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CONTINUUM DAMAGE AND SCALING OF FRACTURE

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ABSTRACT: This paper presents a simple application of continuum damage modelling to failure analysis. An isotropic damage model is described and its non local enhancement discussed. The calibration of the model parameters is addressed with the help of structural size effect data. Such a size effect is a feature of non local models and is also observed experimentally. We show that this scaling law of fracture can be a convenient tool for the purpose of model calibration, and in particular for the determination of the internal length which is introduced in non local models.

1. Introduction

Quasi-brittle materials such as concrete exhibit a non linear stress-strain response due to micro-cracking which may be modelled with continuum damage models. A consequence of such a progressive degradation of the material is strain softening. It is today well established that strain softening induces bifurcation and strain localisation. More importantly, strain softening produces failure without energy dissipation [1]. This is certainly not realistic and modifications of the constitutive relations are among the possible remedies. In this paper, we address these problems with the help of a simple isotropic damage model. After a brief description of the constitutive relations, the non local extension of the damage models is discussed. This modification of the constitutive relations is based on the introduction of an internal length in the material model. It induces a size effect which can serve as a way of calibrating the constitutive parameters.

2. Non local Continuum Damage

Let us consider, for the sake of simplicity, the case of tension dominated mechanical actions and restrict attention to the simplest possible damage model. In the reversible (elastic) domain, the stress-strain relation reads [5]:

$$\varepsilon_{ij} = \frac{1 + \nu_0}{E_0(1-d)} \sigma_{ij} - \frac{\nu_0}{E_0(1-d)} [\sigma_{kk} \delta_{ij}], \quad (1)$$

or
$$\sigma_{ij} = (1 - d)C_{ijkl}^0 \varepsilon_{kl}, \quad (2)$$

where σ_{ij} is the stress component, ε_{kl} is the strain component, E_0 and ν_0 are the Young's modulus and Poisson's ratio of the undamaged isotropic material, and δ_{ij} is the Kronecker symbol. d is the damage variable which varies between 0 and 1. C_{ijkl}^0 is the stiffness of the undamaged material. We shall examine non local (integral and gradient) solutions.

2.1 INTEGRAL CONSTITUTIVE RELATIONS

Consider for instance the case in which damage is a function of the positive strains (which means that it is mainly due to micro cracks opening in mode I). The stress strain relations are those given above and damage is function of the equivalent strain defined by Mazars [6]:

$$\bar{f} = \sqrt{\sum_{i=1}^3 \left(\langle \varepsilon_i \rangle_+ \right)^2}, \quad (3)$$

where ε_i are the principal strains. In classical damage models, the loading function f reads $f(\bar{f}, \chi) = \bar{f} - \chi$. The principle of non local continuum models with local strains is to replace \bar{f} with its average [10]:

$$\bar{\varepsilon}(x) = \frac{1}{V_r(x)} \int_{\Omega} \psi(x-s) \bar{f}(s) ds \quad \text{with} \quad V_r(x) = \int_{\Omega} \psi(x-s) ds, \quad (4)$$

where Ω is the volume of the structure, $V_r(x)$ is the representative volume at point x , and $\psi(x-s)$ is the weight function, for instance:

$$\psi(x-s) = \exp\left(-\frac{4\|x-s\|^2}{l_c^2}\right). \quad (5)$$

l_c is the internal length of the non local continuum. $\bar{\varepsilon}$ replaces the equivalent strain in the evolution of damage which is not detailed in this paper and can be found in Ref. [6]. The loading function becomes also $f(\bar{\varepsilon}, \chi) = \bar{\varepsilon} - \chi$, where χ is the hardening – softening variable.

It should be noticed that this model is easy to implement in the context of explicit, total strain models. Its implementation to plasticity and to implicit incremental relations is awkward. The local tangent stiffness operator relating incremental strains to incremental stresses becomes non symmetric, and its bandwidth can be very large due to non local interactions [9]. This is one of the reasons why gradient damage models, such as those described in the next section, have become popular over the past few years.

The internal length is an additional parameter, which is difficult to obtain directly by experiments. In fact, whenever the strains in specimen are homogeneous, the local damage model and the non local damage model are, by definition, strictly equivalent ($\bar{\varepsilon} = \mathfrak{f}$). An approximation of the internal length was obtained by Bazant and Pijaudier-Cabot [2]. For standard concrete, the internal length lies between $3 d_a$ and $5 d_a$, where d_a is the maximum aggregate size of concrete

2.2 ISOTROPIC GRADIENT DAMAGE MODEL

A simple method to transform the above non local model to a gradient model is to expand the effective strain into Taylor series truncated for instance to the second order:

$$\mathfrak{f}(x + s) = \mathfrak{f}(x) + \frac{\partial \mathfrak{f}(x)}{\partial x} s + \frac{\partial^2 \mathfrak{f}(x)}{\partial x^2} \frac{s^2}{2!} + \dots \quad (6)$$

Substitution in Eq. (4) and integration with respect to variable s yields:

$$\bar{\varepsilon}(x) = \mathfrak{f}(x) + c^2 \nabla^2 \mathfrak{f}(x). \quad (7)$$

c is a parameter which depends on the type of weight function in Eq. (4). Its dimension is L^2 and it can be regarded as the square of an internal length. Substitution of the new expression of the non local effective strain in the non local damage model presented above yields a gradient damage model. Computationally, this model is still delicate to implement because it requires higher continuity in the interpolation of the displacement field. This difficulty can be solved with a simple calculation devised by Peerlings et al. [8]: let us take the Laplacian of the right and left hand-sides of Eq. (4). Because the Taylor expansion in Eq. (6) was truncated to the second order, higher derivatives can be assumed to be negligible. It follows that:

$$\nabla^2 \bar{\varepsilon}(x) = \nabla^2 \mathfrak{f}(x) + \nabla^2 (c^2 \nabla^2 \mathfrak{f}(x)) \approx \nabla^2 \mathfrak{f}(x). \quad (8)$$

Therefore, Eq. (7) becomes $\bar{\varepsilon}(x) - c^2 \nabla^2 \bar{\varepsilon}(x) = \mathfrak{f}(x)$. This implicit equation, which defines the non local effective strain as a function of the local effective strain, is the exact approximation of the integral relation in Eq. (4). The implementation of the gradient damage model becomes similar to the implementation of a thermo-mechanical (local) model in which the non local effective strain replaces the nodal temperatures.

3. Calibration of the mechanical model

The calibration of the model parameters in the integral or gradient damage model is facing the difficulty of getting at the same time the parameters involved in the evolution of damage and the internal length. Theoretically, it could be possible to calibrate the evolution of damage on tests in which the strain distribution remains homogeneous over the specimen. This is in fact impossible because whenever strain softening occurs, the strain and damage distribution happen to become strongly non homogeneous due to

bifurcation and strain localisation during the failure process. It follows that without some simplifying assumptions such as the ones devised by Bazant and Pijaudier-Cabot [2], it is not possible to calibrate the model parameters without considering the computation of a complete boundary value problem up to failure. In the context of non local models, analytical solutions can hardly be derived, and it is the reason why a robust finite element implementation is a crucial stage in the exploitation of non local failure models.

Another characteristic of non local models is size effect. Because the constitutive relations contain a length scale, the response of geometrically similar specimens is not geometrically similar. At least the peak loads for different sizes do not scale with the geometrical similarity factor. In fact, the internal length scale controls the size of the fracture process zone (FPZ), in which micro-cracking occurs during failure. Hence, the size of the FPZ is a model characteristics, which is not dependent on the structure size. It follows that the ratio of the FPZ size to the structural size varies when geometrically similar specimens are tested. This variation produces a size effect, which may be modelled with simple equations proposed by Bazant [3]:

Let us consider the case of three-point bending of geometrically similar notched beams. We tested geometrically similar specimens of various height $D = 80, 160,$ and 320 mm, of length $L = 4D$, and of thickness $b = 40$ mm kept constant for all the specimens. The length-to-height ratio is $L/D = 4$ and the span-to-height ratio is $l/D = 3$. These are mortar specimens, cast with the side of height in the vertical position, using a water-cement ratio of 0.4 and a cement-sand ratio 0.46 (all by weight). The maximal sand grain size is $d_a = 3$ mm. The maximal strength is obtained with the formula:

$$\sigma = \frac{3}{2} \frac{Fl}{bD^2}, \quad (9)$$

where b is the thickness of the beam, D is the height (mm), l the span (mm), and F the maximal load (N). F is obtained for each different size. A simple size effect formula for concrete and others materials is used:

$$\sigma = \frac{Bf_t^{\textcircled{c}}}{\sqrt{1 + D/D_0}}. \quad (10)$$

σ is the maximal strength, D is the size of the beam (height in this case), $f_t^{\textcircled{c}}$ is the tensile strength of the material, D_0 is a characteristic size which corresponds to a change of failure mechanism between strength of materials and fracture mechanics, and B is a geometry – related parameter. In a log – log plot, a strength of material criterion is represented by a horizontal curve and a LEFM (linear elastic fracture mechanics) criterion by a line with the slope $-1/2$. The two lines cut each other at the abscissa $D/D_0=1$. The larger the beam, the lower the relative strength, as observed on Figs. 3 and 5.

This size effect law provides also the fracture energy G_f as a result of the fit. A linear regression is used:

$$\frac{1}{\sigma^2} = aD + c \quad (11)$$

and

$$Gf = \frac{k_0^2}{E} \frac{1}{a}. \quad (12)$$

E is the elastic modulus of the material, and k_0 is a geometry-related parameter.

Let us now look at finite element computations on the three sizes of specimens, and compare the load deflexion curves with the experimental data. Because there are experimental errors, which are inherent to the testing system, there are several sets of model parameters, including the internal length, which provide an acceptable fit of the tests data. Figures 1 and 2 show two of these fits. In the first set, the internal length is $l_c = 40mm$ and it is $l_c = 7mm$ in the second fit. Of course, the other parameters in the evolution law of damage are also quite different between the two fits (they can be found in [4]).

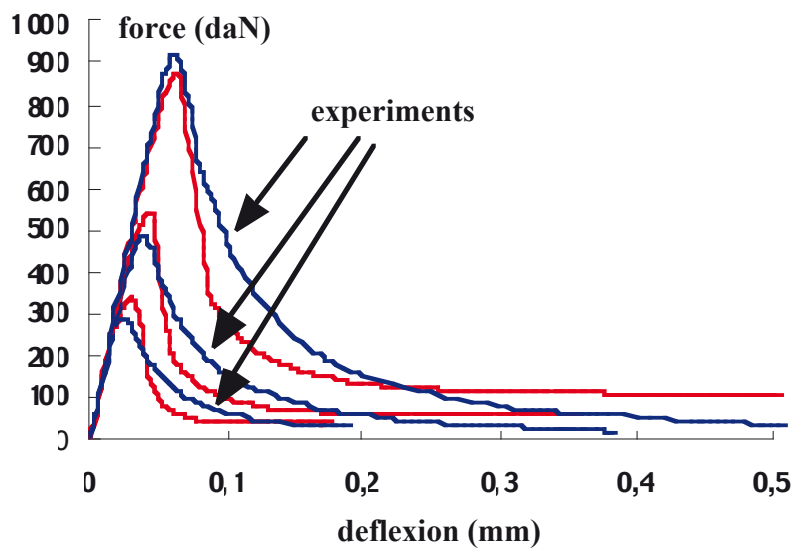


Figure 1. Fit of the mechanical model with the parameter set 1.

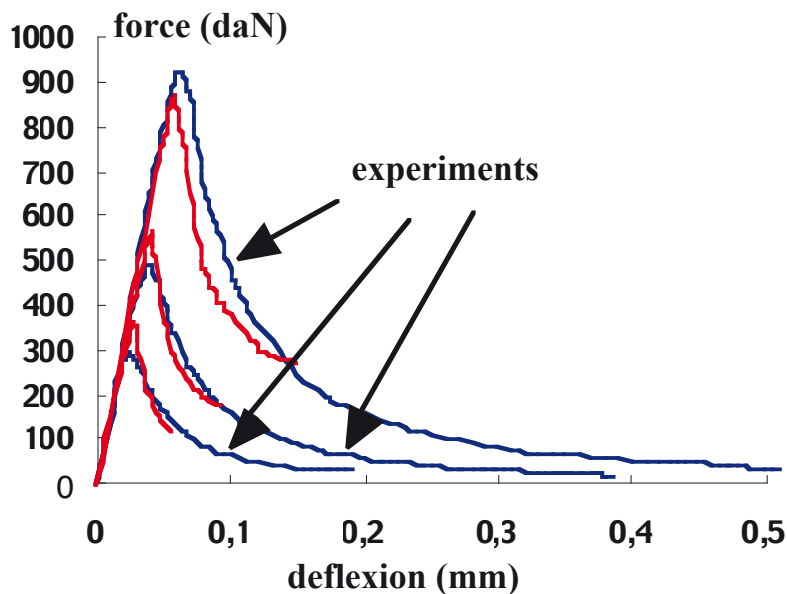


Figure 2. Fit of the mechanical model with the parameter set 2.

Globally these two fits are acceptable. If they were equally good, it would mean that the mechanical model cannot be calibrated objectively because there are several, quite

different sets of parameters which provide identical fits. The difference between these two fits ought to be magnified. For this purpose, the size effect method is quite adequate. Each of this fit can be analysed through a subsequent fit of the size effect formula in Eq. (10). Figure 3 shows the result of such interpretation for several quasi-identical fits. We have also reported on this plot upper and lower experimental bounds (borne sup – borne inf) between which the best fit should be. Note that on this figure, each set of computed data, corresponding to a given set of model parameters, has been fitted separately with the size effect formula. In particular, each set has a different value of D_0 .

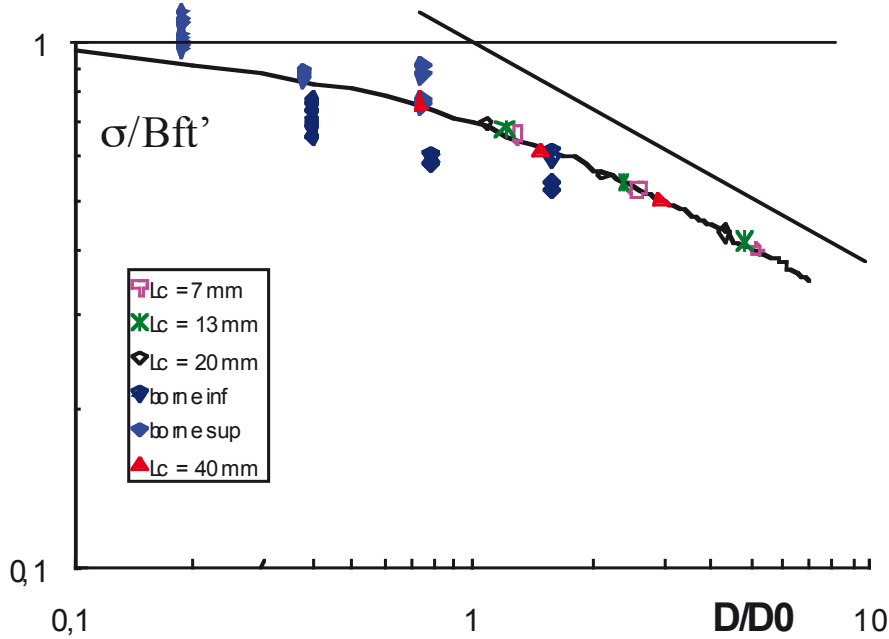


Figure 3. Interpretation of quasi-identical fits on the size effect plot.

The sets of model parameters have quite different internal lengths. It becomes clear on this figure that the best possible fit is the one obtained with an internal length equal to 40 mm. It can be also concluded that the optimum fit has not been obtained since it was not possible to place all the computed points of a single set (3 points corresponding to the three sizes) within the experimental bounds. This result shows that it is quite difficult to calibrate the mechanical model. Even if the model prediction provides good results compared to the experimental data, their *extrapolations* to structures of very different sizes will not be accurate because the size effect law is not well fitted.

In order to provide more insight of the influence of the internal length on the size effect plot, computations can be performed keeping all the model parameters constant, except the internal length. One parameter of importance is D_0 . This parameter is an indication of how brittle (small values of D_0) the structure might be. Figure 4 shows its variation for many sets of model parameters, which differ by the value of the internal length only.

When the internal length increases, the structure becomes more ductile. The case where the internal length is very large compared to the structure size (e.g. larger than 40 mm on this plot), exhibits a secondary effect, which is very seldom. In fact, the fracture process zone is very large when the internal length is very large. It becomes larger than the beam thickness and the beam depth. It is much larger than the specimen notch and in

this range, we believe that the simple size effect law implemented here is not applicable because notch sensitivity is entirely lost.

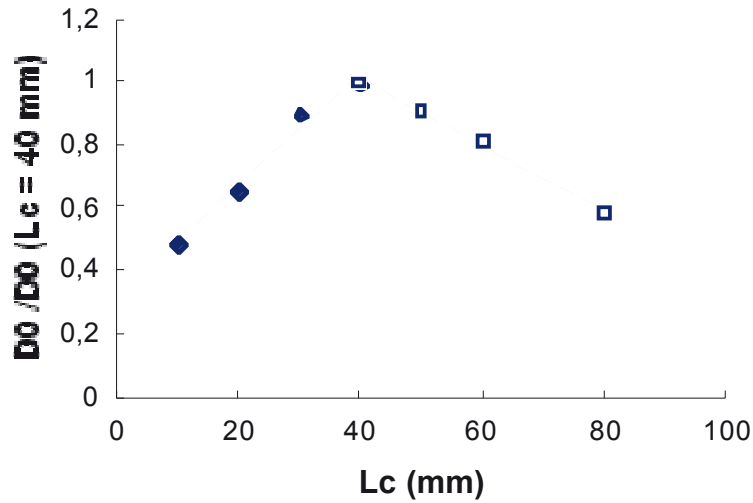


Figure 4 Influence of the internal length on D_0 .

Finally, we could retain for future computations the best fit obtained according to the size effect plot, that is set 1. Figure 5 shows this fit compared with the experimental data. The relatively good agreement is now obtained because the parameters in the size effect formula are *those of the experiments*. In Fig. 3, the size effect formula was fitted with different values (D_0 especially) for each set of (experimental and computed) points. We obtained then a magnification of the discrepancies between the model predictions and the experiments which does not appear any more.

There is one point important to note: with a local model, it is not possible to predict numerically the size effect depicted in this paper. This kind of experiment provides also an indirect proof that an internal length ought to be introduced in the constitutive relations. It can be done with the help of a non local formulation or in the context of a cohesive crack model with the same success in the case of simple mode 1 crack propagation.

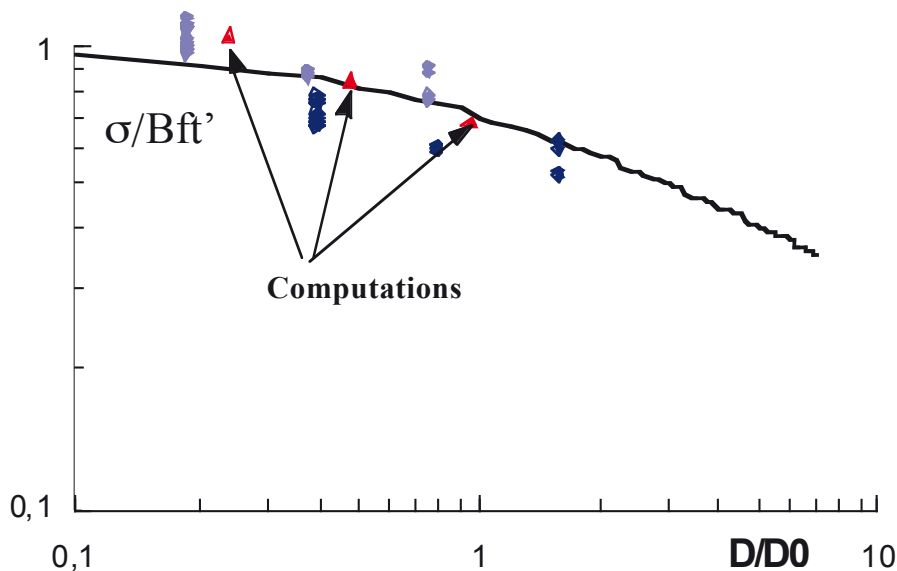


Figure 5. Comparisons between the experiments and the computations on the size effect plot.

4. Closure

The development of non local continuum models with local strains, which has been observed in the literature over the past decade, corresponds to the aim of providing sound and computationally robust failure models. Failure is predicted with a finite – non zero – energy dissipated. Structural size effect is well captured. Furthermore, it is possible to correlate such constitutive models with (non-linear) fracture mechanics [7]. Non local models have been first applied to soils, rocks and concrete. They are currently being applied to alloys too, essentially in the context of micro-systems such as MEMS. Their calibration is facing the usual difficulties involved in inverse analysis. We have shown in this paper that structural scaling of fracture could be a convenient tool, which helps at getting the best possible calibration.

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