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A Regularized Brittle Damage Model Solved by a Level Set Technique

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Abstract. We consider an elastic brittle damage model in which a degradation front is propagating based on a criteria depending both on the energy along the front and the curvature of the front. This model was introduced about 20 years ago but to our knowledge was not yet exploited numerically. The contribution of this paper is to solve this model using a level set technique coupled to the eXtended Finite Element Method (X-FEM).

Key words: brutal damage, front curvature, level set, eXtended Finite Element Method (X-FEM).

1 Introduction

About 20 years ago a paper was published describing a brittle damage model in which a degradation front propagates in a media [1]. The damage is called brittle because a virgin zone (in which the damage is $d = 0$) is separated by a front from a totally damaged zone in which $d = 1$.

The model incorporates a Griffith type criteria. The front is growing provided the energy released is equal to a critical energy plus a term involving the front curvature. The presence of the curvature is essential for the model to avoid spurious localization.

To our knowledge this model has not been tackled numerically. We propose here an algorithm to solve it using a level set technique coupled to the eXtended Finite Element Method.

The fact that some areas in the domain are completely damaged and no longer provide any stiffnesses is taken care of by the eXtended Finite Element Method. Basically a zone with no stiffness behaves as a hole and is treated using the work presented in [2].

Finally, note that in the case of fatigue loading, a level set algorithm was already proposed in [3] to propagate a brittle damage front. In this paper the front speed is only curvature driven.

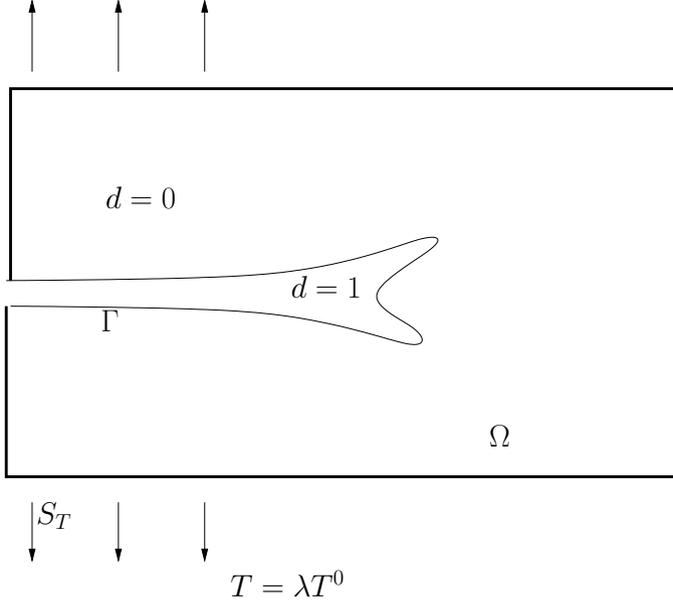


Fig. 1 Model problem showing a fully damaged zone progressing in an undamaged zone.

2 Description of the Mechanical Model

The model considered is the one introduced in [1]. We consider an elastic domain Ω submitted to imposed loads $\mathbf{T}_d(\mathbf{x}, \lambda)$ and displacements $\mathbf{u}_d(\mathbf{x}, \lambda)$ on the part of the boundary S_T and S_u , respectively. The parameter λ is the loading parameter and we assume that the imposed loads and displacements depend linearly on λ , i.e. $\mathbf{T}_d(\mathbf{x}, \lambda) = \lambda \mathbf{T}^0(\mathbf{x})$ and $\mathbf{u}_d(\mathbf{x}, \lambda) = \lambda \mathbf{u}^0(\mathbf{x})$. Note that a single loading parameter is considered. The extension of the level set update detailed below for multiple loading parameters needs more thinking.

The space of admissible displacement field is denoted as U and the space of admissible displacement field to zero is denoted as U_0 .

The complementary part of the boundary delimiting the completely damaged and virgin material is denoted Γ . The potential energy in the system is given by

$$E(\Gamma, \mathbf{u}, \lambda) = \int_{\Omega} e d\Omega - \int_{S_T} \lambda \mathbf{T}^0 \cdot \mathbf{u} dS$$

$$\text{with } e = \frac{1}{2} \boldsymbol{\epsilon}(\mathbf{u}) : \mathbf{E} : \boldsymbol{\epsilon}(\mathbf{u}) \text{ and } \mathbf{u} = \lambda \mathbf{u}^0 \text{ on } S_u \quad (1)$$

where \mathbf{E} is Hooke's tensor. The displacement field $\mathbf{u} \in U$ is obtained through the stationarity of the functional

$$\int_{\Omega} \boldsymbol{\epsilon}(\mathbf{u}) : \mathbf{E} : \boldsymbol{\epsilon}(\mathbf{u}^*) d\Omega = \int_{S_T} \lambda \mathbf{T}^0 \cdot \mathbf{u}^* dS \quad \forall \mathbf{u}^* \in U_0 \quad (2)$$

The solution of which depends on the current degradation front location and load factor:

$$\mathbf{u} = \mathbf{u}(\Gamma, \lambda) \quad (3)$$

The free energy of the system at equilibrium is denoted W

$$W(\Gamma, \lambda) = E(\Gamma, \mathbf{u}(\Gamma, \lambda), \lambda) \quad (4)$$

Assuming a normal velocity q_n modifying the location of the front, the free energy will be altered. Assuming the front is regular, the directional derivative of the free energy with respect to the velocity q_n is

$$DW[q_n] = - \int_{\Gamma} e q_n dS \quad (5)$$

The dual quantity to q_n on the front is the energy release rate e . The brittle propagation law is given by

- If $e < Y_c + \gamma_c/\rho$, the propagation is impossible.
- If $e = Y_c + \gamma_c/\rho$, the propagation is possible.

The above may be rewritten as

$$q_n \geq 0 \quad f = e - \left(Y_c + \frac{\gamma_c}{\rho} \right) \leq 0 \quad f q_n = 0 \quad (6)$$

These relations are very similar to the ones appearing in plasticity (taking q_n as the plastic strain rate and e as for instance the von Mises stress). The degradation front will move forward if the elastic energy on the front is superior to a critical energy Y_c plus a critical surface energy times the front curvature. The second term is essential to avoid spurious localization. The motivation to use the model is that it does not require a specific transition from damage to fracture.

Note that the presentation above is based on the paper by Nguyen et al. [1]. The only difference is that we have considered the curvature term to be part of the propagation law whereas in [1] it is part of the free energy through a surface energy term.

In order to give a geometrical interpretation to the dissipation process, consider a recently created damaged zone in grey in Figure 2. The free energy lost is given by

$$D_{12} = Y_c A + \gamma_c (L_2 - L_1) \quad (7)$$

To give more insight in the dissipation process, we consider in Figure 3 a velocity q_n on the front. This velocity will yield a change in length of the front, \dot{L} , a change in area of the damaged zone, \dot{A} , and a dissipation D :

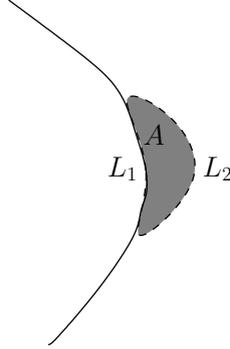


Fig. 2 The grey zone is the new damaged zone. The energy lost in the domain is given by $Y_c A + \gamma_c(L_2 - L_1)$ where A is the newly created area and $L_2 - L_1$ is the change in front length.

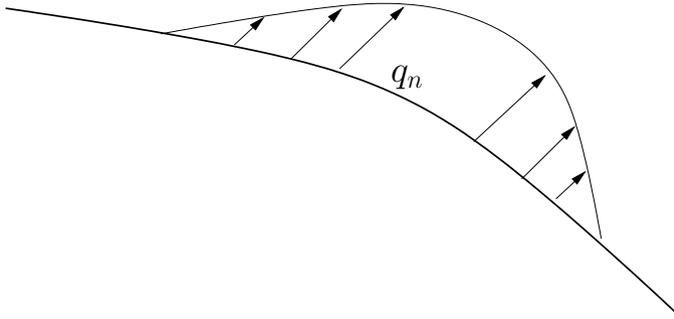


Fig. 3 A virtual velocity q_n on the front.

$$\dot{A} = \int_{\Gamma} q_n dS \quad (8)$$

$$\dot{L} = \int_{\Gamma} \frac{q_n}{\rho} dS \quad (9)$$

$$D = \int_{\Gamma} Y_c q_n + \gamma_c \frac{q_n}{\rho} dS \quad (10)$$

3 Description of the Algorithm

In order to solve the mechanical model, we store the current location of the damaged front by the iso-zero of a level denoted ls . We solve only the elasticity equations in the positive part of the level set. The mesh used does not necessarily have to conform

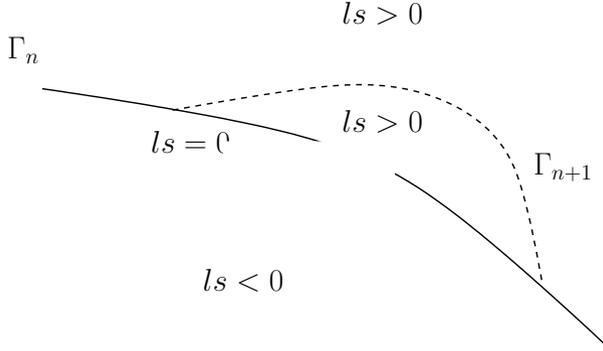


Fig. 4 The converged front (solid line) at time t_n . The damaged zone is located by the negative part of the level set. The front advance at time t_{n+1} is given by the dashed line. The dashed line must lay in the positive zone of the level set because the degradation front may not go backward in between one time step and the next one.

to the location of the damaged front because we use the extended finite element method to solve for the elasticity problem (see [2]). We consider in Figure 4 the location of the front Γ_n at time t_n , we look for the new location of the front Γ_{n+1} at time t_{n+1} and the corresponding loading factor λ_{n+1} . The new location of the front must fulfill the following relations

$$ls(\Gamma_{n+1}) \geq 0 \quad f_{n+1} = e_{n+1} - \left(Y_c + \frac{\gamma_c}{\rho_{n+1}} \right) \leq 0 \quad ls(\Gamma_{n+1})f_{n+1} = 0 \quad (11)$$

$$e_{n+1} = \lambda_{n+1}^2 \bar{e}_{n+1} \quad (12)$$

in which \bar{e}_{n+1} is the front energy for a unit loading parameter.

Note that we do not consider the loading factor as given since the dissipation evolves in the domain, this factor may not evolve monotonically to maintain a stable solution. We monitor the problem using dissipation. We use a dissipation control algorithm generalizing the so-called crack-length control introduced in [4]. The amount of dissipated energy in the system is indeed a monotonically increasing function.

In order to solve the above equations (11–12), we iterate from an initial guess Γ_0 , illustrated in Figure 5. The initial guess is obtained through a predictor step which will be described in a forthcoming paper. The correction step iterates on the front location until equations (11–12) are met. During the iterations, we make sure that the energy in the system is held fixed. In other words, it is the initial guess by its area and length changes which defines the energy lost over the time step $[t_n, t_{n+1}]$. At iteration i , the front velocity is computed on each point of the front using the following equations

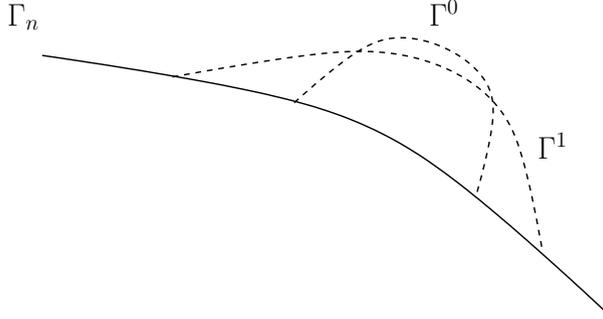


Fig. 5 The converged front (solid line) at time t_n . The initial guess Γ^0 to find the front at t_{n+1} as well as an illustration of the first iterate Γ^1 .

$$\text{If } l_S(\Gamma^i) > 0 \quad q^i = e^i - (Y_c + \frac{\gamma_c}{\rho^i}) \quad (13)$$

$$\text{If } l_S(\Gamma^i) = 0 \quad q^i = \langle e^i - (Y_c + \frac{\gamma_c}{\rho^i}) \rangle_+ \quad (14)$$

$$e^i = (\lambda^i)^2 \bar{e}^i \quad (15)$$

$$\int_{\Gamma^i} q^i (Y_c + \frac{\gamma_c}{\rho^i}) dS = 0 \quad (16)$$

The iterations stop when the velocity is zero everywhere on the front. In practice, we compute the L2 norm and prescribe some tolerance. Equation (14) ensures that the current front will never be inside the previous converged front at time t_n . Equation (16) ensures that the dissipation is zero through the iterations. Note that the system above provides also the corresponding loading factor.

The velocity on the front q^i is used to update the level set location using standard level set procedures.

4 Basic Numerical Experiments

In Figure 6 we consider a plate with a notch under tension (tractions are imposed). We consider only the growth over one time step. The initial guess is circular and we “optimize” its shape through iterations using the equations of the preceding section. The converged shape is shown in Figure 7. In this example, the ratio $l_c = \gamma_c/Y_c = 1$.

The second example, depicted in Figure 8 aims at modeling the branching of a transverse crack in a composite. The notch is located in the soft area. The initial guess is again circular. The final shape is given for $l_c = \gamma_c/Y_c = 1$ and for $l_c = \gamma_c/Y_c = 0.1$. In both cases, the damage zone tends to depart from a straight propagation. We also note that the smaller l_c yields a more abrupt departure.

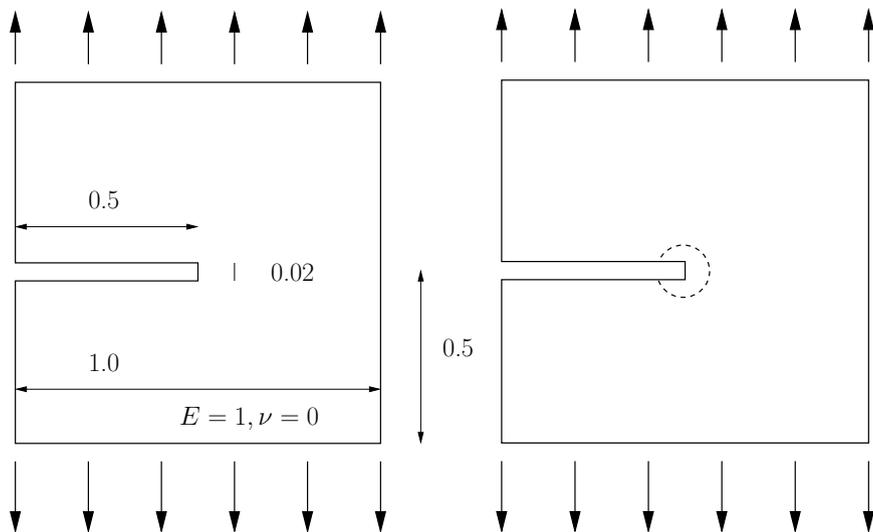


Fig. 6 A notched plate under tension (left) and the initial guess (dashed line on the right) for the degradation front.



Fig. 7 The initial guess for the damage front location (circle) and the converged front location.

5 Conclusions

This paper develops some preliminary work on the resolution of a brittle damage model using a level set formulation. A Griffith type model is considered and solved using an iterative strategy. It is coupled to the extended finite element method allowing one to manage the mesh quite independently of the degradation front location. The interesting feature of the model for the future is to incorporate complex cracking patterns (since a single level set is used to locate the damaged zone). Further studies are also needed to analyze the influence of the parameter l_c ratio between surface and volumetric cracking energies.

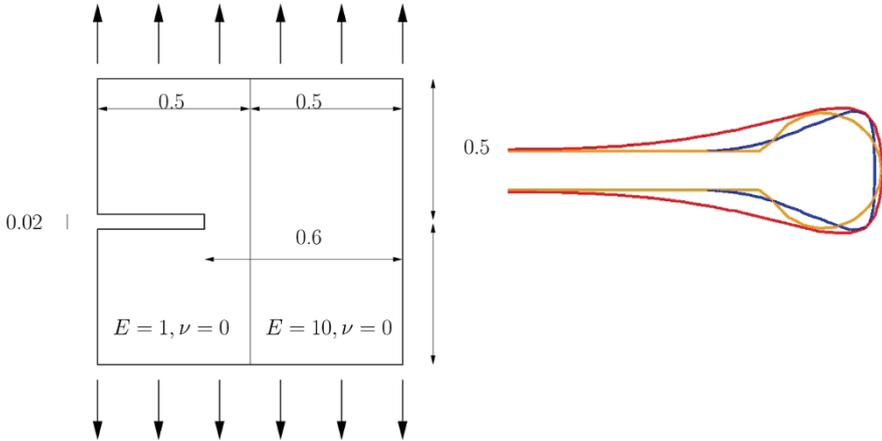


Fig. 8 A notched plate under tension composed of two materials (left). The initial guess is circular. The outer contour is the converged solution for $l_c = 1$ and the inner contour is the converged solution for $l_c = 0.1$ (right).

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