta_i}(r_i)}_{>0}$, $\forall i = 1, \dots, N$. \square

If $N \leq 3$, then Proposition 5 holds; otherwise, checking the diagonal dominance condition is not straightforward.

2) *Optimization of the bilateral trades:* Firstly, we note that the supplies of conventional energy on day-ahead and real-time markets can be expressed as functions of the bilateral trades in real time:

$$s_i^f = \sum_{j \neq i} \lambda_{i \rightarrow j}^f + q_i^f = \sum_{j \neq i} \kappa_{i,j} - S_i + \hat{\Delta}_i - \hat{\omega}_i + r_i$$

$$s_i^0 = \sum_{j \neq i} \lambda_{i \rightarrow j}^0 + (\Delta_i - r_i)_+ = S_i + (\Delta_i - r_i)_+$$

Proposition 6. *The optimal bilateral trade between market i and market k can be expressed as a linear function in r_k , \mathbf{r}_{-k} , $(\mathbb{E}[(\Delta_i - r_i)_+])_{U_i p_i^0 < p_i^0}$ and $\mathbb{E}[(\Delta_k - r_k)_+]$.*

Proof of Proposition 6. The conventional energy producers' expected profit function can be rewritten as:

$$\begin{aligned} \Pi_i = & (\hat{\Delta}_i - \hat{\omega}_i + r_i + \sum_{j \neq i} \kappa_{i,j} - S_i) \frac{1}{B^f} \left(\sum_j (\hat{\Delta}_j - \hat{\omega}_j + r_j) + A^f \right) - \alpha_i^f (\hat{\Delta}_i - \hat{\omega}_i + r_i + \sum_{j \neq i} \kappa_{i,j} - S_i) \\ & - \frac{b_i^f}{2} (\hat{\Delta}_i - \hat{\omega}_i + r_i + \sum_{j \neq i} \kappa_{i,j} - S_i)^2 \\ & + \sum_{j \neq i} (\lambda_{i \rightarrow j}^0)_+ + \mathbb{E} \left[\frac{(\Delta_j - r_j)_+ + A_j^0 + S_j}{B_j^0} \right] \\ & - \frac{b_i^0}{2} \mathbb{E}[(\Delta_i - r_i)_+ + S_i]^2 + \mathbb{E}[(\Delta_i - r_i)_+] \frac{1}{B_i^0} \left((\Delta_i - r_i)_+ \right. \\ & \left. + A_i^0 + S_i \right) - \alpha_i^0 \mathbb{E}[(\Delta_i - r_i)_+ + S_i] \\ & + \mathbb{E}[(S_i - \sum_{j \neq i} (\lambda_{i \rightarrow j}^0)_+) \frac{(\Delta_i - r_i)_+ + A_i^0 + S_i}{B_i^0}] \end{aligned}$$

We suppose that there exists a k in $\{1, \dots, N\}$ such that

$\lambda_{i \rightarrow k}^0 > 0$. We compute:

$$\begin{aligned} \frac{\partial \Pi_i}{\partial \lambda_{i \rightarrow k}^0} &= - \underbrace{\frac{\sum_j (\hat{d}_j - \hat{w}_j + r_j) + A^f}{p^f}}_{p^f} \\ &+ \underbrace{a_i^f + b_i^f \left(\sum_{j \neq i} \kappa_{ij} - S_i + \hat{d}_i - \hat{w}_i + r_i \right)}_{c_i^f(s_i^f)} \\ &- \frac{1}{B_k^0} \lambda_{i \rightarrow j}^0 + \mathbb{E}[p_k^0] + \frac{1}{B_i^0} (S_i - \sum_{j \neq i} (\lambda_{i \rightarrow j})_+) \\ &+ \mathbb{E}[(\Delta_i - r_i)_+] - a_i^0 - b_i^0 \mathbb{E}[(\Delta_i - r_i)_+ + S_i] \end{aligned}$$

We observe that the first line of the above equation vanishes since, by assumption, $p^f = c_i^f(s_i^f)$. Furthermore, since $\mathbb{E}[p_k^0] = \frac{1}{B_k^0} \mathbb{E}[(\Delta_k - r_k)_+] + c_k^0(S_k)$ and after a few simplifications, we obtain: $\frac{\partial \Pi_i}{\partial \lambda_{i \rightarrow k}^0} = -\frac{1}{B_k^0} \lambda_{i \rightarrow k}^0 + \frac{1}{B_k^0} \mathbb{E}[(\Delta_k - r_k)_+] + c_k^0(S_k) - c_i^0(S_i^+) - a_i^0$. Then:

$$\frac{\partial \Pi_i}{\partial \lambda_{i \rightarrow k}^0} = 0 \Leftrightarrow \lambda_{i \rightarrow k}^0 = B_k^0 [c_k^0(S_k) - c_i^0(S_i^+)] + \mathbb{E}[(\Delta_k - r_k)_+] \quad (6)$$

Summing $\lambda_{i \rightarrow k}^0$ over all the k such that $\lambda_{i \rightarrow k}^0 > 0$ i.e., $p_i^0 < p_k^0$, we obtain:

$$S_i^+ = \sum_{k | p_i^0 < p_k^0} (B_k^0 [c_k^0(S_k) - c_i^0(S_i^+)] + \mathbb{E}[(\Delta_k - r_k)_+])$$

Separating the equation in S_i^+ we obtain:

$$\begin{aligned} S_i^+ &= \frac{1}{1 + b_i^0 \sum_{k | p_i^0 < p_k^0} B_k^0} \left\{ \sum_{k | p_i^0 < p_k^0} (B_k c_k^0(S_k)) \right. \\ &+ \left. \mathbb{E}[(\Delta_k - r_k)_+] - a_i^0 \sum_{k | p_i^0 < p_k^0} B_k^0 \right\} \quad (7) \end{aligned}$$

By substitution of Equation (7) in Equation (6), we obtain:

$$\begin{aligned} \lambda_{i \rightarrow k}^0 &= B_k^0 [c_k^0(S_k) \\ &- c_i^0 \left(\frac{1}{1 + b_i^0 \sum_{l | p_i^0 < p_l^0} B_l^0} \left\{ \sum_{l | p_i^0 < p_l^0} (B_l c_l^0(S_l)) \right. \right. \\ &+ \left. \left. \mathbb{E}[(\Delta_l - r_l)_+] - a_i^0 \sum_{l | p_i^0 < p_l^0} B_l^0 \right\} \right)] \\ &+ \mathbb{E}[(\Delta_k - r_k)_+] \end{aligned}$$

This means that $\lambda_{i \rightarrow k}^0$ can be expressed exclusively as a function of S_k , r_k , $(\mathbb{E}[(\Delta_l - r_l)_+]_{l | p_i^0 < p_l^0})$ and $\mathbb{E}[(\Delta_k - r_k)_+]$. But, we proved in Lemma 3 that there exists a linear function $\varphi_k : \mathbb{R}_+^N \rightarrow \mathbb{R}$ such that $S_k = \varphi_k(r_k, \mathbf{r}_{-k})$. As a result, at the optimum, $\lambda_{i \rightarrow k}^0$ can be expressed as a linear function in r_k , \mathbf{r}_{-k} , $(\mathbb{E}[(\Delta_l - r_l)_+]_{l | p_i^0 < p_l^0})$ and $\mathbb{E}[(\Delta_k - r_k)_+]$. \square

D. Signaling Game based on quantity

In Subsection IV-C, the game signal was based on price. In this subsection, we assume that the game signal is based on quantity. The quantity SG assumes that the supplier substitutes the bilateral trades with a linear function of r_i and \mathbf{r}_{-i} , which

is proved in Proposition 7, and optimizes with respect to r_i given \mathbf{r}_{-i} . In this sense, the inter-country trade quantities act as a signal i.e., a mapping of the bilateral trades to r_i and \mathbf{r}_{-i} . The SG based on quantity will also be solved by backward induction. As already mentioned, this implies that the bilevel game timing needs to be reversed i.e., Step (i) becomes (ii) and Step (ii) becomes (i).

The final state of the system is defined by the difference between the real time prices on each couple of markets: $p_i^0 - p_k^0, \forall i, k, i \neq k$. Indeed, its sign determines whether this is an export from market i to market k or, the reverse.

Under backward induction, market i production of conventional energy and prices in real time and in the day ahead can be expressed as linear functions in the sum of its bilateral trades and reserve:

$$\begin{aligned} s_i^0 &= S_i + (\Delta_i - r_i)_+ \\ s_i^f &= \sum_{j \neq i} \kappa_{ij} - S_i + (\hat{d}_i - \hat{w}_i + r_i) \end{aligned}$$

and $p^f = \frac{1}{B^f} [\sum_i (\hat{d}_i - \hat{w}_i + r_i) + A^f]$, $p_i^0 = \frac{1}{B_i^0} [(\Delta_i - r_i)_+ + A_i^0 + S_i]$.

1) *Optimization of the bilateral trades:* Without loss of generality, we assume that: $p_1^0 > p_2^0 > \dots > p_N^0$.

Proposition 7. *The optimal bilateral trades between market i and the other markets can be expressed as linear functions in r_i, \mathbf{r}_{-i} .*

Proof of Proposition 7. We start by assuming that there exists a $k \in \{1, \dots, N\}$ such that $\lambda_{i \rightarrow k}^0 > 0$. Differentiating Π_i with respect to $\lambda_{i \rightarrow k}^0$ we obtain:

$$\frac{\partial \Pi_i}{\partial \lambda_{i \rightarrow k}^0} = 0 \Leftrightarrow \lambda_{i \rightarrow k}^0 = (\Delta_k - r_k)_+ + B_k^0 [c_k^0(S_k) - c_i^0(S_i^+)] \quad (8)$$

Summing this equation over all the $k \in \{1, \dots, N\}$ such that $p_i^0 < p_k^0$ we obtain:

$$\begin{aligned} S_i^+ &= \sum_{k | p_i^0 < p_k^0} (\Delta_k - r_k)_+ + \sum_{k | p_i^0 < p_k^0} B_k^0 c_k^0(S_k) \\ &- c_i^0(S_i^+) \sum_{k | p_i^0 < p_k^0} B_k^0 \end{aligned}$$

which is equivalent to:

$$\begin{aligned} S_i^+ &= \frac{1}{1 + b_i^0 \sum_{k | p_i^0 < p_k^0} B_k^0} \left\{ \sum_{k | p_i^0 < p_k^0} (\Delta_k - r_k)_+ \right. \\ &+ \left. \sum_{k | p_i^0 < p_k^0} B_k^0 c_k^0(S_k) - a_i^0 \sum_{k | p_i^0 < p_k^0} B_k^0 \right\} \quad (9) \end{aligned}$$

Substituting Equation (9) in Equation (8), we obtain:

$$\begin{aligned} \lambda_{i \rightarrow k}^0 &= (\Delta_k - r_k)_+ + B_k^0 \left[c_k^0(S_k) \right. \\ &- \left. c_i^0 \left(\frac{1}{1 + b_i^0 \sum_{l < i} B_l^0} \left\{ \sum_{l < i} (\Delta_l - r_l)_+ \right. \right. \right. \\ &+ \left. \left. \sum_{l < i} B_l^0 c_l^0(S_l) - a_i^0 \sum_{l < i} B_l^0 \right\} \right)] \quad (10) \end{aligned}$$

Note that according to the assumption made on the real-time price ranking, the set $\{l | p_i^0 < p_l^0\}$ is equivalent to the set

$\{l | l < i\}$. Summing Equation (10) over all the $k \neq i$, we obtain:

$$S_i = \omega_i(\mathbf{r}_{-i}) + \sum_{k \neq i} S_k - \beta_i \sum_{l | p_l^0 < p_i^0} S_l$$

where:

$$\begin{aligned} \beta_i &= \frac{b_i^0 \sum_{k \neq i} B_k^0}{1 + b_i^0 \sum_{l | p_l^0 < p_i^0} B_l^0} \\ \omega_i(\mathbf{r}_{-i}) &= \sum_{k \neq i} (\Delta_k - r_k)_+ + \sum_{k \neq i} B_k^0 a_k^0 - a_i^0 \sum_{k \neq i} B_k^0 \\ &\quad - \frac{b_i^0}{1 + b_i^0 \sum_{l | p_l^0 < p_i^0} B_l^0} \left[\sum_{l | p_l^0 < p_i^0} (\Delta_l - r_l)_+ \right. \\ &\quad \left. + \sum_{l | p_l^0 < p_i^0} B_l^0 a_l^0 - a_i^0 \sum_{l | p_l^0 < p_i^0} B_l^0 \right] \sum_{k \neq i} B_k^0 \end{aligned}$$

The above equation can be rewritten as:

$$\begin{aligned} S_1 &= \omega_1(\mathbf{r}_{-1}) + \sum_{k \neq 1} S_k \\ S_i &= \omega_i(\mathbf{r}_{-i}) + \sum_{k \neq i} S_k - \beta_i \sum_{l | p_l^0 < p_i^0} S_l, \forall i \geq 2 \end{aligned}$$

According to the first equation $\sum_{k \neq i} S_k = 2S_1 - \omega_1(\mathbf{r}_{-1}) - S_i$. By substitution in the second equation, we obtain:

$$S_i = \frac{\omega_i(\mathbf{r}_{-i}) - \omega_1(\mathbf{r}_{-1})}{2} + S_1 - \frac{\beta_i}{2} \sum_{l < i} S_l, \forall i \geq 2$$

Considering this equation for any $i \geq 2$, it can be rewritten as a system:

$$\begin{aligned} &\begin{pmatrix} 1 & 0 & 0 & \dots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \frac{\beta_2}{2} & \frac{\beta_2}{2} & 1 & \ddots & 0 \\ \vdots & \vdots & \vdots & \ddots & 0 \\ \frac{\beta_N}{2} & \frac{\beta_N}{2} & \frac{\beta_N}{2} & \dots & 1 \end{pmatrix} \begin{pmatrix} S_2 \\ \vdots \\ S_i \\ \vdots \\ S_N \end{pmatrix} \\ &= \begin{pmatrix} \frac{\omega_2(\mathbf{r}_{-2}) - \omega_1(\mathbf{r}_{-1})}{2} + (1 - \frac{\beta_2}{2})S_1 \\ \vdots \\ \frac{\omega_i(\mathbf{r}_{-i}) - \omega_1(\mathbf{r}_{-1})}{2} + (1 - \frac{\beta_i}{2})S_1 \\ \vdots \\ \frac{\omega_N(\mathbf{r}_{-N}) - \omega_1(\mathbf{r}_{-1})}{2} + (1 - \frac{\beta_N}{2})S_1 \end{pmatrix} \end{aligned}$$

The system matrix is inferior triangular. Therefore, it is invertible using an appropriate algorithm:

$$\begin{aligned} S_2 &= \frac{\omega_2(\mathbf{r}_{-2}) - \omega_1(\mathbf{r}_{-1})}{2} + (1 - \frac{\beta_2}{2})S_1 \\ S_3 &= \frac{\omega_3(\mathbf{r}_{-3}) - \omega_1(\mathbf{r}_{-1})}{2} + (1 - \frac{\beta_3}{2})S_1 - \frac{\beta_2}{2}S_2 \\ &\vdots \\ S_N &= \frac{\omega_N(\mathbf{r}_{-N}) - \omega_1(\mathbf{r}_{-1})}{2} + (1 - \frac{\beta_N}{2})S_1 \\ &\quad - \frac{\beta_N}{2} \underbrace{\sum_{1 < i < N} S_i}_{S_1 - \omega_1(\mathbf{r}_{-1}) - S_N} \end{aligned}$$

The last equation can be rewritten as:

$$S_N = \frac{\omega_N(\mathbf{r}_{-N}) - \omega_1(\mathbf{r}_{-1})}{2} + (1 - \frac{\beta_N}{2})S_1 + \omega_1(\mathbf{r}_{-1}) \frac{\beta_N}{2}$$

Proceeding recursively for $i = N - 1, \dots, 2$, it is possible to express any $S_i, i \geq 2$ as a linear function of S_1 . As a result, there exist vectors $(\zeta_i)_{2 \leq i \leq N}$ and $(\xi_i)_{2 \leq i \leq N}$ such that:

$$\begin{pmatrix} S_2 \\ \vdots \\ S_i \\ \vdots \\ S_N \end{pmatrix} = \begin{pmatrix} \zeta_2 \\ \vdots \\ \zeta_i \\ \vdots \\ \zeta_N \end{pmatrix} + S_1 \begin{pmatrix} \xi_2 \\ \vdots \\ \xi_i \\ \vdots \\ \xi_N \end{pmatrix} \quad (11)$$

To give an example: $\zeta_N = \frac{\omega_N(\mathbf{r}_{-N}) - \omega_1(\mathbf{r}_{-1})}{2} + \omega_1(\mathbf{r}_{-1}) \frac{\beta_N}{2}$ and $\xi_N = 1 - \frac{\beta_N}{2}$.

Summing the above system over all $i \geq 2$, we obtain:

$$\underbrace{\sum_{i \geq 2} S_i}_{S_1 - \omega_1(\mathbf{r}_{-1})} = \sum_{i \geq 2} \zeta_i + S_1 \sum_{i \geq 2} \xi_i$$

which is equivalent to: $S_1 = \frac{\sum_{i \geq 2} \zeta_i + \omega_1(\mathbf{r}_{-1})}{1 - \sum_{i \geq 2} \xi_i}$. By substitution in Equation (11) it is possible to infer $(S_i)_{i \geq 2}$ as a function of $(\zeta_i, \xi_i)_{i \geq 2}$ and $\omega_1(\mathbf{r}_{-1})$ i.e., of r_i, \mathbf{r}_{-i} . \square

Since S_k is linear in $(\Delta_k - r_k)_+, k = 1, \dots, N$ and $p_i^0 = \frac{(\Delta_i - r_i)_+ + A_i^0 + S_i}{B_i^0}$ we obtain that for any market i , there exist coefficients $(\eta_{i,l})_{l=1, \dots, N}$ and $\tilde{\eta}_i$ such that: $p_i^0 = \sum_{l=1, \dots, N} \eta_{i,l} (\Delta_l - r_l)_+ + \tilde{\eta}_i$.

2) Minimization of the suppliers' expected cost:

Proposition 8. *The other markets' optimal reserves being fixed to \mathbf{r}_{-i}^* , we prove that there exists a positive r_i^* minimizing market i expected cost. Furthermore, when $N \leq 3$, there exist values for the marginal cost function parameters $(a_i^0, b_i^0)_i$ guaranteeing the uniqueness of the Nash Equilibrium.*

Proof of Proposition 8. It is provided in Appendix B.

We proved that for both price SG and quantity SG there exists a positive Nash Equilibrium. Although the conditions guaranteeing its uniqueness differ, we observe that in both cases for less than three geographic demand markets, there exist parameter values guaranteeing its uniqueness. There is no guarantee about the existence and uniqueness of a Nash Equilibrium for $N > 3$. From a policy perspective, the existence of multiple Nash Equilibria implies that the outcome of the Market Design cannot be predicted. Computationally, multiple Nash Equilibria imply that the output of the model may depend on the initial conditions of the algorithm.

V. EXPLORING THE MARKOWITZ FRONTIER: WHAT DOES IT LOOK LIKE?

When wind farm investor participates directly in an electricity market i.e., without subsidy-based supporting schemes, it influences the day-ahead positions, and hence the reserves, by putting wind in the system. We assume that the investment

strategy in renewable capacity is defined over a finite horizon $0 < T < +\infty$. In this setting, the price SG, as described in Section IV-C, is repeated over a finite horizon T .

There can be significant year-to-year variations in wind conditions, which would have an impact on profitability, and these may differ between regions [9]. Furthermore, the higher the terrain complexity, the lower the wind predictability. Note that correlation among wind farms decreases with the distance [8]. Therefore, we assume that each market i is clusterized in a subset \mathcal{C}_i of clusters where, over $c \in \mathcal{C}_i$, the estimated demand $\hat{d}_i(c)$ and the wind mean production $\alpha_i(c)$ are supposed constant and such that at each time period t :

$$d_i(c, t) = \hat{d}_i(c) - v_i(c, t) \quad (12)$$

$$w_i(c, t) = \hat{w}_i(c) - \epsilon_i(c, t) \quad (13)$$

Making the parallel with the previous notations, we have: $d_i(t) = \sum_{c \in \mathcal{C}_i} d_i(c, t)$ and $\alpha_i = \sum_{c \in \mathcal{C}_i} \alpha_i(c)$. Clustering may be performed through one of the automatic partitioning algorithms used in Machine Learning [12], as illustrated for Germany in Section VI.

We assume that the geographic market i clusters are defined so that there is no correlation among the cluster forecast errors but that, inside each cluster, there remains a positive correlation between the wind production and the demand forecast error. We remind that $\Delta_i(c, t)$ is the difference between the wind production and the demand forecast errors over market i cluster c , at time period t . Transposing Subsection II-C assumptions to a finer scale (i.e., clusters instead of geographic markets), we assume that $(\Delta_i(c, t))_{c \in \mathcal{C}_i}$ is distributed according to a $|\mathcal{C}_i|$ -dimensional Gaussian density function centered in the zero $|\mathcal{C}_i|$ dimensional vector and with a diagonal variance-covariance matrix having on its principal diagonal all the variances $(\sigma_{\Delta_i(c)}(c))^2, \forall c \in \mathcal{C}_i$ and zeros everywhere else since the covariance between any $\Delta_i(c, t), \Delta_i(c', t), \forall c, c' \in \mathcal{C}_i, c \neq c'$ vanishes. Furthermore, over each cluster $c \in \mathcal{C}_i$, the wind production and the demand forecast errors are correlated due to the assumption that $(\tilde{\epsilon}_i(c, t) \ v_i(c, t))^T$ is a Gaussian random vector centered in $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ and of variance-covariance matrix

$$\begin{pmatrix} (\tilde{\sigma}_i^\epsilon(c))^2 & \mathbb{E}[\tilde{\epsilon}_i(c, t)v_i(c, t)] \\ \mathbb{E}[\tilde{\epsilon}_i(c, t)v_i(c, t)] & (\sigma_i^v(c))^2 \end{pmatrix}.$$

Modern Portfolio Theory is an alternative to the traditional method of analyzing each investment's individual merits. When investors look at each investment's individual merits, they are analyzing one investment without worrying about the way the different investments will perform relative to each other. On the other hand, Modern Portfolio Theory places a large emphasis on the correlation between the investments. Markowitz defines as efficient the portfolios which are characterized by a maximum expected revenue for a fixed risk (or, equivalently, for a minimum risk for a fixed expected revenue) [17]. Risk and volatility are treated as the same thing: Markowitz uses risk as a measurement of the likelihood that an investment still goes up and down in value, and how often and by how much. The theory assumes that investors prefer to minimize risk. The Efficient Frontier, also called the Markowitz Frontier (MF), is then defined as the set

of all the portfolios which are efficient. In this article, the supplier applies Modern Portfolio Theory to determine the wind farm portfolio that maximizes its return (or, equivalently, minimizes its expected long-term cost) while minimizing its risk conditionally to the occurrence of rare events. Indeed as pointed out by Marling and Emanuelson [18], if an investor wants to use the MF model to choose a suitable portfolio then it is suitable to do some complementary computations of the risk conditionally to the occurrence of rare events. This is especially true in the context of this article since the efficiency of wind farm portfolio positioning depends heavily on its capability to cope with rare events caused by a far smaller/larger production of wind power than expected and resulting in large forecast errors. In that case, the reserve might not be sufficient to compensate the forecast error.

In the numerical illustrations we will test two assumptions on the investment function: either it is linear in the number of settled wind farms: $I(\gamma_i(c)) = \text{cost}_{\text{inv}}\gamma_i(c)$ or it is quadratic: $I(\gamma_i(c)) = \text{cost}_{\text{inv}}\gamma_i(c)^2$ with $\text{cost}_{\text{inv}} > 0$ representing the cost of investment for a single wind farm.

A. In the $(\gamma_i(c), \theta_i(c))$ plane

In this subsection, we characterize analytically the MF in the $(\gamma_i(c), \theta_i(c))$ plane.

Theorem 9. *Whatever the Market Design (i.e., two tiered with exogenous prices, two tiered with endogenous prices), the Markowitz Frontier in the $(\gamma_i(c), \theta_i(c))$ plane is always completely described by the following set of equations:*

$$\begin{aligned} \theta_i(c) &= \frac{\ln \frac{\mathbb{E}[\tilde{\epsilon}_i(c)v_i(c)]}{(\tilde{\sigma}_i^\epsilon(c))^2}}{\ln \gamma_i(c)} \\ \frac{\mathbb{E}[\tilde{\epsilon}_i(c)v_i(c)]}{(\tilde{\sigma}_i^\epsilon(c))^2} &\leq \gamma_i(c) \leq \left(\frac{\mathbb{E}[\tilde{\epsilon}_i(c)v_i(c)]}{(\tilde{\sigma}_i^\epsilon(c))^2} \right)^2 \\ \gamma_i(c) &\in \mathbb{N}^* \end{aligned}$$

Theorem 9 states that the optimal concentration of the wind farms $\theta_i(c)$ over cluster $c \in \mathcal{C}_i$ is obtained as the unique output of an explicit function of the number of wind farms $\gamma_i(c)$, provided $\gamma_i(c)$ satisfies the conditions of the second and third equations introduced in the Theorem. This is a strong result as we prove that the form of this explicit function is identical for MD 1 and MD 2. It is worth mentioning that Theorem 9 does not define a unique optimal portfolio but, in fact, a multiplicity of solutions.

The theorem proof is detailed below for the two Market Designs described in Sections III and IV.

Proof of Theorem 9. When optimizing its wind farm portfolio, market i supplier's problem is to determine the optimal number of wind farms $\gamma_i(c) \in \mathbb{N}^*$ and their concentration $\frac{1}{2} \leq \theta_i(c) \leq 1$, over each cluster $c \in \mathcal{C}_i$, such that its expected long-term cost is minimal and the variance of its long-term cost conditionally to the occurrence of rare events is minimal.

MD 1: Over each demand market $i = 1, \dots, N$, the supplier long-term cost is defined as the sum of the cost resulting from its conventional energy demand, repeated T times, and of the cost devoted to the deployment of wind farm portfolio over

its geographic area:

$$\begin{aligned}
\mathcal{J}(i, T) &= \sum_t \left(q_i^f(t) p^f(t) + q_i^0(t) p^0(t) \right) + \sum_{c \in \mathcal{C}_i} I(\gamma_i(c)) \\
&= \sum_t \left(\sum_{c \in \mathcal{C}_i} (\hat{d}_i(c) - \alpha_i(c) \gamma_i(c) + r_i(t)) p^f(t) \right. \\
&\quad + \sum_t \left[\left(\sum_{c \in \mathcal{C}_i} (\tilde{\epsilon}_i(c, t) \gamma_i(c)^{\theta_i(c)} - \nu_i(c, t)) \right. \right. \\
&\quad \left. \left. - r_i(t) \right) p_i^0(t) \right] + \sum_{c \in \mathcal{C}_i} I(\gamma_i(c)) \\
&= \sum_t \sum_{c \in \mathcal{C}_i} (\hat{d}_i(c) - \alpha_i(c) \gamma_i(c) + r_i(t)) p^f(t) \\
&\quad + \sum_t (\Delta_i(t) - r_i(t)) p_i^0(t) + \sum_{c \in \mathcal{C}_i} I(\gamma_i(c)) \quad (14)
\end{aligned}$$

where, we recall: $\Delta_i(t) = \sum_{c \in \mathcal{C}_i} (\epsilon_i(c, t) - \nu_i(c, t)) = \sum_{c \in \mathcal{C}_i} (\gamma_i(c)^{\theta_i(c)} \tilde{\epsilon}_i(c, t) - \nu_i(c, t))$.

We obtain quite easily the analytical expression of the variance of Δ_i :

$$\begin{aligned}
\sigma_{\Delta_i}^2 &= \sum_{c \in \mathcal{C}_i} \sigma_{\Delta_i}^2(c) = \sum_{c \in \mathcal{C}_i} \text{Var}(\epsilon_i(c) - \nu_i(c)) \\
&= \sum_{c \in \mathcal{C}_i} \text{Var}(\gamma_i(c)^{\theta_i(c)} \tilde{\epsilon}_i(c) - \nu_i(c)) \\
&= \sum_{c \in \mathcal{C}_i} \left\{ (\tilde{\sigma}_i^\epsilon(c))^2 \gamma_i(c)^{2\theta_i(c)} \right. \\
&\quad \left. - 2\gamma_i(c)^{\theta_i(c)} \mathbb{E}[\tilde{\epsilon}_i(c) \nu_i(c)] + \sigma_i^\nu(c)^2 \right\} \quad (15)
\end{aligned}$$

Then, we compute the supplier's risk i.e., its variance conditionally to the occurrence of rare events:

$$\begin{aligned}
&\text{Var}\left(\mathcal{J}(i, T) \mid \{\Delta_i(t) \geq r_i(t)\}_t\right) \\
&= \text{Var}\left(\sum_t \Delta_i(t) p_i^0(t) - \sum_t (r_i(t) p_i^0(t)) \right. \\
&\quad \left. + \sum_{c \in \mathcal{C}_i} (\hat{d}_i(c) - \alpha_i(c) \gamma_i(c) + r_i(t)) p^f(t) + \sum_{c \in \mathcal{C}_i} I(\gamma_i(c))\right)
\end{aligned}$$

We note that it is only the first term of this equation that depends on $\Delta_i(t)$. Using properties of the variance operator (i.e., variance of a constant equals zero, variance is a quadratic operator) and the fact that there is no dependence between two time consecutive forecast error differences, we infer that:

$$\begin{aligned}
&\text{Var}\left(\mathcal{J}(i, T) \mid \{\Delta_i(t) \geq r_i(t)\}_t\right) \\
&= \sum_{c \in \mathcal{C}_i} \sigma_{\Delta_i}^2(c) \sum_t p_i^0(t)^2 \\
&= \sum_{c \in \mathcal{C}_i} \left[(\tilde{\sigma}_i^\epsilon(c))^2 \gamma_i(c)^{2\theta_i(c)} \right. \\
&\quad \left. - 2\gamma_i(c)^{\theta_i(c)} \mathbb{E}[\tilde{\epsilon}_i(c) \nu_i(c)] + \sigma_i^\nu(c)^2 \right] \sum_t p_i^0(t)^2
\end{aligned}$$

using Equation (15). In the rest of the article, we let: $\text{RVar}(i, T) \triangleq \text{Var}\left(\mathcal{J}(i, T) \mid \{\Delta_i(t) \geq r_i(t)\}_t\right)$.

The expectation of the supplier's long-term cost is:

$$\begin{aligned}
\mathbb{E}[\mathcal{J}(i, T)] &= \sum_t \left(\sum_{c \in \mathcal{C}_i} \hat{d}_i(c) + \bar{F}_{\Delta_i}^{-1}\left(\frac{p^f(t)}{p_i^0(t)}\right) \right) p^f(t) \\
&\quad - \sum_t \left(\sum_{c \in \mathcal{C}_i} \alpha_i(c) \gamma_i(c) \right) p^f(t) \\
&\quad + \sum_t p_i^0(t) \mathbb{E}[(\Delta_i(t) - \bar{F}_{\Delta_i}^{-1}\left(\frac{p^f(t)}{p_i^0(t)}\right))] \\
&\quad + \sum_{c \in \mathcal{C}_i} I(\gamma_i(c))
\end{aligned}$$

Derivating $\text{RVar}(i, T)$ with respect to $\theta_i(c)$ and solving $\frac{\partial \text{RVar}(i, T)}{\partial \theta_i(c)} = 0$, we obtain:

$$\theta_i(c) = \frac{\ln \frac{\mathbb{E}[\tilde{\epsilon}_i(c) \nu_i(c)]}{(\tilde{\sigma}_i^\epsilon(c))^2}}{\ln \gamma_i(c)} \quad (16)$$

But: $\frac{1}{2} \leq \theta_i(c) \leq 1$ which is equivalent to $\frac{\mathbb{E}[\tilde{\epsilon}_i(c) \nu_i(c)]}{(\tilde{\sigma}_i^\epsilon(c))^2} \leq \gamma_i(c) \leq \left(\frac{\mathbb{E}[\tilde{\epsilon}_i(c) \nu_i(c)]}{(\tilde{\sigma}_i^\epsilon(c))^2}\right)^2$.

Using the fact that $\frac{\mathbb{E}[\tilde{\epsilon}_i(c) \nu_i(c)]}{(\tilde{\sigma}_i^\epsilon(c))^2} \leq \gamma_i(c)$, we check that $\theta_i(c)$ coincides with a minimum for $\text{RVar}(i, T)$ indeed: $\frac{\partial^2 \text{RVar}(i, T)}{\partial \theta_i(c)^2} = 2\gamma_i(c)^{\theta_i(c)} \ln(\gamma_i(c))^2 \left[2(\tilde{\sigma}_i^\epsilon(c))^2 \gamma_i(c) - \mathbb{E}[\tilde{\epsilon}_i(c) \nu_i(c)] \right] > 0$.

Therefore, in case of a two tiered market with exogenous prices, the MF is completely described by the set of equations summarized in the statement of Theorem 9.

MD 2: In Equations (12) and (13), $\hat{d}_i(c)$ and $\hat{w}_i(c)$ do not depend on time. This implies that in the algorithms described in Subsections IV-C and IV-D, the optimal reserves are also independent on time; contrary to MD 1, where the optimal reserves are functions of the day-ahead and real-time price ratio, which is time dependent. Using this observation, we infer that the day-ahead price is time independent; indeed: $p^f = \frac{\sum_j (\hat{d}_j - \hat{w}_j + r_j) + A^f}{B^f}$.

In case of endogenous prices, market i supplier long-term cost over each demand market $i = 1, \dots, N$ takes the following form:

$$\begin{aligned}
\mathcal{J}(i, T) &= \sum_t \left(\underbrace{\sum_{c \in \mathcal{C}_i} (\hat{d}_i(c) - \hat{w}_i(c)) + r_i}_{\hat{d}_i - \hat{w}_i} \right) p^f(t) \\
&\quad + \sum_t \left[\underbrace{\left(\sum_{c \in \mathcal{C}_i} (\tilde{\epsilon}_i(c, t) \gamma_i(c)^{\theta_i(c)} - \nu_i(c, t)) \right)}_{\Delta_i(t)} \right. \\
&\quad \left. - r_i \right] p_i^0(t) + \sum_{c \in \mathcal{C}_i} I(\gamma_i(c)) \quad (17)
\end{aligned}$$

Compared to MD 1, we observe one additional difficulty: at the optimum r_i relies on \mathbf{r}_{-i} and the bilateral trades rely on the decisions of the other markets.

The supplier's variance conditionally to the occurrence of rare events is similar to the one derived for MD 1:

$$\begin{aligned}
\text{RVar}(i, T) &= \text{Var}(\mathcal{J}(i, T) \mid \{\Delta_i(t) \geq r_i\}_t) \\
&= \sigma_{\Delta_i}^2 \sum_t p_i^0(t)^2 \quad (18)
\end{aligned}$$

and the same result about the optimal wind farm concentration, $(\theta_i(c))_{i,c}$ follows. Therefore, in the latter case of a two tiered market with endogenous prices, the MF is completely described by the set of equations summarized in the statement of Theorem 9. \square

B. In the $(\mathbb{E}[J(i, T)], \text{RVar}(i, T))$ plane

The representation of the MF in the $(\mathbb{E}[J(i, T)], \text{RVar}(i, T))$ plane is not straightforward. Indeed, in case of endogenous prices, the optimal number of wind farms to settle on each cluster cannot be computed analytically. Indeed, the optimal reserves and the optimal bilateral trades can only be obtained algorithmically, as explained in Section IV-C (resp. Section IV-D).

Over each cluster $c \in \mathcal{C}_i$, the supplier can deploy a fixed predefined number of wind farms $\gamma_i(c)$. For each combination of $(\gamma_i(c))_{i,c}$, we derive the optimal wind farm portfolio concentration $(\theta_i(c))_{i,c}$ using Theorem 9. We substitute the resulting $(\gamma_i(c), \theta_i(c))_{i,c}$ in $(\sigma_{\Delta_i})_i$ as derived in Equation (15) and in the Nash Equilibrium in the reserves and bilateral trades $(r_i^*, S_i^*)_i$ obtained through the algorithm detailed in Section (IV-C) (resp. Section IV-D).

For any $t = 1, \dots, T$ and $i = 1, \dots, N$, we generate a sequence of forecast error differences such that $\{\Delta_i(t) \geq r_i\}$, according to the Gaussian density function $\mathcal{N}(0; \sigma_{\Delta_i}^2)$. The time elapsed between two consecutive repetitions of the SG being of 24 hours, we assume that, as already mentioned, there is no time dependence between two consecutive samples of forecast error differences. Various simulation techniques can be envisaged to cope with rare events. A straightforward way is to use Monte-Carlo simulation. However, this poses serious problems when the event $\{\Delta_i(t) \geq r_i\}_t$ is a rare event. Indeed, in that case, a large simulation effort is required in order to estimate the rare event accurately i.e., with small relative error or narrow confidence interval [7]. A well-known alternative is to use Importance Sampling. However, it is proved that the optimal Importance Sampling density relies on the rare event probability, which is unknown. The Cross-Entropy (CE) method provides an alternative multi-level approach [7]. Its principle is first to generate randomly a sample according to a specific mechanism and then to update the mechanism parameters by selecting the elements of the sample which are the closest in the sense of the Kullback-Leibler divergence, also known as CE, of the optimal Importance Sampling density function. According to the CE algorithm for rare event simulation, we need to update at each time step the Gaussian density function variance: $\sigma_{\Delta_i}^2$ so that more weight is allocated to rare events. The Algorithm is provided in Appendix C.

We substitute $(r_i^*, S_i^*)_i$ and $(\Delta_i(t))_i$ in the real time price: $p_i^0(t) = \frac{A_i^0 + (\Delta_i(t) - r_i^*)_+ + S_i^*}{B_i^0}$, $\forall i = 1, \dots, N$ derived in Sub-section IV-B. From this, we can infer the supplier's variance conditionally to the occurrence of the rare events $\{\Delta_i(t) \geq r_i\}_t$ using Equation (18).

At the same time, we note that the expectation of market i

supplier's long-term cost can be simplified to give:

$$\begin{aligned} \mathbb{E}[J(i, T)] &= \frac{T}{B_i^f} (\hat{d}_i - \hat{w}_i + r_i) \left\{ \sum_j (\hat{d}_j - \hat{w}_j + r_j) + A^f \right\} \\ &+ \frac{T}{B_i^0} [A_i^0 + \varphi_i(r_i, \mathbf{r}_{-i})] \mathbb{E}[(\Delta_i(t) - r_i)_+] \\ &+ \frac{T}{B_i^0} \mathbb{E}[(\Delta_i(t) - r_i)_+^2] + \sum_{c \in \mathcal{C}_i} I(\gamma_i(c)) \end{aligned}$$

where, as detailed in Appendix A: $\mathbb{E}[(\Delta_i(t) - r_i)_+] = \mathbb{E}[(\Delta_i(t) - r_i) | \Delta_i(t) \geq r_i] = \frac{\sigma_{\Delta_i}}{\sqrt{2\pi}} \exp(-\frac{r_i^2}{2\sigma_{\Delta_i}^2}) - r_i \bar{F}_{\Delta_i}(r_i)$ and $\mathbb{E}[(\Delta_i(t) - r_i)_+^2] = \mathbb{E}[(\Delta_i(t) - r_i)^2 | \Delta_i(t) \geq r_i] = \frac{\sigma_{\Delta_i}^2}{\sqrt{\pi}} \Gamma(\frac{3}{2}) \Gamma_{\text{inc}}(\frac{3}{2}, \frac{r_i^2}{2\sigma_{\Delta_i}^2}) - \frac{\sigma_{\Delta_i}}{\sqrt{2\pi}} \exp(-\frac{r_i^2}{2\sigma_{\Delta_i}^2}) + r_i^2$ with $\Gamma(a)$ the Gamma function evaluated in $a \in \mathbb{R}_+$ and $\Gamma_{\text{inc}}(a, x) = \frac{1}{\Gamma(a)} \int_x^{+\infty} u^{a-1} \exp(-u) du$ the Incomplete Gamma function with lower bound, evaluated in $a, x \in \mathbb{R}_+$.

VI. NUMERICAL ILLUSTRATIONS FOR THREE GEOGRAPHIC DEMAND MARKETS: FRANCE, GERMANY AND BELGIUM

In the numerical illustrations, we consider three geographic demand markets: France, Germany and Belgium. Wind farm portfolio optimization is restricted to the French area, since our energy consumption data focus on this country.

The marginal cost parameters are based on Chao and Peck's six node toy network [5]: for France (Fr) we take $a_{\text{Fr}}^f = 42.5$, $a_{\text{Fr}}^0 = 4250$, $b_{\text{Fr}}^f = b_{\text{Fr}}^0 = 0.025$; for Germany (Ge) we take $a_{\text{Ge}}^f = 15$, $a_{\text{Ge}}^0 = 1500$, $b_{\text{Ge}}^f = b_{\text{Ge}}^0 = 0.05$ and for Belgium (Be) $a_{\text{Be}}^f = 10$, $a_{\text{Be}}^0 = 1000$, $b_{\text{Be}}^f = b_{\text{Be}}^0 = 0.05$.

The equivalent interconnection capacities are set so that: $\kappa_{\text{Be,Ge}} = 2(\text{GW})$, $\kappa_{\text{Be,Fr}} = 6(\text{GW})$ and $\kappa_{\text{Ge,Fr}} = 5(\text{GW})$.

A. Description of the data and clustering of the geographic demand markets

For Germany, our database is made of time series of 75 sensors located all over Germany, providing one year wind speed measures (from 03/19/2013 until 03/18/2014) with one measure per hour [29]. The exact GPS coordinates of the sensors are depicted by circles in Figure 2 (a). We use two Machine Learning techniques to partition the sensors based on the mean and variance of their wind speed time series: firstly, k-Means algorithm clusters data by separating samples in an a priori determined number of groups, minimizing a criterion known as the inertia of the groups. The optimal number of classes for the sensors (four) has been estimated a priori using an unsupervised clustering method known as affinity propagation [32]. Secondly, one-class Support Vector Machine (SVM) can be used as a type of unsupervised learning algorithm, for novelty detection: given a set of samples, it will detect the soft boundary of that set so as to classify new points as belonging to that set or not. Both techniques give identical (or, at least, very close) classes, which are represented in Figure 2 (a) and (b). The convex hull of the sensor classes gives an approximation to the clusters geographic area for Germany.

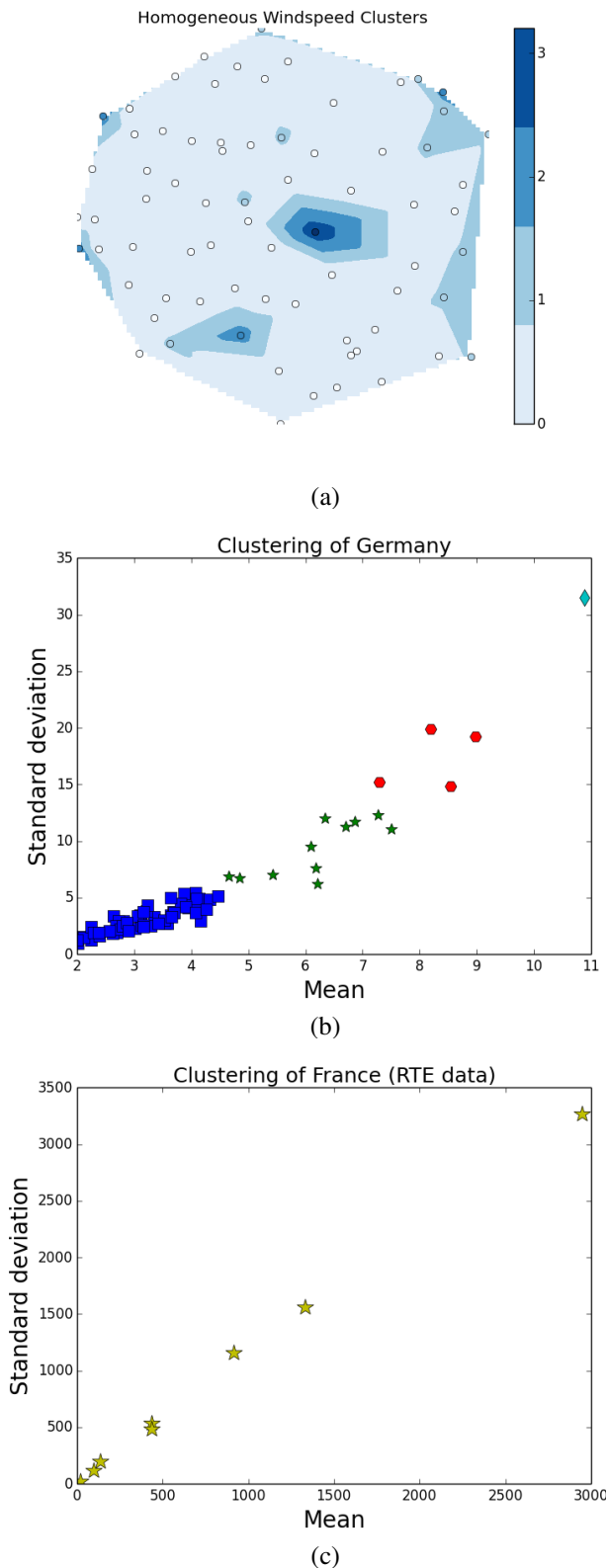


Fig. 2. Partitioning Germany wind speed sensors, using a Machine Learning automatic partitioning algorithm (k-Means or SVM outlier detection) (a). The sensors' locations in GPS coordinates are represented in (a) by circles. In (b), the sensors are represented in the Mean-Variance plane depending on their wind speed time series mean and variance. We partition Germany's wind speed sensors; each marker symbol being associated with a specific class while in (c) we represent each French geographic cluster as a star, in the Mean-Variance plane depending on its wind power production.

Clust. ind.	1	2	3	4	5	6	7	8
Mean prod.	2.94	1.33	0.43	0.02	0.13	0.43	0.92	0.09
Stand. dev.	3.27	1.56	0.54	0.02	0.19	0.49	1.15	0.12

TABLE II
WIND FARM MEAN PRODUCTION (MW) AND STANDARD DEVIATION (MW) FOR FRANCE.

For France and Belgium, we use data provided by RTE report [3]. In this report, France (resp. Belgium) was partitioned in 8 (resp. 2) clusters. Wind speed measurements are performed at various points in the clusters. The measures are then averaged over the clusters and, over each cluster, the average wind power production of a turbine is estimated. Correlation among the clusters is taken into account to evaluate the wind power production. We use hourly forecasts of wind power for year 2015.

In Figure 2 (c), we represent each of the 8 French clusters averaged wind production per turbine in the Mean-Variance plane. These data sets are used to estimate the mean production (MW) and standard deviation (MW) of a single wind farm, over each French cluster (cf. Table II for France). The geographic coordinates of the cluster areas can be found in [3]. We associate an index with each French cluster area.

Estimated wind production for Germany and Belgium is fixed so that: $\hat{w}_{Ge} = 15(\text{GW})$ and $\hat{w}_{Be} = 10(\text{GW})$. The forecast error difference standard deviations are set so that: $\sigma_{\Delta_{Be}} = \sigma_{\Delta_{Ge}} = 5(\text{GW})$. For France, if 10 turbines were placed in each cluster, $\hat{w}_{Fr} = 4.97(\text{GW})$ and the forecast error difference standard deviation would be: $\sigma_{\Delta_{Fr}} = 5.79(\text{GW})$.

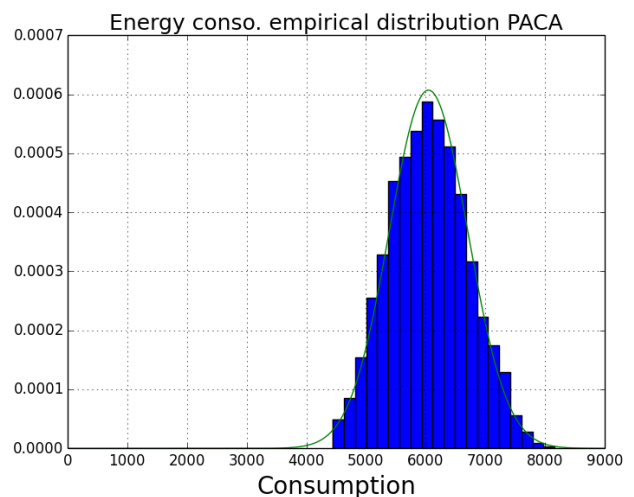


Fig. 3. Energy consumption empirical distribution function (in kWh) compared to the best fit Gaussian density function for PACA region.

For estimating the variance associated to the demand forecast, we use a data base containing one year (2013) power measurements (in kW) for each French region, with a granularity of one measure per 30 minutes [31]. The French regions are then mapped to the French clusters, as defined in RTE report [3]. Then, over each cluster, we take the average of

Clust. ind.	1	2	3	4	5	6	7	8
Mean conso.	19.890	6.043	2.480	2.947	9.104	4.875	3.736	7.098
Conso. stand. dev.	5.048	1.196	0.496	0.654	2.010	1.032	0.953	1.874

TABLE III
MEAN CONSUMPTION (GW) AND CONSUMPTION STANDARD DEVIATION PER FRENCH CLUSTER (GW).

these time series to obtain one value per day. In Figure 3, we compare the empirical distribution function of one French cluster (PACA region) energy consumption with the best fit Gaussian density function. This comparison validates our assumptions on demand forecasting, captured in Equation (12).

For France, the end users' total demand is estimated by: $\hat{d}_{Fr} = 56.173(\text{GW})$; for Germany and Belgium we fix: $\hat{d}_{Ge} = \hat{d}_{Be} = 40(\text{GW})$.

B. Optimal reserves and Markowitz Frontier representations for France

In Figure 4 (a), we plot the optimal reserve for the two tiered market with exogenous prices described in Section III as a function of the exogenous price ratio $\frac{p_{Fr}^0}{p_{Fr}^f}$ and of the forecast error standard deviation $\sigma_{\Delta_{Fr}}$. We observe that the reserve increases parabolically as the real-time price over the day-ahead price ratio increases and as the uncertainty on the difference between the end users' total demand and the renewable production in real time ($d_{Fr} - w_{Fr}$) increases. In Figure 4 (b), we plot the optimal reserve for the two tiered market with endogenous prices and reserves described in Section IV as a function of the wind forecast \hat{w}_{Fr} and of the forecast error standard deviation $\sigma_{\Delta_{Fr}}$. We observe that the reserve increases in the wind forecast and in $d_{Fr} - w_{Fr}$.

In Figure 5, we plot the MF for the wind farm portfolio over each French cluster as a function of the number of turbines and of the concentration of the wind farms over the cluster. These plots are issued from the theoretical relation derived in Theorem 9.

We assume that the supplier can deploy 0, 60 or 120 turbines over each cluster leading to 3^8 combinations for France. This choice of numerical values is justified by the fact that largest US wind farms nowadays have around 600 turbines¹² and by coming down on the scale of France. Assuming that a wind farm counts on the average 60 turbines, the supplier has the choice between deploying 0, 1 or 2 wind farms per cluster. We observe in Figure 6 that the value of cost_{inv} and the form of the investment cost (i.e., linear, quadratic, etc.) deeply influence the MF shape.

VII. CONCLUSION

We developed a methodology for optimizing an investor's wind farm portfolio, using Markowitz Frontier theory, within a Market environment. We considered $N \in \mathbb{N}^*$ interacting

¹²For example, the Roscoe wind farm in Roscoe, Texas, is one of the world's largest capacity wind farms with 634 turbines and a total installed capacity of 781.5 MW.

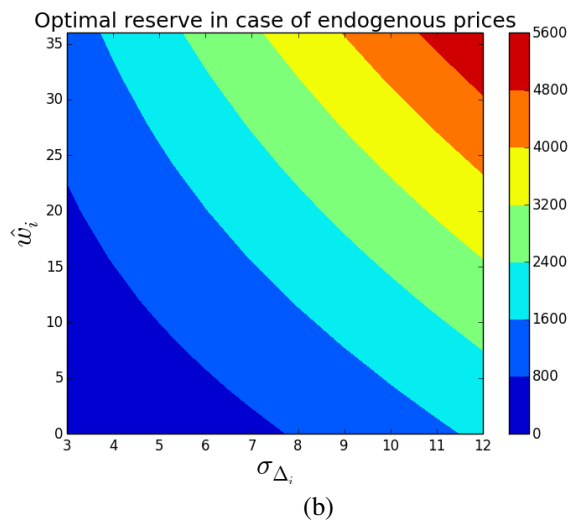
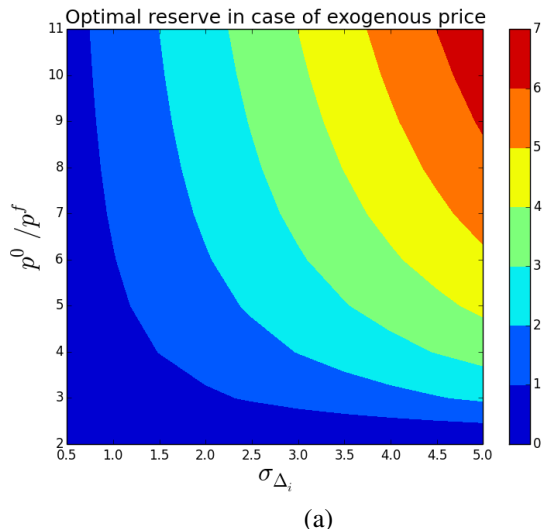


Fig. 4. In (a) (resp. (b)), we represent the optimal reserve r_i in case of exogenous prices (resp. endogenous prices) as a function of the forecast error difference standard deviation σ_{Δ_i} and of the exogenous price ratio ($\frac{p^0}{p^f}$) (resp. of the wind forecast \hat{w}_i).

geographic demand markets optimizing selfishly their reserve and bilateral trades with the others. The novelty, compared to previous works, relies on the fact that we have introduced some competition among the geographic demand markets and that the problem is modeled as a bilevel Signaling Game. The signal i.e., the information shared among the players, was based either on prices or quantities. We proved analytically that first, in both classes of games there exist conditions guaranteeing the existence and the uniqueness of a Nash Equilibrium for $N \leq 3$ and second, that the Markowitz Frontier can be expressed as a function of the number of settled wind farms and of their concentration independently of the Market Design. Finally, we propose an algorithm and simulate, on real case study, the contour of the Markowitz Frontier in the expected cost-conditional variance plane.

The linearity of the cost functions, the inelasticity of demand and day-ahead market coupling are necessary assump-

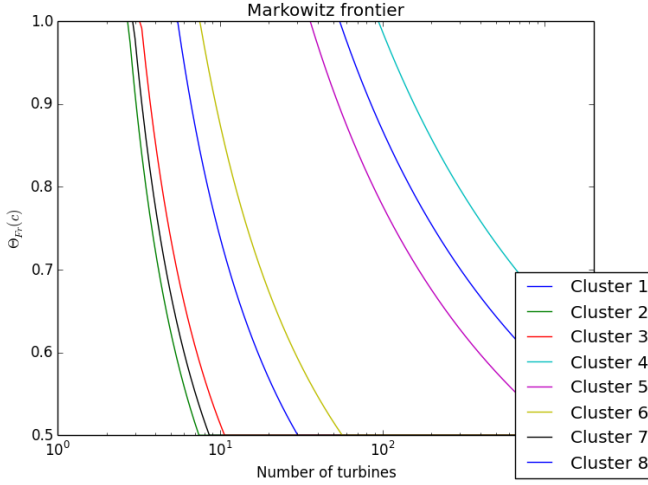


Fig. 5. Over each French cluster, the Markowitz Frontier for the wind farm portfolio is represented as a function of the number of turbines and of the concentration between the wind farms over the cluster. Each of the 8 French clusters is characterized by a specific color.

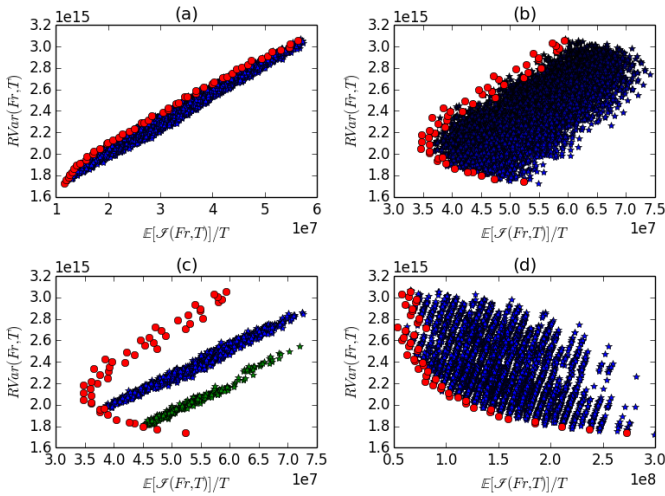


Fig. 6. Markowitz Frontier (MF) in red in the $\mathbb{E}[\mathcal{J}(Fr, T)] - RVar(Fr, T)$ plane. In (a), we represent all the portfolio combinations and the MF for $cost_{inv} = 1$ and a linear investment cost $\sum_{c \in C_{Fr}} \gamma_{Fr}(c)$. In (b), we plot all the portfolio combinations and the MF for $cost_{inv} = 900$ and a linear investment cost. In (c), the same setting holds but we select exclusively the portfolios associated with two investment levels: $\sum_{c \in C_{Fr}} \gamma_i(c) = 24.10^3$ and 36.10^3 . In (d), we plot the portfolio combinations and the MF for $cost_{inv} = 1$ and a quadratic investment cost i.e., $\sum_{c \in C_{Fr}} \gamma_{Fr}(c)^2$.

tions in our analysis. The relaxation of these assumptions is an interesting direction of future research but will lead us to the limits of analytical tractability.

APPENDIX A: PROOF OF PROPOSITION 4

By substitution of the day-ahead and real-time prices at equilibrium obtained in Lemmas 1 and 2 in market i expected procurement cost, we obtain:

$$U_i = q_i^f \frac{\sum_j q_j^f + A^f}{B^f} + \mathbb{E}\left[\frac{(q_i^0)^2}{B_i^0}\right] + \frac{A_i^0 + \sum_{j \neq i} \lambda_{i \rightarrow j}^0}{B_i^0} q_i^0$$

Using Lemma 3 and the fact that $\mathbb{E}[q_i^0] = \mathbb{E}[(\Delta_i - r_i)_+] = \mathbb{E}[(\Delta_i - r_i)|\Delta_i \geq r_i]$, we obtain:

$$\begin{aligned} \frac{\partial U_i}{\partial r_i} &= \frac{\sum_j q_j^f + A^f}{B^f} + \frac{q_i^f}{B^f} \\ &+ \frac{\partial}{\partial r_i} \left(\frac{1}{B_i^0} \mathbb{E}[(\Delta_i - r_i)^2 | \Delta_i \geq r_i] \right) \\ &+ \frac{A_i^0 + \sum_{j \neq i} \lambda_{i \rightarrow j}^0}{B_i^0} \frac{\partial}{\partial r_i} \mathbb{E}[(\Delta_i - r_i) | \Delta_i \geq r_i] \\ &+ b_i^0 \left(\frac{1}{B^f b_i^f} - 1 \right) \mathbb{E}[(\Delta_i - r_i) | \Delta_i \geq r_i] \end{aligned}$$

Since the forecast error differences Δ_i are distributed according to Gaussian distribution functions centered in 0 and of standard deviation σ_{Δ_i} , it is possible to express the first and second derivatives of $(\Delta_i - r_i)$ and $(\Delta_i - r_i)^2$ conditionally to the event $\{\Delta_i \geq r_i\}$ as functions of the Incomplete Gamma function which enables us to derive the following closed forms:

$$\begin{aligned} \mathbb{E}[(\Delta_i - r_i) | \Delta_i \geq r_i] &= \int_{r_i}^{+\infty} \Delta f_{\Delta_i}(\Delta) d\Delta \\ -r_i \int_{r_i}^{+\infty} f_{\Delta_i}(\Delta) d\Delta &= \frac{\sigma_{\Delta_i}}{\sqrt{2\pi}} \exp\left(-\frac{r_i^2}{2\sigma_{\Delta_i}^2}\right) - r_i \bar{F}_{\Delta_i}(r_i) \\ \frac{\partial}{\partial r_i} \mathbb{E}[(\Delta_i - r_i) | \Delta_i \geq r_i] &= -\bar{F}_{\Delta_i}(r_i) \\ \frac{\partial^2}{\partial r_i^2} \mathbb{E}[(\Delta_i - r_i) | \Delta_i \geq r_i] &= f_{\Delta_i}(r_i) \end{aligned} \quad (19)$$

and

$$\begin{aligned} \mathbb{E}[(\Delta_i - r_i)^2 | \Delta_i \geq r_i] &= \int_{r_i}^{+\infty} \Delta^2 f_{\Delta_i}(\Delta) d\Delta \\ - 2r_i \int_{r_i}^{+\infty} \Delta f_{\Delta_i}(\Delta) d\Delta + r_i^2 \int_{r_i}^{+\infty} f_{\Delta_i}(\Delta) d\Delta \\ \frac{\partial}{\partial r_i} \mathbb{E}[(\Delta_i - r_i)^2 | \Delta_i \geq r_i] &= 2[r_i \int_{r_i}^{+\infty} f_{\Delta_i}(\Delta) d\Delta \\ &- \int_{r_i}^{+\infty} \Delta f_{\Delta_i}(\Delta) d\Delta] \\ \frac{\partial^2}{\partial r_i^2} \mathbb{E}[(\Delta_i - r_i)^2 | \Delta_i \geq r_i] &= 2\bar{F}_{\Delta_i}(r_i) \end{aligned} \quad (20)$$

Using Equations (19) and (20), we obtain:

$$\frac{\partial^2 U_i}{\partial r_i^2} = \frac{2}{B^f} + b_i^0 \left(3 - \frac{1}{B^f b_i^f} \right) \bar{F}_{\Delta_i}(r_i) + \frac{A_i^0 + \sum_{j \neq i} \lambda_{i \rightarrow j}^0}{B_i^0} f_{\Delta_i}(r_i)$$

Since $\bar{F}_{\Delta_i}(r_i) > 0$ and $f_{\Delta_i}(r_i) > 0$ for any $r_i \in \mathbb{R}$ and $B^f > 0, B_i^0 > 0$, the sign of $\frac{\partial^2 U_i}{\partial r_i^2}$ for $r_i \in [0; +\infty[$ depends on the sign of $\frac{A_i^0 + \sum_{j \neq i} \lambda_{i \rightarrow j}^0}{B_i^0}$. Two cases are possible:

Case 1: $\frac{A_i^0 + \sum_{j \neq i} \lambda_{i \rightarrow j}^0}{B_i^0} \geq 0 \Leftrightarrow \sum_{j \neq i} \lambda_{i \rightarrow j}^0 \geq -A_i^0$

This first case corresponds to the case where the quantity of imports is not too high compared to the quantity of exports, for market i . In this first case, we infer that $\frac{\partial^2 U_i}{\partial r_i^2} > 0, \forall r_i \geq 0$. Hence U_i is convex in $r_i \geq 0$. Therefore, there exists a unique $r_i \geq 0$ minimizing U_i .

Case 2: $\frac{A_i^0 + \sum_{j \neq i} \lambda_{i \rightarrow j}^0}{B_i^0} < 0 \Leftrightarrow \sum_{j \neq i} \lambda_{i \rightarrow j}^0 < -A_i^0$

This second case corresponds to the case where the quantity of imports is very high compared to the quantity of exports, for geographic market i .

Derivating three times U_i with respect to r_i , we obtain:

$$\begin{aligned} \frac{\partial^3 U_i}{\partial r_i^3} &= -b_i^0 \left(3 - \frac{1}{B^f b_i^f}\right) f_{\Delta_i}(r_i) \\ &- \frac{A_i^0 + \sum_{j \neq i} \lambda_{i \rightarrow j}^0}{B_i^0} \frac{r_i}{\sigma_{\Delta_i}^2} f_{\Delta_i}(r_i) \\ &= -\frac{f_{\Delta_i}(r_i)}{B_i^0} \frac{1}{\sigma_{\Delta_i}^2} \left[\left(3 - \frac{1}{B^f b_i^f}\right) \sigma_{\Delta_i}^2 + r_i (A_i^0 + \sum_{j \neq i} \lambda_{i \rightarrow j}^0) \right] \end{aligned}$$

using the fact that $\frac{df_{\Delta_i}(r_i)}{dr_i} = -\frac{r_i}{\sigma_{\Delta_i}^2} f_{\Delta_i}(r_i), \forall r_i \in \mathbb{R}$ since Δ_i is distributed according to a Gaussian distribution function centered in 0 and of standard deviation σ_{Δ_i} . Then: $\frac{\partial^3 U_i}{\partial r_i^3} =$

$$0 \Leftrightarrow r_i = -\frac{\left(3 - \frac{1}{B^f b_i^f}\right) \sigma_{\Delta_i}^2}{A_i^0 + \sum_{j \neq i} \lambda_{i \rightarrow j}^0}.$$

We set: $r_i^0 \triangleq -\frac{\left(3 - \frac{1}{B^f b_i^f}\right) \sigma_{\Delta_i}^2}{A_i^0 + \sum_{j \neq i} \lambda_{i \rightarrow j}^0}$. Then, we note that:

- If $r_i < r_i^0$ then $r_i < -\frac{\left(3 - \frac{1}{B^f b_i^f}\right) \sigma_{\Delta_i}^2}{A_i^0 + \sum_{j \neq i} \lambda_{i \rightarrow j}^0} \Leftrightarrow \left(3 - \frac{1}{B^f b_i^f}\right) \sigma_{\Delta_i}^2 + r_i (A_i^0 + \sum_{j \neq i} \lambda_{i \rightarrow j}^0) > 0$. This implies in turn that $\frac{\partial^3 U_i}{\partial r_i^3} < 0$.
- Identically, if $r_i > r_i^0$ then $\frac{\partial^3 U_i}{\partial r_i^3} > 0$.

Both of these observations imply that $\frac{\partial^2 U_i}{\partial r_i^2}$ is decreasing on $[0; r_i^0[$ and increasing on $[r_i^0; +\infty[$. Furthermore, the number of points where $\frac{\partial^2 U_i}{\partial r_i^2} = 0$ depend on the value of $\frac{\partial^2 U_i}{\partial r_i^2}|_{r_i=r_i^0}$.

Case 2 (a): $\frac{\partial^2 U_i}{\partial r_i^2}|_{r_i=r_i^0} > 0$

Then $\frac{\partial^2 U_i}{\partial r_i^2} > 0, \forall r_i \geq 0$. This implies that U_i is convex on \mathbb{R}_+ . Therefore it admits a unique minimum on $[0; +\infty[$.

Case 2 (b): $\frac{\partial^2 U_i}{\partial r_i^2}|_{r_i=r_i^0} < 0$

Then two sub-cases should be distinguished depending on the sign of $\frac{\partial^2 U_i}{\partial r_i^2}|_{r_i=0}$.

Case 2 (b) (i): $\frac{\partial^2 U_i}{\partial r_i^2}|_{r_i=0} < 0$ There exists a unique $r_i^1 \in [r_i^0; +\infty[$ such that $\frac{\partial^2 U_i}{\partial r_i^2} < 0$ on $[0; r_i^1[$ and $\frac{\partial^2 U_i}{\partial r_i^2} \geq 0$ on $[r_i^1; +\infty[$. This implies that $\frac{\partial U_i}{\partial r_i}$ is decreasing on $[0; r_i^1[$ and increasing on $[r_i^1; +\infty[$. Hence $\frac{\partial U_i}{\partial r_i} = 0$ in 0, 1 or 2 points for $r_i \geq 0$.

But $\frac{\partial^2 U_i}{\partial r_i^2} > 0$ exclusively for $r_i \in [r_i^1; +\infty[$. Then: either $\frac{\partial U_i}{\partial r_i} > 0, \forall r_i \in [r_i^1; +\infty[$ which implies that U_i is strictly increasing on \mathbb{R}_+ reaching its minimum in $r_i = 0$; or there exists a point $r_i^* \in [r_i^1; +\infty[$ such that $\frac{\partial U_i}{\partial r_i}|_{r_i=r_i^*} = 0$. Since $\frac{\partial^2 U_i}{\partial r_i^2} > 0$ on $[r_i^1; +\infty[$, this implies that r_i^* is the unique minimum of U_i on \mathbb{R}_+ .

Case 2 (b) (ii): $\frac{\partial^2 U_i}{\partial r_i^2}|_{r_i=0} > 0$ There exist $0 \leq r_i^2 \leq r_i^0$ and $r_i^0 \leq r_i^3$ such that $\frac{\partial^2 U_i}{\partial r_i^2} > 0$ on $[0; r_i^2[$, < 0 on $[r_i^2; r_i^3[$ and > 0 on $[r_i^3; +\infty[$.

If $\frac{\partial U_i}{\partial r_i}|_{r_i=0} \geq 0$ then there exists a unique $r_i^* \in [r_i^3; +\infty[$ such that $\frac{\partial U_i}{\partial r_i}|_{r_i=r_i^*} = 0$ and $\frac{\partial^2 U_i}{\partial r_i^2}|_{r_i=r_i^*} > 0$. In this case, r_i^* is the unique minimum of U_i over \mathbb{R}_+ .

Otherwise i.e., if $\frac{\partial U_i}{\partial r_i}|_{r_i=0} < 0$ then: either $\frac{\partial U_i}{\partial r_i}|_{r_i=r_i^2} < 0$ in which case U_i admits a unique minimum over \mathbb{R}_+ belonging to $[r_i^3; +\infty[$; or $\frac{\partial U_i}{\partial r_i}|_{r_i=0} \geq 0$ in which case U_i admits two minima over \mathbb{R}_+ , the first one in $[0; r_i^2[$ and the second one in $[r_i^3; +\infty[$.

The case $\frac{\partial U_i}{\partial r_i}|_{r_i=0} < 0$ and $\frac{\partial^2 U_i}{\partial r_i^2}|_{r_i=0} > 0$ should be avoided since it might give rise to a large number of equilibria (2^N) for Program 3. Therefore, in the case where $\sum_{j \neq i} \lambda_{i \rightarrow j}^0 < -A_i^0$, it might be reasonable to impose some conditions on the problem parameters so that Case 2 (b) (ii) is avoided. In other words:

$$\begin{aligned} \frac{\partial^2 U_i}{\partial r_i^2}|_{r_i=0} &= \frac{2}{B^f} + \frac{1}{2B_i^0} \left(3 - \frac{1}{B^f b_i^f}\right) \\ &+ \frac{A_i^0 + \sum_{j \neq i} \lambda_{i \rightarrow j}^0}{B_i^0} f_{\Delta_i}(0) < 0 \\ \Leftrightarrow f_{\Delta_i}(0) &> -\frac{\frac{2B_i^0}{B^f} + \frac{1}{2} \left(3 - \frac{1}{B^f b_i^f}\right)}{A_i^0 + \sum_{j \neq i} \lambda_{i \rightarrow j}^0} \\ \Leftrightarrow \sigma_{\Delta_i} &< -\frac{1}{\sqrt{2\pi}} \left[\frac{A_i^0 + \sum_{j \neq i} \lambda_{i \rightarrow j}^0}{\frac{2B_i^0}{B^f} + \frac{1}{2} \left(3 - \frac{1}{B^f b_i^f}\right)} \right] \end{aligned}$$

Therefore, to avoid Case 2 (b) (ii), the standard-deviation related to the knowledge of Δ_i should be smaller than $-\frac{1}{\sqrt{2\pi}} \left[\frac{A_i^0 + \sum_{j \neq i} \lambda_{i \rightarrow j}^0}{\frac{2B_i^0}{B^f} + \frac{1}{2} \left(3 - \frac{1}{B^f b_i^f}\right)} \right]$. \square

APPENDIX B: PROOF OF PROPOSITION 8

We recall the analytical expression of market i supplier expected cost:

$$\begin{aligned} U_i &= (\hat{d}_i - \hat{w}_i + r_i) \frac{1}{B^f} \left[\sum_l (\hat{d}_l - \hat{w}_l + r_l) + A^f \right] \\ &+ \mathbb{E}[(\Delta_i - r_i)_+] \frac{1}{B_i^0} \left((\Delta_i - r_i)_+ + A_i^0 + \sum_{j \neq i} \lambda_{i \rightarrow j}^0 \right) \end{aligned}$$

But:

$$\begin{aligned} & \mathbb{E}[(\Delta_i - r_i)_+ \frac{1}{B_i^0} ((\Delta_i - r_i)_+ + A_i^0 + \sum_{j \neq i} \lambda_{i \rightarrow j}^0)] \\ &= b_i^0 \mathbb{E}[(\Delta_i - r_i)_+^2 | \Delta_i \geq r_i] + a_i^0 \mathbb{E}[(\Delta_i - r_i) | \Delta_i \geq r_i] \\ &+ b_i^0 \sum_{j \neq i} \mathbb{E}[\lambda_{i \rightarrow j}^0 (\Delta_i - r_i) | \Delta_i \geq r_i] \end{aligned}$$

Derivating \mathcal{U}_i with respect to r_i gives:

$$\begin{aligned} \frac{\partial \mathcal{U}_i}{\partial r_i} &= \frac{1}{B^f} \left[\sum_l (\hat{d}_l - \hat{w}_l + r_l) + A^f \right] \\ &+ \frac{1}{B^f} (\hat{d}_i - \hat{w}_i + r_i) + 2b_i^0 [r_i \int_{r_i}^{+\infty} f_{\Delta_i}(\Delta) d\Delta \\ &- \int_{r_i}^{+\infty} \Delta f_{\Delta_i}(\Delta) d\Delta] - a_i^0 \bar{F}_{\Delta_i}(r_i) \\ &+ b_i^0 \sum_{j \neq i} \frac{\partial}{\partial r_i} \mathbb{E}[\lambda_{i \rightarrow j}^0 (\Delta_i - r_i) | \Delta_i \geq r_i] \end{aligned}$$

But:

$$\begin{aligned} & \mathbb{E}[\lambda_{i \rightarrow j}^0 (\Delta_i - r_i) | \Delta_i \geq r_i] = \frac{1}{b_j^0} \sum_{l \neq i} (\eta_{j,l} \\ &- \eta_{i,l}) \mathbb{E}[(\Delta_l - r_l) | \Delta_l \geq r_l] \mathbb{E}[(\Delta_i - r_i) | \Delta_i \geq r_i] \\ &+ \frac{1}{b_j^0} (\tilde{\eta}_j - \tilde{\eta}_i) \mathbb{E}[(\Delta_i - r_i) | \Delta_i \geq r_i] \\ &+ \frac{b_i^0 + \eta_{j,i} - \eta_{i,i}}{b_j^0} \mathbb{E}[(\Delta_i - r_i)^2 | \Delta_i \geq r_i] \end{aligned}$$

since $(\Delta_l)_{l \neq i}$ and Δ_i are independent. We remind that the coefficients $(\eta_{j,l})_{j,l}, (\tilde{\eta}_j)_j$ have been introduced following Proposition 7 proof. Explicitly, for any market i , we have proved that there exist coefficients $(\eta_{i,l})_{l=1,\dots,N}$ and $\tilde{\eta}_i$ such that: $p_i^0 = \sum_{l=1,\dots,N} \eta_{i,l} (\Delta_l - r_l)_+ + \tilde{\eta}_i$.

Finally, we obtain:

$$\begin{aligned} \frac{\partial \mathcal{U}_i}{\partial r_i} &= \frac{1}{B^f} \left[\sum_l (\hat{d}_l - \hat{w}_l + r_l) + A^f \right] + \frac{1}{B^f} (\hat{d}_i - \hat{w}_i + r_i) \\ &+ 2b_i^0 [r_i \int_{r_i}^{+\infty} f_{\Delta_i}(\Delta) d\Delta - \int_{r_i}^{+\infty} \Delta f_{\Delta_i}(\Delta) d\Delta] - a_i^0 \bar{F}_{\Delta_i}(r_i) \\ &+ b_i^0 \left\{ - \sum_{j \neq i} \frac{1}{b_j^0} \sum_{l \neq i} (\eta_{j,l} - \eta_{i,l}) \mathbb{E}[\Delta_l - r_l | \Delta_l \geq r_l] \bar{F}_{\Delta_i}(r_i) \right. \\ &\quad \left. - \sum_{j \neq i} \frac{1}{b_j^0} (\tilde{\eta}_j - \tilde{\eta}_i) \bar{F}_{\Delta_i}(r_i) \right. \\ &\quad \left. + 2 \sum_{j \neq i} \frac{b_i^0 + \eta_{j,i} - \eta_{i,i}}{b_j^0} [r_i \int_{r_i}^{+\infty} f_{\Delta_i}(\Delta) d\Delta \right. \\ &\quad \left. - \int_{r_i}^{+\infty} \Delta f_{\Delta_i}(\Delta) d\Delta] \right\} \end{aligned}$$

Derivating twice \mathcal{U}_i with respect to r_i , we obtain:

$$\frac{\partial^2 \mathcal{U}_i}{\partial r_i^2} = \frac{2}{B^f} + \bar{F}_{\Delta_i}(r_i) \mu_1(i) + f_{\Delta_i}(r_i) \mu_2(i)$$

where: $\mu_1(i) = [2b_i^0 (1 + \sum_{j \neq i} \frac{b_i^0 + \eta_{j,i} - \eta_{i,i}}{b_j^0})]$ and $\mu_2(i) = [a_i^0 + \sum_{j \neq i} \frac{b_i^0}{b_j^0} \sum_{l \neq i} (\eta_{j,l} - \eta_{i,l}) \mathbb{E}[\Delta_l - r_l | \Delta_l \geq r_l] + \sum_{j \neq i} \frac{b_i^0}{b_j^0} (\tilde{\eta}_j - \tilde{\eta}_i)]$.

Derivating three times \mathcal{U}_i with respect to r_i :

$$\frac{\partial^3 \mathcal{U}_i}{\partial r_i^3} = -f_{\Delta_i}(r_i) [\mu_1(i) - \frac{r_i}{\sigma_{\Delta_i}^2} \mu_2(i)]$$

Since $f_{\Delta_i}(r_i) > 0, \forall r_i$, $\frac{\partial^3 \mathcal{U}_i}{\partial r_i^3} = 0 \Leftrightarrow r_i = r_i^0 = \sigma_{\Delta_i}^2 \frac{\mu_1(i)}{\mu_2(i)}$.

Furthermore, we observe that: $\frac{\partial^3 \mathcal{U}_i}{\partial r_i^3} |_{r_i=0} = -f_{\Delta_i}(0) \mu_1(i)$. To prove the existence of a unique minimum for \mathcal{U}_i on \mathbb{R}_+ we distinguish between four cases depending on the signs of $\mu_1(i), \mu_2(i)$:

- **Case 1:** $\mu_1(i) > 0$ and $\mu_2(i) > 0$. This implies that $r_i^0 > 0$ and that $\frac{\partial^3 \mathcal{U}_i}{\partial r_i^3} |_{r_i=0} < 0$. Furthermore, we observe that: $\frac{\partial^2 \mathcal{U}_i}{\partial r_i^2} |_{r_i=0} = \frac{2}{B^f} + \bar{F}_{\Delta_i}(0) \mu_1(i) + f_{\Delta_i}(0) \mu_2(i) > 0$ and $\frac{\partial^2 \mathcal{U}_i}{\partial r_i^2} |_{r_i=r_i^0} = \frac{2}{B^f} + \bar{F}_{\Delta_i}(r_i^0) \mu_1(i) + f_{\Delta_i}(r_i^0) \mu_2(i) > 0$. Then either $\frac{\partial \mathcal{U}_i}{\partial r_i} |_{r_i=0} > 0$ in which case $\frac{\partial \mathcal{U}_i}{\partial r_i} > 0, \forall r_i \geq 0$ and \mathcal{U}_i is increasing on \mathbb{R}_+ implying that the minimum is reached in $r_i = 0$; or $\frac{\partial \mathcal{U}_i}{\partial r_i} |_{r_i=0} < 0$ implying that \mathcal{U}_i admits a unique minimum on \mathbb{R}_+ .
- **Case 2:** $\mu_1(i) > 0$ and $\mu_2(i) < 0$. This implies that $r_i^0 < 0$ and that $\frac{\partial^3 \mathcal{U}_i}{\partial r_i^3} |_{r_i=0} < 0$. Then either \mathcal{U}_i admits a unique minimum in $[0; +\infty[$ or \mathcal{U}_i is strictly decreasing on \mathbb{R}_+ in which case the minimum should be reached on the reserve upper bound.
- **Case 3:** $\mu_1(i) < 0$ and $\mu_2(i) > 0$. This implies that $r_i^0 < 0$ and that $\frac{\partial^3 \mathcal{U}_i}{\partial r_i^3} |_{r_i=0} > 0$. Then either \mathcal{U}_i admits a unique minimum in $[0; +\infty[$ or \mathcal{U}_i is strictly increasing on \mathbb{R}_+ in which case the minimum should be reached in $r_i = 0$.
- **Case 4:** $\mu_1(i) < 0$ and $\mu_2(i) < 0$. This implies that $r_i^0 > 0$ and that $\frac{\partial^3 \mathcal{U}_i}{\partial r_i^3} |_{r_i=0} > 0$. Then either \mathcal{U}_i admits a unique minimum in $[0; +\infty[$ or \mathcal{U}_i is increasing and then decreasing on \mathbb{R}_+ in which case the minimum should be reached either in $r_i = 0$ or in the reserve upper bound.

We observe that the Case $\mu_1(i) > 0, \mu_2(i) > 0$ is the only one which guarantees a positive reserve, different from the upper bound. Furthermore, derivating $\frac{\partial \mathcal{U}_i}{\partial r_i}$ with respect to r_j , we obtain: $\frac{\partial^2 \mathcal{U}_i}{\partial r_j \partial r_i} = \frac{1}{B^f} + b_i^0 \sum_{l \neq i} \frac{1}{b_l^0} (\eta_{j,l} - \eta_{i,l}) \bar{F}_{\Delta_j}(r_j) \bar{F}_{\Delta_i}(r_i), \forall j \neq i$. The diagonal dominance condition, introduced in Proposition 5, is checked provided $\mu_1(i) > b_i^0 \sum_{l \neq i} \frac{1}{b_l^0} \sum_{j \neq i} (\eta_{j,j} - \eta_{i,j}) \bar{F}_{\Delta_j}(r_j)$ and $N \leq 3$; for other $\mu_1(i), N$ values it is not straightforward. \square

APPENDIX C: ALGORITHM FOR THE GENERATION OF SEQUENCES OF FORECAST ERROR DIFFERENCES

We let: $\Delta_i^z(t)$ be the z -th sampled realization of the random variable $\Delta_i(t)$. According to the CE algorithm for rare event simulation [7], σ_{Δ_i} update coincides with the solving of the

following stochastic program:

$$\begin{aligned} & \max_{\sigma_{\Delta_i}} \frac{1}{Z} \sum_z \mathbf{1}_{\Delta_i^z(t) \geq \text{threshold}} \ln f_i(\Delta_i^z(t); \sigma_{\Delta_i}^2) \\ \Leftrightarrow & \sigma_{\Delta_i}^2 = \frac{\sum_z \mathbf{1}_{\Delta_i^z(t) \geq \text{threshold}} (\Delta_i^z(t))^2}{\mathbf{1}_{\Delta_i^z(t) \geq \text{threshold}}} \end{aligned}$$

Generation of sequences of forecast error differences $(\Delta_i(t))_{i,t}$

Input:

- $(\gamma_i(c), \theta_i(c))_{i,c}$
- Z sample size
- $\rho = 10^{-2}$ rarity parameter
- initial threshold > 0 value

For each geographic market $i = 1, \dots, N$

Initialization: $t = 1$, $\hat{\sigma}_{\Delta_i}(1) = \sigma_{\Delta_i}$

(1) Generate a sample of size Z such that $(\Delta_i^z(t))_z \sim \mathcal{N}(0; (\hat{\sigma}_{\Delta_i}(t))^2)$

(2) Compute the sample $(1 - \rho)$ -quantile $\hat{\text{threshold}}(t)$ which coincides with the $(1 - \rho)Z$ -th order statistic of the sequence $(\Delta_i^z(t))_s$, provided $\hat{\text{threshold}}(t) < \text{threshold}$. Otherwise set $\hat{\text{threshold}}(t) = \text{threshold}$.

(3) Use the same sample and compute: $\hat{\sigma}_{\Delta_i}(t) = \sqrt{\frac{\sum_z \mathbf{1}_{\Delta_i^z(t) \geq \text{threshold}} (\Delta_i^z(t))^2}{\mathbf{1}_{\Delta_i^z(t) \geq \text{threshold}}}}$.

(4) If $\hat{\text{threshold}}(t) < \text{threshold}$ set $t = t + 1$ and reiterate from (1); else STOP.

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