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# **A 2-PHASE FINITE ELEMENT MODEL TO STUDY CONCURRENT FLUID FLOW AND SOLID DEFORMATION OCCURRING IN MUSHY ZONES DURING THE SOLIDIFICATION OF METALLIC ALLOYS**

Michel Bellet<sup>1</sup>, Steven Le Corre<sup>2</sup>, Victor D. Fachinotti<sup>1</sup>

<sup>1</sup>Ecole des Mines de Paris, CEMEF, UMR CNRS 7635, Sophia Antipolis, France

<sup>2</sup>Ecole Centrale de Nantes, G M, Research Institute in Civil Engng and Mech., Nantes, France

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## **Abstract**

As an approach towards a better modelling of solidification problems, we present the basic assumptions and implementation of a thermo-mechanical two-phase model that considers the solidifying alloy as a binary mixture made of a liquid and a solid phase. Macroscopic mass and momentum balances are obtained considering that, at the microscopic level, the liquid is Newtonian whereas the solid is a power law fluid. Assuming local thermal equilibrium, a single equation for the conservation of the mixture energy is then written. The numerical implementation in a 2D finite element code is then detailed. Lastly, some examples of isothermal simulations of academic tests and application examples are discussed. They particularly enlighten the ability of the formulation to describe the mixture evolution over the whole solidification interval.

## **Introduction**

The macrosegregation of chemical species is a central problem in solidification engineering, because it strongly influences the further processibility of the cast products and the final properties of the parts produced. In most cases, macrosegregation is the result of slow interdendritic flow, driven by thermo-solutal convection. Generally, the influence of the solid motion on the fluid flow is assumed of minor importance and, as a matter of fact, most casting numerical simulations neglect this phenomenon.

However, as demonstrated by Flemings [1], in some cases, the macrosegregation of chemical species highly depends of the deformation of the solid skeleton forming in the mushy zone. This is especially the case in continuous casting (but not exclusively...) where there exists a large mushy zone, which is deformed together with the solid shell because of the bulging phenomenon between the supporting rolls of the caster. The proposed work can be seen as a first step to model and understand these complex phenomena.

The mushy zone is considered here as an effective two-phase continuum. On the one hand, the solid material is considered as an incompressible viscoplastic material, obeying a constitutive equation of power-law type. Invoking homogeneization results, its macroscopic flow rule is viscoplastic, including compressibility, so that the solid continuum can be seen as a deformable compressible porous medium. On the other hand, the liquid phase is intrinsically Newtonian. At the macroscopic scale, its momentum interaction with the solid skeleton is of Darcy type.

A two-dimensional numerical implementation of the corresponding mass, momentum and energy conservation equations has been carried out in a finite element software. The space and

time discretization of equations, as well as the resolution strategy, is presented. Finally, the validation of the formulation is achieved by comparison with an analytical solution and a simple application case is illustrated.

## 1 Macroscopic two-phase model

### General Macroscopic Balance Equations

At the microscopic scale, inside each phase, the thermo-mechanical evolution is assumed to be governed by the usual mass, momentum and energy balances. In this work, the balance equations of the mixture, at the scale of an elementary representative volume (macroscopic scale), are obtained using the spatial averaging method on a fixed control volume  $V_0$ . This method is classical and will not be detailed here; see for example references [2], [3], [4], [5] for further details on its basic principles. The notations used in this paper can be summarized as follows. Denoting  $\chi_k$  the characteristic function of phase  $k$  ( $=1$  in phase  $k$  and  $0$  elsewhere) for any function  $\psi$ , we define:

$$\text{The intrinsic average value : } \psi_k = \langle \psi \rangle^k = \frac{1}{V_k} \int_{V_0} \psi(\mathbf{x}) \chi_k(\mathbf{x}) dV \quad (1)$$

$$\text{The average value : } \langle \psi^k \rangle = \frac{1}{V_0} \int_{V_0} \psi(\mathbf{x}) \chi_k(\mathbf{x}) dV = \frac{V_k}{V_0} \langle \psi \rangle^k = g_k \psi_k$$

The solidifying alloy in the mushy state is considered as a *saturated* two-phase medium, that is to say that both phases volume fractions always satisfy the following relationship:

$$g_s + g_l = 1 \quad (2)$$

Applying the spatial averaging process to microscopic balance equations in each phase  $k$  ( $k = s, l$ ), and using the notations introduced previously, one obtains the following set of macroscopic equations.

$$\begin{aligned} \text{Mass} & \quad \frac{\partial}{\partial t} (g_k \rho_k) + \nabla \cdot (g_k \rho_k \mathbf{v}_k) = \Gamma_k \\ \text{Momentum} & \quad \nabla \cdot (g_k \boldsymbol{\sigma}_k) + \mathbf{M}_k + g_k \rho_k \mathbf{g} = \frac{\partial}{\partial t} (g_k \rho_k \mathbf{v}_k) + \nabla \cdot (g_k \rho_k \mathbf{v}_k \times \mathbf{v}_k) \\ \text{Energy} & \quad \frac{\partial}{\partial t} (g_k \rho_k h_k) + \nabla \cdot (g_k \rho_k h_k \mathbf{v}_k) + \nabla \cdot \langle \mathbf{q}^k \rangle = Q_k \end{aligned} \quad (3)$$

where  $\rho$  denotes the density,  $\mathbf{v}$  the velocity field,  $\boldsymbol{\sigma}$  the stress tensor,  $\mathbf{g}$  the gravity vector,  $h$  the enthalpy per unit of mass,  $\mathbf{q}$  the heat flow vector. The terms  $\Gamma$ ,  $\mathbf{M}$  and  $Q$  are associated with the exchanges of mass, momentum and energy, respectively, between the two phases.

### Mass conservation

The local mass balance at the interface between phases ensures that ensures that  $\Gamma_s + \Gamma_l = 0$  [5]. Then summing equations (3a) for the liquid and solid phase, and assuming that the densities of the two phases remain constant (but different), we get:

$$\rho_s \nabla \cdot (g_s \mathbf{v}_s) + \rho_l \nabla \cdot (g_l \mathbf{v}_l) = \frac{\partial g_s}{\partial t} (\rho_l - \rho_s) \quad (4)$$

or, dividing by  $\rho_l$ ,

$$(1 - \Delta \varepsilon^{tr}) \nabla \cdot (g_s \mathbf{v}_s) + \nabla \cdot (g_l \mathbf{v}_l) = \frac{\partial g_s}{\partial t} \Delta \varepsilon^{tr} \quad (5)$$

where  $\Delta \varepsilon^{tr} = (\rho_l - \rho_s) / \rho_l$  denotes the relative change of volume associated with solidification (negative in general).

### Momentum Conservation

The spatial averaging method used in this paper is efficient to obtain in a simple way the macroscopic governing equations of the semi-solid alloy but does not enable to go further in the specifications of the macroscopic model. Reliable constitutive equations would require more sophisticated approaches such as homogenization [6], [7] associated with numerical simulation at the microscopic scale, but this is not in the scope of this work. The full definition of the two-phase model will simply be based on further constitutive assumptions.

### **Macroscopic constitutive equation for the fluid phase**

At the microscopic scale, we assume that the liquid metal behaves as an incompressible Newtonian fluid. The incompressibility hypothesis is valid as long as the temperature range of the solidification interval remains narrow enough. We therefore can write:

$$\boldsymbol{\sigma} = s - p\mathbf{I} \quad \text{with} \quad s = 2\mu_l \dot{\boldsymbol{\epsilon}}(\mathbf{v}) \quad (6)$$

where  $p$  denotes the hydrostatic pressure,  $s$  the deviatoric part of  $\boldsymbol{\sigma}$ ,  $\mu_l$  the viscosity of the liquid, and  $\dot{\boldsymbol{\epsilon}}(\mathbf{v})$  the strain rate tensor. As suggested by Ganesan & Poirier [8] and Rappaz *et al.* [5], we adopt the following model for the macroscopic deviatoric stress tensor:

$$\boldsymbol{\Sigma}^l = \langle \mathbf{s}^l \rangle = 2\mu_l g_l \text{dev}(\dot{\boldsymbol{\epsilon}}(\mathbf{v}_l)) \quad (7)$$

where dev denotes the deviatoric part.

### **Macroscopic constitutive equation for the solid phase**

Experimental studies on the behavior of metallic alloys at high temperature show that the behavior of the solid phase is well described by constitutive equations of the Norton-Hoff type:

$$\boldsymbol{\sigma} = s - p\mathbf{I} \quad \text{with} \quad s = 2K(\sqrt{3}\dot{\boldsymbol{\epsilon}}_{eq})^{m-1} \dot{\boldsymbol{\epsilon}}(\mathbf{v}) \quad (8)$$

where  $K$  and  $m$  denote the consistency and the strain rate sensitivity and  $\dot{\boldsymbol{\epsilon}}_{eq}$  the von Mises equivalent strain rate. The behavior is then defined by the relation between the equivalent stress (in the von Mises sense) and the equivalent strain rate:

$$\sigma_{eq} = K(\sqrt{3})^{m+1} \dot{\boldsymbol{\epsilon}}_{eq}^m \quad (9)$$

For solid fractions above the coherency fraction, using the analysis of Geindreau and Auriault [7], we get that the effective stress tensor  $\boldsymbol{\Sigma}^s = \langle \mathbf{s}^s \rangle - \langle p_s \rangle \mathbf{I} + g_s p_l \mathbf{I}$  is a degree  $m$  homogeneous function with respect to the strain rate tensor  $\langle \dot{\boldsymbol{\epsilon}} \rangle^s = \dot{\boldsymbol{\epsilon}}(\mathbf{v}^s)$ . This property shows that the solid phase can be modelled as a compressible power law fluid. We therefore adopt a compressible viscoplastic formalism [9], [10]. Constitutive equations are the ones described by (9), but with equivalents defined as

$$\begin{aligned} \Sigma_{eq}^s &= \left[ A \text{dev}(\boldsymbol{\Sigma}^s) : \text{dev}(\boldsymbol{\Sigma}^s) + B \left( \text{tr} \langle \boldsymbol{\Sigma}^s \rangle \right)^2 \right]^{1/2} \\ \langle \dot{\boldsymbol{\epsilon}} \rangle_{eq}^s &= \frac{1}{A} \langle \dot{\boldsymbol{\epsilon}} \rangle^s : \langle \dot{\boldsymbol{\epsilon}} \rangle^s + \left( \frac{1}{9B} - \frac{1}{3A} \right) \left( \text{tr} \langle \dot{\boldsymbol{\epsilon}} \rangle^s \right)^2 \end{aligned} \quad (10)$$

Those equations require two rheological functions  $A$  and  $B$  that depend on the solid volume fraction and for which several models can be found [9], [10]. The constitutive equations of the solid phase at the macroscopic scale therefore read

$$\boldsymbol{\Sigma}^s = 3K(\sqrt{3}\langle \dot{\boldsymbol{\epsilon}} \rangle_{eq}^s)^{m-1} \left( \frac{1}{A} \langle \dot{\boldsymbol{\epsilon}} \rangle^s + \left( \frac{1}{9B} - \frac{1}{3A} \right) \text{tr} \langle \dot{\boldsymbol{\epsilon}} \rangle^s \mathbf{I} \right) \quad (11)$$

This model is simple and permits a first introduction of the compressibility of the solid skeleton in the mechanical behavior of a muhy zone. The interested reader can refer to the recent work of Ludwig et al. [11], who suggest to complement the previous model by introducing a state variable representing the cohesion of the skeleton. An evolution law of this variable is also proposed in order to represent the resistance of the inter-granular liquid films and solid bridges to tension. These modifications permit a better response of the model for small deformations, such as those encountered in continuous casting.

For lower solid fractions, the solid phase will be supposed to be stress free.

### ***Exchange of Momentum***

According to the work of Ni & Beckermann [3],  $\mathbf{M}_k$  can be partitioned as:

$$\mathbf{M}_k = \mathbf{M}_k^d + \mathbf{M}_k^p \quad (12)$$

the first part being the contribution of deviatoric stresses and the second one the contribution of the isotropic part, and it can easily be shown that

$$\mathbf{M}_l^d + \mathbf{M}_s^d = 0 \quad \text{and} \quad \mathbf{M}_l^p + \mathbf{M}_s^p = 0 \quad (13)$$

The liquid being a Newtonian incompressible fluid with a very low viscosity, we will assume that the pressure equilibrium in the liquid phase is almost instantaneous. Subsequently, the interfacial pressures in both phases ( $p_k^*$ ) equal the intrinsic average value of liquid pressure, that is its microscopic value:

$$p_l^* = p_s^* = p_l \quad (14)$$

$\mathbf{M}_k^p$  can therefore be expressed as follows [3]:

$$\mathbf{M}_s^p = -\mathbf{M}_l^p = -p_l \nabla g_l = p_l \nabla g_s \quad (15)$$

Depending on the solid fraction, the dissipative terms  $\mathbf{M}_k^d$  are generally interpreted either as the drag force exerted by the liquid on the isolated solid grains [3] or the filtration force exerted by the liquid flowing through the solid, considered as a rigid porous medium [7], [5]. In both cases,  $\mathbf{M}_k^d$  can be modelled by a law of the generic type:

$$\mathbf{M}_s^d = -\mathbf{M}_l^d = g_l C(g_l) (\mathbf{v}_l - \mathbf{v}_s) \quad (16)$$

where the factor  $C$  may be defined through the usual permeability factor  $K$  as:

$$C = \frac{g_l \mu_l}{K} \quad (17)$$

### **Energy Conservation**

At the microscopic level, the Fourier law is used. Given  $\lambda$  the thermal conductivity, we have:

$$\mathbf{q} = -\lambda \nabla T \quad (18)$$

Our approach is based on the local thermal equilibrium assumption, which is valid because the liquid and the solid do not exhibit too different thermal properties nor strong thermal interfacial barriers. Therefore, at the macroscopic scale, their average temperature is the same:

$$T_s = T_l = T \quad (19)$$

The enthalpies of both phases can then be written as a function of the temperature  $T$ . Assuming for simplicity a constant specific heat  $c_p$ , and a constant latent heat per unit of mass  $L$ , we have:

$$h_s = \int_{T_0}^T c_p(\tau) d\tau \quad \text{and} \quad h_l = h_s + L \quad (20)$$

Because of the thermal equilibrium hypothesis, the energy conservation is written as a single equation which is the sum of the energy balance on both phases:

$$\frac{\partial}{\partial t} \langle \rho h \rangle + \nabla \cdot \langle \rho h \mathbf{v} \rangle + \nabla \cdot \langle \mathbf{q} \rangle = 0 \quad (21)$$

where

$$\begin{aligned} \langle \rho h \rangle &= \langle (\rho h)^l \rangle + \langle (\rho h)^s \rangle = g_l \rho_l h_l + g_s \rho_s h_s \\ \langle \rho h \mathbf{v} \rangle &= \langle (\rho h \mathbf{v})^l \rangle + \langle (\rho h \mathbf{v})^s \rangle = g_l \rho_l h_l \mathbf{v}_l + g_s \rho_s h_s \mathbf{v}_s \\ \langle \mathbf{q} \rangle &= \langle \mathbf{q}_s \rangle + \langle \mathbf{q}_l \rangle = -(g_s \lambda_s + g_l \lambda_l) \nabla T = -\langle \lambda \rangle \nabla T \end{aligned} \quad (22)$$

After elementary calculations, we obtain:

$$\langle \rho \rangle \frac{\partial}{\partial t} \langle h \rangle + c_p \nabla T \cdot \langle \rho \mathbf{v} \rangle + L \left( \rho_l \nabla \cdot \langle \mathbf{v}^l \rangle - g_l \nabla \cdot \langle \rho \mathbf{v} \rangle \right) - \nabla \cdot (\langle \lambda \rangle \nabla T) = 0 \quad (23)$$

### Final Equation Set Adopted

The set of equations (24) sums up the adopted equations set for this two-phase formulation.

$$\begin{aligned} \text{Liquid/Momentum} \quad \nabla \cdot \boldsymbol{\Sigma}^l - g_l \nabla p_l + \mathbf{M}_l^d + g_l \rho_l \mathbf{g} &= \rho_l \frac{\partial}{\partial t} (g_l \mathbf{v}_l) + \rho_l \nabla \cdot (g_l \mathbf{v}_l \times \mathbf{v}_l) \\ \text{Solid/Momentum} \quad \nabla \cdot \boldsymbol{\Sigma}^s - g_s \nabla p_l - \mathbf{M}_l^d + g_s \rho_s \mathbf{g} &= \rho_s \frac{\partial}{\partial t} (g_s \mathbf{v}_s) + \rho_s \nabla \cdot (g_s \mathbf{v}_s \times \mathbf{v}_s) \\ \text{Mixture/Mass} \quad (1 - \Delta \varepsilon^{tr}) \nabla \cdot (g_s \mathbf{v}_s) + \nabla \cdot (g_l \mathbf{v}_l) &= \frac{\partial g_s}{\partial t} \Delta \varepsilon^{tr} \\ \text{Mixture/Energy} \quad \langle \rho \rangle \frac{\partial \langle h \rangle}{\partial t} + c_p \nabla T \cdot \langle \rho \mathbf{v} \rangle + L (\rho_l \nabla \cdot \langle \mathbf{v}^l \rangle - g_l \nabla \cdot \langle \rho \mathbf{v} \rangle) - \nabla \cdot (\langle \lambda \rangle \nabla T) &= 0 \end{aligned} \quad (24)$$

## 2 Numerical Implementation

This formulation was implemented in the two-dimensional code R2SOL, finite element code using linear triangles and based on the P1+/P1 mixed formulation [12], [13]. Up to now, the two-phase approach has been validated for isothermal situations only, so only the implementation of the isothermal mechanical problem will be discussed here.

In what follows,  $\mathbf{v}_s$  will be denoted  $\mathbf{u}$ ,  $\mathbf{v}_l$  will be denoted  $\mathbf{v}$ , and  $p_l$  will be denoted  $p$ . The boundary conditions of the mechanical problem are:

$$\begin{aligned} \mathbf{u} = \mathbf{v} = \mathbf{V}_{imp} \quad \text{on } \partial \Omega_u \\ \langle \boldsymbol{\sigma}^s \rangle \mathbf{n} = (\boldsymbol{\Sigma}^s - g_s p \mathbf{I}) \mathbf{n} = \mathbf{T}_{imp}, \quad \langle \boldsymbol{\sigma}^l \rangle \mathbf{n} = \mathbf{T}_{imp} \quad \text{on } \partial \Omega_s \end{aligned} \quad (25)$$

If  $V$  is the space of “kinematically admissible” velocity fields and  $V_0$  is the space of “zero kinematically admissible” velocity fields. The virtual power principle states that the solution of the problem  $(\mathbf{u}, \mathbf{v}, p) \in V \times L^2(\Omega)$  must fulfil:

$$\forall (\mathbf{u}^*, \mathbf{v}^*, p^*) \in V_0 \times L^2(\Omega)$$

$$\begin{aligned} \int_{\Omega} \boldsymbol{\Sigma}^s : \nabla \mathbf{u}^* - \int_{\Omega} p \nabla \cdot (g_s \mathbf{u}^*) - \int_{\partial\Omega} \langle \boldsymbol{\sigma}^s \rangle \mathbf{n} \cdot \mathbf{u}^* - \int_{\Omega} g_l^2 \mu_l \mathbf{K}^{-1} (\mathbf{v} - \mathbf{u}) \cdot \mathbf{u}^* - \int_{\Omega} g_s \rho_s \mathbf{g} \cdot \mathbf{u}^* \\ + \int_{\Omega} \rho_s \left( \frac{\partial}{\partial t} (g_s \mathbf{v}_s) + \nabla \cdot (g_s \mathbf{v}_s \times \mathbf{v}_s) \right) \cdot \mathbf{u}^* = 0 \\ \int_{\Omega} \boldsymbol{\Sigma}^l : \nabla \mathbf{v}^* - \int_{\Omega} p \nabla \cdot (g_l \mathbf{v}^*) - \int_{\partial\Omega} \langle \boldsymbol{\sigma}^l \rangle \mathbf{n} \cdot \mathbf{v}^* + \int_{\Omega} g_l^2 \mu_l \mathbf{K}^{-1} (\mathbf{v} - \mathbf{u}) \cdot \mathbf{v}^* - \int_{\Omega} g_l \rho_l \mathbf{g} \cdot \mathbf{v}^* \\ + \int_{\Omega} \rho_l \left( \frac{\partial}{\partial t} (g_l \mathbf{v}_l) + \nabla \cdot (g_l \mathbf{v}_l \times \mathbf{v}_l) \right) \cdot \mathbf{v}^* = 0 \end{aligned} \quad (26)$$

$$\int_{\Omega} p^* \left( (1 - \Delta \varepsilon^{tr}) \nabla \cdot (g_s \mathbf{v}_s) + \nabla \cdot (g_l \mathbf{v}_l) - \frac{\partial g_s}{\partial t} \Delta \varepsilon^{tr} \right) = 0$$

It is a mixed velocity-pressure formulation involving two velocity fields that requires interpolation functions satisfying the Brezzi-Babuska conditions. Using the previous formulation developed in the one-phase case, we adopt a “(P1+)<sup>2</sup>/P1” formulation. On each finite element, the velocity fields  $\mathbf{u}$  and  $\mathbf{v}$  are approximated as follows:

$$\begin{aligned} \mathbf{u} \approx \mathbf{u}_h + \mathbf{a} &= \sum_{i=1}^3 N^i \mathbf{U}^i + N^b \mathbf{A} \\ \mathbf{v} \approx \mathbf{v}_h + \mathbf{b} &= \sum_{i=1}^3 N^i \mathbf{V}^i + N^b \mathbf{B} \end{aligned} \quad (27)$$

$\mathbf{A}$  and  $\mathbf{B}$  are the additional bubble degrees of freedom located at the center of each element. The bubble interpolation function  $N^b$  is linear on each sub-triangle, and is constructed such as  $N^b=0$  on the edges of the element and  $N^b=1$  at the center of the triangle. The interstitial pressure  $p$  is approximated by a classical linear interpolation. Test functions  $\mathbf{u}^*$ ,  $\mathbf{v}^*$  and  $p^*$  are approximated in the same way. Thanks to the particular shape of the bubble functions, the additional degrees of freedom  $\mathbf{A}$  and  $\mathbf{B}$  can be eliminated from the discrete system at the elements level. This is obtained by the same process as the one described in [12] and [13] for a one-phase problem, except that the size of the local system to solve is double, due to the two velocity fields. The resulting non linear discretized system is then solved by the means of a Newton-Raphson iterative method.

### 3 Application tests

#### Validation: comparison with an exact solution

The two-phase mechanical solver was first validated with respect to an analytical solution. The latter was calculated from homogeneous simple compression problem depicted in Figure 1. In this test, both the liquid and the solid phases behavior are assumed linear and compressible. For calculation convenience, we take:

$$\boldsymbol{\Sigma}^s = K_s \left( \alpha \text{tr} \langle \dot{\boldsymbol{\varepsilon}}^s \rangle \mathbf{I} + \beta \langle \dot{\boldsymbol{\varepsilon}}^s \rangle \right) \quad (28)$$

$$\boldsymbol{\Sigma}^l = K_l \left( a \text{tr} \langle \dot{\boldsymbol{\varepsilon}}^l \rangle \mathbf{I} + b \langle \dot{\boldsymbol{\varepsilon}}^l \rangle \right) \quad (29)$$

$g_l$  is assumed constant over the whole domain. This produces a simple compression like kinematics for both phases with an imposed strain rate  $\dot{\boldsymbol{\varepsilon}} = V_{imp} / h$ . The normal stress on each

phase is supposed to be null on the right face. The resolution leads to the following expressions for the phases velocity and pressure fields:

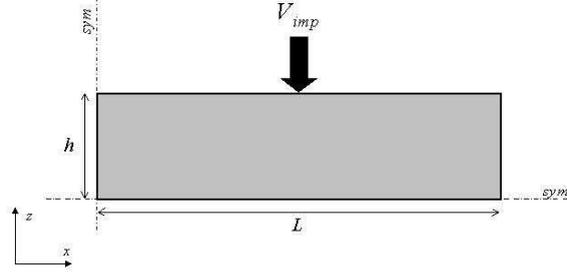


Figure 1. Two-phase simple compression test geometry.

$$\begin{cases} v_{s,x}(x) = -\dot{\epsilon}x + a_0 \sinh(rx) \\ \langle v^l \rangle_x(x) = -\dot{\epsilon}x - g_s v_{s,x}(x) \\ p(x) = p_0 + (K_s(\alpha + \beta) - g_s K_l(a + b)) \frac{dv_{s,x}(x)}{dx} \end{cases} \quad (30)$$

with

$$\begin{cases} r = \sqrt{\frac{C}{\tilde{\mu}}} \\ \tilde{\mu} = g_l K_s(\alpha + \beta) + g_s^2 K_l(a + b) \\ a_0 = \frac{g_l [K_s \beta - g_s K_l b]}{\tilde{\mu} r \cosh(rL)} \dot{\epsilon} \\ p_0 = (K_s \alpha - K_l(g_s a + b)) \dot{\epsilon} \end{cases} \quad (31)$$

Results exposed in Figure 2 correspond to a test performed at  $V_{imp} = -1 \text{ m.s}^{-1}$  on a sample with  $L = 5 \text{ m}$ ,  $h = 1 \text{ m}$ , and:  $g_s = 0.8$ ,  $K_s = 100 \text{ Pa.s}$ ,  $K_l = 0.1 \text{ Pa.s}$ ,  $\alpha = 0.189$ ,  $\beta = 0.1$ ,  $a = -0.667$ ,  $b = 2$ ,  $C = 20$ .

As visible from Figure 2, numerical results perfectly match the analytical solution both in terms of relative velocity  $v_l - v_s$  and of interstitial pressure. Such results were obtained for several sets of rheological parameters and remain valid for any value of the interaction coefficient  $C$ .

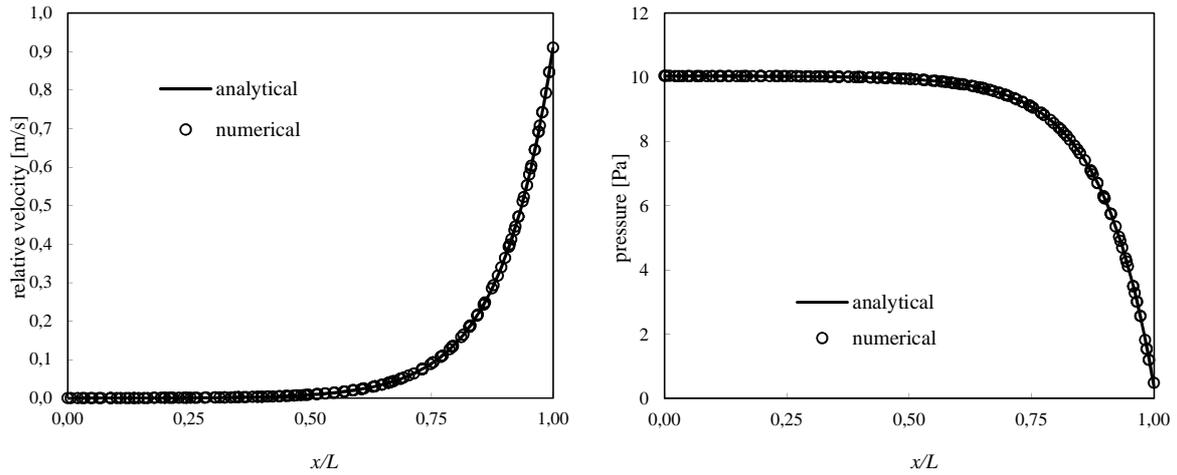


Figure 2. Relative velocity and interstitial pressure along the  $x$  direction, numerical vs analytical, 1D test case.

### Application to Liquid Segregation

This test aims at simulating a situation frequently encountered in continuous casting problems. It consists in imposing a deformation at the surface of a partially solidified alloy on a small part of its surface (Figure 3). It thus simulates the action of a roll. Symmetry conditions are imposed on all the boundaries except on the upper face, which is a free surface. We perform here again an isothermal calculation, but imposing an initial distribution of liquid fraction along  $y$  described in Figure 3. Here, the solid phase rheology is still somewhat arbitrary, but it is now non-linear, with a strain rate sensitivity of 0.2. The interaction coefficient is modelled by a classical Carman-Kozeny law and reaches very high value in the solidified zone.

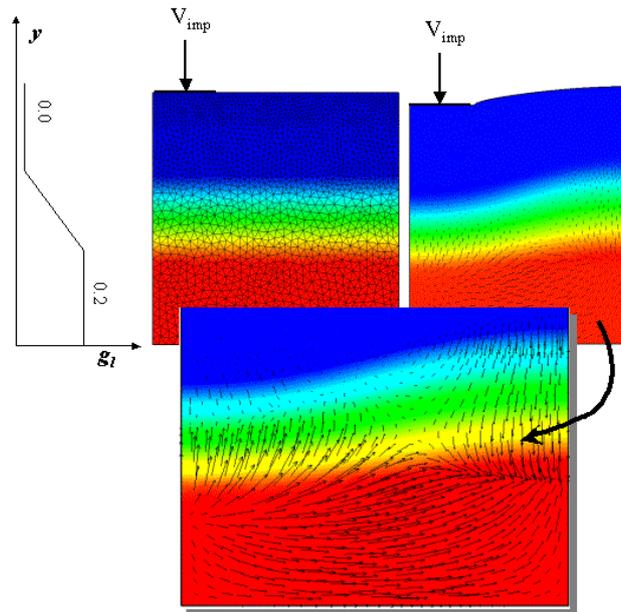


Figure 3. Redistribution of the liquid inside a semi-solid domain with an initial fraction distribution.

Black arrows plotted in the zoom of Figure 3 show the relative average velocity vectors  $\langle \mathbf{v}^l \rangle - g_l \mathbf{v}_s$ . Results show that the applied pressure leads to a redistribution of the liquid with respect to the solid. In the zone where the solid is under pressure (left side), it undergoes densification and the liquid is rejected to the right side of the sample.

## Conclusion

In this paper, the macroscopic conservation equations for a two-phase continuum have been briefly discussed and summarized. A two-dimensional finite element resolution has been proposed and validated on an analytical test. The ability of the formulation to represent the deformation of a saturated solid skeleton, using a “sponge”-like model has been demonstrated. In the future, the model will be extended to solidification, involving mass exchange between the two phases and transport of chemical species, which is of great importance in some casting processes.

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## References

1. Flemings M.C., Our understanding of macrosegregation: past and present, *ISIJ International* 40, 833-841 (2000)
2. Hassanizadeh M., and Gray W.G., General conservation equations for multi-phase systems: 1. Averaging procedure, *Adv. in Water Resources*, vol. 2, 131-144 (1979)
3. Ni J., and Beckermann C., A volume-averaged two-phase model for transport phenomena during solidification, *Metall. Trans. B*, vol. 22, 349-361 (1991)
4. Wang C.Y., Beckermann C., Equiaxed dendritic solidification with convection: Part 1. Multiscale/multiphase modeling, *Metall. and Mat. Trans. A* 27, 2754-2764 (1996)
5. Rappaz M., and Bellet M., and Deville M., *Numerical modelling in materials science and engineering*, New-York, Springer Verlag (2003)
6. J.-L. Auriault, E. Sanchez-Palencia, Etude du comportement macroscopique d’un milieu poreux saturé déformable, *J. de Mécanique* 16, 575-603 (1977)
7. Geindreau C., Auriault J.-L., Investigation of the viscoplastic behaviour of alloys in the semi-solid state by homogenization, *Mechanics of Materials* 31, 535-551 (1999)
8. Ganesan S. and Poirier D.R., Conservation of mass and momentum for the flow of interdendritic liquid during solidification, *Metall. Trans. B* 21, 173-181 (1990)
9. Nguyen T. G., Favier D., Suery M., Theoretical and experimental study of the isothermal mechanical behaviour of alloys in the semi-solid state, *Int. J. Plasticity* 10, 663-693 (1994)
10. Abouaf M., Chenot J.-L., Raïsson G., Bauduin P., Finite element simulation of hot isostatic pressing of metal powders, *Int. J. Numer. Methods Engrg* 25, 191-212 (1988)
11. Ludwig O., Commet B., Drezet J.M., Martin C.L., Suéry M., Rheological behavior of partially solidified Al-Cu alloys : experimental and numerical study, *Proc. MCWASP X, 10<sup>th</sup> Int. Conf. on Modeling of Casting, Welding and Advanced Solidification Processes*, Destin (Florida, USA), May 25-30, 2003, D.M. Stefanescu, J.A. Warren, M.R. Jolly & M.J.M. Krane (eds.), The Minerals, Metals & Materials Society, Warrendale, Pennsylvania, USA, 183-190, 2003.
12. Perchat E., Fourment L., Coupez T., *Mixed formulation and iterative solver for the parallelisation of a forging simulation software by mesh partitioning*, 3<sup>rd</sup> Euro Conf. On Parallel and Distributed Computing for Computational Mechanics, Weimar, Germany, B.H.V. Topping (ed.), *Developments in Computational Mechanics with High Performance Computing*, Civil-Comp Press, Edimburgh, 67-72 (1999)
13. A. Heinrich, Modélisation thermomécanique de la coulée continue d’acier en deux dimensions (Two-dimensional thermomechanical modelling of steel continuous casting), Thèse de Doctorat en Sciences et Génie des Matériaux (in french), Ecole des Mines de Paris (2003)