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# Wind Farm Portfolio Optimization under Network Capacity Constraints

Hélène Le Cadre, Anthony Papavasiliou and Yves Smeers

**Abstract**—In this article, we provide a new methodology for optimizing a portfolio of wind farms in a Market Coupling organization, for two Market Designs (exogenous prices and endogenous prices). Our model is built on an agent based representation of a certain number of interacting geographic demand markets, each facing a bilevel program to optimize its production level and bilateral trades with the others while anticipating the grid congestion. The Nash Equilibria resulting from this Signaling Game are characterized using Algorithmic Game Theory. The Markowitz Frontier, containing the set of efficient wind farm portfolios, is derived theoretically as a function of the number of wind farms and of their concentration. Finally, using France-Germany-Belgium real life data, we simulate the Markowitz Frontier contour in the expected cost-conditional variance plane.

**Index Terms**—Algorithmic Game Theory, Market Design, Wind Farm Concentration, Markowitz Frontier.

## I. INTRODUCTION

THE most advanced market design in the restructuring of the European electricity market is Market Coupling. As several other electricity markets, Market Coupling is organized as a two tiered system with a day ahead market and a real time system. European power exchanges operate as zonal markets that ignore Kirchoff’s laws and assume no congestion within zones. Zonal models can lead to dispatch that violates transmission constraints. Counter-trading is required in order to redispatch the system such that transmission constraints are not violated. In their models, Smeers et al. aggregated nodes into zones and ignored Kirchoff’s laws [26]. They determined the flows over interconnections and the total amount of power production (and consumption). The clearing of their zonal market followed by counter-trading might be sub-optimal because there is no congestion anticipation in the day ahead market and no representation of the uncertainty associated to the integration of renewable supply.

In comparison with other systems, Market Coupling relies on a separation of the energy market (the power exchanges) and the transmission system organized by the Transmission System Operator (TSO) [26]. Moreover Market Coupling is progressively moving from a decentralized to a more centralized organization. This trend may become more and more relevant with increasing wind power penetration, which is a result of both European and national policies that complicates this comparison [23]. The comparison between centralized

and decentralized organization in the presence of wind is the subject of the approach developed in [16]. The impact of information asymmetry is also quantified and the price of information (regarding the certification of the renewable supplies) is derived as a function of the required confidence level.

Oggioni et al. compared the effect of two wind policies (“priority dispatch” under which the TSO must accommodate all wind energy produced and the “no priority dispatch” under which the TSO can decide not to inject all potential wind power in the grid in order to limit congestion problems) in a context of Market Coupling organization [23]. The authors showed, using stochastic programming models depending on the different wind penetration levels, that “no priority dispatch” removes most of the problems resulting from Market Coupling organization. However the relevance of this conclusion relies on the strong assumption that the power exchanges and the TSOs are perfectly coordinated among the zones. While Oggioni et al. focused on the day ahead modeling, Nair et al. explicitly characterized the impact of growing wind power penetration, assuming that conventional energy may be procured in three stages (i.e., day ahead, intra day and real time) to balance supply and demand [21]. Our model extends the approach of Nair et al. [21] by taking into account the European market coupling and congestion anticipation. Furthermore, we model power exchange interactions thereby capturing the competition among national energy markets which has not been considered so far.

Accurate short-term forecasts of wind farms power output over the next few hours to days are important factors for secure and low cost operations of power systems with high wind power penetration [20], [24]. According to Girard et al. [8], it is difficult to quantify the economic benefit of increasing predictability. The recent literature dealing with the placement of wind turbines concludes that the aggregation of wind farms can produce significant effects in terms of variability and cost reduction [8] since forecast errors might compensate each other. Furthermore a portfolio of wind farms is likely to give better results in terms of the trade-off between cost and profit and its variability than relying on a single wind farm [9]. Considering both problems of wind farm expansion and optimal wind farm portfolio generation, Girard et al. checked, using West Denmark real life data, that the power producer’s revenue is linear in the wind farm capacity factor and that the predictability of the site i.e., the level of accuracy of short-term wind power productions, has only a very small impact on it. However, as raised by the authors, their results do not quantify the benefit of predictability from the global system

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point of view. Adopting a more systemic approach, Green showed, using 18 years of hourly wind speed data coming from 120 sites around Great Britain, that careful market analysis is needed if investors are to build optimal portfolios of wind stations [9]. Baringo and Conejo already made the link, dealing with the optimization of a strategic wind power unit owner's investment while selling his wind power in a two sided market (including a day ahead and a balancing market) [1].

In this article, we propose to tackle the difficult problem of providing a methodology for optimizing a portfolio of wind farms in a Market Coupling organization. As in [8], the revenue of a wind farm portfolio corresponds to the real time value of its produced energy and not to a subsidy (such as those provided through feed-in tariff systems). A large share of current installed capacity in Europe is supported by feed-in tariffs. As wind power penetration increases, States are switching to direct market participation mechanisms for wind farms [8].

According to the literature mentioned above, a careful (simplified) modeling of Market Coupling is the most crucial modeling aspect. We consider a certain number of geographic demand markets, described in Section II. Since we want to characterize the agents' general behaviors, we do not consider explicitly Kirchoff's laws (though they might be used to provide us with our virtual representation of the grid equivalent capacities) and aggregate the supply and the demand at the market level. The originality of our approach relies on its capability to cope with competition among the energy markets which was ignored in the previously cited models. After having defined the agents' roles in Subsection II-A, we assume that over each geographic market, a bilevel optimization problem occurs in the day ahead market. Its timing is described in Subsection II-B. It is based on the anticipation of what will happen in the real time market. The game is solved by backward induction. The link between day ahead and real time markets is guaranteed by the existence of a reserve, defined here as a quantity of energy purchased in the day ahead, to compensate for the uncertainty of supply in real time. The bilevel optimization problem is solved for two Market Designs: two tiered with exogenous prices in Section III and two tiered with endogenous prices in Section IV. The way this bilevel optimization problem is solved depends on the received signals i.e., which information is shared among the agents. These signals can come from price (real time prices, in Subsection IV-C) or from quantity (reserves, in Subsection IV-D)<sup>1</sup>. Efficient wind farm portfolios are then characterized on the basis of the Markowitz Frontier definition. Its exploration is detailed in Section V. Contrary to traditional approaches, it is computed in a context of rare events, guaranteeing the robustness of the wind farm distribution. Illustrations based on real life wind speed and energy consumption data for France, Germany and Belgium are provided in Section VI.

## II. THE MARKET MODEL

We consider suppliers (distributors or utility companies) with long term contracts for renewable energy. Given such

a long term contract, the suppliers participate in a two tiered market for conventional energy production. It consists of a day ahead market occurring at  $t_f > 0$  and of a real time market, occurring at  $t_0 > t_f$ ; meaning that  $t_0$  occurs after  $t_f$ .

In the European Union (EU), the real time markets introduced in this article can be assimilated to intra day markets [12], though the design and timing of the latter is still debated, or to the EU balancing mechanism where the imbalance price settlement mechanism [8], [15] would be designed so that no compensation would be provided in case of reserve over-provisioning. This choice of modeling can be justified by the fact that our model aims at determining how the suppliers share the risk of under-provisioning between the day ahead and the real time markets.

Market design cannot be separated from the physical transmission constraints resulting from the country interconnection [9]. The super grid is the network backbone enabling the power flow exchanges between the  $N \in \mathbb{N}^*$  interconnected markets. The interconnection among the markets is performed through  $L \in \mathbb{N}^*$  links. Each link  $l = 1, \dots, L$  is defined by its Available Transmission Capacity (ATC) which can be reduced due to losses occurring through the transmission lines. Losses are all the more important as renewable sources of production are located at the extremities of the power network [14] resulting in significant losses.

Inside the economic system formed by the power markets, bilateral trades occur among the markets. We let  $t_{i \rightarrow j}^f$  (resp.  $t_{i \rightarrow j}^0$ ) be the bilaterally traded flow of energy between market  $i$  and market  $j$  in the day ahead market (resp. real time market). Depending on the sign of the traded flow, it can be an import from  $j$  to  $i$  in case where it is negative and, an export from  $i$  to  $j$ , in case where it is positive. Throughout the article, we will use the following conventions:  $(t_{i \rightarrow j}^0)_+ \triangleq \max\{0; t_{i \rightarrow j}^0\}$  and  $(t_{i \rightarrow j}^f)_+ \triangleq \max\{0; t_{i \rightarrow j}^f\}$  and the simplifying notations:  $S_i \triangleq \sum_{j \neq i} t_{i \rightarrow j}^0$  and  $S_i^+ \triangleq \sum_{j \neq i} (t_{i \rightarrow j}^0)_+$ . The power transfer distribution factor (PTDF) matrix, which depends non linearly on the impedances of the transmission lines, enables the linear scaling of the bilateral trades among the markets to physical flows running along the super grid lines [10], [22].

Market Coupling clears energy and transmission in the day ahead and attempts to align prices in real time under the so called "implicit auction" [10], [22]. In practice, the markets having the smaller prices export toward the markets having higher prices until a common price is reached or congestion occurs due to limited ATCs at the interconnection points [6], [14], [26]. If congestion occurs between market  $i$  and market  $j$ , they are split; otherwise they become coupled through a unique integrated market. Congestion management remains a controversial issue in the restructured European electricity sector. Congestion occurs when the infrastructure constrains transactions and long term modification of generation and consumption [6].

Our network model is based on a simplified representation of the European area: we assume that each market is represented as a virtual geographic area and fully interconnected with the other markets, as depicted in Figure 1 for three markets. This assumption is consistent with the French Energy

<sup>1</sup>It is classical to separate non-cooperative games in quantity based ones (Cournot) and in price based ones (Bertrand).

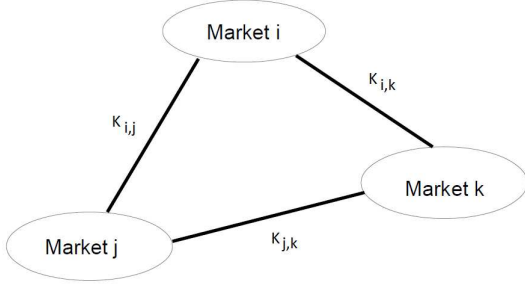


Fig. 1. Equivalent interconnection capacity between the split markets.

Transport Operator (RTE)'s ambition to simplify the European grid modeling by cutting it in zones and by defining an equivalent network between these zones [4]. As in the classical ATCs/PTDFs model, the interconnections among the virtual geographic areas are limited by the super grid line capacity. We introduce the equivalent interconnection capacity between market  $i$  and market  $j$ :  $\kappa_{i,j} \in \mathbb{R}$  such that  $\kappa_{i,j} < +\infty, i \neq j$ . The equivalent interconnection capacities are currently being evaluated by RTE using tests performed throughout the super grid [4]. According to Hutcheon and Bialek's representation of the copper plate [11], for any  $i, j = 1, \dots, N, i \neq j$ ,  $\kappa_{i,j}$  can be either positive or negative; furthermore, we have the relation:  $\kappa_{i,j} = -\kappa_{j,i}$ . Suppose we consider three split markets. In case of congestion,  $t_{i \rightarrow j}^f + t_{i \rightarrow j}^0 = \kappa_{i,j}, \forall i, j = 1, 2, 3, i \neq j$  because, otherwise, the markets which can still export would export until being coupled, leading to two or less split markets.

#### A. The agents

There is a certain number of geographic<sup>2</sup> demand markets, each characterized by a price insensitive<sup>3</sup> and random demand. This demand is unknown in day ahead but revealed in real time. Similarly, wind generation in each geographic market is price insensitive and random: it is unknown in day ahead and revealed in real time. We now describe the different categories of agents interacting over each market.

- Suppliers (distributors or utility companies) deliver energy to consumers characterized by their aggregated demand. They are price takers in the first Market Design (MD 1) detailed in Section III. In contrast with standard assumptions, they are not in the second Market Design (MD 2), described in Section IV. In this latter Market Design, they are aware that their decisions modify prices and take that knowledge into account to minimize their procurement cost. Because the consumers' demand is price insensitive they do not exercise market power with respect to the final demand.

<sup>2</sup>The term "geographic" will be understood in the rest of the article.

<sup>3</sup>We do not consider demand side management in the present article. Models dealing with decentralized demand response integration through distributed learning approaches can be found in [15]. In [17], the end user's demand is price responsive and storage is possible either at the end users' level, for instance through the battery of their electric vehicles, or at the microgrid aggregator's level.

- Conventional energy producers are characterized by their aggregated profit function. There is conventional generation in each market. Marginal costs are higher in real time than in day ahead. We will assume that the conventional energy market in each geographic demand market is perfect, meaning that generators cannot exercise market power and charge a margin on top of marginal cost. As a result, we will assume that suppliers buy electricity at marginal cost.

Finally, an investor (independent power producers, wind farm developers, aggregators, virtual power plant operator) is introduced into the markets. The investor has to decide how to compose an optimal wind farm portfolio for participating in the electricity market.

#### B. Timing

We make the assumption that a clearing price is reached at  $t_f$ . Because the transfers are limited by the equivalent interconnection capacity, it will be harder to align the market prices at  $t_0$ . But to optimize the bilateral trades in the real time, the agents need to forecast what will happen in the real time market. For that purpose, they can consider two scenarios:

**Congestion anticipation:** The markets anticipate the potential congestion of the lines. They are then split in  $N \in \mathbb{N}^*$  geographic areas. Note that two or more countries can be grouped in the same geographic area

**Coupling anticipation:** The markets are myopic and do not anticipate potential congestion, meaning that they are coupled. The mathematical analysis of this scenario will not be detailed in this article; all analytical and numerical details can be found in [16]

We now describe the bilevel program faced by each market. Over each market  $i$ , at time instant  $t_f$ :

- Anticipating what will happen on the real time market i.e., at time instant  $t_0$ , the conventional producers optimize independently and simultaneously the bilateral trades with the other markets so as to maximize their expected profit under equivalent capacity constraints
- The suppliers optimize independently and simultaneously their purchase of conventional energy so as to minimize their expected cost while ensuring that the total purchased quantity satisfies the residual demand

Under congestion anticipation, which is the scenario that we will follow throughout the article, the trades at  $t_f$  are linked to the optimal trades at  $t_0$  according to the relation:  $t_{i \rightarrow j}^f = \kappa_{i,j} - t_{i \rightarrow j}^0, \forall i, j, i \neq j$ . The usual way to solve the bilevel problem described above is to proceed by backward induction. Proceeding backward, the optimization of conventional energy purchases depends on the expectations of the market prices which themselves depend on the bilateral trades concluded by the producers on the power markets. Such games are called Signaling Games (SGs) [27]. Here the signal (the shared information) is based on price but it might also be possible to consider that the game signal is based on quantity. Under this latter assumption, it will be more appropriate to invert Steps (i) and (ii) in the bilevel problem described above. The SG based

on quantity will also be solved by backward induction. Note that the SG based on quantity backward solving coincides with the SG based on price forward solving.

Justifying which form of SG occurs in practice depends on which information is shared among the agents i.e., is the information shared based on market prices or on quantities? In the extreme case where no information is shared, the bilevel game becomes a simultaneous game i.e., Steps (i) and (ii) occur simultaneously. In this case, the conventional energy purchase is optimized through the method described in Subsection IV-C1 and the bilateral trades are optimized using the results established in Subsection IV-D1.

### C. Description of the markets

Market  $i$  is defined by:

- $d_i$  the end users' total demand of energy at time  $t_0$ . It satisfies the relation:  $d_i = \hat{d}_i - v_i$  where  $\hat{d}_i$  is the forecast made at  $t_f$  of the end users' total demand of energy at  $t_0$ .  $v_i$  is a random variable, representing the forecast error made on the demand prediction, and distributed according to a Gaussian density function centered in 0 and of standard deviation  $\sigma_i^v$ :  $v_i \sim \mathcal{N}(0; (\sigma_i^v)^2)$ .
- $w_i$  the energy produced at time  $t_0$  by the market renewable energy producers. It satisfies the relation:  $w_i = \hat{w}_i - \epsilon_i$  where  $\hat{w}_i$  is the forecast made at  $t_f$  of the quantity of renewable energy that market  $i$  producer will produce at  $t_0$ .  $\epsilon_i$  is a random variable, representing the forecast error made on the prediction of the renewable production, distributed according to a Gaussian density function centered in 0 and of standard deviation  $\sigma_i^\epsilon$ :  $\epsilon_i \sim \mathcal{N}(0; (\sigma_i^\epsilon)^2)$ . The forecast error on the production of a *single* wind farm will be denoted  $\tilde{\epsilon}_i$ . Being consistent with the assumption made on  $\epsilon_i$  generation, it is distributed according to a Gaussian density function<sup>4</sup> centered in 0 and of standard deviation  $\tilde{\sigma}_i^\epsilon$ :  $\tilde{\epsilon}_i \sim \mathcal{N}(0; (\tilde{\sigma}_i^\epsilon)^2)$ . The relation between  $\epsilon_i$  and  $\tilde{\epsilon}_i$  will be discussed explicitly in Subsection II-E.
- The forecast error vector for wind production and demand:  $(\tilde{\epsilon}_i \ v_i)^T$ <sup>5</sup> is also supposed to be a Gaussian random vector. According to Sinden [25], wind power output in the United Kingdom (UK) has a weak positive correlation to current electricity demand patterns i.e.,  $\frac{\mathbb{E}[\tilde{\epsilon}_i v_i]}{\sigma_i^v \tilde{\sigma}_i^\epsilon} > 0$ <sup>6</sup>. This implies that  $\Delta_i \triangleq \epsilon_i - v_i$ , which is the difference between renewable production and demand forecast errors, is distributed according to a Gaussian distribution function centered in 0 and of variance  $\sigma_{\Delta_i}^2 = (\sigma_i^\epsilon)^2 - 2\mathbb{E}[\epsilon_i v_i] + (\sigma_i^v)^2$ . In the rest of the article, we will let:  $\Delta_i \sim f_{\Delta_i} \equiv \mathcal{N}(0; \sigma_{\Delta_i}^2)$ ;  $\bar{F}_{\Delta_i}$  will represent the associated complementary cumulative distribution function.  $\Delta_i$  is supposed to be independent

<sup>4</sup>Other density functions might be considered without adding any changes in the derived theoretical results except in the numerical illustrations where the Gaussian assumption greatly simplified the computations.

<sup>5</sup> $T$  stands for the transpose of the vector.

<sup>6</sup>Data analysis ran on 66 onshore weather recording sites for the period 1970 – 2003 in the UK showed a correlation of 0.28 [25]. This is the value that we will use in the simulations.

of any  $\Delta_j, \forall j = 1, \dots, N, j \neq i$  i.e., the prediction errors made on one geographic market are independent of the ones made on the other geographic markets.

- $s_i^f$  (resp.  $s_i^0$ ) market  $i$  supply of conventional energy in day ahead (resp. real time) markets.
- $c_i^f(s_i^f) = a_i^f + b_i^f s_i^f$  (resp.  $c_i^0(s_i^0) = a_i^0 + b_i^0 s_i^0$ ) the marginal cost function of conventional energy produced by market  $i$  and purchased at  $t_f$  (resp.  $t_0$ ), with  $a_i^0 > a_i^f > 0$  and  $b_i^0 > b_i^f > 0$  guaranteeing that the marginal cost on the real time market remains larger than in the day ahead market.
- $q_i^f$  (resp.  $q_i^0$ ) market  $i$  demand of conventional energy in day ahead (resp. real time) markets.

The amounts of energy purchased by market  $i$  at  $t_f$  and at  $t_0$  are defined as follows:  $q_i^f = (\hat{d}_i - \hat{w}_i + r_i)_+$  and  $q_i^0 = (d_i - w_i - q_i^f)_+$  where  $r_i$  is a reserve of energy purchased in day ahead (lower cost) market because of uncertainty of supply at  $t_0$ . Reserve  $r_i$  is determined by the energy supplier in market  $i$  for the consumers' demand  $d_i$  to be satisfied at  $t_0$  at the lowest possible cost. Market  $i$  knows  $\hat{d}_i$  and  $\hat{w}_i$ . Hence it is equivalent for the supplier to determine  $q_i^f$  or  $r_i$ . The hypothesis that  $q_i^f > 0$  holds as long as the demand exceeds the average wind capacity. In the rest of the article, we will assume that:  $q_i^f \triangleq \hat{d}_i - \hat{w}_i + r_i$ .

### D. Suppliers' expected cost and producers' expected profits

We define  $U_i$ , as the expected cost at  $t_f$ , that the supplier has to pay for his end user energy consumption:

$$U_i = q_i^f p^f + \mathbb{E}[q_i^0 p_i^0] \quad (1)$$

We let  $\Pi_i$  be the expected profit at  $t_f$  of market  $i$  conventional energy producer. It is defined as the difference between the price paid by all the markets for the purchase of conventional energy and the cost of the energy<sup>7</sup>. We assume that all the supply is sold at each time. Then:

$$\begin{aligned} \Pi_i &= \sum_{j \neq i} (t_{i \rightarrow j}^f)_+ p^f + [s_i^f - \sum_{j \neq i} (t_{i \rightarrow j}^f)_+] p^f \\ &\quad - \int_0^{s_i^f} c_i^f(s) ds + \mathbb{E}\left[\sum_{j \neq i} (t_{i \rightarrow j}^0)_+ p_j^0\right] \\ &\quad + \mathbb{E}\left[(s_i^0 - \sum_{j \neq i} (t_{i \rightarrow j}^0)_+) p_i^0\right] - \mathbb{E}\left[\int_0^{s_i^0} c_i^0(s) ds\right] \\ &= s_i^f p^f - \int_0^{s_i^f} c_i^f(s) ds + \mathbb{E}\left[\sum_{j \neq i} (t_{i \rightarrow j}^0)_+ p_j^0\right] \\ &\quad + \mathbb{E}\left[(s_i^0 - \sum_{j \neq i} (t_{i \rightarrow j}^0)_+) p_i^0\right] - \mathbb{E}\left[\int_0^{s_i^0} c_i^0(s) ds\right] \quad (2) \end{aligned}$$

### E. Renewable energy modeling

The renewable wind energy production of market  $i$  is a function of the number of wind farms and of their concentration which is characterized by their spatial distribution over

<sup>7</sup>The cost of transport of power in the bilateral trades will not be considered since we focus on a market scale.

market  $i$  geographic area. To determine the renewable energy procurement for market  $i$ , we use the model of Nair et al. [21]. For market  $i$ , we introduce:

- $\alpha_i$  the average wind production of a single wind farm over the geographic area of market  $i$
- $\gamma_i$  the number of wind farms over market  $i$  geographic area
- $\theta_i \in [\frac{1}{2}; 1]$  (resp.  $1 - \theta_i \in [0; \frac{1}{2}]$ ) a constant capturing the concentration (resp. the scattering) of the wind farm locations over market  $i$  geographic area. The more (resp. the less) concentration, the more (resp. the less) correlation there is between the wind farm productions

We suppose that, at  $t_f$ ,  $\alpha_i$  is the best forecast of wind energy procurement of a wind farm [21]. Then:  $\hat{w}_i(\gamma_i) = \alpha_i \gamma_i$ . The forecast error will depend on the wind penetration too, and we choose the coefficient  $\theta_i$  so that  $\epsilon_i(\gamma_i) = \gamma_i^{\theta_i} \tilde{\epsilon}_i$  where  $\tilde{\epsilon}_i$ , as already introduced, represents the forecast error for the production of a single wind farm. We propose the following interpretation for the scaling of  $\theta_i$ : If the wind farms are co-located they will all produce the same quantity of energy at the same time i.e., their productions are strongly correlated. This is the case when  $\theta_i = 1$ . This implies in turn that:  $\epsilon_i = \gamma_i \tilde{\epsilon}_i$  and that:  $\hat{w}_i = w_i + \gamma_i \tilde{\epsilon}_i$ . On the contrary, if they are spatially distributed so that their productions are independent from one another i.e., uncorrelated, and under the assumption that the forecast errors are distributed according to Gaussian distribution functions, the Central Limit Theorem tells us that:  $\sigma_i^\epsilon = \sqrt{\gamma_i} \tilde{\sigma}_i^\epsilon$  [21]. Therefore, the wind farm productions are independent from one another if, and only if,  $\theta_i = \frac{1}{2}$ . Note that in case of more general forecast error distribution functions, it can be interpreted as an approximation for  $\gamma_i$  large enough. Finally, in case where  $\theta_i \in ]\frac{1}{2}; 1[$ , the wind farms are randomly located over the market geographic area and their spatial distribution is intermediate between perfect independence and co-location. With these notations, we obtain:

$$\begin{aligned} w_i(\gamma_i) &= \hat{w}_i(\gamma_i) - \epsilon_i(\gamma_i) = \alpha_i \gamma_i - \gamma_i^{\theta_i} \tilde{\epsilon}_i \\ \sigma_i^\epsilon(\gamma_i) &= \gamma_i^{\theta_i} \tilde{\sigma}_i^\epsilon \\ \epsilon_i(\gamma_i) &= \gamma_i^{\theta_i} \tilde{\epsilon}_i \end{aligned}$$

In the rest of the article, for the sake of simplicity, the dependence of  $w_i$ ,  $\sigma_i^\epsilon$  and  $\epsilon_i$  on  $\gamma_i$  will be omitted.

In the following sections, we derive the suppliers' optimal reserves and real time prices, for two Market Designs.

### III. MD 1: TWO TIERED WITH EXOGENOUS PRICES

In this first Market Design, the prices are supposed exogenous and such that:

$$0 < p^f < p_i^0, \forall i = 1, \dots, N$$

Market  $i$  supplier's expected cost takes the form:

$$\begin{aligned} U_i &= q_i^f p^f + \mathbb{E}[q_i^0 p_i^0] \\ &= (\hat{d}_i - \hat{w}_i + r_i) p^f + p_i^0 \mathbb{E}[(\Delta_i - r_i)_+] \end{aligned}$$

We solve the bilevel Program described in Subsection II-B by backward induction. In Step (ii), each market  $i$  supplier determines independently and simultaneously the quantity of

energy to purchase,  $q_i^f$ , or, equivalently, his reserve,  $r_i$ , so as to minimize his expected procurement cost<sup>8</sup>:

$$\min_{r_i \geq 0} U_i \quad (3)$$

We made the assumption that  $r_i \geq 0$  since otherwise this means that a supplier could be short in the day ahead, something that one may find unrealistic given that conventional plants are more expensive in real time.

Derivating market  $i$  expected cost with respect to  $r_i$ , we obtain:  $\frac{\partial U_i}{\partial r_i} = p^f + p_i^0 \frac{\partial}{\partial r_i} \mathbb{E}[\Delta_i - r_i | \Delta_i \geq r_i] = p^f - p_i^0 \bar{F}_{\Delta_i}(r_i)$ . Then:

$$\frac{\partial U_i}{\partial r_i} |_{r_i=r_i^*} = 0 \Leftrightarrow r_i^* = \bar{F}_{\Delta_i}^{-1}\left(\frac{p^f}{p_i^0}\right) \quad (4)$$

Since  $\frac{\partial^2 U_i}{\partial r_i^2} = p_i^0 f_{\Delta_i}(r_i) > 0$ , it coincides with a minimum for  $U_i$ . Furthermore market  $i$ 's optimal reserve being independent of the other markets' optimal reserves, Equation (4) leads to a unique Nash equilibrium for the market reserves.

Note that inverting the bilateral game steps leads exactly to the same optimal reserve since it depends exclusively on exogenous market prices ( $p_i^0, p^f$ ) and on the forecast error difference standard deviation  $\sigma_{\Delta_i}$ . The investor's total cost, which will be detailed in Section V and used to optimize his wind farm portfolio, depends only on the market prices and on the reserves. Market prices and the optimal reserves being independent of the bilateral trades, we do not give here the detail of their computation.

### IV. MD 2: TWO TIERED WITH ENDOGENOUS PRICES

In the following subsections, we derive analytically the endogenous prices in the day ahead and in the real time markets.

The global day ahead market is characterized by the equilibrium between the supply and the demand:  $q_{\text{tot}}^f(N) = \sum_{i=1, \dots, N} q_i^f = \sum_{i=1, \dots, N} s_i^f$  which is the global quantity of conventional energy exchanged on day ahead markets. Furthermore, for any market  $i$ , we suppose, at  $t_0$ , that the difference between the supply and the demand for conventional energy over market  $i$  coincides with the sum of bilateral trades with the other markets:

$$\sum_{j=1, \dots, N, j \neq i} t_{i \rightarrow j}^0 = s_i^0 - q_i^0 \Leftrightarrow S_i = s_i^0 - q_i^0 \quad (5)$$

We make the assumption that the prices  $p_i^f$  and  $p_i^0$  paid by market  $i$  suppliers for the energy purchased at  $t_f$  and  $t_0$  respectively equal the marginal costs. This assumption has been justified in Subsection II-A. It implies that:  $p_i^f = c_i^f(s_i^f)$  and  $p_i^0 = c_i^0(s_i^0)$ . Furthermore we assume that a clearing price is reached at  $t_f$  i.e.,  $p_i^f = p_j^f \triangleq p^f, \forall i, j = 1, \dots, N, i \neq j$  meaning that all the markets are integrated in a single one at that time. Because the transfers are limited by the available transmission capacities, it will be harder to align the market prices at  $t_0$ : if the markets clear then  $p_i^0 = p_j^0 \triangleq p^0, \forall i, j =$

<sup>8</sup>The reserve is used by the suppliers to compensate for their forecast errors in the real time. So, it is quite natural to assume that they are responsible for their procurements.

$1, \dots, N, i \neq j$ ; otherwise there exists at least one market  $i \in \{1, \dots, N\}$  in which the supplier pays  $p_i^0 \neq p_j^0$  for  $j \in \{1, \dots, N\}$  and  $j \neq i^0$ .

#### A. Derivation of the coupling price

We set:  $A^f \triangleq \sum_{i=1, \dots, N} \frac{a_i^f}{b_i^f}$  and  $B^f \triangleq \sum_{i=1, \dots, N} \frac{1}{b_i^f} > 0$ .

**Lemma 1.** *The coupling price for the day ahead market is:*

$$p^f = \frac{\sum_{i=1, \dots, N} q_i^f + A^f}{B^f}.$$

Proof of Lemma 1. Using the assumption of the supply and demand equilibrium guaranteed by the day ahead market rules, we have:

$$\begin{aligned} q_{\text{tot}}^f(N) &= \sum_{i=1, \dots, N} q_i^f = \sum_{i=1, \dots, N} s_i^f \\ &= \sum_{i=1, \dots, N} \frac{p^f - a_i^f}{b_i^f} \\ &\text{under the assumption that } p_i^f = c_i^f \\ &= \sum_{i=1, \dots, N} \frac{p^f - a_i^f}{b_i^f} \\ &\text{since the } N \text{ markets are coupled at } t_f \\ &= p^f \left( \sum_{i=1, \dots, N} \frac{1}{b_i^f} \right) - \sum_{i=1, \dots, N} \frac{a_i^f}{b_i^f} \end{aligned}$$

We infer from the following equations the day ahead price on the coupling zone:  $p^f = \frac{\sum_{i=1, \dots, N} q_i^f + A^f}{B^f}$ .  $\square$

We set:  $A^0 \triangleq \sum_{i=1, \dots, N} \frac{a_i^0}{b_i^0}$  and  $B^0 \triangleq \sum_{i=1, \dots, N} \frac{1}{b_i^0} > 0$ .

We proved in [16] that the  $N$  markets being coupled at time  $t_0$ , the coupling price for the real time market is:

$$p^0 = \frac{\sum_{i=1, \dots, N} q_i^0 + A^0}{B^0}.$$

#### B. Derivation of the split market prices

We set:  $A_i^0 \triangleq \frac{a_i^0}{b_i^0}$  and  $B_i^0 \triangleq \frac{1}{b_i^0} > 0$ . As in the proof of Lemma 1, we infer the real time price on the  $i$ -th split market:

**Lemma 2.** *The  $N$  markets being split in  $N$  geographic areas, at time  $t_0$ , market  $i$  price for the real time market is:*

$$p_i^0 = \frac{q_i^0 + A_i^0 + \left[ \sum_{j=1, \dots, N, j \neq i} t_{i \rightarrow j}^0 \right]}{B_i^0}.$$

Proof of Lemma 2. Using the real time market rules defined through Equation (5), we have:  $q_i^0 = s_i^0 - \sum_{j=1, \dots, N, j \neq i} t_{i \rightarrow j}^0$ . This implies that:  $q_i^0 + \sum_{j=1, \dots, N, j \neq i} t_{i \rightarrow j}^0 =$

<sup>9</sup>In case where  $p_i^0 \neq p_j^0$ , a congestion rent  $CR_{i,j} = (p_i^0 - p_j^0)t_{i \rightarrow j}^0$  is paid to the transmission operator.  $CR_{i,j}$  is: positive if the lower price market is exporting energy to the higher price market; null if the interconnection lines, binding market  $i$  to market  $j$ , are not congested and  $p_i^0 = p_j^0 = p^0$ ; negative if the lower price market is importing energy from the higher price market.

	Day ahead price	Real time price
<b>Coupling anticipation</b>	$\frac{1}{B^f} \left[ \sum_{i=1, \dots, N} q_i^f + A^f \right]$	$\frac{1}{B^0} \left[ \sum_{i=1, \dots, N} q_i^0 + A^0 \right]$
<b>Congestion anticipation</b>	$\frac{1}{B^f} \left[ \sum_{i=1, \dots, N} q_i^f + A^f \right]$	$\frac{1}{B_i^0} \left[ q_i^0 + A_i^0 + \sum_{j=1, \dots, N, j \neq i} t_{i \rightarrow j}^0 \right]$

TABLE I  
ENDOGENOUS DAY AHEAD PRICE ( $p^f$ ) AND REAL TIME PRICE ( $p_i^0$ ).

$p_i^0 \frac{1}{b_i^0} - \frac{a_i^0}{b_i^0}$  using the fact that  $c_i^0 = p_i^0$ . Then:  $p_i^0 = \frac{q_i^0 + \frac{a_i^0}{b_i^0} + \left[ \sum_{j=1, \dots, N, j \neq i} t_{i \rightarrow j}^0 \right]}{\frac{1}{b_i^0}} = \frac{q_i^0 + A_i^0 + \left[ \sum_{j=1, \dots, N, j \neq i} t_{i \rightarrow j}^0 \right]}{B_i^0}$

using the simplifying notations.  $\square$

In Table I, we summarize the analytical expressions of market  $i$  endogenous prices in day ahead and in real time, depending on whether we are in coupling or in congestion.

#### C. Signaling Game based on price

In this subsection, the signal received by the suppliers is based on price. We assume that congestion occurs at  $t_0$ . We solve the bilevel game described in Subsection II-B proceeding by backward induction. Reserves and bilateral trades cannot be derived analytically. However we provide conditions guaranteeing the existence and uniqueness of a Nash equilibrium for the reserves and detail algorithmically how reserves and bilateral trades at the optimum should be computed.

1) *Minimization of the suppliers' expected cost:* We determined the analytical expressions of the endogenous coupling price for the integrated day ahead market in Subsection IV-A and of the endogenous prices for the split markets in real time in Subsection IV-B. Substituting these values in the suppliers' expected costs, each market  $i$  supplier determines independently and simultaneously the quantity of energy to purchase,  $q_i^f$ , or, equivalently, his reserve,  $r_i$ , so as to minimize his expected procurement cost, as described in optimization Program 3.

Market  $i$  supplier determines the best answer,  $r_i^{BA}(\mathbf{r}_{-i})$ , where  $\mathbf{r}_{-i}$  is a  $N - 1$  dimensional vector containing the reserves of all the suppliers except market  $i$  supplier, which minimizes his expected procurement cost. The decentralized program output is a Nash Equilibrium,  $(r_i^*)_{i=1, \dots, N}$ , defined by:  $r_i^* = r_i^{BA}(\mathbf{r}_{-i}^*), \forall i = 1, \dots, N$ .

**Lemma 3.** *The sum of the bilateral trades of market  $i$  in the real time market,  $S_i$ , can be expressed as a linear function in  $r_i$  and  $\mathbf{r}_{-i}$ .*

Proof of Lemma 3. By definition:  $q_i^f = \sum_{j \neq i} t_{i \rightarrow j}^f + s_i^f$  and  $p^f = a_i^f + b_i^f s_i^f$ . This implies, in turn, that:  $\sum_{j \neq i} t_{i \rightarrow j}^f = q_i^f - s_i^f$  and  $s_i^f = \frac{p^f - a_i^f}{b_i^f}$ . This implies that:  $\sum_{j \neq i} t_{i \rightarrow j}^f = q_i^f - \frac{p^f - a_i^f}{b_i^f} = q_i^f - \frac{\sum_j q_j^f + A^f}{B^f b_i^f} + \frac{a_i^f}{b_i^f}$  using the definition of  $p^f$  under congestion anticipation. Then:  $\sum_{j \neq i} t_{i \rightarrow j}^f =$

$$\begin{aligned} \sum_{j \neq i} \kappa_{i,j} - \sum_{j \neq i} t_{i \rightarrow j}^f &= \sum_{j \neq i} \kappa_{i,j} - (q_i^f - \frac{\sum_j q_j^f + A^f}{B^f b_i^f} + \frac{a_i^f}{b_i^f}) = \\ &= (\frac{1}{B^f b_i^f} - 1)(\hat{d}_i - \hat{w}_i + r_i) + \frac{1}{B^f b_i^f} \sum_{j \neq i} (\hat{d}_j - \hat{w}_j + r_j) + \\ &+ \sum_{j \neq i} \kappa_{i,j} - \frac{1}{b_i^f} (a_i^f - \frac{A^f}{B^f}). \end{aligned}$$

□

This means that there exists a linear function  $\varphi_i : \mathbb{R}_+^N \rightarrow \mathbb{R}$  such that  $S_i = \varphi_i(r_i, \mathbf{r}_{-i})$ .

As a corollary of Lemma 3, we obtain:  $\frac{\partial S_i}{\partial r_i} = \frac{\partial}{\partial r_i} (\sum_{j \neq i} t_{i \rightarrow j}^0) = \frac{1}{B^f b_i^f} - 1$ .

**Proposition 4.** *If  $\sum_{j \neq i} t_{i \rightarrow j}^0 > -A_i^0$  (i.e., low quantity of imports), there exists a positive Nash Equilibrium solution of Program 3. Otherwise, the result still holds provided the standard deviation of  $\Delta_i$  is smaller than  $-\frac{1}{\sqrt{2\pi}} [\frac{A_i^0 + \sum_{j \neq i} t_{i \rightarrow j}^0}{\frac{2B_i^0}{B^f} + \frac{1}{2}(3 - \frac{1}{B^f b_i^f})}]$*

(i.e., small variance for the difference between the error associated to renewable production forecast and the error related to demand forecast).

Proof of Proposition 4. It is provided in Appendix A.

Using the methodology described in Proposition 4 proof, market  $i$  determines the best answer:  $r_i^* = r_i^{B^f A}(\mathbf{r}_{-i}^*)$  which minimizes its expected cost. Going a step further in the computations detailed in the proof of Proposition 4, we prove that this best answer is obtained as the solution of a fixed point equation:  $\bar{F}_{\Delta_i}(r_i^*) = B_i^0 \{A_i^0 + (\frac{1}{B^f b_i^f} - 1)(\hat{d}_i - \hat{w}_i + r_i^*) + \frac{1}{B^f b_i^f} \sum_{j \neq i} (\hat{d}_j - \hat{w}_j + r_j^*) + \sum_{j \neq i} \kappa_{ij} - \frac{1}{b_i^f} (a_i^f - \frac{A^f}{B^f}) - 2r_i^*\} - 1 \left[ \frac{\sum_j (\hat{d}_j - \hat{w}_j) + \sum_{j \neq i} r_j^* + r_i^* + A^f}{B^f} + \frac{\hat{d}_i - \hat{w}_i + r_i^*}{B^f} + (\frac{1}{B^f b_i^f} + 1) \frac{\sigma_{\Delta_i}}{B_i^0 \sqrt{2\pi}} \exp(-\frac{(r_i^*)^2}{2\sigma_{\Delta_i}^2}) \right]$ . This fixed point equation is solved simultaneously by all the markets. Nash Equilibria are obtained at the intersections of the best answers.

To show uniqueness of the resulting Nash Equilibrium, we apply the contraction mapping approach. Due to Bertsekas [2], it is sufficient to show that the Hessian of the expected cost functions fulfills the diagonal dominance condition i.e.,  $\sum_{j \neq i} |\frac{\partial^2 U_i}{\partial r_i \partial r_j}| < |\frac{\partial^2 U_i}{\partial r_i^2}|, \forall i = 1, \dots, N$ . We show, below, that the uniqueness of the Nash Equilibrium heavily relies on the number of interacting markets:

**Proposition 5.** *Providing there is a low quantity of imports (i.e.,  $S_i > -A_i^0, \forall i = 1, \dots, N$  according to Proposition 4), if  $\frac{1}{B^f} (3 - N) + b_i^0 (3 - N) \bar{F}_{\Delta_i}(r_i) + \frac{A_i^0 + S_i}{B_i^0} f_{\Delta_i}(r_i) > 0, \forall i = 1, \dots, N$  then there exists a unique Nash Equilibrium solution of Program 3.*

Proof of Proposition 5. Derivating first  $U_i$  with respect to  $r_i$  and then, a second time, with respect to  $r_j, j \neq i$ , we obtain:  $\frac{\partial^2 U_i}{\partial r_i \partial r_j} = \frac{1}{B^f} + \frac{b_i^0}{b_i^f B^f} \bar{F}_{\Delta_i}(r_i) > 0$ . The assumption that  $S_i > -A_i^0$  implies, according to Proposition 4:  $\frac{\partial^2 U_i}{\partial r_i^2} - \sum_{j \neq i} \frac{\partial^2 U_i}{\partial r_i \partial r_j} = \frac{1}{B^f} (3 - N) + b_i^0 (3 - \frac{N}{b_i^f B^f}) \bar{F}_{\Delta_i}(r_i) + \frac{A_i^0 + S_i}{B_i^0} f_{\Delta_i}(r_i) > \frac{1}{B^f} (3 - N) + b_i^0 (3 - N) \bar{F}_{\Delta_i}(r_i) + \frac{A_i^0 + S_i}{B_i^0} f_{\Delta_i}(r_i), \forall i = 1, \dots, N$ . □

If  $N \leq 3$ , then Proposition 5 holds ; otherwise checking the

diagonal dominance condition is not straightforward.

2) *Optimization of the bilateral trades:* First, we note that the supplies of conventional energy on day ahead and real time markets can be expressed as functions of the bilateral trades in real time:

$$\begin{aligned} s_i^f &= \sum_{j \neq i} t_{i \rightarrow j}^f + q_i^f = \sum_{j \neq i} \kappa_{i,j} - S_i + \hat{d}_i - \hat{w}_i + r_i \\ s_i^0 &= \sum_{j \neq i} t_{i \rightarrow j}^0 + (\Delta_i - r_i)_+ = S_i + (\Delta_i - r_i)_+ \end{aligned}$$

**Proposition 6.** *The optimal bilateral trade between market  $i$  and market  $k$  can be expressed as a linear function in  $r_k, \mathbf{r}_{-k}, (\mathbb{E}[(\Delta_i - r_i)_+])_{|p_i^0 < p_i^0}$  and  $\mathbb{E}[(\Delta_k - r_k)_+]$ .*

Proof of Proposition 6. The conventional energy producers' expected profit function can be rewritten as:

$$\begin{aligned} \Pi_i &= (\hat{d}_i - \hat{w}_i + r_i + \sum_{j \neq i} \kappa_{i,j} - S_i) \frac{1}{B^f} \left( \sum_j (\hat{d}_j - \hat{w}_j + r_j) + A^f \right) - a_i^f (\hat{d}_i - \hat{w}_i + r_i + \sum_{j \neq i} \kappa_{i,j} - S_i) \\ &\quad - \frac{b_i^f}{2} (\hat{d}_i - \hat{w}_i + r_i + \sum_{j \neq i} \kappa_{i,j} - S_i)^2 \\ &\quad + \sum_{j \neq i} (t_{i \rightarrow j}^0)_+ \mathbb{E} \left[ \frac{(\Delta_j - r_j)_+ + A_j^0 + S_j}{B_j^0} \right] \\ &\quad - \frac{b_i^0}{2} \mathbb{E} [((\Delta_i - r_i)_+ + S_i)^2] + \mathbb{E} [(\Delta_i - r_i)_+] \frac{1}{B_i^0} \left( (\Delta_i - r_i)_+ \right. \\ &\quad \left. + A_i^0 + S_i \right) - a_i^0 \mathbb{E} [(\Delta_i - r_i)_+ + S_i] \\ &\quad + \mathbb{E} [S_i - \sum_{j \neq i} (t_{i \rightarrow j}^0)_+] \frac{(\Delta_i - r_i)_+ + A_i^0 + S_i}{B_i^0} \end{aligned}$$

We suppose that there exists a  $k$  in  $\{1, \dots, N\}$  such that  $t_{i \rightarrow k}^0 > 0$ . We compute:

$$\begin{aligned} \frac{\partial \Pi_i}{\partial t_{i \rightarrow k}^0} &= - \frac{\sum_j (\hat{d}_j - \hat{w}_j + r_j) + A^f}{B^f} \\ &\quad + \underbrace{a_i^f + b_i^f \left( \sum_{j \neq i} \kappa_{ij} - S_i + \hat{d}_i - \hat{w}_i + r_i \right)}_{c_i^f(s_i^f)} \\ &\quad - \frac{1}{B_k^0} t_{i \rightarrow j}^0 + \mathbb{E}[p_k^0] + \frac{1}{B_i^0} (S_i - \sum_{j \neq i} (t_{i \rightarrow j}^0)_+ \\ &\quad + \mathbb{E}[(\Delta_i - r_i)_+]) - a_i^0 - b_i^0 \mathbb{E}[(\Delta_i - r_i)_+ + S_i] \end{aligned}$$

We observe that the first line of the above equation vanishes since, by assumption,  $p^f = c_i^f(s_i^f)$ . Furthermore, since  $\mathbb{E}[p_k^0] = \frac{1}{B_k^0} \mathbb{E}[(\Delta_k - r_k)_+] + c_k^0(S_k)$  and after a few simplifications, we obtain:  $\frac{\partial \Pi_i}{\partial t_{i \rightarrow k}^0} = -\frac{1}{B_k^0} t_{i \rightarrow k}^0 + \frac{1}{B_k^0} \mathbb{E}[(\Delta_k - r_k)_+] + c_k^0(S_k) - c_i^0(S_i^+) - a_i^0$ . Then:

$$\frac{\partial \Pi_i}{\partial t_{i \rightarrow k}^0} = 0 \Leftrightarrow t_{i \rightarrow k}^0 = B_k^0 [c_k^0(S_k) - c_i^0(S_i^+)] + \mathbb{E}[(\Delta_k - r_k)_+] \quad (6)$$



Summing  $t_{i \rightarrow k}^0$  over all the  $k$  such that  $t_{i \rightarrow k}^0 > 0$  i.e.,  $p_i^0 < p_k^0$ , we obtain:

$$S_i^+ = \sum_{k|p_i^0 < p_k^0} (B_k^0 [c_k^0(S_k) - c_i^0(S_i^+)] + \mathbb{E}[(\Delta_k - r_k)_+])$$

Separating the equation in  $S_i^+$  we obtain:

$$S_i^+ = \frac{1}{1 + b_i^0 \sum_{k|p_i^0 < p_k^0} B_k^0} \left\{ \sum_{k|p_i^0 < p_k^0} (B_k c_k^0(S_k) + \mathbb{E}[(\Delta_k - r_k)_+]) - a_i^0 \sum_{k|p_i^0 < p_k^0} B_k^0 \right\} \quad (7)$$

By substitution of Equation (7) in Equation (6), we obtain:

$$\begin{aligned} t_{i \rightarrow k}^0 &= B_k^0 [c_k^0(S_k) - c_i^0(S_i^+)] \\ &- c_i^0 \left( \frac{1}{1 + b_i^0 \sum_{l|p_i^0 < p_l^0} B_l^0} \left\{ \sum_{l|p_i^0 < p_l^0} (B_l c_l^0(S_l) + \mathbb{E}[(\Delta_l - r_l)_+]) - a_i^0 \sum_{l|p_i^0 < p_l^0} B_l^0 \right\} \right) \\ &+ \mathbb{E}[(\Delta_k - r_k)_+] \end{aligned}$$

This means that  $t_{i \rightarrow k}^0$  can be expressed exclusively as a function of  $S_k$ ,  $r_k$ ,  $(\mathbb{E}[(\Delta_l - r_l)_+])_{l|p_i^0 < p_l^0}$  and  $\mathbb{E}[(\Delta_k - r_k)_+]$ . But, we proved in Lemma 3 that there exists a linear function  $\varphi_k : \mathbb{R}_+^N \rightarrow \mathbb{R}$  such that  $S_k = \varphi_k(r_k, \mathbf{r}_{-k})$ . As a result, at the optimum,  $t_{i \rightarrow k}^0$  can be expressed as a linear function in  $r_k$ ,  $\mathbf{r}_{-k}$ ,  $(\mathbb{E}[(\Delta_l - r_l)_+])_{l|p_i^0 < p_l^0}$  and  $\mathbb{E}[(\Delta_k - r_k)_+]$ .  $\square$

#### D. Signaling Game based on quantity

In Subsection IV-C, the game signal was based on price. In this subsection, we assume that the game signal is based on quantity and that congestion occurs at  $t_0$ . As already mentioned, this implies that the bilevel game timing needs to be reversed i.e., Step (i) becomes (ii) and Step (ii) becomes (i). Similarly to the previous section, the resulting SG is solved by backward induction.

The final state of the system is defined by the difference between the real time prices on each couple of markets:  $p_i^0 - p_k^0, \forall i, k, i \neq k$ . Indeed, its sign determines whether this is an export from market  $i$  to market  $k$  or, the reverse.

Under backward induction, market  $i$  production of conventional energy and prices in real time and in day ahead can be expressed as linear functions in the sum of its bilateral trades and in its reserve:

$$\begin{aligned} s_i^0 &= S_i + (\Delta_i - r_i)_+ \\ s_i^f &= \sum_{j \neq i} \kappa_{i,j} - S_i + (\hat{d}_i - \hat{w}_i + r_i) \end{aligned}$$

and  $p^f = \frac{1}{B^f} [\sum_i (\hat{d}_i - \hat{w}_i + r_i) + A^f]$ ,  $p_i^0 = \frac{1}{B_i^0} [(\Delta_i - r_i)_+ + A_i^0 + S_i]$ .

1) *Optimization of the bilateral trades:* Without loss of generality, we assume that:  $p_1^0 > p_2^0 > \dots > p_N^0$ .

**Proposition 7.** *The optimal bilateral trades between market  $i$  and the other markets can be expressed as linear functions in  $r_i, \mathbf{r}_{-i}$ .*

*Proof of Proposition 7.* We start by assuming that there exists a  $k$  in  $1, \dots, N$  such that  $t_{i \rightarrow k}^0 > 0$ . Derivating  $\Pi_i$  with respect to  $t_{i \rightarrow k}^0$  we obtain:

$$\frac{\partial \Pi_i}{\partial t_{i \rightarrow k}^0} = 0 \Leftrightarrow t_{i \rightarrow k}^0 = (\Delta_k - r_k)_+ + B_k^0 [c_k^0(S_k) - c_i^0(S_i^+)] \quad (8)$$

Summing this equation over all the  $k \in \{1, \dots, N\}$  such that  $p_i^0 < p_k^0$  we obtain:

$$\begin{aligned} S_i^+ &= \sum_{k|p_i^0 < p_k^0} (\Delta_k - r_k)_+ + \sum_{k|p_i^0 < p_k^0} B_k^0 c_k^0(S_k) \\ &- c_i^0(S_i^+) \sum_{k|p_i^0 < p_k^0} B_k^0 \end{aligned}$$

which is equivalent with:

$$\begin{aligned} S_i^+ &= \frac{1}{1 + b_i^0 \sum_{k|p_i^0 < p_k^0} B_k^0} \left\{ \sum_{k|p_i^0 < p_k^0} (\Delta_k - r_k)_+ + \sum_{k|p_i^0 < p_k^0} B_k^0 c_k^0(S_k) - a_i^0 \sum_{k|p_i^0 < p_k^0} B_k^0 \right\} \quad (9) \end{aligned}$$

Substituting Equation (9) in Equation (8), we obtain:

$$\begin{aligned} t_{i \rightarrow k}^0 &= (\Delta_k - r_k)_+ + B_k^0 [c_k^0(S_k) - c_i^0(S_i^+)] \\ &- c_i^0 \left( \frac{1}{1 + b_i^0 \sum_{l < i} B_l^0} \left\{ \sum_{l < i} (\Delta_l - r_l)_+ + \sum_{l < i} B_l^0 c_l^0(S_l) - a_i^0 \sum_{l < i} B_l^0 \right\} \right) \quad (10) \end{aligned}$$

Note that according to the assumption made on the real time price ranking, the set  $\{l|p_i^0 < p_l^0\}$  is equivalent with the set  $\{l|l < i\}$ . Summing Equation (10) over all the  $k \neq i$ , we obtain:

$$S_i = \omega_i(\mathbf{r}_{-i}) + \sum_{k \neq i} S_k - \beta_i \sum_{l|p_i^0 < p_l^0} S_l$$

where:

$$\begin{aligned} \beta_i &= \frac{b_i^0 \sum_{k \neq i} B_k^0}{1 + b_i^0 \sum_{l|p_i^0 < p_l^0} B_l^0} \\ \omega_i(\mathbf{r}_{-i}) &= \sum_{k \neq i} (\Delta_k - r_k)_+ + \sum_{k \neq i} B_k^0 a_k^0 - a_i^0 \sum_{k \neq i} B_k^0 \\ &- \frac{b_i^0}{1 + b_i^0 \sum_{l|p_i^0 < p_l^0} B_l^0} \left[ \sum_{l|p_i^0 < p_l^0} (\Delta_l - r_l)_+ + \sum_{l|p_i^0 < p_l^0} B_l^0 a_l^0 - a_i^0 \sum_{l|p_i^0 < p_l^0} B_l^0 \right] \sum_{k \neq i} B_k^0 \end{aligned}$$

The above equation can be rewritten as:

$$\begin{aligned} S_1 &= \omega_1(\mathbf{r}_{-1}) + \sum_{k \neq 1} S_k \\ S_i &= \omega_i(\mathbf{r}_{-i}) + \sum_{k \neq i} S_k - \beta_i \sum_{l|p_i^0 < p_l^0} S_l, \forall i \geq 2 \end{aligned}$$

According to the first equation  $\sum_{k \neq i} S_k = 2S_1 - \omega_1(\mathbf{r}_{-1}) - S_i$ . By substitution in the second equation, we obtain:

$$S_i = \frac{\omega_i(\mathbf{r}_{-i}) - \omega_1(\mathbf{r}_{-1})}{2} + S_1 - \frac{\beta_i}{2} \sum_{l < i} S_l, \forall i \geq 2$$

Considering this equation for any  $i \geq 2$ , it can be rewritten as a system:

$$\begin{pmatrix} 1 & 0 & 0 & \dots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \frac{\beta_2}{2} & \frac{\beta_2}{2} & 1 & \ddots & 0 \\ \vdots & \vdots & \vdots & \ddots & 0 \\ \frac{\beta_N}{2} & \frac{\beta_N}{2} & \frac{\beta_N}{2} & \dots & 1 \end{pmatrix} \begin{pmatrix} S_2 \\ \vdots \\ S_i \\ \vdots \\ S_N \end{pmatrix} = \begin{pmatrix} \frac{\omega_2(\mathbf{r}_{-2}) - \omega_1(\mathbf{r}_{-1})}{2} + (1 - \frac{\beta_2}{2})S_1 \\ \vdots \\ \frac{\omega_i(\mathbf{r}_{-i}) - \omega_1(\mathbf{r}_{-1})}{2} + (1 - \frac{\beta_i}{2})S_1 \\ \vdots \\ \frac{\omega_N(\mathbf{r}_{-N}) - \omega_1(\mathbf{r}_{-1})}{2} + (1 - \frac{\beta_N}{2})S_1 \end{pmatrix}$$

The system matrix is inferior triangular. Therefore, it is invertible using an appropriate algorithm:

$$\begin{aligned} S_2 &= \frac{\omega_2(\mathbf{r}_{-2}) - \omega_1(\mathbf{r}_{-1})}{2} + (1 - \frac{\beta_2}{2})S_1 \\ S_3 &= \frac{\omega_3(\mathbf{r}_{-3}) - \omega_1(\mathbf{r}_{-1})}{2} + (1 - \frac{\beta_3}{2})S_1 - \frac{\beta_2}{2}S_2 \\ &\vdots \\ S_N &= \frac{\omega_N(\mathbf{r}_{-N}) - \omega_1(\mathbf{r}_{-1})}{2} + (1 - \frac{\beta_N}{2})S_1 \\ &\quad - \frac{\beta_N}{2} \underbrace{\sum_{1 < i < N} S_i}_{S_1 - \omega_1(\mathbf{r}_{-1}) - S_N} \end{aligned}$$

The last equation can be rewritten as:

$$S_N = \frac{\omega_N(\mathbf{r}_{-N}) - \omega_1(\mathbf{r}_{-1})}{2} + (1 - \frac{\beta_N}{2})S_1 + \omega_1(\mathbf{r}_{-1}) \frac{\beta_N}{2}$$

Proceeding recursively for  $i = N - 1, \dots, 2$ , it is possible to express any  $S_i, i \geq 2$  as a linear function of  $S_1$ . As a result, there exist vectors  $(\zeta_i)_{2 \leq i \leq N}$  and  $(\xi_i)_{2 \leq i \leq N}$  such that:

$$\begin{pmatrix} S_2 \\ \vdots \\ S_i \\ \vdots \\ S_N \end{pmatrix} = \begin{pmatrix} \zeta_2 \\ \vdots \\ \zeta_i \\ \vdots \\ \zeta_N \end{pmatrix} + S_1 \begin{pmatrix} \xi_2 \\ \vdots \\ \xi_i \\ \vdots \\ \xi_N \end{pmatrix} \quad (11)$$

To give an example:  $\zeta_N = \frac{\omega_N(\mathbf{r}_{-N}) - \omega_1(\mathbf{r}_{-1})}{2} + \omega_1(\mathbf{r}_{-1}) \frac{\beta_N}{2}$  and  $\xi_N = 1 - \frac{\beta_N}{2}$ .

Summing the above system over all  $i \geq 2$ , we obtain:

$$\underbrace{\sum_{i \geq 2} S_i}_{S_1 - \omega_1(\mathbf{r}_{-1})} = \sum_{i \geq 2} \zeta_i + S_1 \sum_{i \geq 2} \xi_i$$

which is equivalent with:  $S_1 = \frac{\sum_{i \geq 2} \zeta_i + \omega_1(\mathbf{r}_{-1})}{1 - \sum_{i \geq 2} \xi_i}$ . By substitution in Equation (11) it is possible to infer  $(S_i)_{i \geq 2}$  as a function of  $(\zeta_i, \xi_i)_{i \geq 2}$  and  $\omega_1(\mathbf{r}_{-1})$  i.e., of  $\mathbf{r}_i, \mathbf{r}_{-i}$ .  $\square$

Since  $S_k$  is linear in  $(\Delta_k - r_k)_+, k = 1, \dots, N$  and  $p_i^0 = \frac{(\Delta_i - r_i)_+ + A_i^0 + S_i}{B_i^0}$  we obtain that for any market  $i$ , there exist coefficients  $(\eta_{i,l})_{l=1, \dots, N}$  and  $\bar{\eta}_i$  such that:  $p_i^0 = \sum_{l=1, \dots, N} \eta_{i,l} (\Delta_l - r_l)_+ + \bar{\eta}_i$ .

2) *Minimization of the suppliers' expected cost:*

**Proposition 8.** *The other markets' optimal reserves being fixed to  $\mathbf{r}_{-i}^*$ , we prove that there exists a positive  $r_i^*$  minimizing market  $i$  expected cost. Furthermore, when  $N \leq 3$ , there exist values for the marginal cost function parameters  $(a_i^0, b_i^0)_i$  guaranteeing the uniqueness of the Nash equilibrium.*

Proof of Proposition 8. It is provided in Appendix B.

We proved that for both SG based on price and SG based on quantity there exists a positive Nash equilibrium. Although the conditions guaranteeing its uniqueness differ, we observe that in both cases for less than three geographic markets, there exist parameter values guaranteeing its uniqueness.

Identifying conditions guaranteeing the uniqueness of the Nash equilibrium is fundamental. Indeed the presence of multiple Nash equilibria might introduce instability in the system behavior. To illustrate this instability threat, it has been reported that the Baltic-Nordic-Polish area crashed in August 13, 2012 because the Nord Pool spot was unable to include Poland<sup>10</sup>.

## V. EXPLORING THE MARKOWITZ FRONTIER: WHAT DOES IT LOOK LIKE?

Power system and market operators may want to incite wind farm investors to design portfolios that increase predictability so that the suppliers' costs is minimized while minimizing the deployment cost.

There can be significant year-to-year variations in wind conditions, which would have an impact on profitability, and these may differ between regions [9]. Furthermore, the higher the terrain complexity, the lower the wind predictability and correlation among wind farms decreases with the distance [8]. Therefore we assume that each market  $i$  is clusterized in a subset  $\mathcal{C}_i$  of clusters where, over  $c \in \mathcal{C}_i$ , the estimated demand  $\hat{d}_i(c)$  and the wind mean production  $\alpha_i(c)$  are supposed constant and such that:

$$d_i(c, t) = \hat{d}_i(c) - v_i(c, t) \quad (12)$$

$$w_i(c, t) = \hat{w}_i(c) - \epsilon_i(c, t) \quad (13)$$

Making the parallel with the previous notations, we have:  $d_i(t) = \sum_{c \in \mathcal{C}_i} d_i(c, t)$  and  $\alpha_i = \sum_{c \in \mathcal{C}_i} \alpha_i(c)$ . Clustering may be performed through one of the automatic partitioning algorithms used in Machine Learning [13], as illustrated for Germany in Section VI.

We assume that the geographic market  $i$  clusters are defined so that there is no correlation among the cluster forecast

<sup>10</sup>Source of this crash reporting is <http://houmollerconsulting.dk>.

errors but that, inside each cluster, there remains a positive correlation between the wind production and the demand forecast error. Transposing Subsection II-C assumptions to a finer scale (i.e., clusters instead of geographic markets), we assume that  $(\Delta_i(c))_{c \in \mathcal{C}_i}$  is distributed according to a  $|\mathcal{C}_i|$ -dimensional Gaussian density function centered in the zero  $|\mathcal{C}_i|$  dimensional vector and with a diagonal variance-covariance matrix having on its principal diagonal all the variances  $(\sigma_{\Delta_i}(c))^2, \forall c \in \mathcal{C}_i$  and zeros everywhere else since the covariance between any  $\Delta_i(c), \Delta_i(c'), \forall c, c' \in \mathcal{C}_i, c \neq c'$  vanishes. Furthermore, over each cluster  $c \in \mathcal{C}_i$ , the wind production and the demand forecast errors are correlated due to the assumption that  $(\tilde{\epsilon}_i(c) \nu_i(c))^T$  is a Gaussian random vector centered in  $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$  and of variance-covariance matrix  $\begin{pmatrix} (\tilde{\sigma}_i^\epsilon(c))^2 & \mathbb{E}[\tilde{\epsilon}_i(c)\nu_i(c)] \\ \mathbb{E}[\tilde{\epsilon}_i(c)\nu_i(c)] & (\sigma_i^\nu(c))^2 \end{pmatrix}$ .

Modern Portfolio Theory is an alternative to the traditional method of analyzing each investment's individual merits. When investors look at each investment's individual merits, they are analyzing one investment without worrying about the way the different investments will perform relative to each other. On the other hand, Modern Portfolio Theory places a large emphasis on the correlation between the investments. Markowitz defines as efficient the portfolios which are characterized by a maximum expected revenue for a fixed risk (or, equivalently, for a minimum risk for a fixed expected revenue) [18]. Risk and volatility are treated as the same thing: Markowitz uses risk as a measurement of the likelihood that an investment still goes up and down in value, and how often and by how much. The theory assumes that investors prefer to minimize risk. The Efficient Frontier, also called the Markowitz Frontier (MF), is then defined as the set of all the portfolios which are efficient. In this article, the investor applies Modern Portfolio Theory to determine the wind farm portfolio that maximizes his return (expected cost) while minimizing his risk conditionally to the occurrence of rare events. Indeed as raised by Marling and Emanuelson [19], if an investor wants to use the MF model to choose a suitable portfolio then it is suitable to do some complementary computations of the risk conditionally to the occurrence of rare events. This is especially true in the context of this article since the efficiency of our wind farm portfolio positioning depends heavily on its capability to cope with rare events caused by a far smaller/larger production of wind power than expected and resulting in large forecast errors. In that case, the reserve might not be sufficient to compensate the forecast error.

In the numerical illustrations we will test two assumptions on the investment function: either it is linear in the number of settled wind farms:  $I(\gamma_i(c)) = \text{cost}_{\text{inv}}\gamma_i(c)$  or it is quadratic:  $I(\gamma_i(c)) = \text{cost}_{\text{inv}}\gamma_i(c)^2$  with  $\text{cost}_{\text{inv}} > 0$  representing the cost of investment for a single wind farm.

#### A. In the $(\gamma_i(c), \theta_i(c))_{i,c}$ plane

In this subsection, we characterize analytically the MF in the  $(\gamma_i(c), \theta_i(c))_{i,c}$  plane.

**Theorem 9.** *Whatever the Market Design (i.e., two tiered with exogenous prices, two tiered with endogenous prices), the Markowitz Frontier in the  $\gamma_i(c), \theta_i(c)$  plane is always completely described by the following set of equations:*

$$\begin{aligned} \theta_i(c) &= \frac{\ln \frac{\mathbb{E}[\tilde{\epsilon}_i(c)\nu_i(c)]}{(\tilde{\sigma}_i^\epsilon(c))^2}}{\ln \gamma_i(c)} \\ \frac{\mathbb{E}[\tilde{\epsilon}_i(c)\nu_i(c)]}{(\tilde{\sigma}_i^\epsilon(c))^2} &\leq \gamma_i(c) \leq \left( \frac{\mathbb{E}[\tilde{\epsilon}_i(c)\nu_i(c)]}{(\tilde{\sigma}_i^\epsilon(c))^2} \right)^2 \\ \gamma_i(c) &\in \mathbb{N}^* \end{aligned}$$

This is a strong result as it means that the optimal concentration for the wind farm positioning is independent of what happens on the markets.

The theorem proof is detailed below for the two Market Designs described in Sections III and IV.

The investment strategy for the optimal wind farm portfolio deployment is defined over a finite horizon  $0 < T < +\infty$ . The SG based on price as described in Section IV-C is repeated over a finite horizon  $T$ . The investor's problem is to determine the optimal number of wind farms  $\gamma_i(c) \in \mathbb{N}^*$  and their concentration  $\frac{1}{2} \leq \theta_i(c) \leq 1$ , over each cluster  $c \in \mathcal{C}_i$  of the geographic demand market  $i$ , such that his expected cost is minimal and the variance of his cost conditionally to the occurrence of rare events is minimal.

The wind power predictability level is measured by the cost devoted to the generation of conventional energy: the smaller this cost is, the higher the wind power predictability. It is introduced in the investor's total cost to give him incentives to design portfolios increasing wind power predictability and therefore, minimizing the suppliers' expected costs.

**MD 1:** Market  $i$ 's contribution to the investor's total cost is defined as the sum of the cost resulting from its conventional energy demand, repeated  $T$  times, and of the cost devoted to the deployment of wind farm portfolios over its geographic area:

$$\begin{aligned} \mathcal{J}(i, T) &= \sum_t \left( \sum_{c \in \mathcal{C}_i} (\hat{d}_i(c) - \alpha_i(c)\gamma_i(c)) + r_i(t) \right) p^f(t) \\ &+ \sum_t \left[ \left( \sum_{c \in \mathcal{C}_i} (\tilde{\epsilon}_i(c, t)\gamma_i(c)^{\theta_i(c)} - \nu_i(c, t)) \right. \right. \\ &\left. \left. - r_i(t) \right)_+ p_i^0(t) \right] + \sum_{c \in \mathcal{C}_i} I(\gamma_i(c)) \\ &= \sum_t \sum_{c \in \mathcal{C}_i} (\hat{d}_i(c) - \alpha_i(c)\gamma_i(c) + r_i(t)) p^f(t) \\ &+ \sum_t (\Delta_i(t) - r_i(t))_+ p_i^0(t) + \sum_{c \in \mathcal{C}_i} I(\gamma_i(c)) \end{aligned}$$

where, we recall:  $\Delta_i(t) = \sum_{c \in \mathcal{C}_i} (\epsilon_i(c, t) - \nu_i(c, t)) = \sum_{c \in \mathcal{C}_i} (\gamma_i(c)^{\theta_i(c)} \tilde{\epsilon}_i(c, t) - \nu_i(c, t))$ .

We obtain quite easily the analytical expression of the

variance of  $\Delta_i$ :

$$\begin{aligned}
\sigma_{\Delta_i}^2 &= \sum_{c \in \mathcal{C}_i} \sigma_{\Delta_i}^2(c) = \sum_{c \in \mathcal{C}_i} \text{Var}(\epsilon_i(c) - \nu_i(c)) \\
&= \sum_{c \in \mathcal{C}_i} \text{Var}(\gamma_i(c)^{\theta_i(c)} \tilde{\epsilon}_i(c) - \nu_i(c)) \\
&= \sum_{c \in \mathcal{C}_i} \left\{ (\tilde{\sigma}_i^\epsilon(c))^2 \gamma_i(c)^{2\theta_i(c)} \right. \\
&\quad \left. - 2\gamma_i(c)^{\theta_i(c)} \mathbb{E}[\tilde{\epsilon}_i(c)\nu_i(c)] + \sigma_i^\nu(c)^2 \right\} \quad (14)
\end{aligned}$$

Then we compute the investor's risk i.e., his variance conditionally to the occurrence of rare events:

$$\begin{aligned}
&\text{Var}(\mathcal{J}(T) | \cap_i \{\Delta_i(t) \geq r_i\}_t) \\
&= \sum_i \text{Var}(\mathcal{J}(i, T) | \{\Delta_i(t) \geq r_i\}_t) \\
&= \sum_i \sigma_{\Delta_i}^2 \sum_t p_i^0(t)^2 \\
&= \sum_i \sum_{c \in \mathcal{C}_i} \sigma_{\Delta_i}^2(c) \sum_t p_i^0(t)^2 \\
&= \sum_i \sum_{c \in \mathcal{C}_i} \left[ (\tilde{\sigma}_i^\epsilon(c))^2 \gamma_i(c)^{2\theta_i(c)} \right. \\
&\quad \left. - 2\gamma_i(c)^{\theta_i(c)} \mathbb{E}[\tilde{\epsilon}_i(c)\nu_i(c)] + \sigma_i^\nu(c)^2 \right] \sum_t p_i^0(t)^2
\end{aligned}$$

using Equation (14). In the rest of the article, we will let:  $\text{RVar}(T) \triangleq \text{Var}(\mathcal{J}(T) | \cap_i \{\Delta_i(t) \geq r_i\}_t)$  and  $\text{RVar}(i, T) \triangleq \text{Var}(\mathcal{J}(i, T) | \{\Delta_i(t) \geq r_i\}_t)$ .

The expectation of the investor's total cost is:

$$\begin{aligned}
\mathbb{E}[\mathcal{J}(T)] &= \sum_i \sum_t \left( \sum_{c \in \mathcal{C}_i} \hat{d}_i(c) + \bar{F}_{\Delta_i}^{-1} \left( \frac{p^f(t)}{p_i^0(t)} \right) \right) p^f(t) \\
&\quad - \sum_i \sum_t \left( \sum_{c \in \mathcal{C}_i} \alpha_i(c) \gamma_i(c) \right) p^f(t) \\
&\quad + \sum_i \sum_t p_i^0(t) \mathbb{E}[(\Delta_i(t) - \bar{F}_{\Delta_i}^{-1} \left( \frac{p^f(t)}{p_i^0(t)} \right))_+] \\
&\quad + \sum_i \sum_{c \in \mathcal{C}_i} I(\gamma_i(c))
\end{aligned}$$

Derivating  $\text{RVar}(T)$  with respect to  $\theta_i(c)$  and solving  $\frac{\partial \text{RVar}(T)}{\partial \theta_i(c)} = 0$ , we obtain:

$$\theta_i(c) = \frac{\ln \frac{\mathbb{E}[\tilde{\epsilon}_i(c)\nu_i(c)]}{(\tilde{\sigma}_i^\epsilon(c))^2}}{\ln \gamma_i(c)} \quad (15)$$

But:  $\frac{1}{2} \leq \theta_i(c) \leq 1$  which is equivalent with  $\frac{\mathbb{E}[\tilde{\epsilon}_i(c)\nu_i(c)]}{(\tilde{\sigma}_i^\epsilon(c))^2} \leq \gamma_i(c) \leq \left( \frac{\mathbb{E}[\tilde{\epsilon}_i(c)\nu_i(c)]}{(\tilde{\sigma}_i^\epsilon(c))^2} \right)^2$ .

Using the fact that  $\frac{\mathbb{E}[\tilde{\epsilon}_i(c)\nu_i(c)]}{(\tilde{\sigma}_i^\epsilon(c))^2} \leq \gamma_i(c)$ , we check that  $\theta_i(c)$  coincides with a minimum for  $\text{RVar}(T)$  indeed:  $\frac{\partial^2 \text{RVar}(T)}{\partial \theta_i(c)^2} = 2\gamma_i(c)^{\theta_i(c)} \ln(\gamma_i(c))^2 \left[ 2(\tilde{\sigma}_i^\epsilon(c))^2 \gamma_i(c) - \mathbb{E}[\tilde{\epsilon}_i(c)\nu_i(c)] \right] > 0$ .

Therefore, in case of a two tiered market with exogenous prices, the MF is completely described by the set of equations summarized in the statement of Theorem 9.

**MD 2:** According to Equations (12) and (13),  $\hat{d}_i(c)$  and  $\hat{w}_i(c)$  are time independent. Then using the algorithms described in Subsections IV-C1 and IV-D2 to compute the optimal reserves, we infer that they are price independently too, contrary to MD 1, where the reserve changes are indexed on the exogenous price variations (and are therefore price dependent). Using this observation, we infer that the day ahead price is time independent ; indeed:  $p^f = \frac{\sum_j (\hat{d}_j - \hat{w}_j + r_j) + A^f}{B^f}$ .

In case of endogenous prices, the contribution of market  $i$  to the investor's total cost is:

$$\begin{aligned}
\mathcal{J}(i, T) &= \sum_t \left( \underbrace{\sum_{c \in \mathcal{C}_i} (\hat{d}_i(c) - \hat{w}_i(c)) + r_i}_{\hat{d}_i - \hat{w}_i} \right) p^f(t) \\
&\quad + \sum_t \left[ \underbrace{\left( \sum_{c \in \mathcal{C}_i} (\tilde{\epsilon}_i(c, t) \gamma_i(c)^{\theta_i(c)} - \nu_i(c, t)) \right)}_{\Delta_i(t)} \right. \\
&\quad \left. - r_i \right] + \sum_{c \in \mathcal{C}_i} I(\gamma_i(c)) \quad (16)
\end{aligned}$$

and his utility:  $\mathcal{J}(T) = \sum_i \mathcal{J}(i, T)$ .

Compared to MD 1, we observe one additional difficulty: at the optimum  $r_i$  relies on  $r_{-i}$  and the bilateral trades rely on the decisions of the other markets.

The investor's variance conditionally to the occurrence of rare events is similar to the one derived for MD 1:

$$\begin{aligned}
\text{RVar}(T) &= \sum_i \text{Var}(\mathcal{J}(i, T) | \{\Delta_i(t) \geq r_i\}_t) \\
&= \sum_i \sigma_{\Delta_i}^2 \sum_t p_i^0(t)^2 \quad (17)
\end{aligned}$$

and the same result about the optimal wind farm concentration,  $(\theta_i(c))_{i,c}$  follows. Therefore, in the latter case of a two tiered market with endogenous prices, the MF is completely described by the set of equations summarized in the statement of Theorem 9.

### B. In the $\mathbb{E}[\mathcal{J}(T)], \text{RVar}(T)$ plane

The representation of the MF in the  $\mathbb{E}[\mathcal{J}(T)], \text{RVar}(T)$  plane is not straightforward. Indeed, in case of endogenous prices, the optimal number of wind farms to settle on each cluster cannot be computed analytically. Indeed, the optimal reserves and the optimal bilateral trades can only be obtained algorithmically, as explained in Section IV-C (resp. Section IV-D).

Over each cluster  $c \in \mathcal{C}_i$ , the investor can deploy a fixed predefined number of wind farms  $\gamma_i(c)$ . For each combination of  $(\gamma_i(c))_{i,c}$ , we derive the optimal wind farm portfolio concentration  $(\theta_i(c))_{i,c}$  using Theorem 9. We substitute the resulting  $(\gamma_i(c), \theta_i(c))_{i,c}$  in  $(\sigma_{\Delta_i})_i$  as derived in Equation (14) and in the Nash equilibrium in the reserves and bilateral trades  $(r_i^*, S_i^*)_i$  obtained through the algorithm detailed in Section (IV-C) (resp. Section IV-D).

For any  $t = 1, \dots, T$  and  $i = 1, \dots, N$ , we generate a sequence of forecast error differences such that  $\{\Delta_i(t) \geq r_i\}$ , according to the Gaussian density function  $\mathcal{N}(0; \sigma_{\Delta_i}^2)$ . The time elapsed between two consecutive repetitions of the SG being of 24 hours, we assume that there is no time dependence

between two consecutive samples of forecast error differences. Various simulation techniques can be envisaged to cope with rare events. A straightforward way is to use Monte-Carlo simulation. However, this poses serious problems when the event  $\{\Delta_i(t) \geq r_i\}_t$  is a rare event. Indeed, in that case, a large simulation effort is required in order to estimate the rare event accurately i.e., with small relative error or narrow confidence interval [7]. A well-known alternative is to use Importance Sampling. However, it is proved that the optimal Importance Sampling density relies on the rare event probability, which is unknown. The Cross-Entropy (CE) method provides an alternative multi-level approach [7]. Its principle is first to generate randomly a sample according to a specific mechanism and then to update the mechanism parameters by selecting the elements of the sample which are the closest in the sense of the Kullback-Leibler divergence, also known as CE, of the optimal Importance Sampling density function. According to the CE algorithm for rare event simulation, we need to update at each time step the Gaussian density function variance:  $\sigma_{\Delta_i}^2$  so that more weight is allocated to rare events. The Algorithm is provided in Appendix C.

We substitute  $(r_i^*, S_i^*)_i$  and  $(\Delta_i(t))_i$  in the real time price:  $p_i^0(t) = \frac{A_i^0 + (\Delta_i(t) - r_i^*) + S_i^*}{B_i^0}, \forall i = 1, \dots, N$  derived in Subsection IV-B. From this, we can infer the investor's variance conditionally to the occurrence of the rare events  $\cap_i \{\Delta_i(t) \geq r_i\}$  using Equation (17).

At the same time, we note that the expectation of the contribution of market  $i$  to the investor's utility can be simplified to give:

$$\begin{aligned} \mathbb{E}[J(i, T)] &= \frac{T}{B_i^0} (\hat{d}_i - \hat{w}_i + r_i) \left\{ \sum_j (\hat{d}_j - \hat{w}_j + r_j) + A^f \right\} \\ &+ \frac{T}{B_i^0} [A_i^0 + \varphi_i(r_i, \mathbf{r}_{-i})] \mathbb{E}[(\Delta_i(t) - r_i)_+] \\ &+ \frac{T}{B_i^0} \mathbb{E}[(\Delta_i(t) - r_i)_+^2] + \sum_{c \in \mathcal{C}_i} I(\gamma_i(c)) \end{aligned}$$

where, as detailed in Appendix A:  $\mathbb{E}[(\Delta_i(t) - r_i)_+] = \mathbb{E}[(\Delta_i(t) - r_i) | \Delta_i(t) \geq r_i] = \frac{\sigma_{\Delta_i}}{\sqrt{2\pi}} \exp(-\frac{r_i^2}{2\sigma_{\Delta_i}^2}) - r_i \bar{F}_{\Delta_i}(r_i)$  and  $\mathbb{E}[(\Delta_i(t) - r_i)_+^2] = \mathbb{E}[(\Delta_i(t) - r_i)^2 | \Delta_i(t) \geq r_i] = \frac{\sigma_{\Delta_i}^2}{\sqrt{\pi}} \Gamma(\frac{3}{2}) \Gamma_{inc}(\frac{3}{2}, \frac{r_i^2}{2\sigma_{\Delta_i}^2}) - \frac{\sigma_{\Delta_i}}{\sqrt{2\pi}} \exp(-\frac{r_i^2}{2\sigma_{\Delta_i}^2}) + r_i^2$  with  $\Gamma(a)$  the Gamma function evaluated in  $a \in \mathbb{R}_+$  and  $\Gamma_{inc}(a, x) = \frac{1}{\Gamma(a)} \int_x^{+\infty} u^{a-1} \exp(-u) du$  the Incomplete Gamma function with lower bound, evaluated in  $a, x \in \mathbb{R}_+$ .

## VI. NUMERICAL ILLUSTRATIONS FOR THREE GEOGRAPHIC DEMAND MARKETS: FRANCE, GERMANY AND BELGIUM

In the numerical illustrations, we consider three geographic demand markets: France, Germany and Belgium. Wind farm portfolio optimization is restricted to the French area, since our energy consumption data focus on this country.

The marginal cost parameters are based on Chao and Peck's six node toy network [5]: for France (Fr) we take  $a_{Fr}^f = 42.5$ ,  $a_{Fr}^0 = 4250$ ,  $b_{Fr}^f = b_{Fr}^0 = 0.025$ ; for Germany (Ge) we take  $a_{Ge}^f = 15$ ,  $a_{Ge}^0 = 1500$ ,  $b_{Ge}^f = b_{Ge}^0 = 0.05$  and for Belgium (Be)  $a_{Be}^f = 10$ ,  $a_{Be}^0 = 1000$ ,  $b_{Be}^f = b_{Be}^0 = 0.05$ .

The equivalent interconnection capacities are set so that:  $\kappa_{Be, Ge} = 2(\text{GW})$ ,  $\kappa_{Be, Fr} = 6(\text{GW})$  and  $\kappa_{Ge, Fr} = 5(\text{GW})$ .

### A. Description of the data and clustering of the geographic demand markets

For Germany, our database is made of time series of 75 sensors located all over Germany (cf. Figure 2 (c)) providing one year wind speed measures (from 03/19/2013 until 03/18/2014) with one measure per hour [29]. The exact GPS coordinates of the sensors are depicted by circles in Figure 2 (a). We use two Machine Learning techniques to partition the sensors based on the mean and variance of their wind speed time series: first, k-Means algorithm clusters data by separating samples in an a priori determined number of groups, minimizing a criterion known as the inertia of the groups (cf. Figure 2 (a)). The optimal number of classes for the sensors (four) has been estimated a priori using an unsupervised clustering method known as affinity propagation. Second, one-class Support Vector Machine (SVM) can be used as a type of unsupervised learning algorithm, for novelty detection, that is, given a set of samples, it will detect the soft boundary of that set so as to classify new points as belonging to that set or not (cf. Figure 2 (b)). Both techniques give identical (or, at least, very close) classes. Then the convex hull of the sensor classes gives an approximation to the clusters geographic area for Germany.

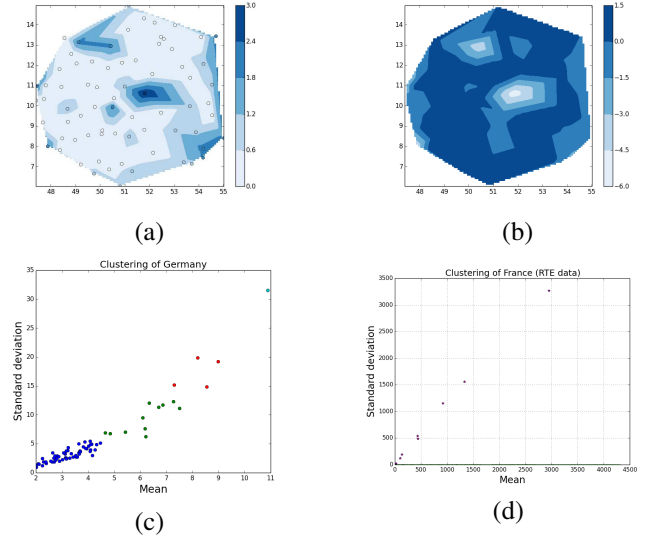


Fig. 2. Partitioning Germany's wind speed sensors, using k-Means (a) and Support Vector Machine outlier detection (b). The sensors' locations in GPS coordinates are represented in (a) by circles. In (c), the sensors are represented in the Mean-Variance plane depending on their wind speed time series mean and variance. We partition Germany's wind speed sensors; each color being associated with a specific class while in (d) we represent each French geographic cluster as a star, in the Mean-Variance plane depending on its wind power production.

For France and Belgium, we use data provided by the Optimate project [3]. In this project, France (resp. Belgium) was partitioned in 8 (resp. 2) clusters. Wind speed measurements are performed at various points in the clusters. The measures

Clust. ind.	1	2	3	4	5	6	7	8
Mean prod.	2.94	1.33	0.43	0.02	0.13	0.43	0.92	0.09
Stand. dev.	3.27	1.56	0.54	0.02	0.19	0.49	1.15	0.12

TABLE II  
WIND FARM MEAN PRODUCTION (MW) AND STANDARD DEVIATION (MW) FOR FRANCE.

are then averaged over the clusters and, over each cluster, the average wind power production of a turbine is estimated. Correlation among the clusters is taken into account to evaluate the wind power production. We use hourly forecasts of wind power for year 2015.

In Figure 2 (c) right, we represent each of the 8 French clusters averaged wind production per turbine in the Mean-Variance plane. These data sets are used to estimate the mean production (MW) and standard deviation (MW) of a single wind farm, over each French cluster (cf. Table II for France). The geographic coordinates of the cluster areas can be found in [3]; here, each cluster area will be referenced with an index.

Estimated wind production for Germany and Belgium is fixed so that:  $\hat{w}_{Ge} = 15(\text{GW})$  and  $\hat{w}_{Be} = 10(\text{GW})$ . The forecast error difference standard deviations are set so that:  $\sigma_{\Delta_{Be}} = \sigma_{\Delta_{Ge}} = 5(\text{GW})$ . For France, if 1000 turbines were placed in each cluster,  $\hat{w}_{Fr} = 6.3(\text{GW})$  and the forecast error difference standard deviation would be:  $\sigma_{\Delta_{Fr}} = 21.2(\text{GW})$ .

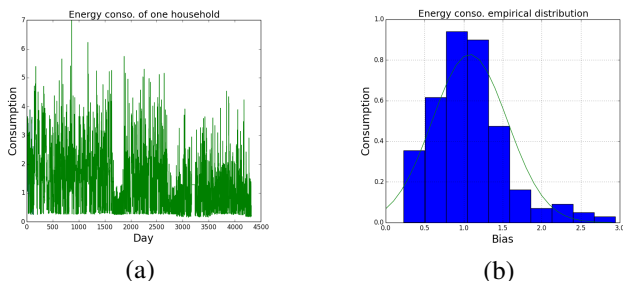


Fig. 3. Energy consumption of one household over one year (in kW), the value being averaged over a day (a) and its associated empirical distribution function compared to the best fit Gaussian density function (b).

For estimating the variance associated to the demand forecast, we use a data base containing one year power measurement (in kW) for an individual household, with a granularity of one measure per second [30]. We first take the average of this time serie to obtain one value per day. Then we calculate the average and the standard deviation for the whole year and we multiply these values by the number of inhabitants per French cluster divided by two<sup>11</sup> (cf. Table III for France). In Figure 3 left, we represent the time serie of one household power consumption (in kW), over a year, data being averaged over a day. In Figure 3 right, we compare the empirical distribution of the household energy consumption time series with the best

<sup>11</sup>According to INSEE statistics, on average, a household is composed of two inhabitants <http://www.insee.fr/>.

Clust. ind.	1	2	3	4	5	6	7	8
Households (Million)	5.3	2.6	1	1.4	3.9	2.1	2	3.5
Mean conso.	5.77	2.81	1.11	1.50	4.25	2.32	2.19	3.83
Conso. stand. dev.	4.75	2.31	0.91	1.23	3.49	1.91	1.81	3.16

TABLE III  
MILLION OF HOUSEHOLDS, MEAN CONSUMPTION (GW) AND CONSUMPTION STANDARD DEVIATION PER FRENCH CLUSTER.

fit Gaussian density function which validates our assumptions on the demand forecast error generation.

For France, the end users' total demand is estimated by:  $\hat{d}_{Fr} = 23.8(\text{GW})$ ; for Germany and Belgium we fix:  $\hat{d}_{Ge} = \hat{d}_{Be} = 40(\text{GW})$ .

### B. Optimal reserves and Markowitz Frontier representations for France

In Figure 4 (a), we plot the optimal reserve for the two tiered market with exogenous prices described in Section III as a function of the exogenous price ratio  $\frac{p_{Fr}^0}{p_{Fr}}$  and of the forecast error standard deviation  $\sigma_{\Delta_{Fr}}$ . We observe that the reserve increases parabolically as the real time price over the day ahead price ratio increases and as the uncertainty on the difference between the end users' total demand and the renewable production in real time ( $d_{Fr} - w_{Fr}$ ) increases. In Figure 4 (b), we plot the optimal reserve for the two tiered market with endogenous prices and reserves described in Section IV as a function of the wind forecast  $\hat{w}_{Fr}$  and of the forecast error standard deviation  $\sigma_{\Delta_{Fr}}$ . We observe that the reserve increases in the wind forecast and in  $d_{Fr} - w_{Fr}$ .

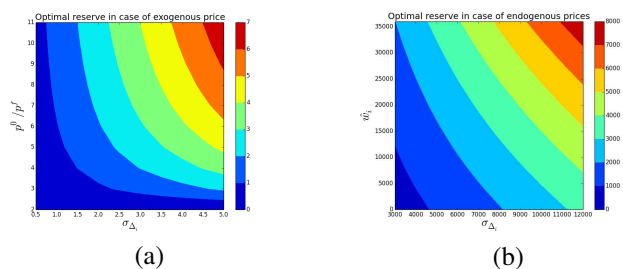


Fig. 4. In (a) (resp. (b)), we represent the optimal reserve  $r_i$  in case of exogenous prices (resp. endogenous prices) as a function of the forecast error difference standard deviation  $\sigma_{\Delta_i}$  and of the exogenous price ratio ( $\frac{p_{Fr}^0}{p_{Fr}}$ ) (resp. of the wind forecast  $\hat{w}_i$ ).

In Figure 5, we plot the MF for the wind farm portfolio over each French cluster as a function of the number of turbines and of the concentration of the wind farms over the cluster. These plots are issued from the theoretical relation derived in Theorem 9.

We assume that the investor can deploy 0, 3000 or 6000 turbines over each cluster leading to  $3^8$  combinations for France. This choice of numerical values is justified by the fact that largest wind farms nowadays have around 6000



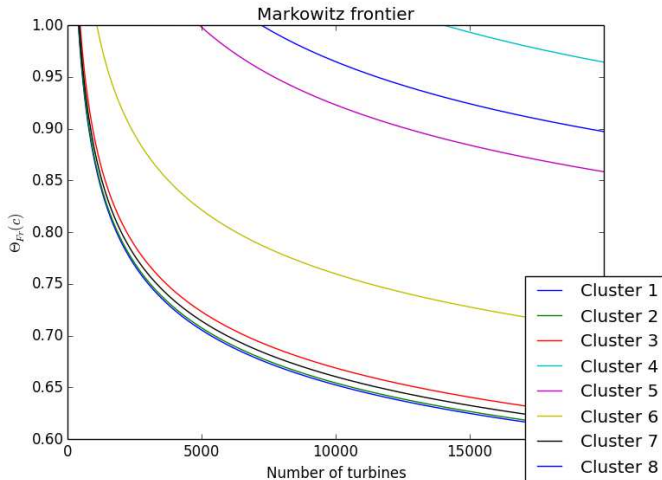


Fig. 5. Over each French cluster, the Markowitz Frontier for the wind farm portfolio is represented as a function of the number of turbines and of the concentration between the wind farms over the cluster. Each of the 8 French clusters is characterized by a specific color.

turbines. Assuming that a wind farm counts on the average 3000 turbines, the investor has the choice between deploying 0, 1 or 2 wind farms per cluster. We observe in Figure 6 that the value of  $\text{cost}_{\text{inv}}$  and the form of the investment cost (i.e., linear, quadratic, etc.) deeply influence the MF shape.

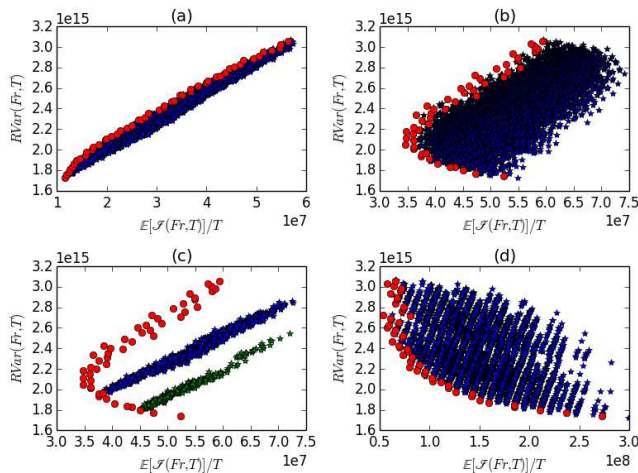


Fig. 6. Markowitz Frontier (MF) in red in the  $\frac{\mathbb{E}[\mathcal{J}(Fr, T)]}{T} - R\text{Var}(Fr, T)$  plane. In (a), we represent all the portfolio combinations and the MF for  $\text{cost}_{\text{inv}} = 1$  and a linear investment cost  $\sum_{c \in \mathcal{C}_{Fr}} \gamma_{Fr}(c)$ . In (b), we plot all the portfolio combinations and the MF for  $\text{cost}_{\text{inv}} = 900$  and a linear investment cost. In (c), the same setting holds but we select exclusively the portfolios associated with two investment levels:  $\sum_{c \in \mathcal{C}_{Fr}} \gamma_i(c) = 24 \cdot 10^3$  and  $36 \cdot 10^3$ . In (d), we plot the portfolio combinations and the MF for  $\text{cost}_{\text{inv}} = 1$  and a quadratic investment cost i.e.,  $\sum_{c \in \mathcal{C}_{Fr}} \gamma_{Fr}(c)^2$ .

## VII. CONCLUSION

We developed a methodology for optimizing an investor's wind farm portfolio, using Markowitz Frontier theory, in

a Market Coupling organization. We considered  $N \in \mathbb{N}^*$  interacting geographic demand markets optimizing selfishly their reserve and bilateral trades with the others. The novelty, compared to previous works, relies on the fact that we have introduced some competition among the geographic demand markets and that the problem is modeled as a bilevel Signaling Game. The signal i.e., the information shared among the players, was based either on prices or on quantities. We proved analytically that first, in both classes of games there exist conditions guaranteeing the existence and the uniqueness of a Nash Equilibrium for  $N \leq 3$  and second, that the Markowitz Frontier can be expressed as a function of the number of settled wind farms and of their concentration independently of the Market Design. Finally we propose an algorithm and simulate, on real life data sets, the contour of the Markowitz Frontier in the expected cost-conditional variance plane.

The modeling of the end users' demand and of the renewable production dynamic evolutions were voluntarily simplified to enable the derivation of analytical results, enabling a better understanding of the market system behavior. Bootstrap-based Extreme Learning Machine [28] appears as very promising to perform online learning on erratic processes such as renewable productions. We plan to compare its performance with expert advise fusion on the basis of real life large and distributed data bases of wind and solar data.

## APPENDIX A: PROOF OF PROPOSITION 4

By substitution of the day ahead and real time prices at equilibrium obtained in Lemmas 1 and 2 in market  $i$  expected procurement cost, we obtain:

$$U_i = q_i^f \frac{\sum_j q_j^f + A^f}{B^f} + \mathbb{E}\left[\frac{(q_i^0)^2}{B_i^0}\right] + \frac{A_i^0 + \sum_{j \neq i} t_{i \rightarrow j}^0}{B_i^0} q_i^0$$

Using Lemma 3 and the fact that  $\mathbb{E}[q_i^0] = \mathbb{E}[(\Delta_i - r_i)_+] = \mathbb{E}[(\Delta_i - r_i)|\Delta_i \geq r_i]$ , we obtain:

$$\begin{aligned} \frac{\partial U_i}{\partial r_i} &= \frac{\sum_j q_j^f + A^f}{B^f} + \frac{q_i^f}{B^f} \\ &+ \frac{\partial}{\partial r_i} \left( \frac{1}{B_i^0} \mathbb{E}[(\Delta_i - r_i)^2 | \Delta_i \geq r_i] \right) \\ &+ \frac{A_i^0 + \sum_{j \neq i} t_{i \rightarrow j}^0}{B_i^0} \frac{\partial}{\partial r_i} \mathbb{E}[(\Delta_i - r_i) | \Delta_i \geq r_i] \\ &+ b_i^0 \left( \frac{1}{B^f b_i^f} - 1 \right) \mathbb{E}[(\Delta_i - r_i) | \Delta_i \geq r_i] \end{aligned}$$

Since the forecast error differences  $\Delta_i$  are distributed according to Gaussian distribution functions centered in 0 and of standard deviation  $\sigma_{\Delta_i}$ , it is possible to express the first and second derivatives of  $(\Delta_i - r_i)$  and  $(\Delta_i - r_i)^2$  conditionally to the event  $\{\Delta_i \geq r_i\}$  as functions of the incomplete gamma function which enables us to derive the following closed forms:

$$\begin{aligned} \mathbb{E}[(\Delta_i - r_i) | \Delta_i \geq r_i] &= \int_{r_i}^{+\infty} \Delta f_{\Delta_i}(\Delta) d\Delta \\ -r_i \int_{r_i}^{+\infty} f_{\Delta_i}(\Delta) d\Delta &= \frac{\sigma_{\Delta_i}}{\sqrt{2\pi}} \exp\left(-\frac{r_i^2}{2\sigma_{\Delta_i}^2}\right) - r_i \bar{F}_{\Delta_i}(r_i) \end{aligned}$$

$$\begin{aligned}\frac{\partial}{\partial r_i} \mathbb{E}[(\Delta_i - r_i) | \Delta_i \geq r_i] &= -\bar{F}_{\Delta_i}(r_i) \\ \frac{\partial^2}{\partial r_i^2} \mathbb{E}[(\Delta_i - r_i) | \Delta_i \geq r_i] &= f_{\Delta_i}(r_i)\end{aligned}\quad (18)$$

and

$$\begin{aligned}\mathbb{E}[(\Delta_i - r_i)^2 | \Delta_i \geq r_i] &= \int_{r_i}^{+\infty} \Delta^2 f_{\Delta_i}(\Delta) d\Delta \\ &- 2r_i \int_{r_i}^{+\infty} \Delta f_{\Delta_i}(\Delta) d\Delta + r_i^2 \int_{r_i}^{+\infty} f_{\Delta_i}(\Delta) d\Delta \\ \frac{\partial}{\partial r_i} \mathbb{E}[(\Delta_i - r_i)^2 | \Delta_i \geq r_i] &= 2[r_i \int_{r_i}^{+\infty} f_{\Delta_i}(\Delta) d\Delta \\ &- \int_{r_i}^{+\infty} \Delta f_{\Delta_i}(\Delta) d\Delta] \\ \frac{\partial^2}{\partial r_i^2} \mathbb{E}[(\Delta_i - r_i)^2 | \Delta_i \geq r_i] &= 2\bar{F}_{\Delta_i}(r_i)\end{aligned}\quad (19)$$

Using Equations (18) and (19), we obtain:

$$\frac{\partial^2 \mathcal{U}_i}{\partial r_i^2} = \frac{2}{B^f} + b_i^0 \left(3 - \frac{1}{B^f b_i^f}\right) \bar{F}_{\Delta_i}(r_i) + \frac{A_i^0 + \sum_{j \neq i} t_{i \rightarrow j}^0}{B_i^0} f_{\Delta_i}(r_i)$$

Since  $\bar{F}_{\Delta_i}(r_i) > 0$  and  $f_{\Delta_i}(r_i) > 0$  for any  $r_i \in \mathbb{R}$  and  $B^f > 0, B_i^0 > 0$ , the sign of  $\frac{\partial^2 \mathcal{U}_i}{\partial r_i^2}$  for  $r_i \in [0; +\infty[$  depends on the sign of  $\frac{A_i^0 + \sum_{j \neq i} t_{i \rightarrow j}^0}{B_i^0}$ . Two cases are possible:

**Case 1:**  $\frac{A_i^0 + \sum_{j \neq i} t_{i \rightarrow j}^0}{B_i^0} \geq 0 \Leftrightarrow \sum_{j \neq i} t_{i \rightarrow j}^0 \geq -A_i^0$

This first case corresponds to the case where the quantity of imports is not too high compared to the quantity of exports, for market  $i$ . In this first case, we infer that  $\frac{\partial^2 \mathcal{U}_i}{\partial r_i^2} > 0, \forall r_i \geq 0$ . Hence  $\mathcal{U}_i$  is convex in  $r_i \geq 0$ . Therefore, there exists a unique  $r_i \geq 0$  minimizing  $\mathcal{U}_i$ .

**Case 2:**  $\frac{A_i^0 + \sum_{j \neq i} t_{i \rightarrow j}^0}{B_i^0} < 0 \Leftrightarrow \sum_{j \neq i} t_{i \rightarrow j}^0 < -A_i^0$

This second case corresponds to the case where the quantity of imports is very high compared to the quantity of exports, for geographic market  $i$ .

Derivating three times  $\mathcal{U}_i$  with respect to  $r_i$ , we obtain:

$$\begin{aligned}\frac{\partial^3 \mathcal{U}_i}{\partial r_i^3} &= -b_i^0 \left(3 - \frac{1}{B^f b_i^f}\right) f_{\Delta_i}(r_i) \\ &- \frac{A_i^0 + \sum_{j \neq i} t_{i \rightarrow j}^0}{B_i^0} \frac{r_i}{\sigma_{\Delta_i}^2} f_{\Delta_i}(r_i) \\ &= -\frac{f_{\Delta_i}(r_i)}{B_i^0} \frac{1}{\sigma_{\Delta_i}^2} \left[ \left(3 - \frac{1}{B^f b_i^f}\right) \sigma_{\Delta_i}^2 + r_i \left( A_i^0 + \sum_{j \neq i} t_{i \rightarrow j}^0 \right) \right]\end{aligned}$$

using the fact that  $\frac{df_{\Delta_i}(r_i)}{dr_i} = -\frac{r_i}{\sigma_{\Delta_i}^2} f_{\Delta_i}(r_i), \forall r_i \in \mathbb{R}$  since  $\Delta_i$  is distributed according to a Gaussian distribution function centered in 0 and of standard deviation  $\sigma_{\Delta_i}$ . Then:  $\frac{\partial^3 \mathcal{U}_i}{\partial r_i^3} =$

$$0 \Leftrightarrow r_i = -\frac{\left(3 - \frac{1}{B^f b_i^f}\right) \sigma_{\Delta_i}^2}{A_i^0 + \sum_{j \neq i} t_{i \rightarrow j}^0}.$$

We set:  $r_i^0 \triangleq -\frac{\left(3 - \frac{1}{B^f b_i^f}\right) \sigma_{\Delta_i}^2}{A_i^0 + \sum_{j \neq i} t_{i \rightarrow j}^0}$ . Then, we note that:

- If  $r_i < r_i^0$  then  $r_i < -\frac{\left(3 - \frac{1}{B^f b_i^f}\right) \sigma_{\Delta_i}^2}{A_i^0 + \sum_{j \neq i} t_{i \rightarrow j}^0} \Leftrightarrow \left(3 - \frac{1}{B^f b_i^f}\right) \sigma_{\Delta_i}^2 + r_i \left( A_i^0 + \sum_{j \neq i} t_{i \rightarrow j}^0 \right) > 0$ . This implies in turn that  $\frac{\partial^2 \mathcal{U}_i}{\partial r_i^2} < 0$ .
- Identically, if  $r_i > r_i^0$  then  $\frac{\partial^3 \mathcal{U}_i}{\partial r_i^3} > 0$ .

Both of these observations imply that  $\frac{\partial^2 \mathcal{U}_i}{\partial r_i^2}$  is decreasing on  $[0; r_i^0[$  and increasing on  $[r_i^0; +\infty[$ . Furthermore, the number of points where  $\frac{\partial^2 \mathcal{U}_i}{\partial r_i^2} = 0$  depend on the value of  $\frac{\partial^2 \mathcal{U}_i}{\partial r_i^2} |_{r_i=r_i^0}$ .

**Case 2 (a):**  $\frac{\partial^2 \mathcal{U}_i}{\partial r_i^2} |_{r_i=r_i^0} > 0$

Then  $\frac{\partial \mathcal{U}_i}{\partial r_i} > 0, \forall r_i \geq 0$ . This implies that  $\mathcal{U}_i$  is convex on  $\mathbb{R}_+$ . Therefore it admits a unique minimum on  $[0; +\infty[$ .

**Case 2 (b):**  $\frac{\partial^2 \mathcal{U}_i}{\partial r_i^2} |_{r_i=r_i^0} < 0$

Then two sub-cases should be distinguished depending on the sign of  $\frac{\partial^2 \mathcal{U}_i}{\partial r_i^2} |_{r_i=0}$ .

**Case 2 (b) (i):**  $\frac{\partial^2 \mathcal{U}_i}{\partial r_i^2} |_{r_i=0} < 0$  There exists a unique

$r_i^1 \in [r_i^0; +\infty[$  such that  $\frac{\partial^2 \mathcal{U}_i}{\partial r_i^2} < 0$  on  $[0; r_i^1[$  and  $\frac{\partial^2 \mathcal{U}_i}{\partial r_i^2} \geq 0$  on  $[r_i^1; +\infty[$ . This implies that  $\frac{\partial \mathcal{U}_i}{\partial r_i}$  is decreasing on  $[0; r_i^1[$  and increasing on  $[r_i^1; +\infty[$ . Hence  $\frac{\partial \mathcal{U}_i}{\partial r_i} = 0$  in 0, 1 or 2 points for  $r_i \geq 0$ . But  $\frac{\partial^2 \mathcal{U}_i}{\partial r_i^2} > 0$  exclusively for  $r_i \in [r_i^1; +\infty[$ . Then: either  $\frac{\partial \mathcal{U}_i}{\partial r_i} > 0, \forall r_i \in [r_i^1; +\infty[$  which implies that  $\mathcal{U}_i$  is strictly increasing on  $\mathbb{R}_+$  reaching its minimum in  $r_i = 0$ ; or there exists a point  $r_i^* \in [r_i^1; +\infty[$  such that  $\frac{\partial \mathcal{U}_i}{\partial r_i} |_{r_i=r_i^*} = 0$ . Since  $\frac{\partial^2 \mathcal{U}_i}{\partial r_i^2} > 0$  on  $[r_i^1; +\infty[$ , this implies that  $r_i^*$  is the unique minimum of  $\mathcal{U}_i$  on  $\mathbb{R}_+$ .

**Case 2 (b) (ii):**  $\frac{\partial^2 \mathcal{U}_i}{\partial r_i^2} |_{r_i=0} > 0$  There exist  $0 \leq r_i^2 \leq r_i^0$  and  $r_i^0 \leq r_i^3$  such that  $\frac{\partial^2 \mathcal{U}_i}{\partial r_i^2} > 0$  on  $[0; r_i^2[$ ,  $< 0$  on  $[r_i^2; r_i^3[$  and  $> 0$  on  $[r_i^3; +\infty[$ .

If  $\frac{\partial \mathcal{U}_i}{\partial r_i} |_{r_i=0} \geq 0$  then there exists a unique  $r_i^* \in [r_i^3; +\infty[$  such that  $\frac{\partial \mathcal{U}_i}{\partial r_i} |_{r_i=r_i^*} = 0$  and  $\frac{\partial^2 \mathcal{U}_i}{\partial r_i^2} |_{r_i=r_i^*} > 0$ . In this case,  $r_i^*$  is the unique minimum of  $\mathcal{U}_i$  over  $\mathbb{R}_+$ .

Otherwise i.e., if  $\frac{\partial \mathcal{U}_i}{\partial r_i} |_{r_i=0} < 0$  then: either  $\frac{\partial \mathcal{U}_i}{\partial r_i} |_{r_i=r_i^2} < 0$  in which case  $\mathcal{U}_i$  admits a unique minimum over  $\mathbb{R}_+$  belonging to  $[r_i^3; +\infty[$ ; or  $\frac{\partial \mathcal{U}_i}{\partial r_i} |_{r_i=0} \geq 0$  in which case  $\mathcal{U}_i$  admits two minima over  $\mathbb{R}_+$ , the first one in  $[0; r_i^2[$  and the second one in  $[r_i^3; +\infty[$ .

The case  $\frac{\partial \mathcal{U}_i}{\partial r_i} |_{r_i=0} < 0$  and  $\frac{\partial^2 \mathcal{U}_i}{\partial r_i^2} |_{r_i=0} > 0$  should be avoided since it might give rise to a large number of equilibria ( $2^N$ ) for Program 3. Therefore, in the case where  $\sum_{j \neq i} t_{i \rightarrow j}^0 < -A_i^0$ , it might be reasonable to impose some conditions on the problem parameters so that Case 2 (b) (ii)



is avoided. In other words:

$$\begin{aligned} \frac{\partial^2 U_i}{\partial r_i^2} \Big|_{r_i=0} &= \frac{2}{B^f} + \frac{1}{2B_i^0} \left(3 - \frac{1}{B^f b_i^f}\right) \\ &+ \frac{A_i^0 + \sum_{j \neq i} t_{i \rightarrow j}^0}{B_i^0} f_{\Delta_i}(0) < 0 \\ \Leftrightarrow f_{\Delta_i}(0) &> -\frac{\frac{2B_i^0}{B^f} + \frac{1}{2} \left(3 - \frac{1}{B^f b_i^f}\right)}{A_i^0 + \sum_{j \neq i} t_{i \rightarrow j}^0} \\ \Leftrightarrow \sigma_{\Delta_i} &< -\frac{1}{\sqrt{2\pi}} \left[ \frac{A_i^0 + \sum_{j \neq i} t_{i \rightarrow j}^0}{\frac{2B_i^0}{B^f} + \frac{1}{2} \left(3 - \frac{1}{B^f b_i^f}\right)} \right] \end{aligned}$$

Therefore, to avoid Case 2 (b) (ii), the standard-deviation related to the knowledge of  $\Delta_i$  should be smaller than  $-\frac{1}{\sqrt{2\pi}} \left[ \frac{A_i^0 + \sum_{j \neq i} t_{i \rightarrow j}^0}{\frac{2B_i^0}{B^f} + \frac{1}{2} \left(3 - \frac{1}{B^f b_i^f}\right)} \right]$ .  $\square$

#### APPENDIX B: PROOF OF PROPOSITION 8

$$\begin{aligned} U_i &= (\hat{d}_i - \hat{w}_i + r_i) \frac{1}{B^f} \left[ \sum_l (\hat{d}_l - \hat{w}_l + r_l) + A^f \right] \\ &+ \mathbb{E}[(\Delta_i - r_i)_+ \frac{1}{B_i^0} ((\Delta_i - r_i)_+ + A_i^0 + \sum_{j \neq i} t_{i \rightarrow j}^0)] \end{aligned}$$

But:

$$\begin{aligned} &\mathbb{E}[(\Delta_i - r_i)_+ \frac{1}{B_i^0} ((\Delta_i - r_i)_+ + A_i^0 + \sum_{j \neq i} t_{i \rightarrow j}^0)] \\ &= b_i^0 \mathbb{E}[(\Delta_i - r_i)_+^2 | \Delta_i \geq r_i] + a_i^0 \mathbb{E}[(\Delta_i - r_i) | \Delta_i \geq r_i] \\ &+ b_i^0 \sum_{j \neq i} \mathbb{E}[t_{i \rightarrow j}^0 (\Delta_i - r_i) | \Delta_i \geq r_i] \end{aligned}$$

Derivating  $U_i$  with respect to  $r_i$  gives:

$$\begin{aligned} \frac{\partial U_i}{\partial r_i} &= \frac{1}{B^f} \left[ \sum_l (\hat{d}_l - \hat{w}_l + r_l) + A^f \right] \\ &+ \frac{1}{B^f} (\hat{d}_i - \hat{w}_i + r_i) + 2b_i^0 [r_i \int_{r_i}^{+\infty} f_{\Delta_i}(\Delta) d\Delta \\ &- \int_{r_i}^{+\infty} \Delta f_{\Delta_i}(\Delta) d\Delta] - a_i^0 \bar{F}_{\Delta_i}(r_i) \\ &+ b_i^0 \sum_{j \neq i} \frac{\partial}{\partial r_i} \mathbb{E}[t_{i \rightarrow j}^0 (\Delta_i - r_i) | \Delta_i \geq r_i] \end{aligned}$$

But:

$$\begin{aligned} &\mathbb{E}[t_{i \rightarrow j}^0 (\Delta_i - r_i) | \Delta_i \geq r_i] = \frac{1}{b_j^0} \sum_{l \neq i} (\eta_{j,l} \\ &- \eta_{i,l}) \mathbb{E}[(\Delta_l - r_l) | \Delta_l \geq r_l] \mathbb{E}[(\Delta_i - r_i) | \Delta_i \geq r_i] \\ &+ \frac{1}{b_j^0} (\tilde{\eta}_j - \tilde{\eta}_i) \mathbb{E}[(\Delta_i - r_i) | \Delta_i \geq r_i] \\ &+ \frac{b_i^0 + \eta_{j,i} - \eta_{i,i}}{b_j^0} \mathbb{E}[(\Delta_i - r_i)^2 | \Delta_i \geq r_i] \end{aligned}$$

since  $(\Delta_l)_{l \neq i}$  and  $\Delta_i$  are independent.

Finally, we obtain:

$$\begin{aligned} \frac{\partial U_i}{\partial r_i} &= \frac{1}{B^f} \left[ \sum_l (\hat{d}_l - \hat{w}_l + r_l) + A^f \right] + \frac{1}{B^f} (\hat{d}_i - \hat{w}_i + r_i) \\ &+ 2b_i^0 [r_i \int_{r_i}^{+\infty} f_{\Delta_i}(\Delta) d\Delta - \int_{r_i}^{+\infty} \Delta f_{\Delta_i}(\Delta) d\Delta] - a_i^0 \bar{F}_{\Delta_i}(r_i) \\ &+ b_i^0 \left\{ - \sum_{j \neq i} \frac{1}{b_j^0} \sum_{l \neq i} (\eta_{j,l} - \eta_{i,l}) \mathbb{E}[\Delta_l - r_l | \Delta_l \geq r_l] \bar{F}_{\Delta_i}(r_i) \right. \\ &\quad \left. - \sum_{j \neq i} \frac{1}{b_j^0} (\tilde{\eta}_j - \tilde{\eta}_i) \bar{F}_{\Delta_i}(r_i) \right. \\ &\quad \left. + 2 \sum_{j \neq i} \frac{b_i^0 + \eta_{j,i} - \eta_{i,i}}{b_j^0} [r_i \int_{r_i}^{+\infty} f_{\Delta_i}(\Delta) d\Delta \right. \\ &\quad \left. - \int_{r_i}^{+\infty} \Delta f_{\Delta_i}(\Delta) d\Delta] \right\} \end{aligned}$$

Derivating twice  $U_i$  with respect to  $r_i$ , we obtain:

$$\frac{\partial^2 U_i}{\partial r_i^2} = \frac{2}{B^f} + \bar{F}_{\Delta_i}(r_i) \mu_1(i) + f_{\Delta_i}(r_i) \mu_2(i)$$

where:  $\mu_1(i) = [2b_i^0(1 + \sum_{j \neq i} \frac{b_i^0 + \eta_{j,i} - \eta_{i,i}}{b_j^0})]$  and  $\mu_2(i) = [a_i^0 + \sum_{j \neq i} \frac{b_i^0}{b_j^0} \sum_{l \neq i} (\eta_{j,l} - \eta_{i,l}) \mathbb{E}[\Delta_l - r_l | \Delta_l \geq r_l] + \sum_{j \neq i} \frac{b_i^0}{b_j^0} (\tilde{\eta}_j - \tilde{\eta}_i)]$ .

Derivating three times  $U_i$  with respect to  $r_i$ :

$$\frac{\partial^3 U_i}{\partial r_i^3} = -f_{\Delta_i}(r_i) [\mu_1(i) - \frac{r_i}{\sigma_{\Delta_i}^2} \mu_2(i)]$$

Since  $f_{\Delta_i}(r_i) > 0, \forall r_i$ ,  $\frac{\partial^3 U_i}{\partial r_i^3} = 0 \Leftrightarrow r_i = r_i^0 = \sigma_{\Delta_i}^2 \frac{\mu_1(i)}{\mu_2(i)}$ . Furthermore, we observe that:  $\frac{\partial^3 U_i}{\partial r_i^3} \Big|_{r_i=0} = -f_{\Delta_i}(0) \mu_1(i)$ . To prove the existence of a unique minimum for  $U_i$  on  $\mathbb{R}_+$  we distinguish between four cases depending on the signs of  $\mu_1(i), \mu_2(i)$ :

- **Case 1:**  $\mu_1(i) > 0$  and  $\mu_2(i) > 0$ . This implies that  $r_i^0 > 0$  and that  $\frac{\partial^3 U_i}{\partial r_i^3} \Big|_{r_i=0} < 0$ . Furthermore, we observe that:  $\frac{\partial^2 U_i}{\partial r_i^2} \Big|_{r_i=0} = \frac{2}{B^f} + \bar{F}_{\Delta_i}(0) \mu_1(i) + f_{\Delta_i}(0) \mu_2(i) > 0$  and  $\frac{\partial^2 U_i}{\partial r_i^2} \Big|_{r_i=r_i^0} = \frac{2}{B^f} + \bar{F}_{\Delta_i}(r_i^0) \mu_1(i) + f_{\Delta_i}(r_i^0) \mu_2(i) > 0$ . Then either  $\frac{\partial U_i}{\partial r_i} \Big|_{r_i=0} > 0$  in which case  $\frac{\partial U_i}{\partial r_i} > 0, \forall r_i \geq 0$  and  $U_i$  is increasing on  $\mathbb{R}_+$  implying that the minimum is reached in  $r_i = 0$ ; or  $\frac{\partial U_i}{\partial r_i} \Big|_{r_i=0} < 0$  implying that  $U_i$  admits a unique minimum on  $\mathbb{R}_+$ .
- **Case 2:**  $\mu_1(i) > 0$  and  $\mu_2(i) < 0$ . This implies that  $r_i^0 < 0$  and that  $\frac{\partial^3 U_i}{\partial r_i^3} \Big|_{r_i=0} < 0$ . Then either  $U_i$  admits a unique minimum in  $[0; +\infty[$  or  $U_i$  is strictly decreasing on  $\mathbb{R}_+$  in which case the minimum should be reached on the reserve upper bound.
- **Case 3:**  $\mu_1(i) < 0$  and  $\mu_2(i) > 0$ . This implies that  $r_i^0 < 0$  and that  $\frac{\partial^3 U_i}{\partial r_i^3} \Big|_{r_i=0} > 0$ . Then either  $U_i$  admits a unique minimum in  $[0; +\infty[$  or  $U_i$  is strictly increasing on  $\mathbb{R}_+$  in which case the minimum should be reached in  $r_i = 0$ .
- **Case 4:**  $\mu_1(i) < 0$  and  $\mu_2(i) < 0$ . This implies that  $r_i^0 > 0$  and that  $\frac{\partial^3 U_i}{\partial r_i^3} \Big|_{r_i=0} > 0$ . Then either  $U_i$  admits a

unique minimum in  $[0; +\infty[$  or  $U_i$  in increasing and then decreasing on  $\mathbb{R}_+$  in which case the minimum should be reached either in  $r_i = 0$  or in the reserve upper bound.

We observe that the Case  $\mu_1(i) > 0, \mu_2(i) > 0$  is the only one which guarantees a positive reserve, different from the upper bound. Furthermore, derivating  $\frac{\partial U_i}{\partial r_i}$  with respect to  $r_j$ , we obtain:  $\frac{\partial^2 U_i}{\partial r_j \partial r_i} = \frac{1}{B^r} + b_i^0 \sum_{l \neq i} \frac{1}{b_i^0} (\eta_{j,j} - \eta_{i,j}) \bar{F}_{\Delta_j}(r_j) \bar{F}_{\Delta_i}(r_i), \forall j \neq i$ . The diagonal dominance condition, introduced in Proposition 5, is checked provided  $\mu_1(i) > b_i^0 \sum_{l \neq i} \frac{1}{b_i^0} \sum_{j \neq i} (\eta_{j,j} - \eta_{i,j}) \bar{F}_{\Delta_j}(r_j)$  and  $N \leq 3$ ; for other  $\mu_1(i), N$  values it is not straightforward.  $\square$

#### APPENDIX C: ALGORITHM FOR THE GENERATION OF SEQUENCES OF FORECAST ERROR DIFFERENCES

We let:  $\Delta_i^s(t)$  be the  $s$ -th sampled realization of the random variable  $\Delta_i(t)$ . According to the CE algorithm for rare event simulation [7],  $\sigma_{\Delta_i}$  update coincides with the solving of the following stochastic program:

$$\begin{aligned} & \max_{\sigma_{\Delta_i}} \frac{1}{S} \sum_s \mathbf{1}_{\Delta_i^s(t) \geq \text{threshold}} \ln f_i(\Delta_i^s(t); \sigma_{\Delta_i}^2) \\ \Leftrightarrow & \sigma_{\Delta_i}^2 = \frac{\sum_s \mathbf{1}_{\Delta_i^s(t) \geq \text{threshold}} (\Delta_i^s(t))^2}{\mathbf{1}_{\Delta_i^s(t) \geq \text{threshold}}} \end{aligned}$$

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#### Generation of sequences of forecast error differences $(\Delta_i(t))_t$

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##### Input:

- $(\gamma_i(c), \theta_i(c))_{i,c}$
- $S$  sample size
- $\rho = 10^{-2}$  rarity parameter
- initial threshold  $> 0$  value

For each geographic market  $i = 1, \dots, N$

Initialization:  $\hat{\sigma}_{\Delta_i}(1) = \sigma_{\Delta_i}$

(1) Generate a sample of size  $S$  such that  $(\Delta_i^s(t))_s \sim \mathcal{N}(0; (\hat{\sigma}_{\Delta_i}(t))^2)$

(2) Compute the sample  $(1 - \rho)$ -quantile  $\hat{\text{threshold}}(t)$  which coincides with the  $(1 - \rho)S$ -th order statistic of the sequence  $(\Delta_i^s(t))_s$ , provided  $\hat{\text{threshold}}(t) < \text{threshold}$ . Otherwise set  $\hat{\text{threshold}}(t) = \text{threshold}$ .

(3) Use the same sample and compute:  $\sigma_{\Delta_i}^{\hat{}}(t) = \sqrt{\frac{\sum_s \mathbf{1}_{\Delta_i^s(t) \geq \hat{\text{threshold}}(t)} (\Delta_i^s(t))^2}{\mathbf{1}_{\Delta_i^s(t) \geq \hat{\text{threshold}}(t)}}}$ .

(4) If  $\hat{\text{threshold}}(t) < \text{threshold}$  set  $t = t + 1$  and reiterate from (1); else STOP.

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