

A thick level-set approach to damage mechanics.

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Outline

- Motivations.
- A sharp level set damage model.
- Numerical experiments (work in progress).
- Improved model : Thick level set damage Model.
- Conclusions and further work.

Outline and motivations

Sharp Level set
Front damage
model

Thick Level set
damage model

Motivations

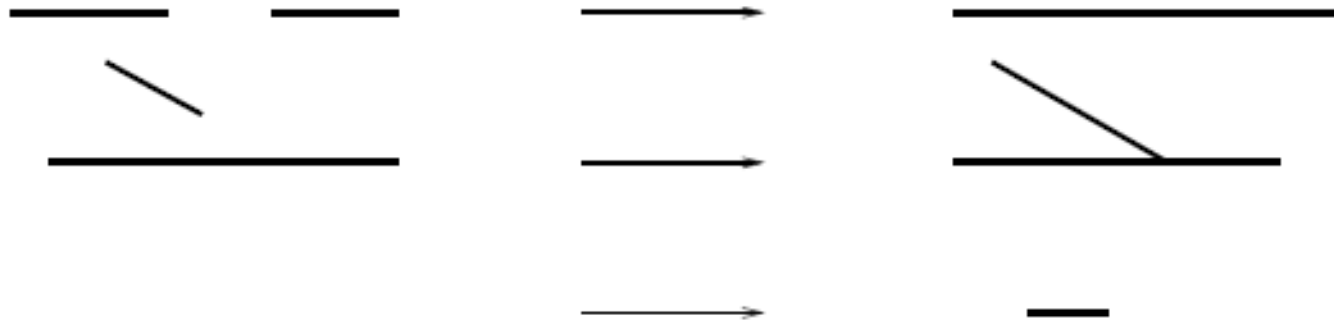
Outline and motivations

From a numerical point of view, certain topological changes in crack are hard to tackle in the context of fracture mechanics, even with X-FEM and level sets.

Sharp Level set Front damage model



Thick Level set damage model



Applications : Crack in composites, Intersecting shear bands, faults in soil ..

Motivations and bibliography

Outline and motivations

Sharp Level set Front damage model

Thick Level set damage model

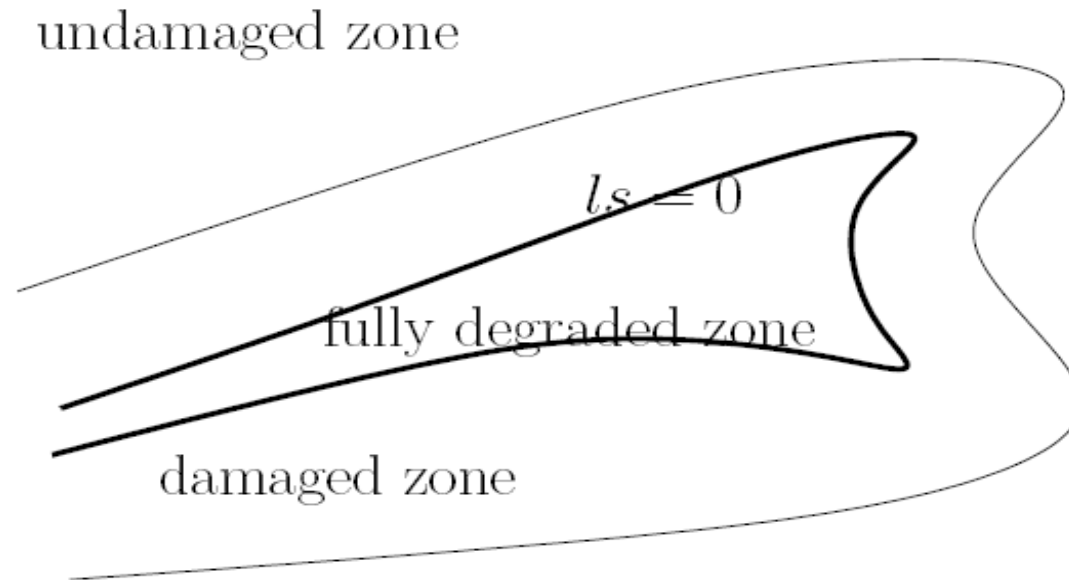
- In these situations, fracture mechanics models are limited : damage type models are needed.
- The difficulty with damage models is how to place the crack. Yet good progress for instance (Perego et al 2007)

Avoiding spurious localization in damage model.

- Non local damage model (Bazant, Belytschko, Chang, 1984), (Pijaudier-Cabot 1987).
- Higher order gradient models (Aifantlis 1984), (Triantafyllis and Aifantis 1986), (Shreyer and Chen 1986).
- Micro-polar continuum model (Mühlhaus and Vardoulakis 1987).
- Phase- field Approach (Hakim and Karma 2005), (Karma, Kessler and Levine 2001)
- Variational approach (Francfort and Marigo 1998), (Bourdin, Francfort and Marigo, 2000, 2008)

Level Set approach to damage

- Idea : model a degradation front with a single level set inside the damaged zone. One level set whatever the number of zones.



Some relations to G. Allaire and Goethem 2007 and Salac and Lu 2007

Sharp front model

Outline and motivations

Sharp Level set Front damage model

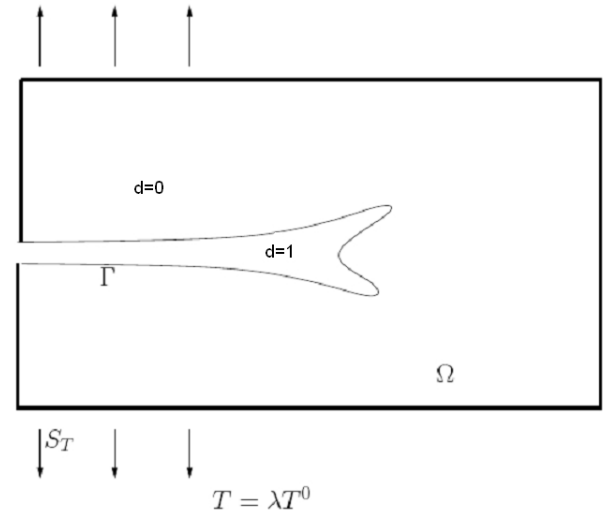
Thick Level set damage model

Potential energy E

$$E(\Gamma, \mathbf{u}, \lambda) = \int_{\Omega} e d\Omega - \int_{S_T} \lambda \mathbf{T}^0 \cdot \mathbf{u} dS$$

$$\mathbf{u} = \lambda \mathbf{u}^0 \text{ on } S_u$$

$$e = \frac{1}{2} \boldsymbol{\epsilon}(\mathbf{u}) : \mathbf{E} : \boldsymbol{\epsilon}(\mathbf{u})$$



Minimizing E yields $\mathbf{u} = \mathbf{u}(\Gamma, \lambda)$

$$W(\Gamma, \lambda) = E(\Gamma, \mathbf{u}(\Gamma, \lambda), \lambda) \quad DW[v_n] = - \int_{\Gamma} e v_n dS$$

Propagation law (Nguyen , Pradeilles and Stolz 1989)

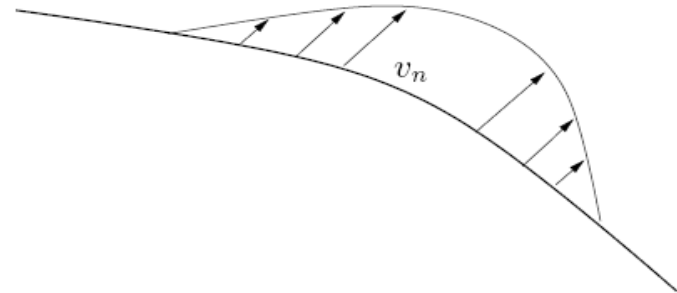
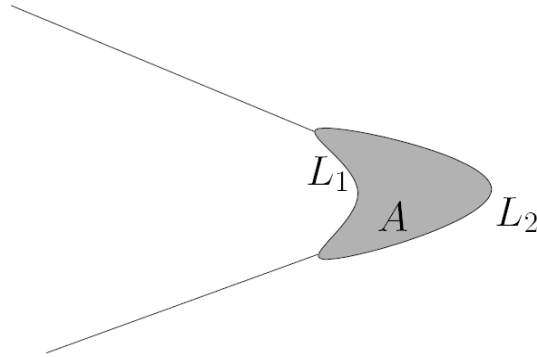
$$v_n \geq 0 \quad f = e - \left(Y_c + \frac{\gamma_c}{\rho} \right) \leq 0 \quad f v_n = 0$$

Geometrical interpretation of the dissipation

Outline and motivations

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Dissipation from 1 to 2

$$D_{12} = Y_c A + \gamma_c (L_2 - L_1)$$

Change in area

$$\dot{A} = \int_{\Gamma} v_n dS$$

Change in length

$$\dot{L} = \int_{\Gamma} \frac{v_n}{\rho} dS$$

Change in dissipation

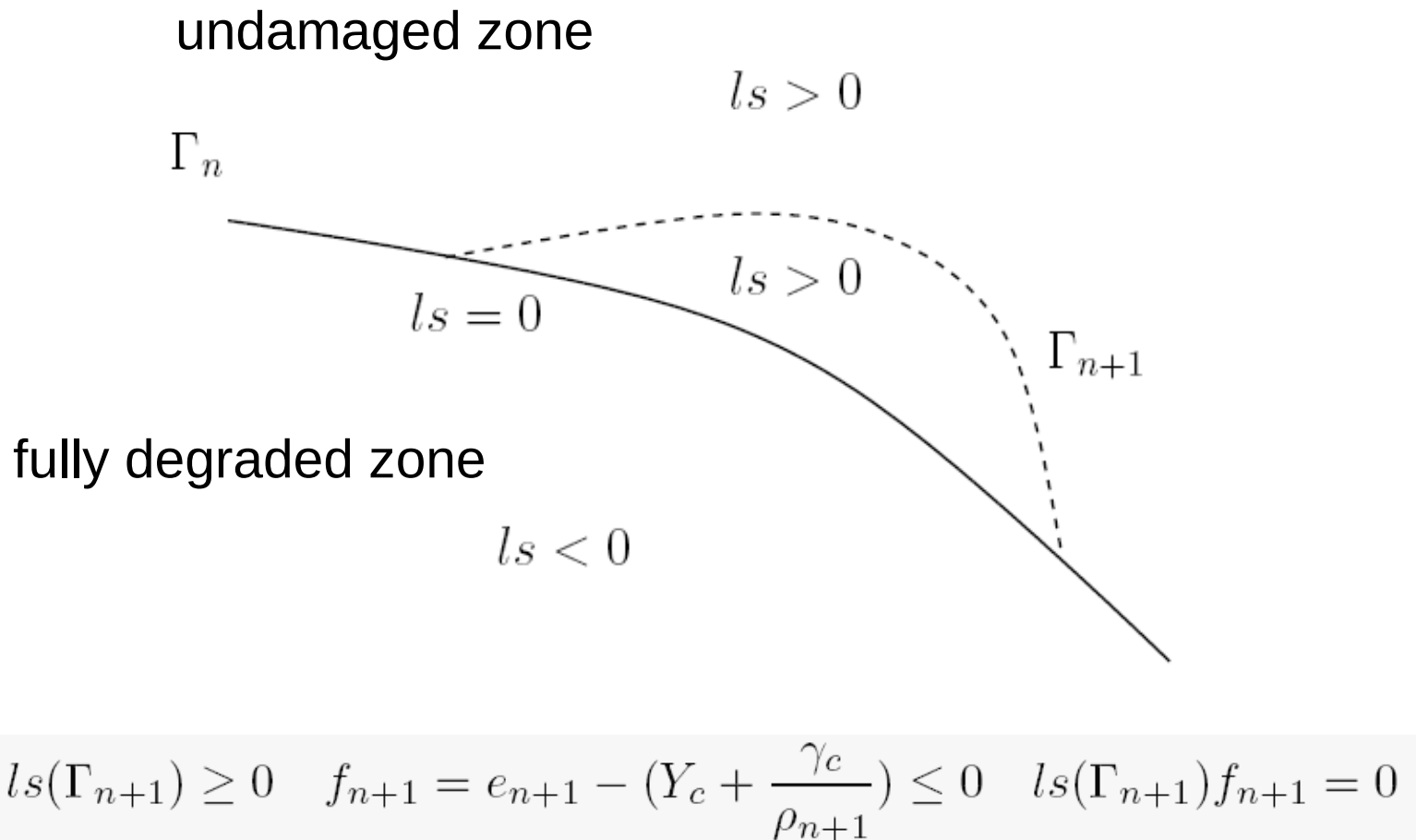
$$\dot{D} = \int_{\Gamma} Y_c v_n + \gamma_c \frac{v_n}{\rho} dS$$

Time discretization

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$$e_{n+1} = \lambda_{n+1}^2 e_{n+1}^0$$

Minimum principle

Outline and motivations

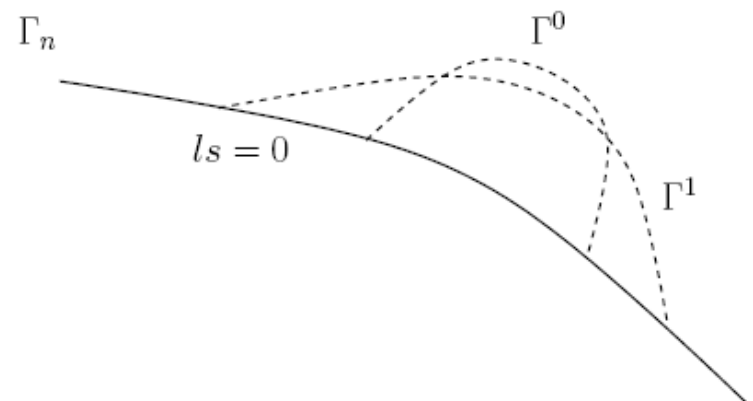
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$$\inf_{u_{n+1}^0 \in U^0} \inf_{\Gamma_{n+1} \geq \Gamma_n} \sup_{\lambda_{n+1}} \int_{\Omega_{n+1}} \epsilon(u_{n+1}^0) d\Omega - \int_{S_T} T^0 \cdot u_{n+1}^0 dS + \frac{1}{\lambda_{n+1}^2} (\text{meas}(\Omega_{n+1}/\Omega_n) Y_c + \text{meas}(\Gamma_{n+1}/\Gamma_n) \gamma_c - \Delta D)$$

From an initial guess (predictor), optimize the shape while maintaining the same dissipation.

Dissipation control algorithm. No arc-length.



Projected gradient technique

Outline and motivations

$$\text{If } l s^i > 0 \quad v_n^i = e^i - (Y_c + \frac{\gamma_c}{\rho^i})$$

$$\text{If } l s^i = 0 \quad v_n^i = \langle e^i - (Y_c + \frac{\gamma_c}{\rho^i}) \rangle_+$$

$$\int_{\Gamma^i} v_n^i (Y_c + \frac{\gamma_c}{\rho^i}) dS = 0 \quad \text{Preservation of the dissipation}$$

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Front damage
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Non-linear equation to find the proper loading factor

$$\frac{\partial l s}{\partial t} + (e^i - Y_c - \gamma_c \operatorname{div}(\frac{\nabla l s}{\|\nabla l s\|})) \|\nabla l s\| = 0$$

level set equation

Numerical Experiments

Outline and motivations

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Preliminary experiments to test the different ingredients :

- Extended finite element: the mesh is not conforming to the front.
- Octree – type mesh adaptation.
- Level set algorithm.
- Test limited for now to explicit fatigue algorithm :

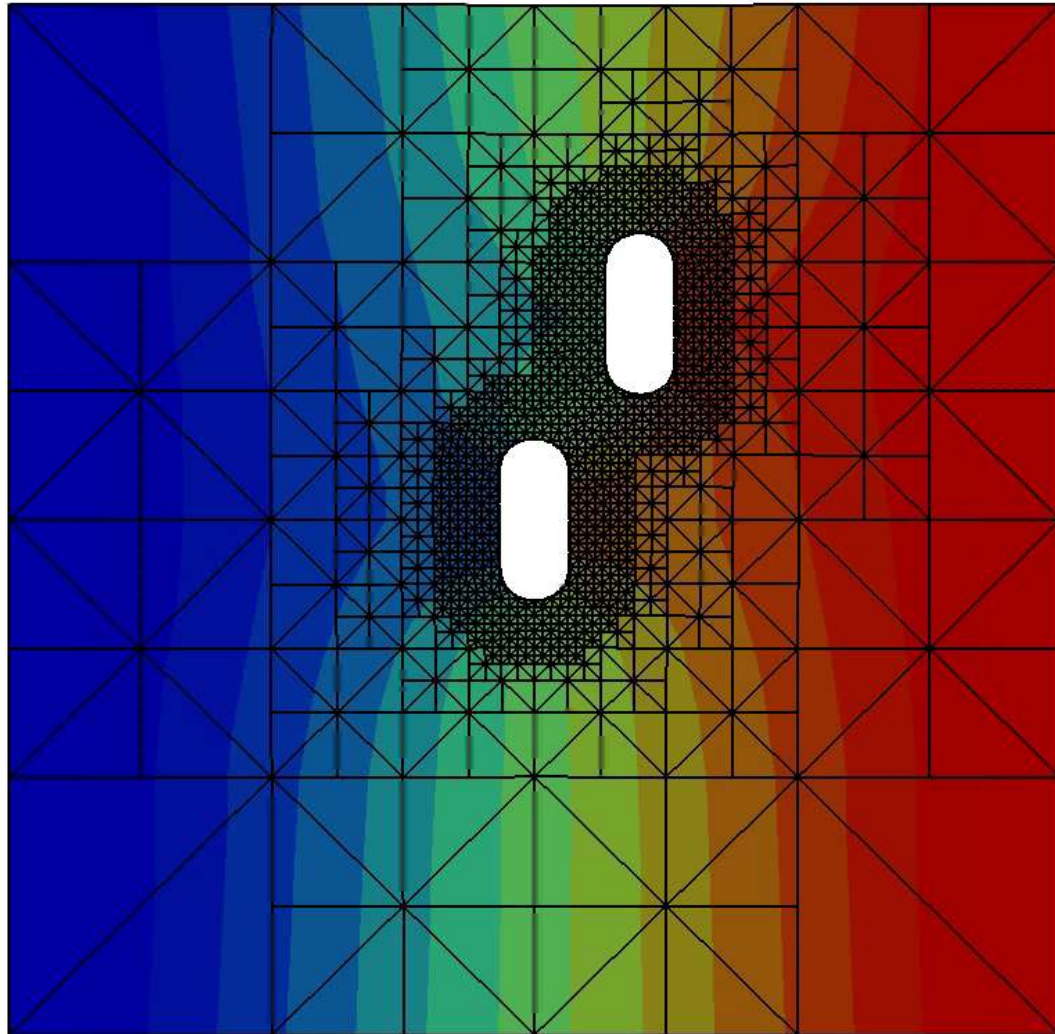
$$v = \langle e - Y_c - \gamma_c / \rho \rangle_+$$

Merging Benchmark

Outline and motivations

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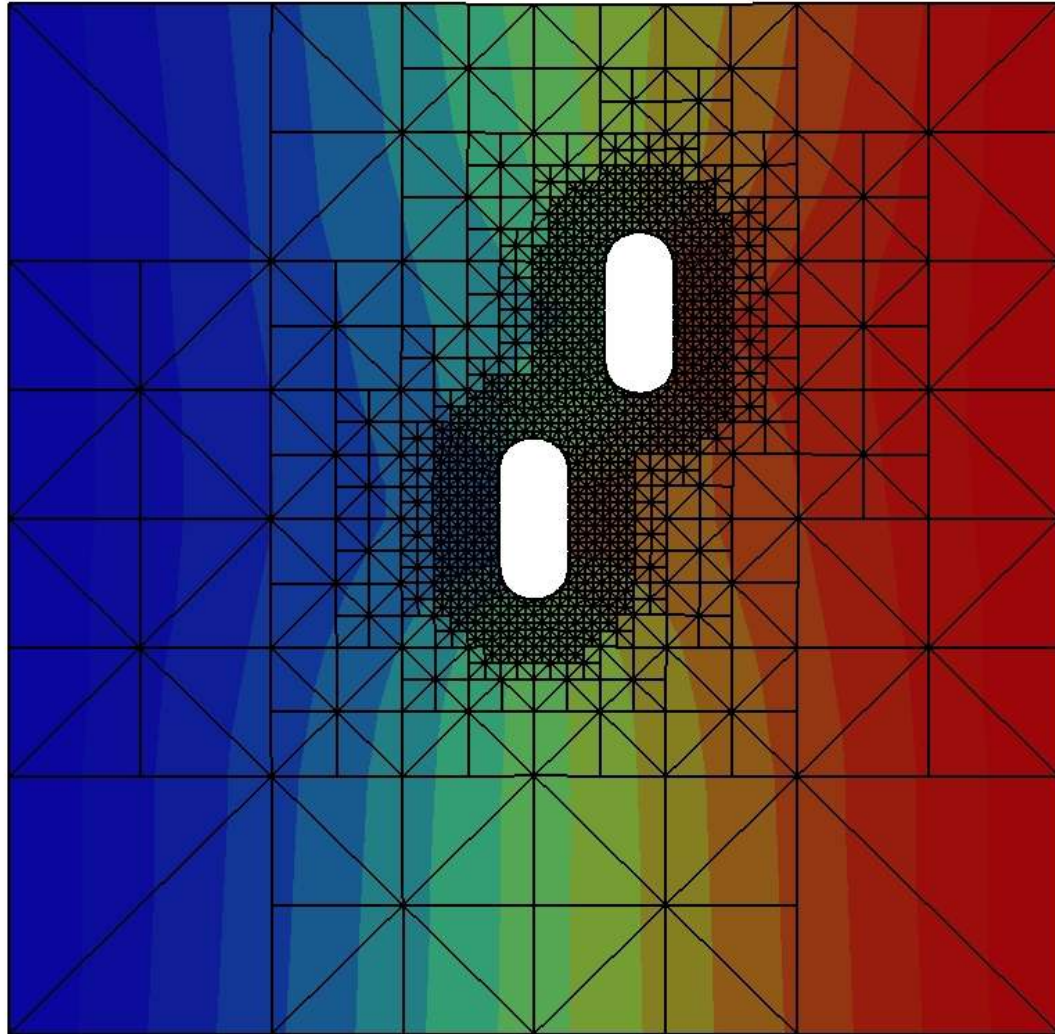
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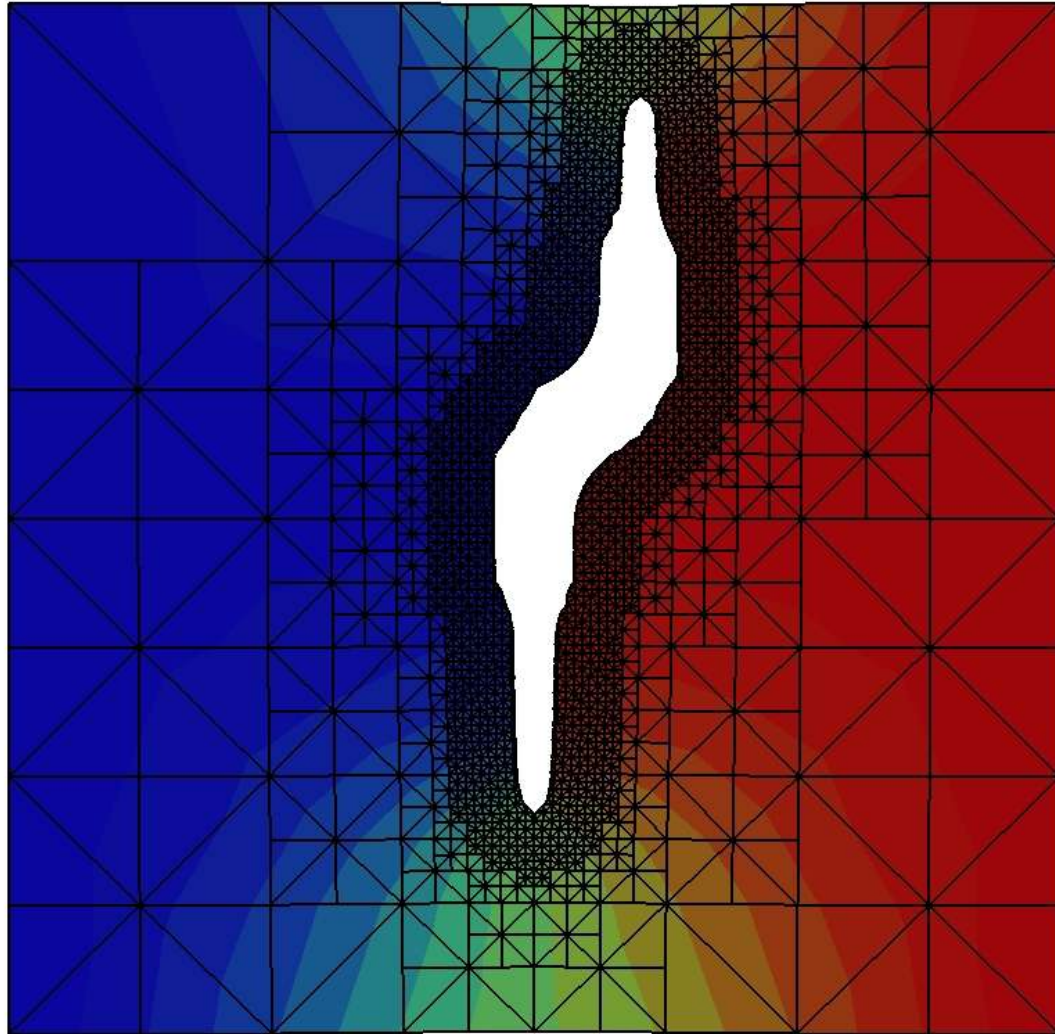
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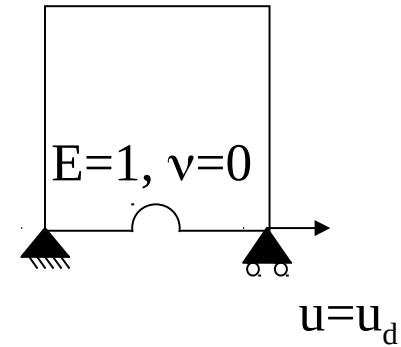
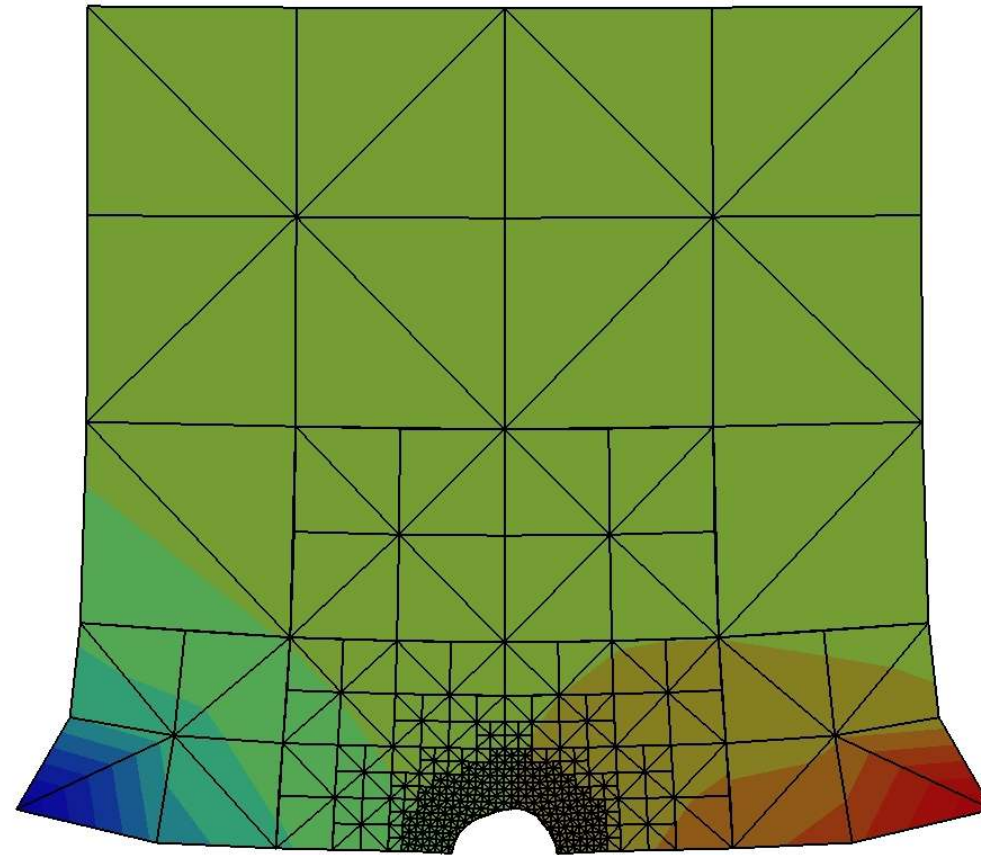
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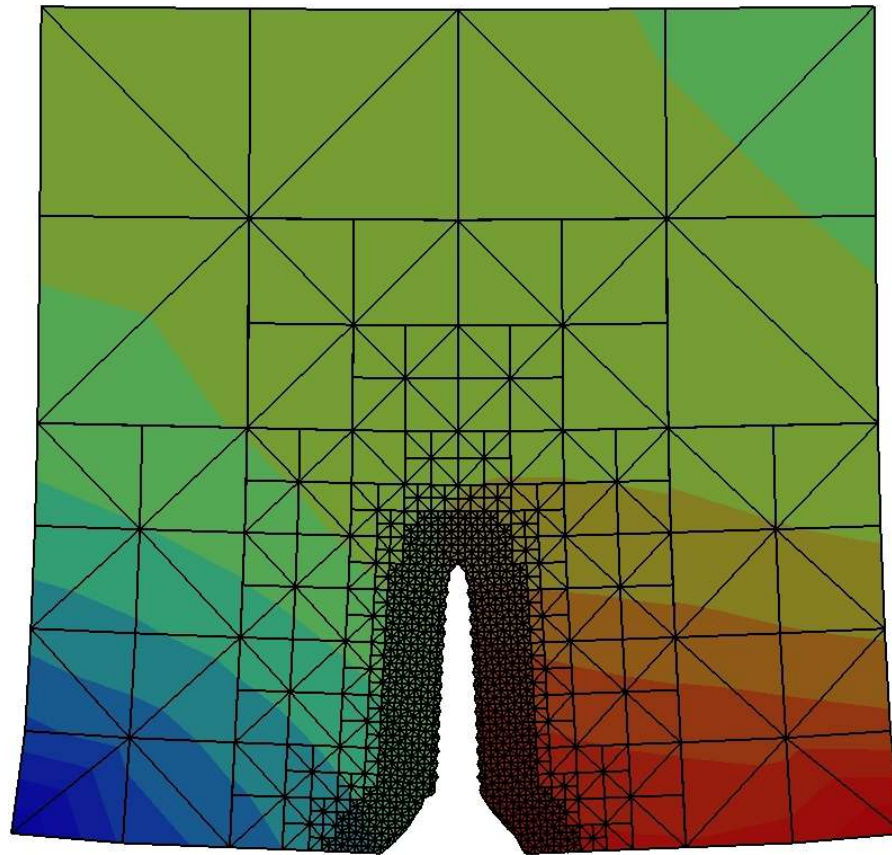
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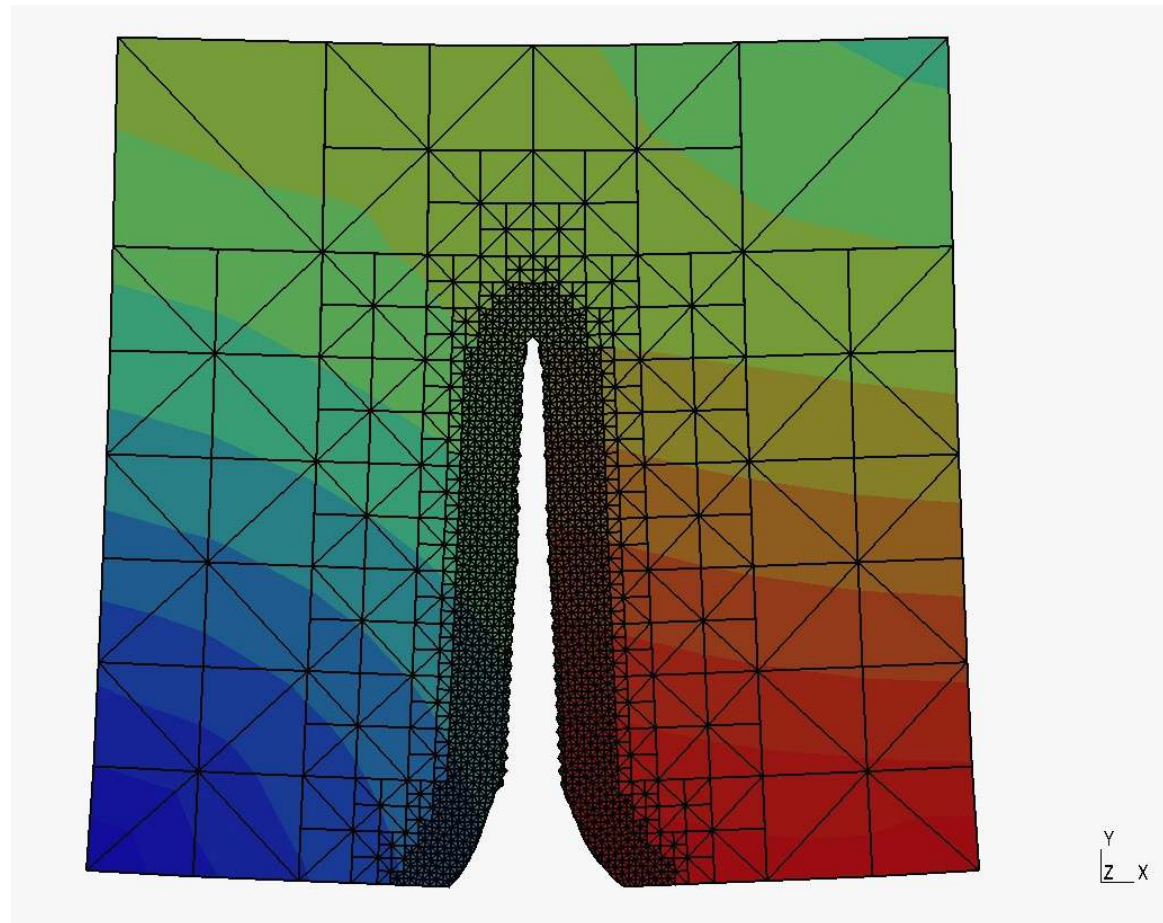
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Encouraging results, but :

- Whatever l_c , the damage zone tend to a crack with a thickness that depends on the mesh size.
- Might be solved with the full implicit algorithm.
- Quick fix : make the curvature term more important, for example :

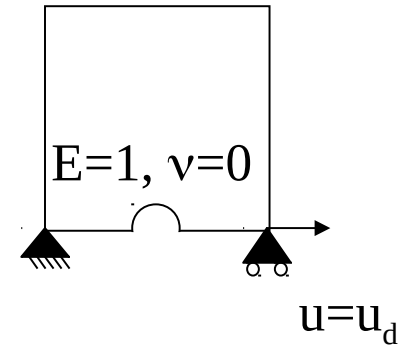
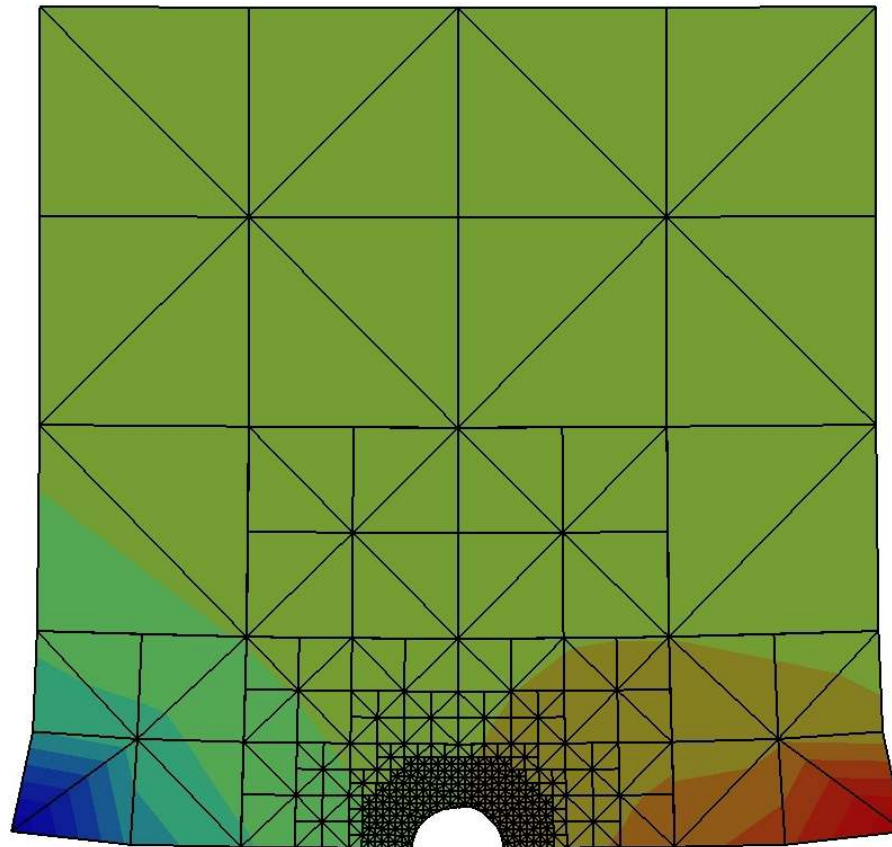
$$v = \langle e - Y_c - \text{sign}(\rho) Y_c l_c^2 / \rho^2 \rangle_+$$

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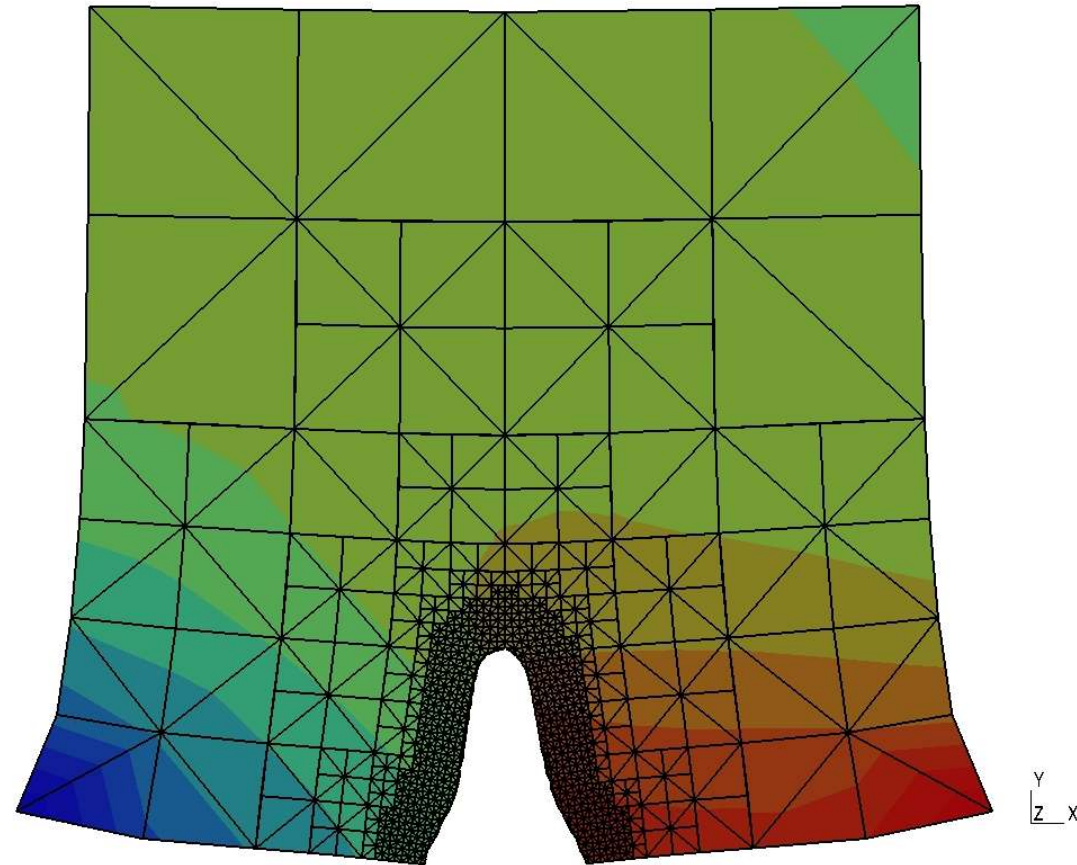


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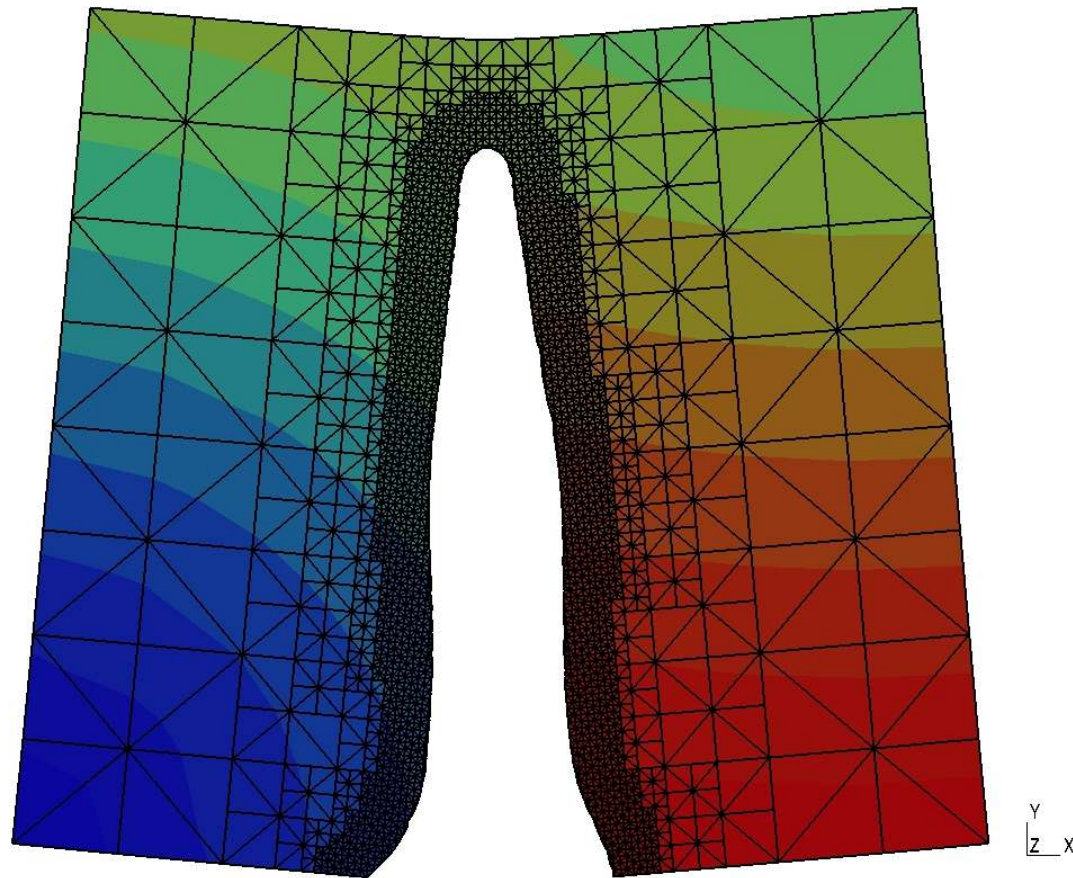
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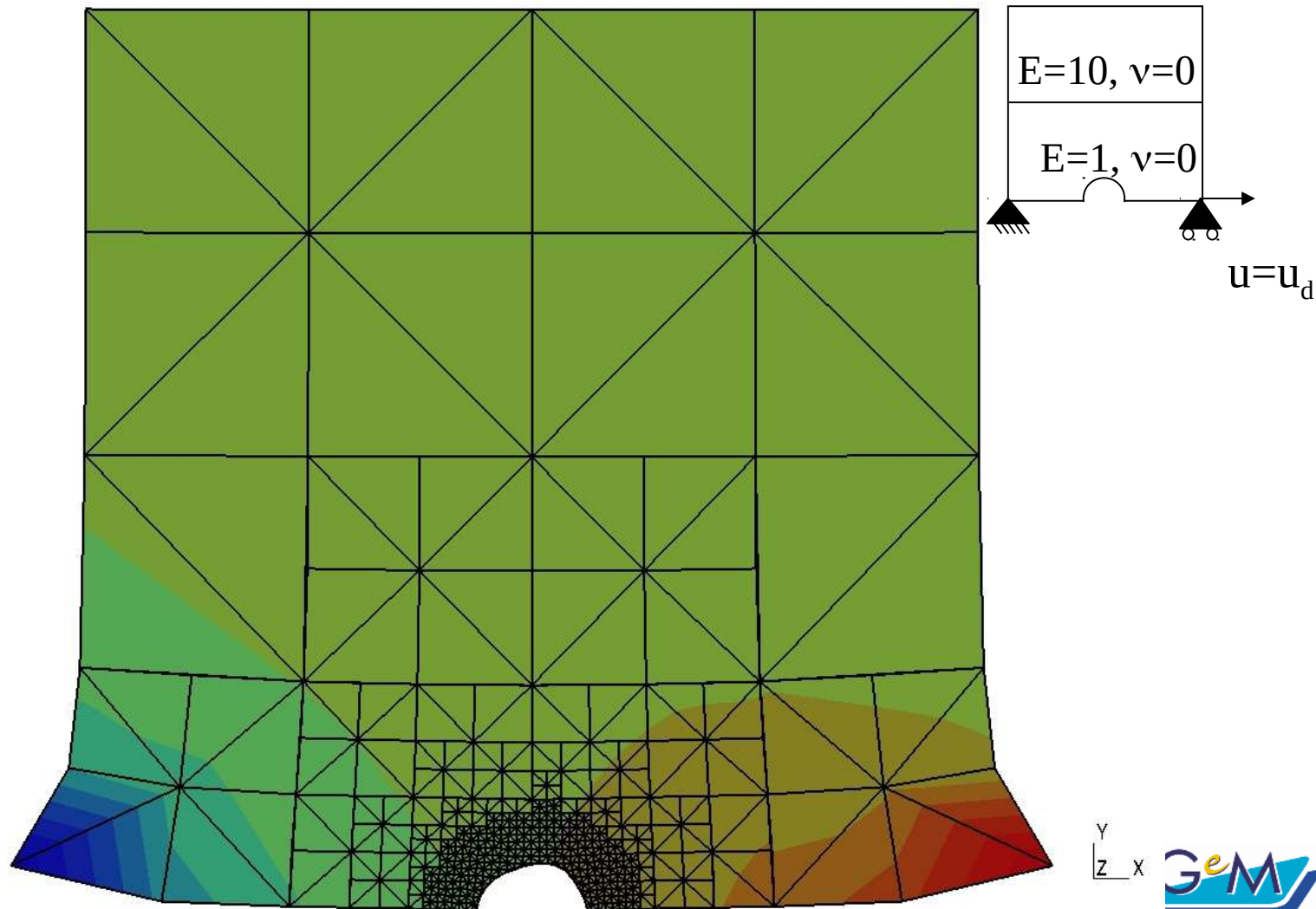
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Branching benchmark

Outline and motivations

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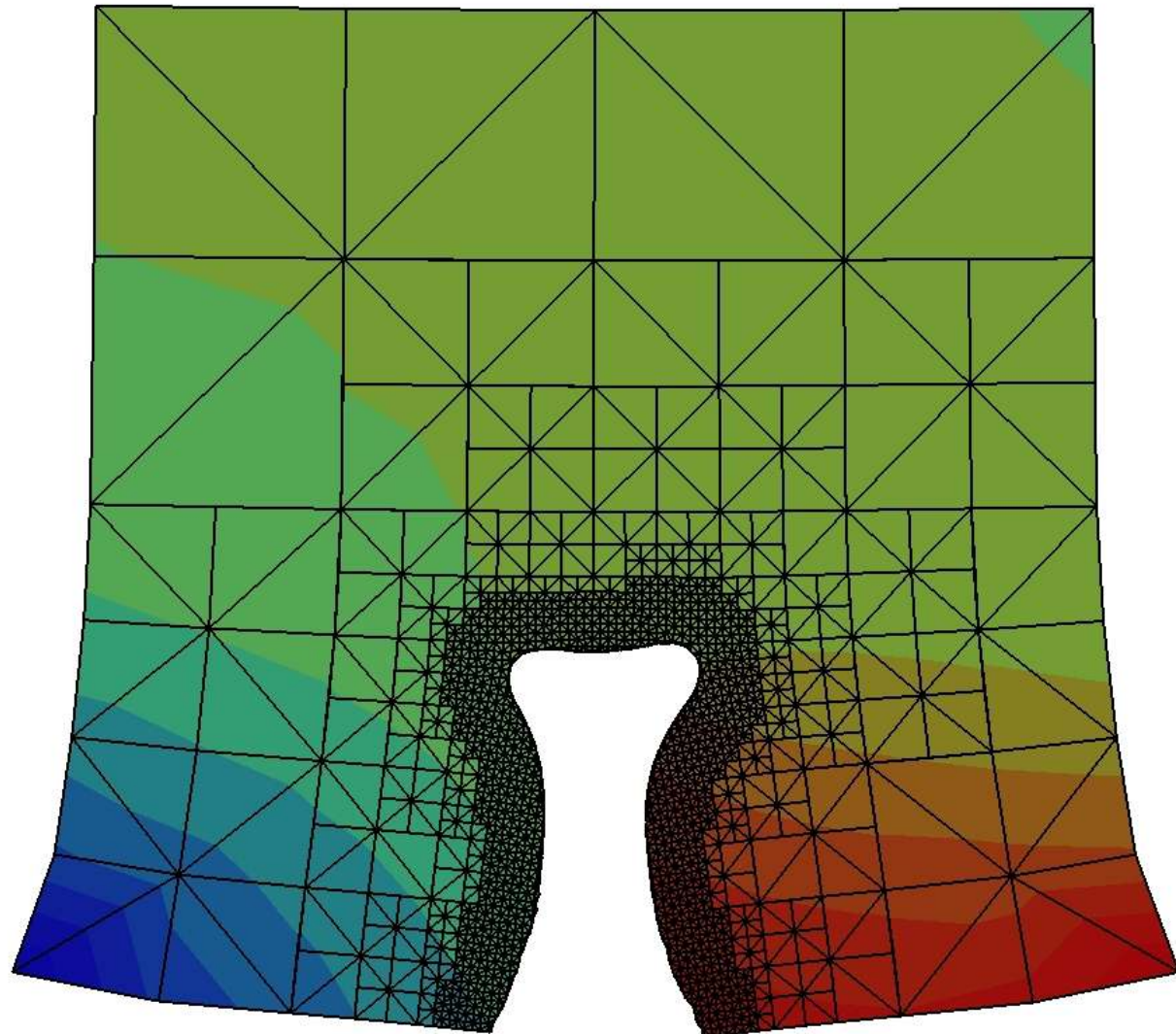
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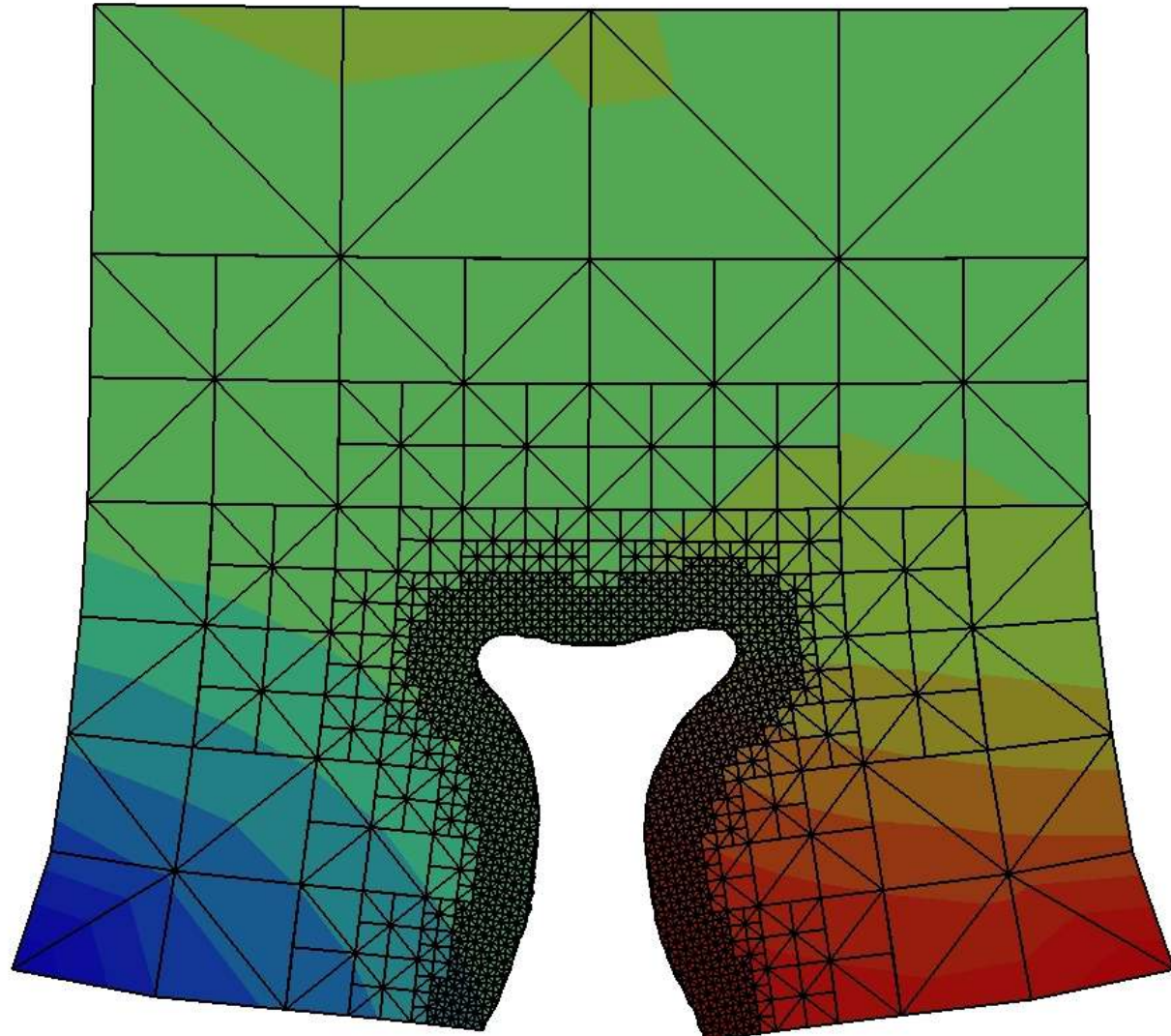
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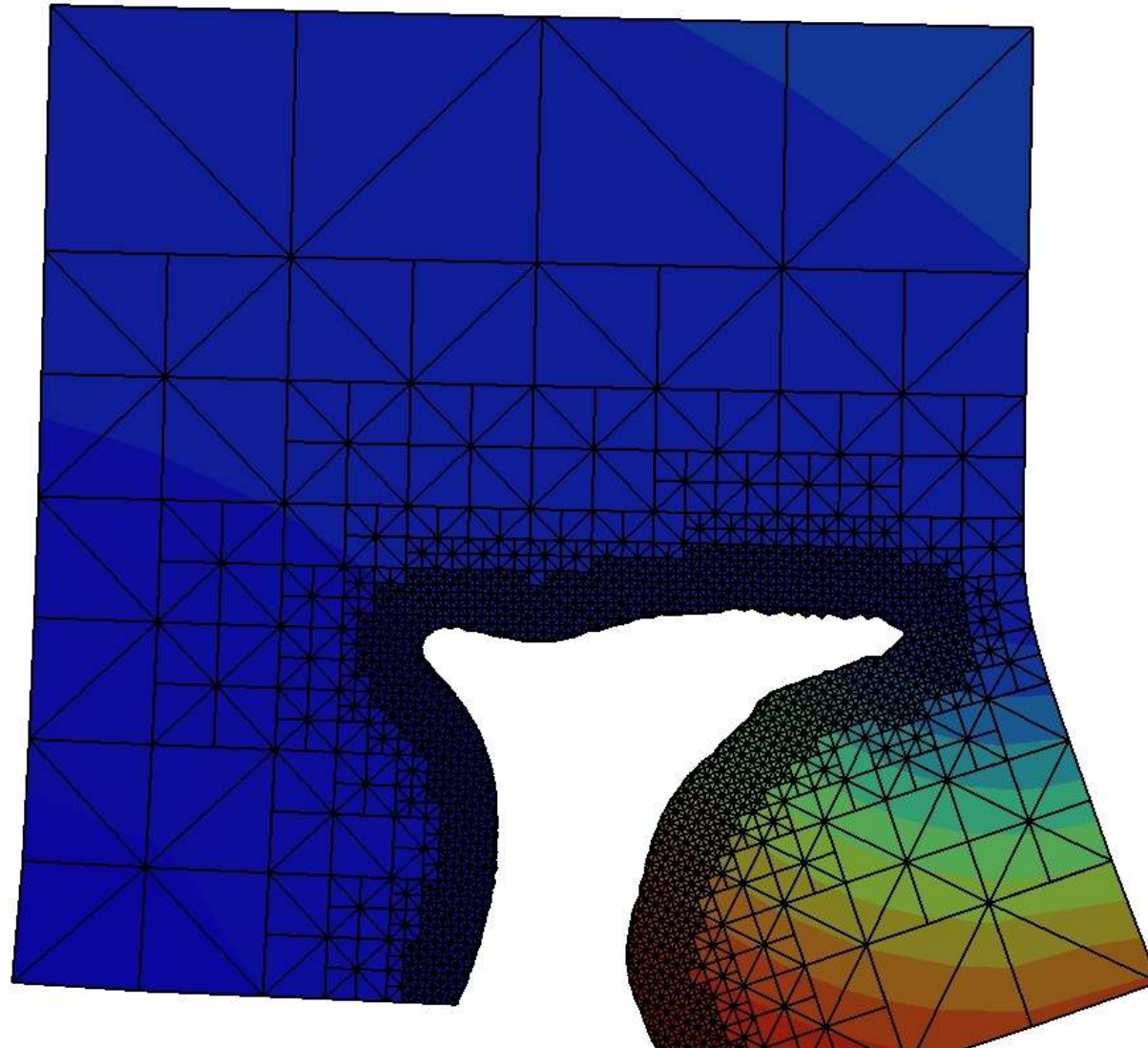
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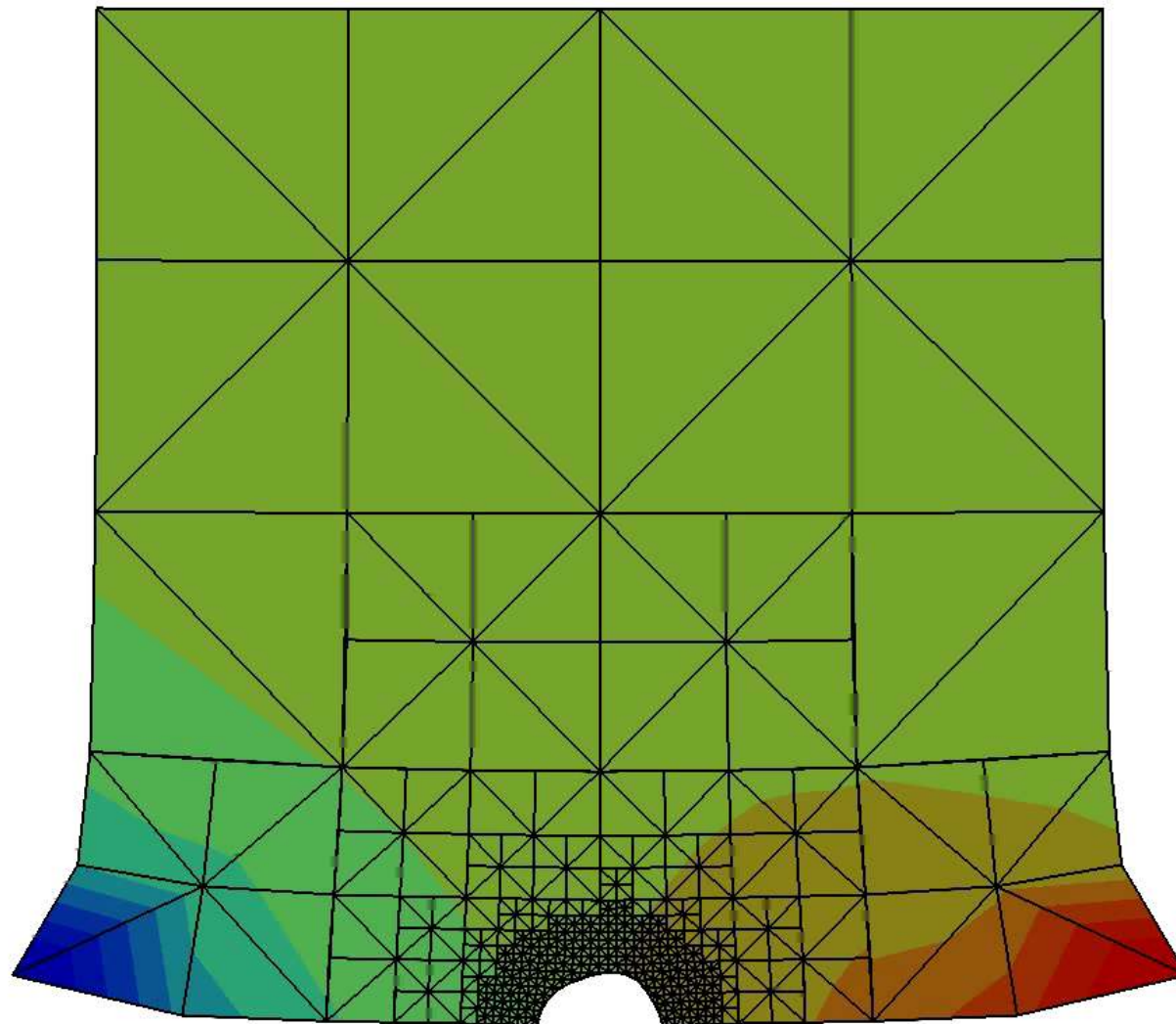
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Advantages and Limitations

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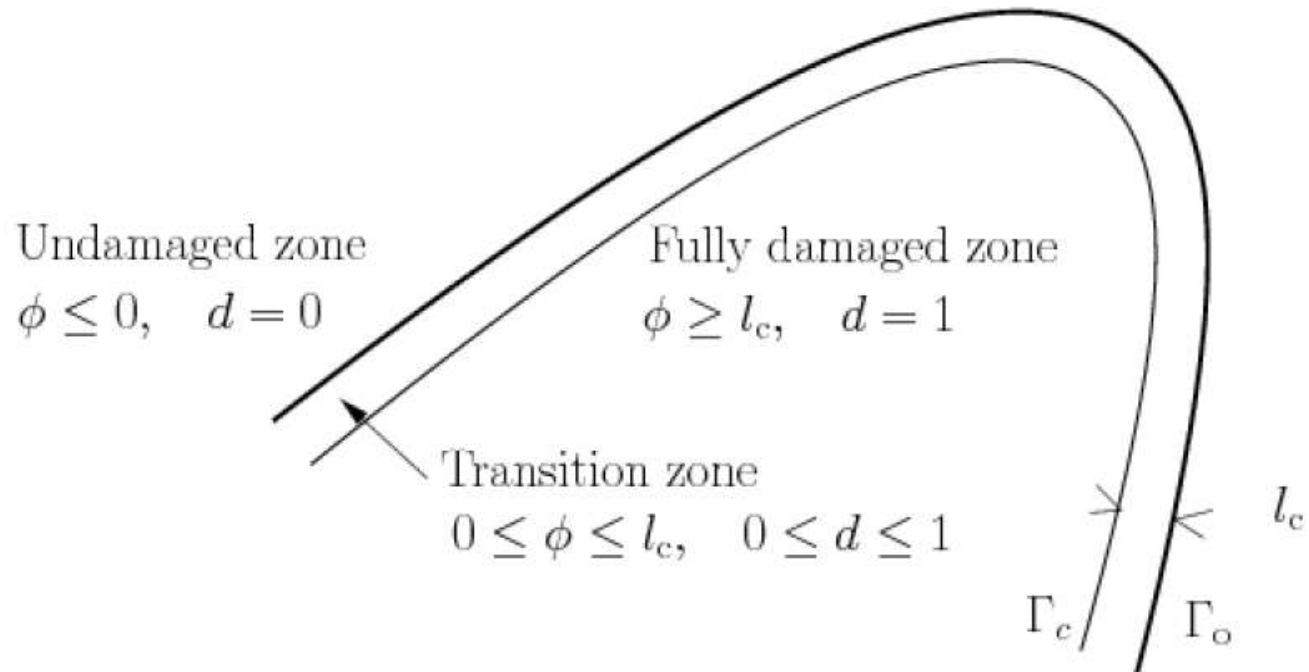
- + Allowing topological changes
- + Length scale is introduced through surface energy .
- + No need for second order gradient in displacement or non local fields.
- + Dissipation control algorithm avoiding arc length.
- - No gradual damage, the shift from $d=0$ to $d=1$ is brutal.
- - It turns out that the load to initiate a hole is infinite. Indeed the criteria $Y_c + \gamma_c / \rho$ goes to infinity as ρ goes to zero.

A new thick level set approach

Outline and motivations

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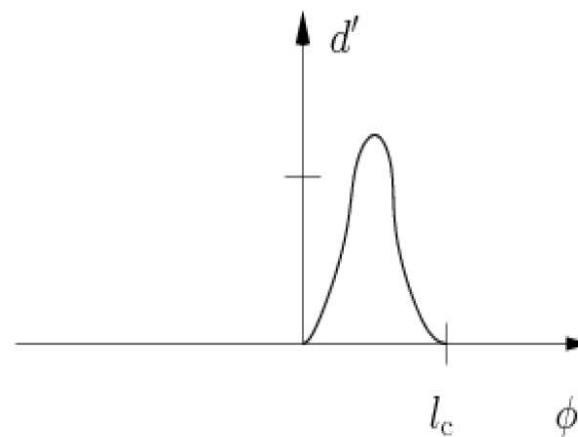
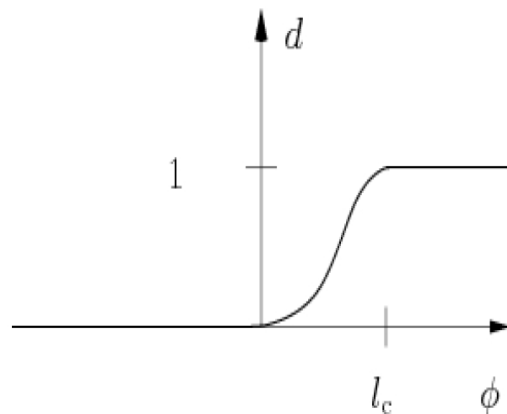


Relation between damage and level set.

Outline and motivations

- The variation of the damage with the distance to the front is a parameter in the model.

Sharp Level set Front damage model



$$d = d(\phi), \quad d'(\phi) \geq 0$$

From local quantities (d, Y) to front quantities (a, g)

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Local damage model

$$\varphi = \varphi(\boldsymbol{\epsilon}, d) = \frac{1}{2}(1 - d)\boldsymbol{\epsilon} : \mathbf{E} : \boldsymbol{\epsilon}$$

$$\boldsymbol{\sigma} = \frac{\partial \varphi}{\partial \boldsymbol{\epsilon}} = (1 - d)\mathbf{E} : \boldsymbol{\epsilon}$$

$$Y = -\frac{\partial \varphi}{\partial d} = \frac{1}{2}\boldsymbol{\epsilon} : \mathbf{E} : \boldsymbol{\epsilon}$$

$$\dot{d} = \frac{\partial \psi^*(Y)}{\partial Y} \quad Y = \frac{\partial \psi(\dot{d})}{\partial \dot{d}}$$

Damage evolution law

$$\psi(\dot{d}) \text{ convex, } \psi(\dot{d}) \geq 0, \quad \psi(0) = 0$$

Ensure a positive
dissipation

$$\psi^*(Y) + \psi(\dot{d}) - Y\dot{d} \geq 0 \quad \text{And } = 0 \text{ if the damage evolution law is verified}$$

From local quantities (d, Y) to front quantities (a, g)

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Potential Energy of the system

$$E(\mathbf{u}, \phi) = \int_{\Omega(\phi \leq l_c)} \varphi(\boldsymbol{\epsilon}(\mathbf{u}), d(\phi)) d\Omega - \int_{\Gamma_T} \mathbf{T} \cdot \mathbf{u} d\Gamma$$

Admissible fields and variations.

$$\begin{aligned} A_d &= \{(\mathbf{u}, \phi) : \mathbf{u} \in A_d^u, \phi \in A_d^\phi\} & A_0 &= \{(\delta\mathbf{u}, \delta\phi) : \delta\mathbf{u} \in A_0^u, \delta\phi \in A_0^\phi\} \\ A_d^u &= \{\mathbf{u} : \mathbf{u} = \mathbf{u}_d \text{ on } \Gamma_u\} & A_0^u &= \{\delta\mathbf{u} : \delta\mathbf{u} = 0 \text{ on } \Gamma_u\} \\ A_d^\phi &= \{\phi : \|\nabla\phi\| = 1 \text{ on } \Omega\} & A_0^\phi &= \{\delta\phi : \nabla\delta\phi \cdot \nabla\phi = 0 \text{ on } \Omega\} \end{aligned}$$

$$\delta\phi = \delta\phi(s) = \delta a(s)$$

From local quantities (d, Y) to front quantities (a, g)

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$$DE(\mathbf{u}, \phi)[(\delta \mathbf{u}, 0)] = \int_{\Omega(\phi \leq l_c)} \boldsymbol{\sigma} : \boldsymbol{\epsilon}(\delta \mathbf{u}) d\Omega - \int_{\Gamma_T} \mathbf{T}_d \cdot \delta \mathbf{u} d\Gamma = 0 \quad \forall \delta \mathbf{u} \in A_0^u$$

Equilibrium equation.

$$\begin{aligned} DE(\mathbf{u}, \phi)[(0, \delta \phi)] &= - \int_{\Omega_{oc}} Y d'(\phi) \delta \phi d\Omega \\ &= - \int_{\Gamma_0} \int_0^l Y(\phi, s) d'(\phi) \delta a(s) \frac{\rho(\phi, s)}{\rho(0, s)} d\phi ds \\ &= - \int_{\Gamma_o} g(s) \delta a(s) ds = - \int_{\Gamma_o} g \delta a d\Gamma \end{aligned}$$

The local variables d and Y are now linked to the homogenized variables a and g.

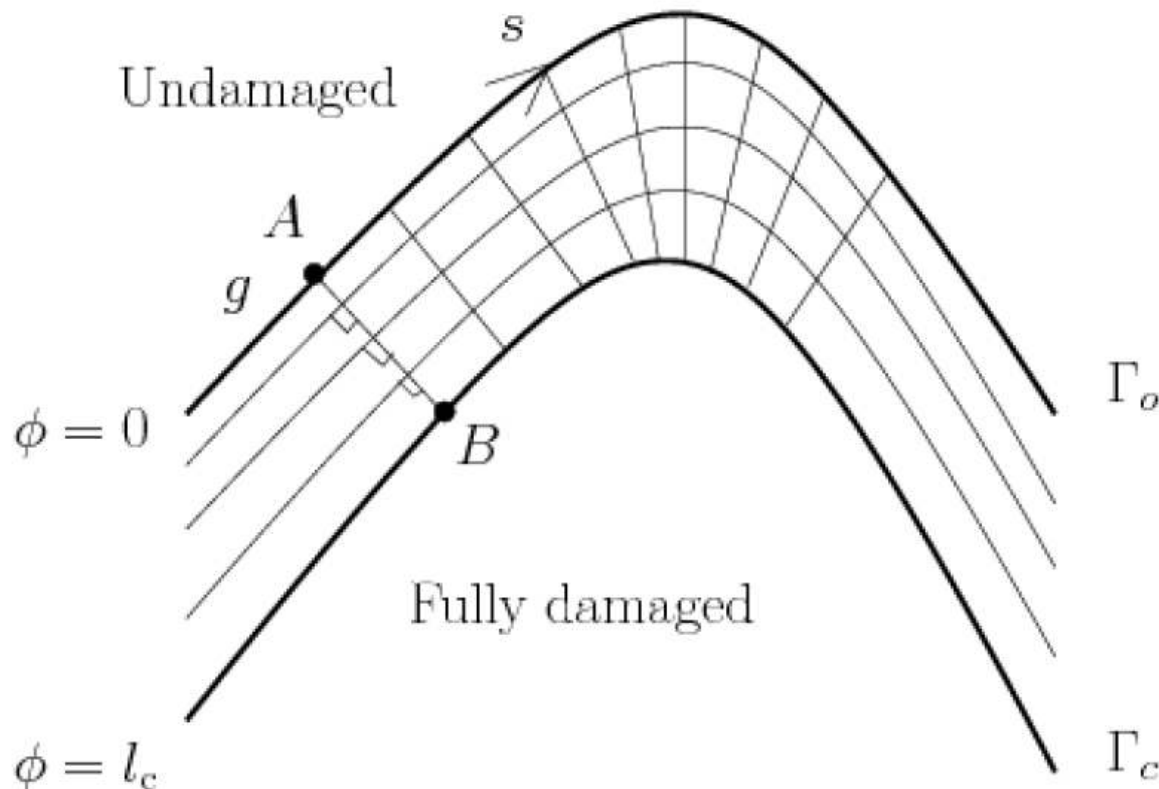
The front driving force is a weighted average of the local driving force.

Outline and motivations

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$$g(A) = \int_{AB} Y d'(\phi) \frac{\rho(\phi)}{\rho(A)} d\phi$$

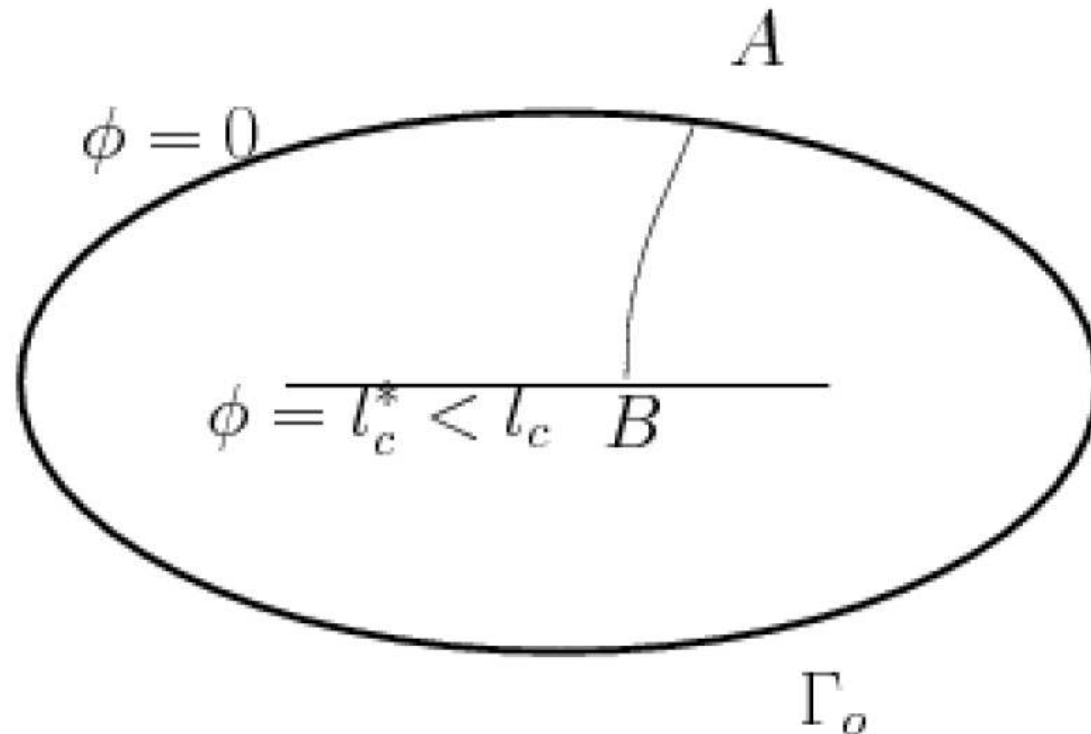


Case Where the damage zone is not fully developed

Outline and motivations

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From local quantities (d, Y) to front quantities (a, g)

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Dissipation is now :

$$-\frac{dE(\mathbf{u}, \phi)}{dt} = \int_{\Omega_{oc}} Y \dot{d} d\Omega = \int_{\Gamma_o} g \dot{a} d\Gamma$$

To close the model, an evolution law is needed, relating the front speed and the configuration force.

- Choose a phenomenological law
- Or homogenize the local evolution law, under the constrain :

$$\dot{d} = d'(\phi) \dot{a}$$

$$\inf_{\dot{d}=d'(\phi)\dot{a}} \int_{\Omega(\phi \leq l_c)} \psi^*(Y) + \psi(\dot{d}) - Y \dot{d} d\Omega$$

From local quantities (d, Y) to front quantities (a, g)

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$$\int_{\Gamma_o} \bar{\psi}(\dot{a}, s) d\Gamma = \int_{\Omega_{oc}} \psi(d'(\phi)\dot{a}) d\Omega$$

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$$\int_{\Omega(\phi \leq l_c)} Y d'(\phi)\dot{a} d\Omega = \int_{\Gamma_o} g\dot{a} d\Gamma$$

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$$-\inf_{\dot{a}} \int_{\Gamma_o} \bar{\psi}(\dot{a}, s) - g\dot{a} d\Gamma = \sup_{\dot{a}} \int_{\Gamma_o} g\dot{a} - \bar{\psi}(\dot{a}, s) d\Gamma = \int_{\Gamma_o} \bar{\psi}^*(g, s) d\Gamma$$

$$\dot{a} = \frac{\partial \bar{\psi}^*(g, s)}{\partial g}, \quad \Leftrightarrow g = \frac{\partial \bar{\psi}(\dot{a}, s)}{\partial \dot{a}}, \quad \Leftrightarrow \bar{\psi}^*(g, s) + \bar{\psi}(\dot{a}, s) - g\dot{a} = 0$$

$$\bar{\psi}(\dot{a}) \text{ convex, } \bar{\psi}(\dot{a}) \geq 0, \quad \bar{\psi}(0) = 0$$

Strength of the new model

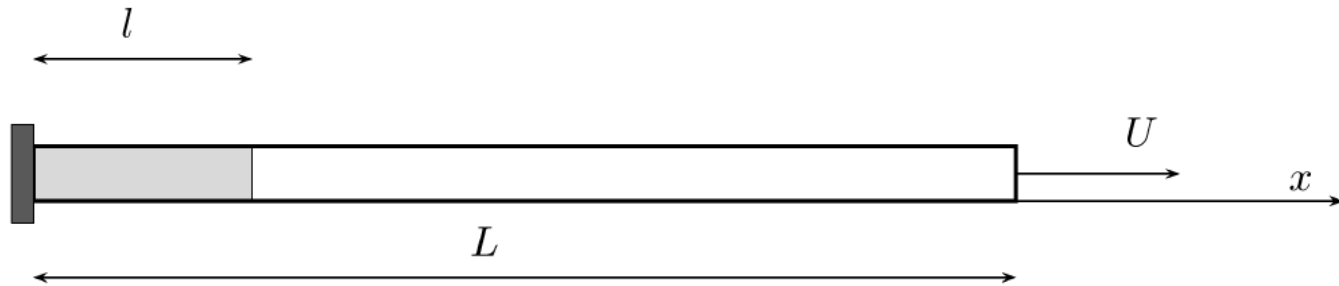
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- The “non-locality is local”, numerical treatment is in the thickness of the front.
- The model embeds the case $d=1$.
- Initiation is based on a local criteria and thus if there is an initial crack, damage starts at the tip and not further away as in non-local formulation (Simone, Askes, Sluys, 2004).
- The model is thermodynamically sound and does not suffer problems of non-local formulation (Lorentz et Andrieux 2003). Duality is not corrupted.
- Any local model may be used as an input.
- If the radius of curvature of the front is large with respect to l_c the sharp front model and the thick front model are related.

One dimensional example



$$\varphi(\epsilon, d) = \frac{1}{2}(1 - d)E\epsilon^2$$

$$d(\phi) = \phi/l_c$$

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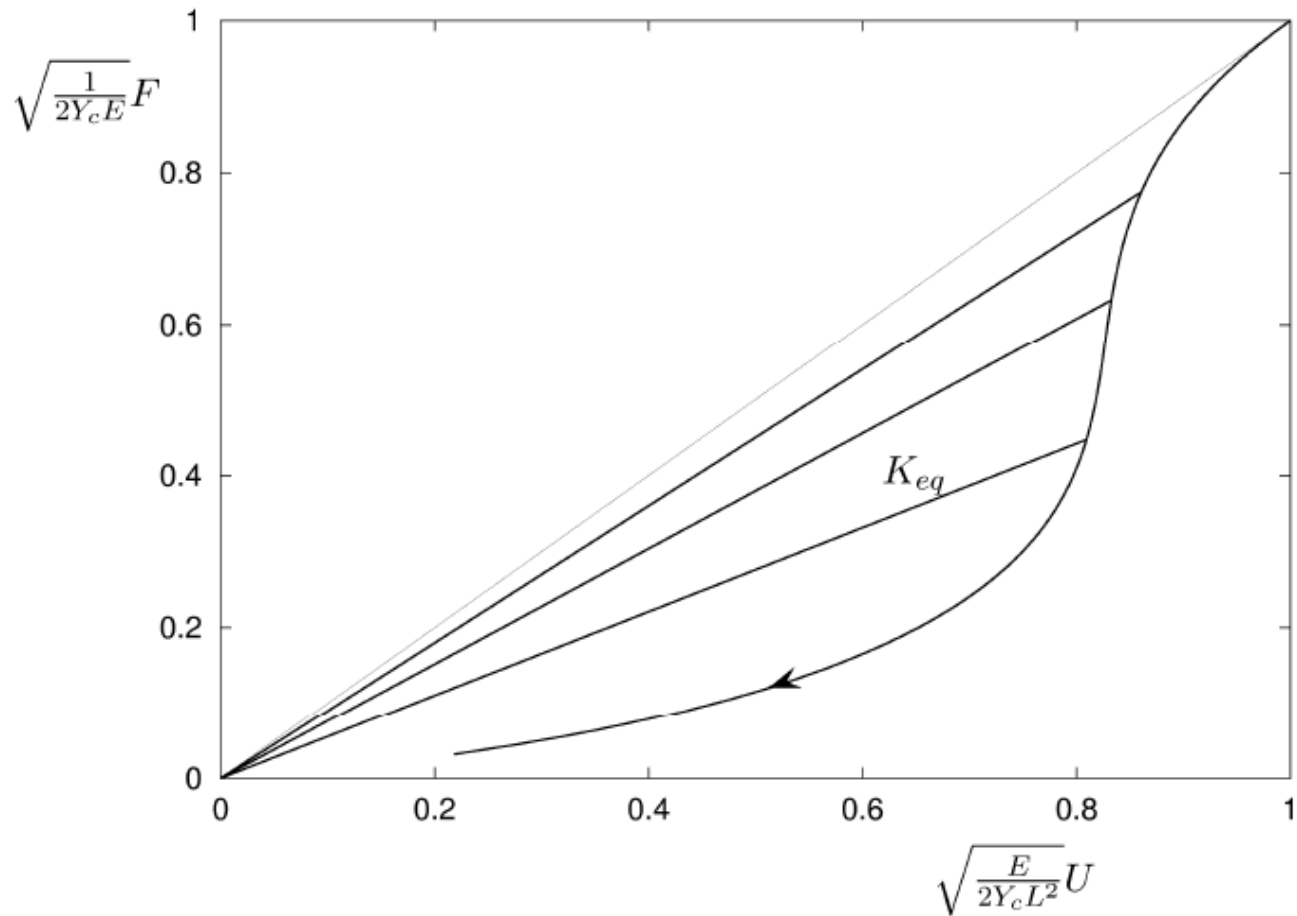
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One dimensional example

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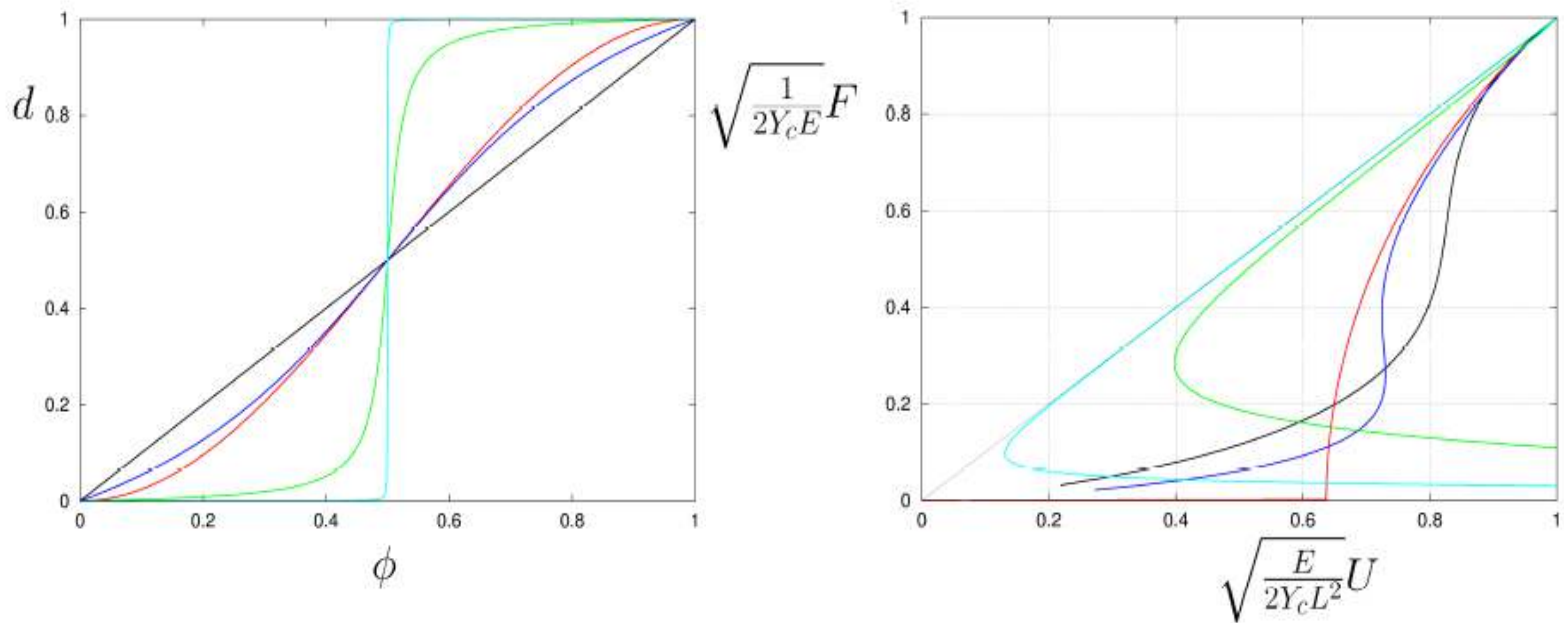


One dimensional example

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Influence of the damage shape in the damaged zone.

Preliminary numerical examples

Outline and motivations

Sharp Level set Front damage model

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As for the sharp damage model, the following examples are limited here to fatigue situation : the speed of the front is computed explicitly from the field.

The purpose is to test some of the numeric tools developed to cope with this model :

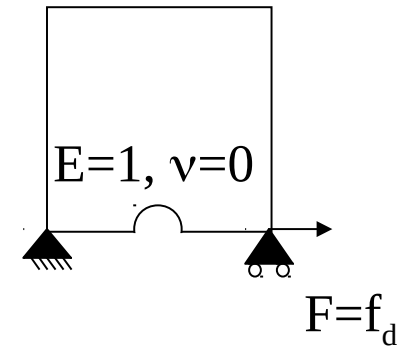
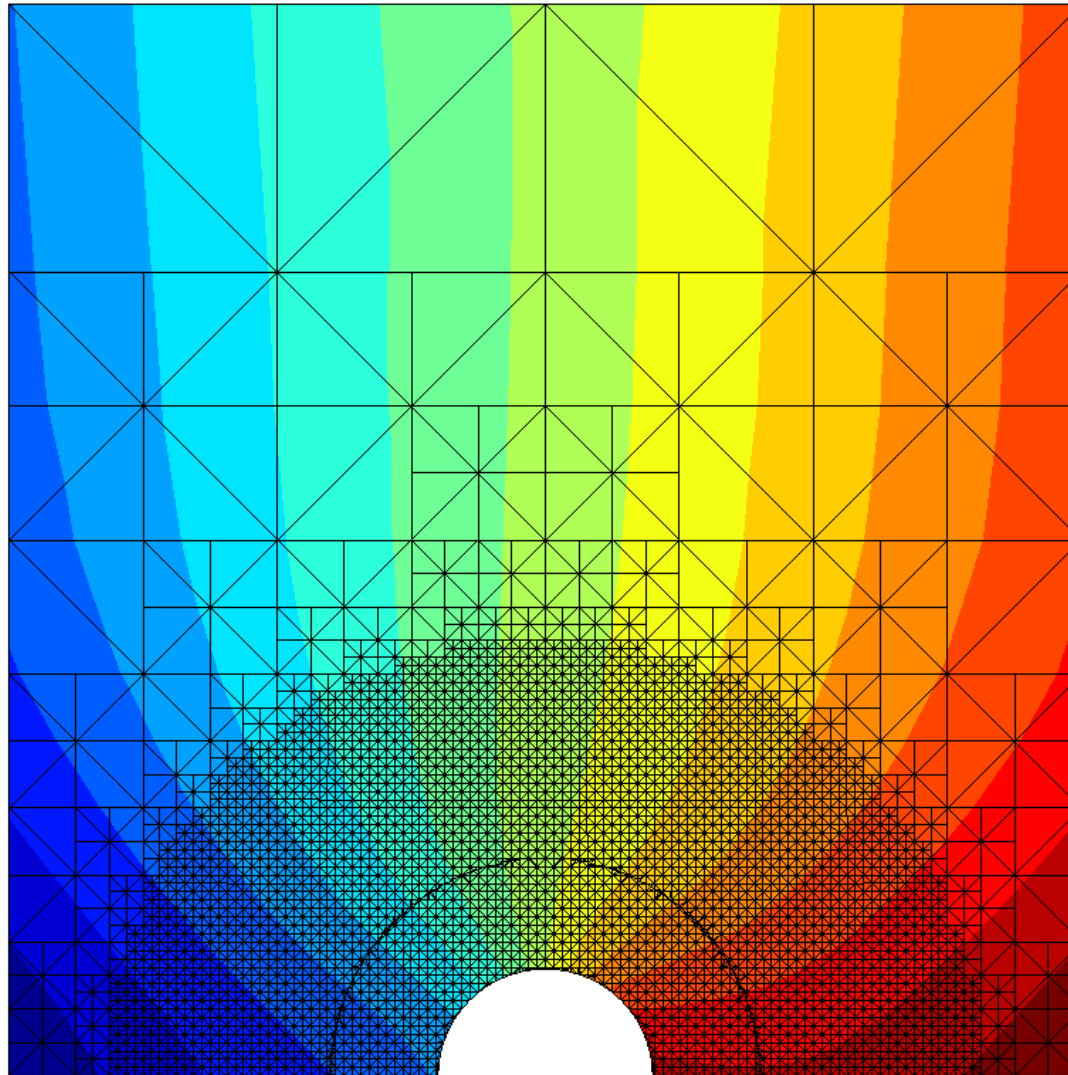
- Definition of the band, and its parameterization.
- Integration of the local quantities (Y_d) over lines along the gradient of the level set.

Mode I benchmark

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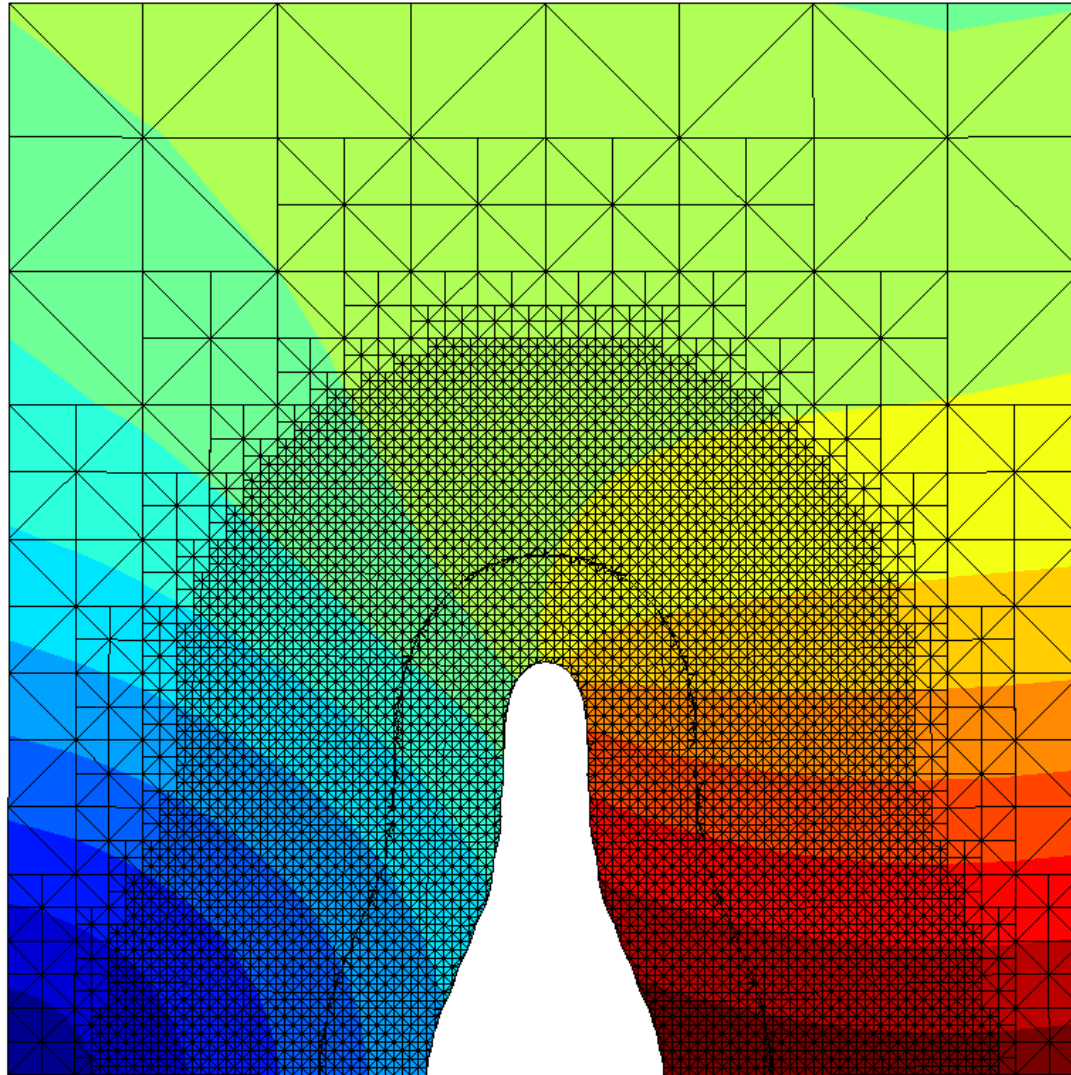


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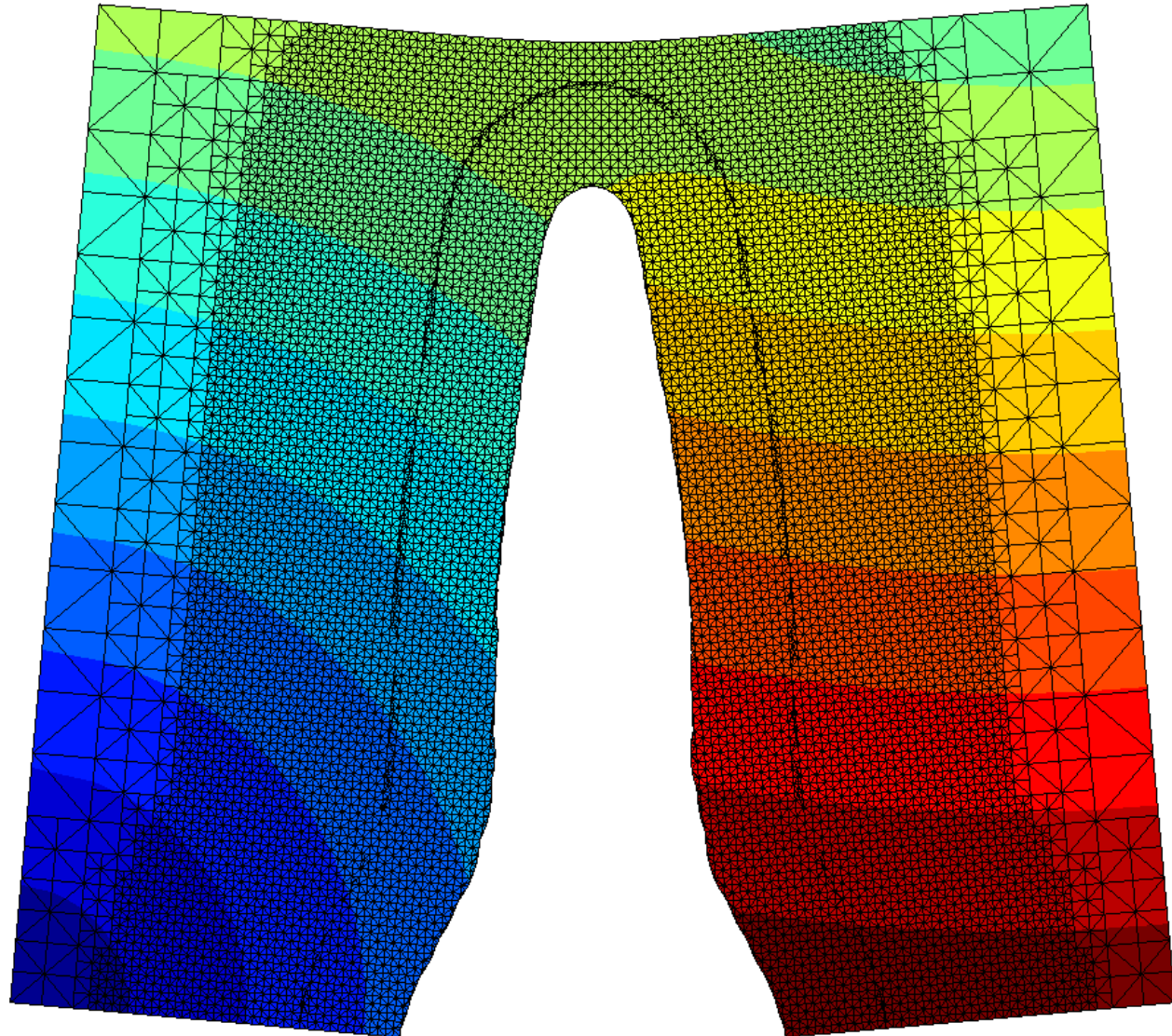
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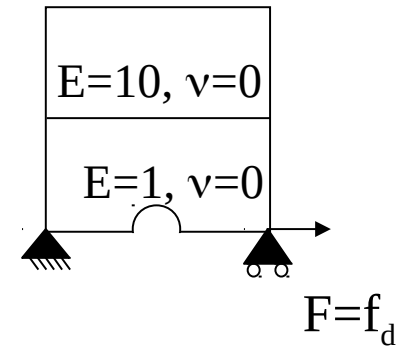
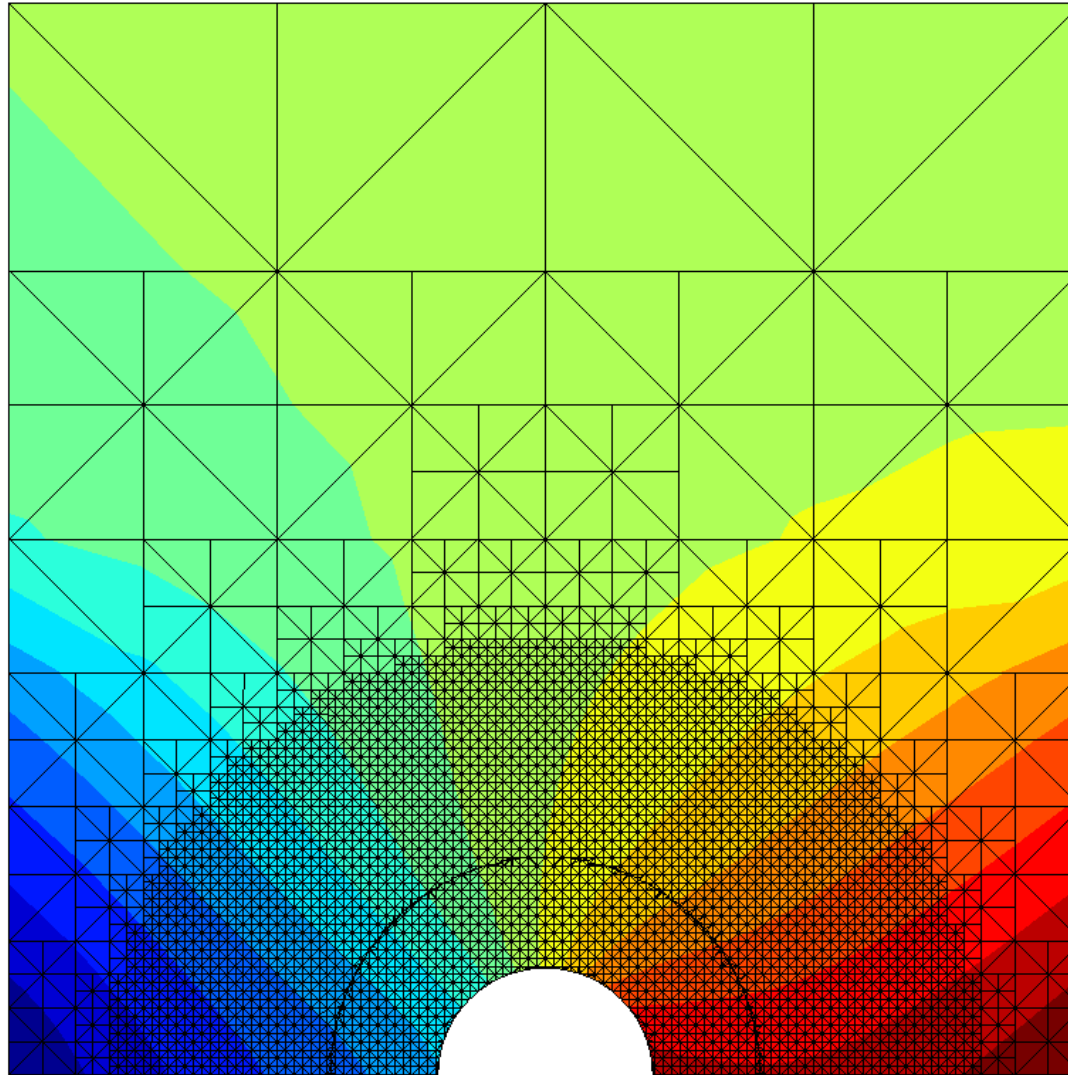
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Thick Level set
damage model

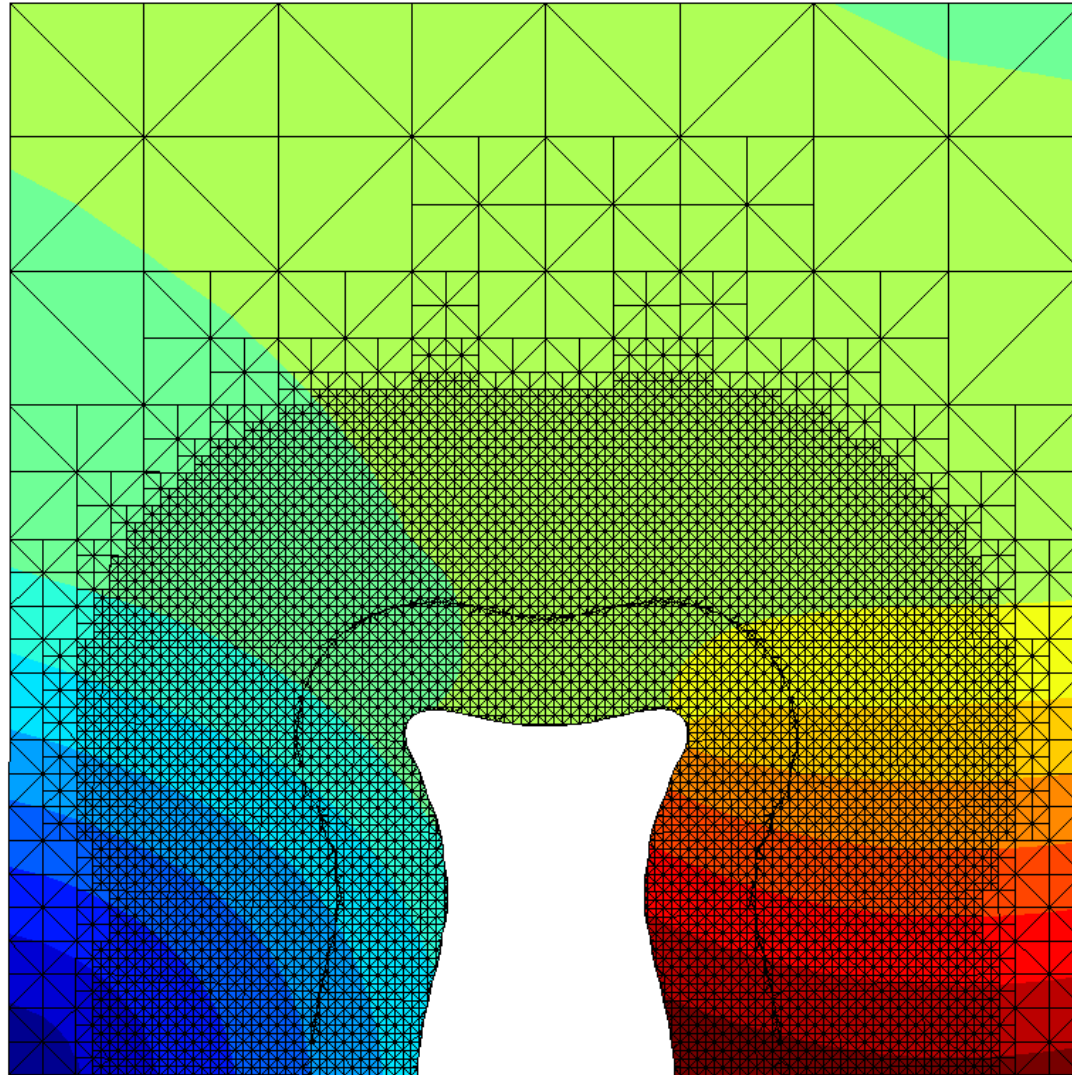


Branching benchmark

Outline and motivations

Sharp Level set
Front damage
model

Thick Level set
damage model



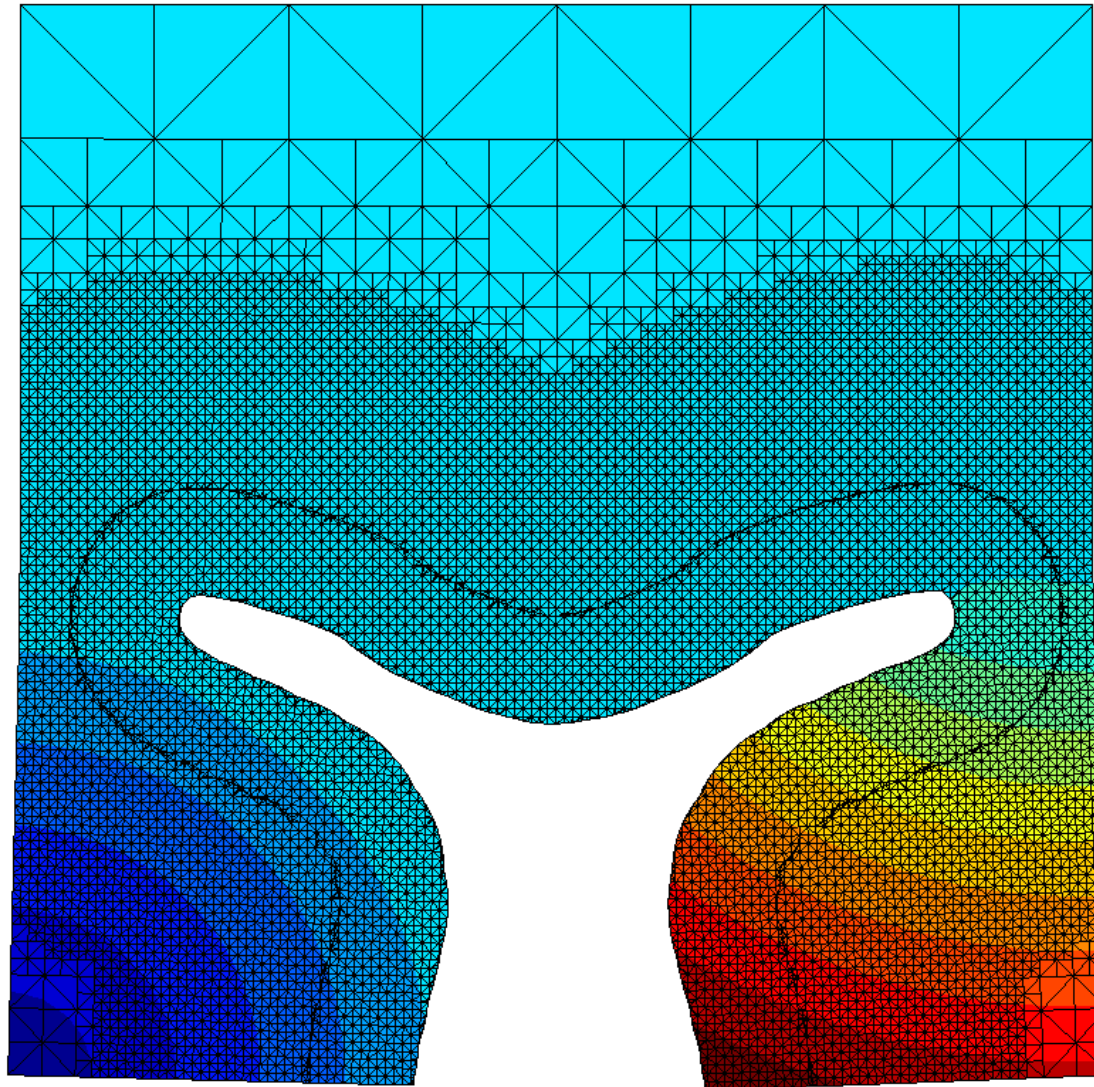
ISDMM09,
Trento, July 5-9,
2009

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Conclusion and future work.

Outline and motivations

Sharp Level set
Front damage model

Thick Level set
damage model

- We started the development of two related models that seems interesting as a middle ground between fracture mechanics and damage mechanics. More work is left to be done :

- Fully implement the implicit version to be able to cope with brittle damage problem with the two models, and better understand the stability properties.
- Compare solution with other non-local model on well defined benchmarks.