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An elastoplastic model for granular materials taking into account grain breakage

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Abstract – Granular materials are constituted of an assembly of particles. In spite of the simplicity of this assembly, its mechanical behaviour is complex. In the first stage we propose a framework to establish correlations between parameters of the supposedly continuous medium and grain properties which are assumed to be constant. However, this hypothesis is no longer valid in the case where physical (shape, size...) or mechanical properties (Young modulus E_g , Poisson's ratio ν_g ...) of grains evolve during loading, causing the behaviour of the assembly to modify. We study the influence of the physical and mechanical parameters on grain breakage. We subsequently propose a way to model the influence of the grain breakage on granular materials and we introduce this influence in an elastoplastic constitutive model. Validations are made on two kinds of sands under isotropic and triaxial loading. Since the results of numerical computations corresponded well with the experimental data, we believe that the new model is capable of accurately simulating the behaviour of granular materials under a wide range of stresses and of taking into account, through new parameters, the individual strength of grains.

elastoplastic model / grain breakage / high pressure

1. Introduction

The mechanical behaviour of granular materials is dependent on the properties of the grains of which they are constituted. Hence, it is natural to search for correlations between the parameters of the constitutive model of a supposedly continuous medium and the properties of the grains (form, size, granulometry...). In order for these correlations to have meaning it would be necessary to introduce a system of understanding from the discontinuous to the continuous medium. This methodological work has notably been developed by Biarez and Favre (1977) and Biarez and Hicher (1994). It has been applied for determining the parameters of an elastoplastic model. Correlations between the parameters of the grain, on one hand, and the parameters of the elastoplastic model, on the other hand, have been proposed and validated in the case of sands which possess diverse grain size distributions (Hicher and Rahma, 1994).

The purpose of this article is to re-analyse, in the spirit of this approach, the influence of the granulometric characteristics on the behaviour of the grain assembly, particularly in the case where the above would evolve in the course of mechanical loading under the effect of grain ruptures. The aim is to determine the mechanical characteristics of the medium considered as continuous which would become integrated within the constitutive model by using complementary knowledge about the properties of the discontinuous medium.

We shall therefore analyse the evolution of behaviour under the effect of granulometric change. We shall then present a method that takes into account progressive grain ruptures through a systematic actualisation of the parameters of the constitutive model, by making them dependent on the evolution of the physical properties of the discontinuous medium.

2. Methodology for classifying parameters: from a discontinuous to a continuous medium

If we consider a grain assembly in which only the grains are treated as continuous media, the equilibrium of the assembly submitted to exterior forces could be calculated by the traditional method in continuum mechanics (Biarez and Hicher, 1994). The solution to the problem provides mechanical properties of the granular material considered as a continuous medium. By use of this method, we can deduce that the mechanical properties of the granular medium will depend on the following two groups:

- the mechanical properties of the grains themselves,
- boundary conditions of a solid formed by grains which depend on the geometry of grains and on the geometry of their arrangement. The geometry of grains may be defined by the parameters of size, form, and grain size distribution. The geometry of the arrangement consists of a parameter which expresses compacity and parameters which describe its anisotropy.

In all cases where grain rupture may be considered as negligible, the irreversible deformations are due only to a change in the arrangement of particles and not to a change of the particles themselves. The first group of parameters will therefore be constant, only the parameters of the arrangement will evolve.

Under these conditions, we can distinguish two classes of parameters:

Class I: constant parameters independent of the arrangement also called nature parameters.

(a) parameters of the constitutive law of the grains:

- of the grains themselves, e.g., the linear elasticity E_g ,
- of the contact between the grains: interparticulate friction angle φ_g , bonds between the grains C_g ;

(b) parameters of the geometry of the grains:

- size, grain size distribution. . . ,
- shape: angularity. . . ,
- surface state.

Class II: variable parameters describing the arrangement geometry

- the isotropic part: void ratio (e), compacity I_D ,
- anisotropy: its description requires in general a tensorial representation.

This analysis of the discontinuous / continuous medium passage allows us to establish a framework of expression for the correlations between parameters. It will hence be possible to obtain correlations either within the same class, or among classes by using the following scheme:

$$\begin{array}{ccccccc}
 \mathbf{General} & & \mathbf{Constitutive\ law} & & \mathbf{Boundary} & \Rightarrow & \mathbf{Solution} \\
 \mathbf{equations} & + & \mathbf{of\ grains} & + & \mathbf{conditions} & & \\
 \text{balance} & & & & \text{(geometry of} & & \text{(parameters of} \\
 \text{equations . . .} & & & & \text{arrangement)} & & \text{continuous medium)} \\
 & & & & & & \\
 & & & & & & \text{(1)}
 \end{array}$$

and in the absence of grain rupture:

$$\mathbf{Nature\ (I) + Compacity\ (II) \Rightarrow Rheology\ of\ equivalent\ continuous\ material.} \quad \text{(2)}$$

The above relations allow us to build a framework for the correlations among the parameters representative of the grains and the parameters of a constitutive law of the continuous medium (Biarez and Hicher, 1994) and (Hicher and Rahma, 1994).

3. Influence of grain ruptures on the behaviour of granular materials

Grain breakage is an important subject in Soil Mechanics. Fukumoto (1992) qualified it as a subject of major importance as the soil undergoes significant changes due to this phenomenon. There are many practical problems related to grain breakage such as the stability of earth dams, the bearing capacity of piles. . . . We may cite several investigations on this subject: (Roberts and De Souza, 1958; Marsal, 1967; Lee and Farhoomand, 1967; Lee and Seed, 1967; Vesic and Clough, 1968). . . and more recently (Colliat-Dangus, 1986; Fukumoto, 1990, 1992; Hagerty et al., 1993; Coop, 1993; Kim, 1995; Yamamuro and Lade, 1996a, 1996b; Lade and Yamamuro, 1996; Mc Dowell et al., 1996; Leung et al., 1996; Mc Dowell and Bolton, 1998).

In the preceding chapter we emphasised the hypothesis of invariance of the parameters which characterise the geometry of the grains. When significant grain ruptures occur during mechanical loading, this hypothesis is no longer valid. Grain rupture may be classified according to three modes as showing in *figure 1* (Guyon and Troadec, 1994):

- fracture: a grain breaks into smaller grains of similar sizes,
- attrition: a grain breaks into one grain of a slightly smaller size and several much smaller ones,
- abrasion: the result is that the granulometry remains almost constant but with a production of fine particles.

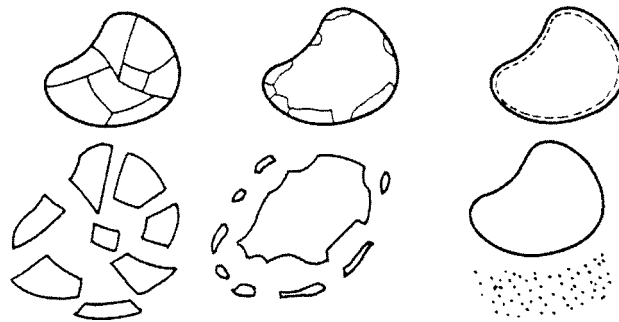


Figure 1. Different modes of grain breakage.

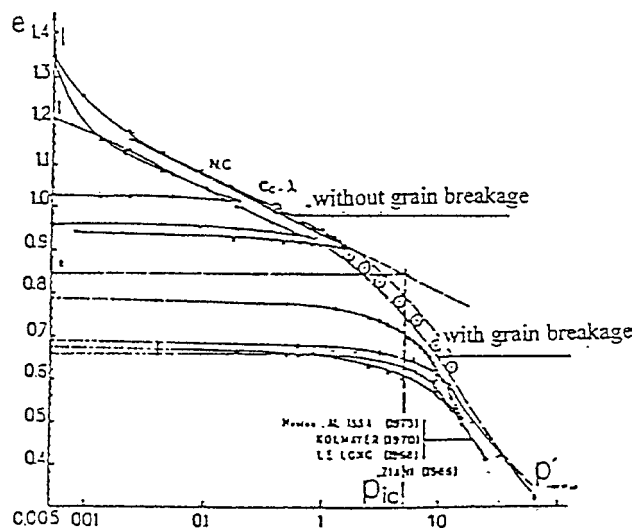


Figure 2. Grain ruptures in one-dimensional compression tests for poorly graded sands.

Hence, the rupture of sharp angles may be classed in the mode of rupture by attrition, whereas the rupture of micro-asperities during the sliding of the grains may be classed in the mode of abrasion rupture. This may occur in the absence of fracture or attrition notably, in the case of cyclic tests of small stress intensity.

In this study, we have concentrated on the two first modes of rupture. They both lead to a significant evolution of the grain size distribution curve. This phenomenon brings about an evolution of the mechanical properties of the granular material, which we may summarise as follows:

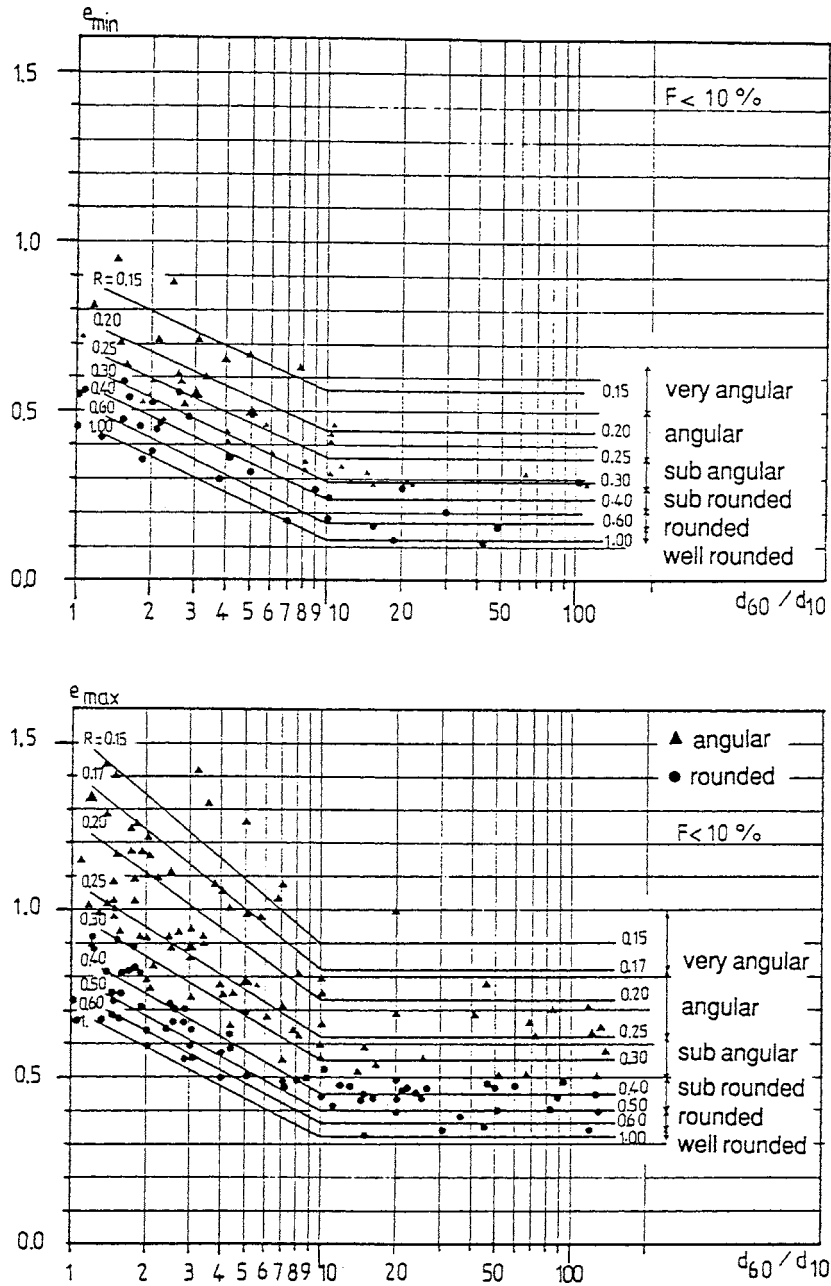


Figure 3. Evolution of the minimal and the maximal void ratio as a function of grain size distribution and the angularity of the grains.

Grain ruptures on an isotropic or one-dimensional compression path produce an increase of compressibility, which is represented by a slope increase in the $e-\log p$ plane (e is the void ratio and p the mean effective stress). In the case of identical grains such as an assembly of glass balls of the same diameter, the compressibility increase is very abrupt since the ruptures occur simultaneously. The phenomenon is more progressive in sand on account of the grain's size and shape differences, which produces a less homogeneous division between the contact forces. In *figure 2* we can see the superposition (in the $e-\log p$ plane) of several one-dimensional compression tests on a sand for stresses up to 100 MPa. We obtain first for very loose samples a compression index (slope C_c in the $e-\log p$ plane) in the range of 0.15 to 0.20. The value of this slope increases for values of the mean effective stress $p > 1$ MPa as a result of grain crushing. We obtain then a slope of apparent

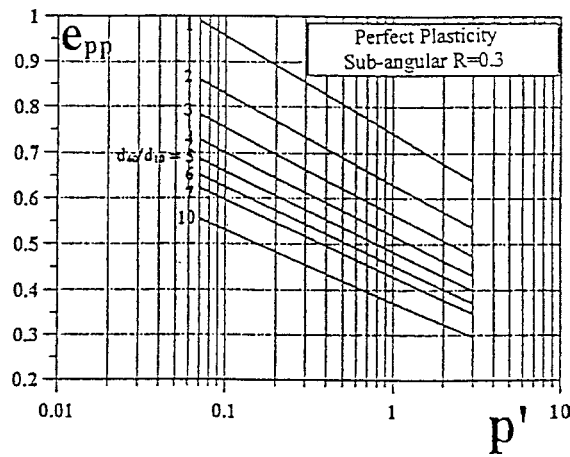


Figure 4. Perfect plasticity relation as a function of the grain size distribution.

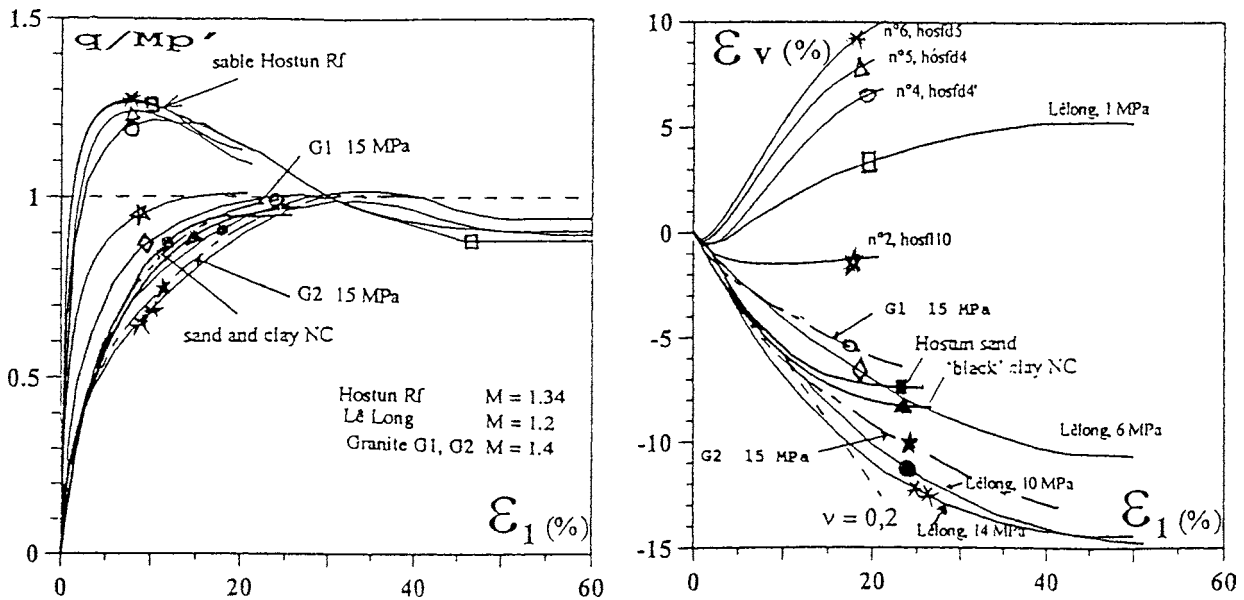


Figure 5. Grain breakage influence on stress-strain relationship on Hostun sand and crushed granite in comparison with standard behaviours.

Table I. Characteristics of materials used to determine a network (without grain breakage) compared to tests under elevated stresses (with grain breakage).

	Le Long	Colliat-Dangus	Ziani	Kim	
Nature	Quartzic sand (Hostun)	Quartzic sand (Hostun)	Quartzic sand (Hostun)	Granite	
				G1	G2
$C_u = d_{60}/d_{10}$	1.62	1.70	/	10	2
e_{\min}	0.580	0.656	0.620	0.430	0.430
e_{\max}	0.866	1.000	0.840	0.800	1.230
e_0	0.680	0.673	1.190	0.467	0.870
$I_D(\%)$	65	95*	(very loose)**	90	90

*: $e_0 = 0.890$ and $I_D = 32\%$ for $\sigma'_3 = 300$ KPa.

** : $e_0 > e_{\max}$ as $w = 5\%$.

consolidation whose value can double or triple in relation to standard values. This slope increase depends clearly on the amplitude of grain rupture (Biarez and Hicher, 1994).

On a deviatoric path, for example a compression triaxial test, we observe first of all a significantly bigger contractancy. This may be explained by the displacement of the perfect plasticity relation in the e - $\log p$ plane. The explanation is schematically given in *figure 3* and *figure 4* from (Biarez and Hicher, 1997). We can see in *figure 3* that the values of the minimal (e_{\min}) and maximal (e_{\max}) void ratio decrease when the grain size distribution (C_u) increases (until $C_u \approx 10$). For a given shape, we obtain correlations between the position of the critical state line and nature parameters (e_{\min} , e_{\max} , grain angularity, $C_u = d_{60}/d_{10}$, where d_n represents the diameter corresponding to $n\%$ of material being smaller by weight) are drawn (*figure 4*). One can see that an increase of C_u due to grain ruptures will produce a slip of the critical state line towards smaller void ratios. In order to reach the critical state during a triaxial test, a bigger volume change will be needed in case of significant grain breakage. This phenomenon will also result in a change of the stress-strain relationship and in particular in an increase of the axial strain corresponding to the maximum strength. Therefore a significant change takes place in the global behaviour of granular media due to grain breakage, which needs to be taken into account in a constitutive model.

This may be verified by comparing the results obtained from a sand under small stresses (negligible ruptures of grains) and under elevated stresses. We observe a very strong increase in the volume change. In the adimensional plane $q/Mp - \varepsilon_1$ (*figure 5*), where q is the second invariant of the stress tensor and M a constant of the material (value of q/p at large deformations), the test curves under elevated stresses are shifted towards the right in relation to the network which corresponds to the behaviour of a loose contractant material without grain breakage (characteristics of materials tested by Le Long (1968), Colliat-Dangus (1986), Ziani (1987) and Kim (1995) are given in *table I*).

4. Parameters influencing grain rupture

We shall briefly analyse and classify the parameters which are likely to influence grain rupture. Two groups of parameters may be extracted:

- nature parameters, that is to say parameters related to characterising the discontinuous medium. These are parameters of class I, as defined above.

- mechanical parameters, that is to say parameters characterising the mechanical state of a medium supposed to be continuous: stress and strain tensors.

4.1. Influence of nature parameters

Several studies (Vesic and Clough, 1968; Colliat-Dangus, 1986; Bard, 1993; Kim, 1995)...have clearly shown the influence of nature parameters of the grains: the mechanical resistance of the grains, the form, size, and grain size distribution.

The mechanical resistance of grains may be linked, on one hand, to its mineralogical nature and on the other hand to its state of alteration. It therefore depends on the history of how the natural deposits were formed, the history of its fabrication (crushing...) for the artificial mixtures. The influence of the shape may be linked to the preceding point in that the statistical distribution of faults (grain joints, fissures...) depends on the shape of the particles, the smallest being more resistant than the large ones (scale effect).

Grain breakage augments with their angularity. This may be attributed to a greater fragility of the contact points of small curvature radius.

Another important factor is the grain size distribution and different studies appear to agree that a badly-graded grain size distribution enhances grain rupture. This effect may be explained by a better distribution of intergranular forces in a well-graded medium. A recent experimental study realised by Hicher et al. (1995) provides proof of this influence with the help of two mixtures made from crushed granite. Tested in the same mechanical conditions and for identical nature parameters except for the value of d_{60}/d_{10} , the well graded mixture G1 ($d_{60}/d_{10} = 10$) presented very slight evolution in its grain size distribution in contrast to the badly graded mixture G2 ($d_{60}/d_{10} = 2$). Some examples of evolution on a one-dimensional compression path are presented in *figure 6*. It is interesting to note that it is the totality of the grain size distribution which has evolved: all the grain sizes are concerned by the rupture phenomenon and not only the largest among them.

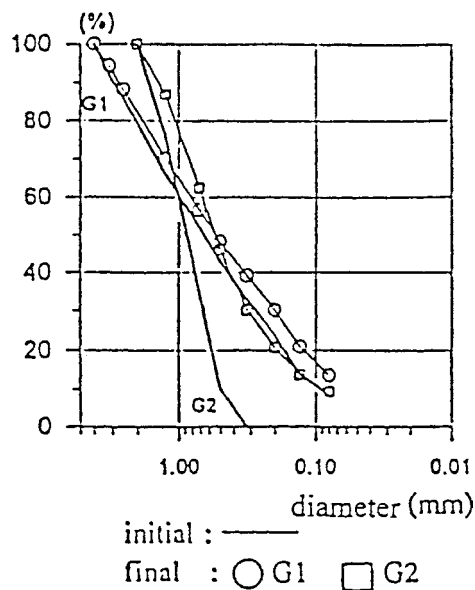


Figure 6. Evolution of the grain size distribution curve in a one-dimensional compression test. Influence of the initial distribution.

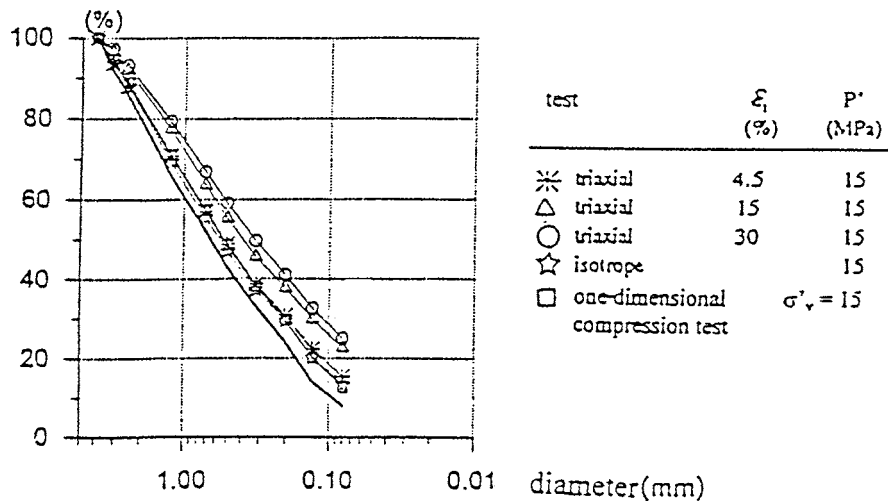


Figure 7. Influence of stress and strain paths on grain breakage.

4.2. Influence of stress–strain history

It is evident that the intensity of applied stresses is a preponderant factor on the development of grain rupture. The level of stress from which they appear in a significant manner depends on the preceding parameters. However, the stress path also plays an important role. Experimental studies on this subject allow us to conclude that the stress deviator plays a major role and that the triaxial tests, for example, generate more ruptures than the isotropic or one-dimensional compression tests of the same average stress (see *figure 7*) (Hicher et al., 1995).

This may very well be explained by the relative displacements of particles which encourage grain rupture. *Figure 7* shows this phenomenon in the case of crushed granite as seen above. We observe that for a triaxial test of a lateral stress constant and equal to 15 MPa, there is an evolution of the grain size distribution between the axial strains of 15% and 30%, while the axial stress has attained a practically constant value. These results permit us to illustrate the effect of mechanical parameters associated with the continuous medium: the level and path of stress, the amplitude of strains during mechanical loading.

5. A brief review of existing relations

Most of the investigations cited in section 3 are experimental ones. They permit us to understand the influence of grain crushing on the behaviour of granular materials as well as the parameters influencing this breakage. Some authors have proposed relations that we can schematically divide into four categories:

- (i) factors which allow the degree of grain breakage to be quantified: Marsal (1967), Lee and Farhoomand (1967), Hardin (1985) and Lade and Yamamuro (1996);
- (ii) relations to determine the evolution of the grain size distribution: Fukumoto (1990, 1992), Mc Dowell et al. (1996) and Mc Dowell and Bolton (1998);
- (iii) relations between the maximal friction angle and the friction angle at critical state or the interparticulate friction angle: Vesic and Clough (1968), Kørner (1970), Barton and Kjærnsli (1981), Bolton (1986), Naylor et al. (1986), Maksimovic (1996);
- (iv) models to provide the evolution of the stress-strain relationship during loading: Loret (1982), Simonini (1996), Mc Dowell et al. (1996) and Mc Dowell and Bolton (1998).

Among the different grain breakage factors proposed (i) Hardin's factor (1985) is the most widely used. More recently, Lade and Yamamuro (1996) have proposed a new interesting factor based on the effective grain size. Unfortunately, these factors are computed after the tests and do not give information on the grain size distribution during loading nor on the stress–strain relationship.

It might be interesting to know the grain size distribution after grain crushing occurs (ii). For example, in order to evaluate, albeit with a gross approximation, the permeability k of the filter of earth dams, we can use the well-known Hazen's relation:

$$k = 100(d_{10})^2 \quad \text{where } k \text{ is given in cm/s and } d_{10} \text{ in cm.} \quad (3)$$

Fukumoto (1992) proposed a relation allowing us to plot the grain size distribution after testing, regardless of the type of test performed (one-dimensional or triaxial compression tests, gyratory test or compression test) or of the stress level or the number of blows applied. He assumed that the percentage in weight of a given size of grain can be expressed by a geometric progression. However, it is necessary to obtain the grain size distribution after testing in order to determine the parameters of the relation. Mc Dowell et al. (1996) and Mc Dowell and Bolton (1998) have also proposed a relation between the percentage in weight passing through a given sieve and the grain size. For that, they assumed that the grain size distribution has a fractal character. However, the resulting curves are only qualitative.

Vesic and Clough (1968) have plotted the secant friction angle as a function of the mean stress for several sands (iii). This angle corresponds to the maximal circle in the Mohr plane and is given by (see next section for notations):

$$\phi_s = \arcsin\left(\frac{\sigma_1 - \sigma_3}{\sigma_1 + \sigma_3}\right). \quad (4)$$

The value of this secant angle decreases with the increase of the mean stress and tends to a minimal value. Vesic and Clough (1968) have proposed that the measured shear strength can be expressed as the strength due to sliding friction + dilatancy effects + crushing and rearranging effects. K erner (1970) has proposed the same expression in terms of friction angles. Bolton (1986) stated that the friction angle can be decomposed in two terms: the friction angle at the critical state and the dilatancy angle. Moreover, he proposed, for a given type of test, a relation between the maximum angle of friction and the maximum dilatancy rate. These relations are widely used. The relation proposed by Naylor et al. (1986) connects the angle of friction to the confining pressure. Maksimovic (1996) also related the friction angle to the state of stress.

The last category (iv) concerns the models that can provide the evolution of the stress–strain relationship. Simonini (1996) proposed a model composed of two yield surfaces. A first nonlinear Mohr–Coulomb criterion is used when non-significant grain crushing occurs and a second criterion is proposed by Zienkiewicz and Humpheson (1977) (cited by the author) when it is not. In both cases, the flow rules are nonassociated. Simonini also used Bolton's relation (1986) and computes Hardin's factor to quantify the degree of grain crushing. It is obvious that, in the transition zone, the model cannot estimate accurately the state of stresses and strains. The results obtained with the plot of the constant Hardin's factor are interesting even though no results in terms of stress or strain are given. In his elastoplastic model, Loret (1982) has modified the well-known relation proposed by Roscoe et al. (1963) and Schofield and Wroth (1968):

$$q \, d\varepsilon_d^p + p \, d\varepsilon_v^p = Mp \, d\varepsilon_d^p \quad (5)$$

widely used for cohesionless materials or clays. The right-hand side of relation (5) was identified as the internal friction dissipation. Loret added a term to the right-hand side that corresponds to the breakdown work. Thus,

M does not remain constant but is a function of the mean stress p . Mc Dowell et al. (1996) and Mc Dowell and Bolton (1998) proposed a relation between the void ratio and the logarithm of the stress which can be used for one-dimensional compression tests. They also added, to the right hand side of the relation (5), a term that took into account the increase of the surface due to grain breakage and the surface energy.

As we can see, very little has been proposed concerning the modelling of grain breakage influence on the stress–strain relationship of granular materials. We present in the following an elastoplastic model developed for the modelling of soil behaviour. Inside the framework of this model, we have introduced a special treatment concerning grain breakage influence in order to enlarge the capabilities of the model.

6. An elastoplastic constitutive model for granular media

The constitutive model we used is an elastoplastic model which combines four mechanisms: three deviatoric mechanisms of plane strain and an isotropic mechanism. We shall present it here succinctly in the case of monotonic loading. For more information, our readers may refer to the following studies: Aubry et al. (1982) and Hujeux (1985).

6.1. Notations

Assuming that σ , $\boldsymbol{\varepsilon}$ and $\dot{\boldsymbol{\varepsilon}}$ are respectively the tensors of effective stress, strain and strain rate, we can write the following:

$$\begin{aligned} p &= \frac{1}{3} \text{Tr}(\boldsymbol{\sigma}), & \varepsilon_v &= \text{Tr}(\boldsymbol{\varepsilon}), \\ \mathbf{s} &= \boldsymbol{\sigma} - p\mathbf{I}, & \mathbf{e} &= \boldsymbol{\varepsilon} - \frac{\varepsilon_v}{3}\mathbf{I}, \\ q &= \left(\frac{3}{2} s_{ij} s_{ij} \right)^{1/2}, & \dot{\varepsilon}_d &= \left(\frac{2}{3} \dot{\varepsilon}_{ij} \dot{\varepsilon}_{ij} \right)^{1/2}. \end{aligned} \quad (6a)$$

In the k -plane, we can define effective stress, strain and strain rate tensors respectively $\boldsymbol{\sigma}_k$, $\boldsymbol{\varepsilon}_k$ and $\dot{\boldsymbol{\varepsilon}}_k$. If ii represents the component “ ii ” of the tensor (no summation on i) then:

$$\begin{aligned} p_k &= \frac{1}{2} \text{Tr}(\boldsymbol{\sigma}_k) = \frac{1}{2} (\sigma_{ii} + \sigma_{jj}), & (\varepsilon_v)_k &= \text{Tr}(\boldsymbol{\varepsilon}_k) = \varepsilon_{ii} + \varepsilon_{jj}, \\ q_k &= \left[\frac{1}{4} (\sigma_{ii} - \sigma_{jj})^2 + \sigma_{ij}^2 \right]^{1/2}, & \dot{\gamma}_k &= [(\dot{\varepsilon}_{ii} - \dot{\varepsilon}_{jj})^2 + 4\dot{\varepsilon}_{ij}^2]^{1/2}, \\ s_k &= \begin{pmatrix} s_{k1} \\ s_{k2} \end{pmatrix} = \begin{pmatrix} \frac{\sigma_{ii} - \sigma_{jj}}{2} \\ \sigma_{ij} \end{pmatrix}, \end{aligned} \quad (6b)$$

p_a is the atmospheric pressure, \mathbf{I} is the unit tensor.

As in the theory of elastoplasticity, we assume that the strain rate tensor ($\dot{\boldsymbol{\varepsilon}}$) can be decomposed into an elastic strain rate tensor ($\dot{\boldsymbol{\varepsilon}}^e$) and an irrecoverable strain rate tensor ($\dot{\boldsymbol{\varepsilon}}^p$):

$$\dot{\boldsymbol{\varepsilon}} = \dot{\boldsymbol{\varepsilon}}^e + \dot{\boldsymbol{\varepsilon}}^p. \quad (7)$$

6.2. Elastic part

The elastic part of the model is isotropic, non-linear and depends on the mean stress:

$$\dot{\varepsilon}_v^e = \frac{1}{K(p)} \dot{p}, \quad (8)$$

$$\dot{\varepsilon}_d^e = \frac{1}{3G(p)} \dot{q}, \quad (9)$$

$$\begin{aligned} K(p) &= K_a p_a \left(\frac{p}{p_a} \right)^n, \\ G(p) &= G_a p_a \left(\frac{p}{p_a} \right)^n, \end{aligned} \quad (10)$$

where K and G are respectively the bulk and shear modules and n is a constant ($0 \leq n \leq 1$).

6.3. Plastic part

We assume that the plastic part of the deformation is governed by three deviatoric plane strain mechanisms in three orthogonal planes and a purely volumetric mechanism.

6.3.1. Deviatoric mechanisms

The equations of each mechanism k ($k = 1, 2, 3$) are identical. We assume that the mechanism k , in the ij -plane, produce plane strain and strain rates.

The yield surface, for each k -plane, is given by:

$$f_k(p, p_k, \mathbf{s}_k, p_c, r_k) = \|\tilde{\mathbf{s}}_k\| - r_k, \quad (11)$$

where the components of $\tilde{\mathbf{s}}_k$ are:

$$\begin{aligned} \tilde{s}_{k1} &= \frac{\mathbf{s}_{k1}}{F_k(p, p_k, p_c)}, \\ \tilde{s}_{k2} &= \frac{\mathbf{s}_{k2}}{F_k(p, p_k, p_c)} \end{aligned} \quad (12)$$

and $F_k(p, p_k, p_c)$, a factor of normalisation, is given by:

$$F_k(p, p_k, p_c) = \sin \phi p_k \left(1 - b \text{Log} \frac{p}{p_c} \right), \quad (13)$$

r_k is an internal variable allowing a progressive mobilisation of the mechanism k :

$$r_k = r_k^{\text{el}} + \frac{\gamma_k^p}{1 + \gamma_k^p}. \quad (14)$$

Its evolution law is given by:

$$\dot{r}_k = \lambda_k^p \frac{(1 - r_k)^2}{a(r_k)}. \quad (15)$$

p_c is the critical pressure and is related to the plastic volumetric strain ε_v^p by the following equation:

$$p_c = p_{c0} \exp(\beta \varepsilon_v^p) \quad (16)$$

where $1/\beta$ is the slope of the critical state line in the ε_v^p - $\log p$ plane and

$$a(r_k) = a_{cyc} + (a_m - a_{cyc})\alpha_k, \quad (17)$$

$a_m, a_{cyc}, b, r_k^{el}, p_{co}, \phi$ and α_k are constant parameters of the model.

In the k -plane, the plastic deviatoric strain rate tensor \dot{e}_k^p and the plastic volumetric strain rate $(\dot{\varepsilon}_v^p)_k$ are defined as:

$$\begin{aligned} \dot{e}_{k_1}^p &= (\dot{\varepsilon}_{ii}^p)_k - (\dot{\varepsilon}_{jj}^p)_k = \lambda_k^p \Psi_{k_1}^d, \\ \dot{e}_{k_2}^p &= 2(\dot{\varepsilon}_{ij}^p)_k = \lambda_k^p \Psi_{k_2}^d, \end{aligned} \quad (18)$$

$$(\dot{\varepsilon}_v^p)_k = (\dot{\varepsilon}_{ii}^p)_k + (\dot{\varepsilon}_{jj}^p)_k = \lambda_k^p \Psi_k^v,$$

where λ_k^p is the plastic multiplier and Ψ_k gives the direction of plastic strains, assuming that this tensor can be decomposed into a deviatoric and a volumetric part.

We make the hypothesis that, in the normalised deviatoric plane, the law is associated, which defines Ψ_k^d :

$$\Psi_k^d = \partial_{\mathbf{s}_k} f_k = \frac{\mathbf{s}_k}{\|\mathbf{s}_k\|}. \quad (19)$$

The plastic potential relationship permitting contractancy and dilatancy is given by:

$$\Psi_k^v = \frac{d\varepsilon_{v_k}^p}{d\varepsilon_{d_k}^p} \mathbf{I} = \alpha_k \left(\sin \phi - \frac{\mathbf{s}_k : \Psi_k^d}{p_k} \right) \mathbf{I}. \quad (20)$$

6.3.2. Isotropic mechanism

Experimental results exhibit plastic strains during an isotropic compression. Because they are not activated under this stress path, the deviatoric mechanisms cannot generate purely volumetric strains. An isotropic mechanism is thus introduced.

$$f_4(p, p_c, r_4) = |\tilde{p}| - r_4, \quad (21)$$

where

$$\tilde{p} = \frac{p}{F_4(p_c)} \quad (22)$$

and $F_4(p_c)$ is a factor of normalisation given by:

$$F_4(p_c) = dp_c, \quad (23)$$

r_4 is the degree of mobilisation of this mechanism. Its value increases continuously from r_4^{el} (elastic) to 1 and its evolution law is as hyperbolic form given by:

$$\dot{r}_4 = \lambda_p^4 \frac{(1 - r_4)^2 p_a}{c p_c}. \quad (24)$$

The mechanism generates plastic volumetric strains:

$$(\dot{\varepsilon}_{ij}^p)_4 = \lambda_4^p \Psi_4^v \frac{\delta_{ij}}{3}. \quad (25)$$

c and d are constant parameters of the model.

During loading, the four mechanisms can be activated and are coupled by the density hardening p_c as follows:

$$\varepsilon_v^p = \sum_{m=1}^4 (\varepsilon_v^p)_m. \quad (26)$$

The equation of compatibility, which permits us to determine the plastic multipliers, is given for each of these mechanisms by:

$$\dot{f}(\sigma, \varepsilon_v^p, r) = \partial_\sigma f : \dot{\sigma} + \partial_{\varepsilon_v^p} f \dot{\varepsilon}_v^p + \partial_r f \dot{r}. \quad (27)$$

Several authors (Lee and Seed (1967), Vesic and Clough (1968), Ramamurthy et al. (1974), Colliat-Dangus (1986), Lade and Yamamuro (1996), Yamamuro and Lade (1996a, 1996b)) observed that the compressibility of granular medium increases with grain breakage. Thus, the dilatancy decreases and tends to vanish. Lee and Seed (1967) have observed: “The stress–strain volume change characteristic of dense sand at high confining pressures are not unlike those of low pressures”. We believe that the concept of critical state in sands can then be used (see also Been et al., 1991).

The methodology for connecting the parameters of the discontinuous medium and the equivalent fictitious continuous medium has been applied in order to propose links, by means of correlations, between the parameters of the elastoplastic model and the nature and compacity parameters of granular media. The approach was developed by Hicher and Rahma (1994), who used a database called Modelisol (Favre et al., 1991) which contained experimental results on 21 different sands. They first assumed that the previous studies gave enough elements for the statistical determination of elastic parameters E , ν , n and perfect plasticity (or critical state) parameters: ϕ , β , d , p_{co} (Biarez and Favre, 1977; Biarez and Hicher, 1994). For p_{co} , the initial density influence was introduced by means of the density index $I_D = \frac{e_{\max} - e_0}{e_{\max} - e_{\min}}$ taking into account the values of the two nature parameters e_{\max} and e_{\min} as well as the value of the initial void ratio e_0 .

They concentrated their study on the other parameters, specific to Hujeux’s model: r^{el} , a , b , α which appear in the constitutive equations (11) to (27). For this purpose, the principal component analysis method was chosen. Using the methodology of connections between parameters of discontinuous medium described above, the principal component analysis expresses the fact that the parameters describing the discontinuous medium can be presented in factorial space either by parameters describing the nature and the compacity of the granular material (FSPD) or by the parameters of the constitutive model (FSPC). We can say that the space FSPD is the solution, by means of the correlations and statistical links of the space FSPC. Thus the values of the constitutive model parameters are the result of the projection of FSPD on FSPC.

It was therefore possible to determine the variables which were strongly correlated in order to propose, by means of multiple regression, mathematical links between model parameters and nature parameters.

The following results were obtained: if Y_i is a parameter of the elastoplastic model, Y_i can be written:

$$Y_i = g_i(y_j), \quad j = 1, \dots, n, \quad (28)$$

where y_j are the parameters describing the properties of the granular medium (nature and compacity parameters). In fact, only E and p_{co} are depending on the compacity, the other parameters are constant for a given medium and therefore, depend only on the nature parameters.

At the present stage, these correlation models can be considered as very valuable, not for the final determination of the model parameters, but more likely for the definition of an initial set of parameters, realistic enough to permit an optimal optimisation process to be undertaken.

7. Accounting for grain breakage in the constitutive model

The first step was to propose a method for quantifying the amount of grain breakage. Several proposals which consider generally the evolution of different parameters representative of the grain size distribution have already been made. Having tested some of them (Kim, 1995), we decided to use a parameter which could be representative of the whole evolution of the grain size distribution in agreement with Fukumoto (1992). For this reason, we select the surface S intercepted by the initial and the actual grain size distribution curves (see *figure 8*).

As we saw above, the evolution of S occurred for two kind of parameters: nature and mechanical parameters. A convenient way of taking into account both stress and strain is to consider the plastic work:

$$W^P = \int \sigma_{ij} d\varepsilon_{ij}^P. \quad (29)$$

Experimental results show that triaxial tests produce more grain breakage than isotropic or one-dimensional compression tests. In the first expression, as axial and radial strains have a different sign (the first is negative while the second is positive), the plastic work, for a given stress and axial strain, is smaller for a triaxial test than for an one-dimensional compression test. By introducing the absolute value of the strain increment, we can correct this point. So we decided to use the following expression:

$$W^{*P} = \int \sigma_{ij} |d\varepsilon_{ij}^P|. \quad (30)$$

The whole surface S can be given by a function of W^{*P} , which depends on nature parameters and can be written:

$$S = \psi_{ph}(W^{*P}), \quad (31)$$

ψ_{ph} is a function depending only on the initial granular material.

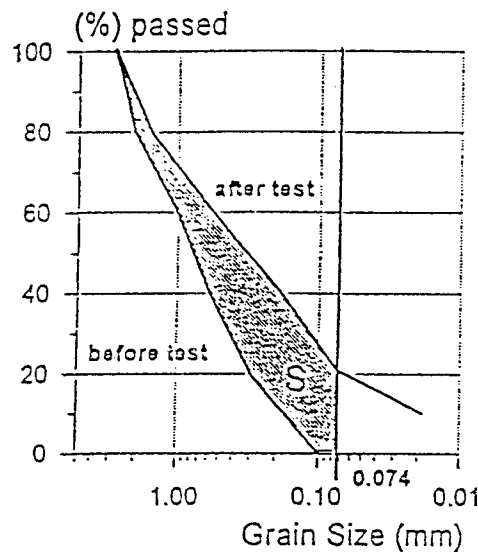


Figure 8. Definition of the surface S .

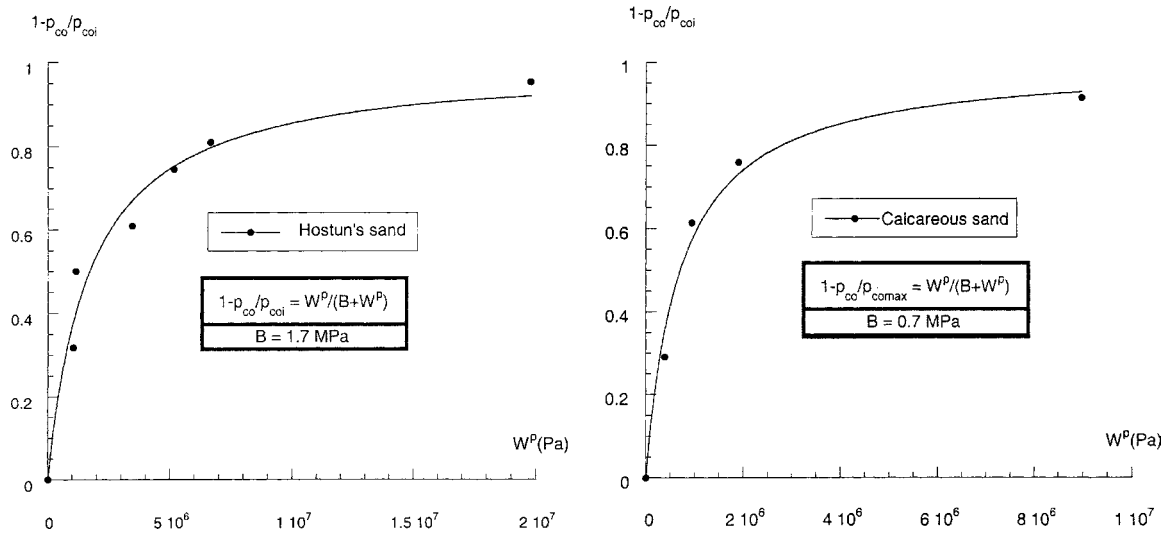


Figure 9. Correlation between the critical reference pressure and plastic work for Hostun sand and a calcareous sand.

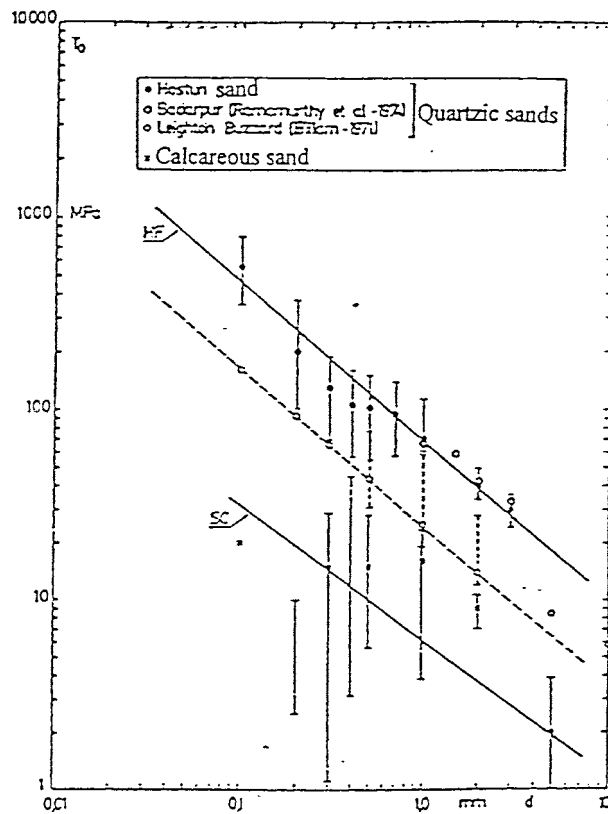


Figure 10. Tensile strength of individual particles as a function of their diameter.

With reference to Hujieux's model, we can now introduce this function to take into account the material changes due to grain breakage.

The equation (28) was proposed assuming that the nature parameters were constant along any stress and strain path. Should this hypothesis no longer be valid, as in the case of significant grain ruptures, the equations have to be modified in order to follow these changes.

We can assume that y_j are modified by grain ruptures in the following manner:

$$y_j = h_j(S). \quad (32)$$

The model parameters can therefore be written:

$$Y_i = g_i(h_j(S)). \quad (33)$$

A study of different factors of influence has shown that the principal parameters to be taken into account were those which control the position of the perfect plasticity relation, that is for the model p_{co} and β which control hardening in density by the relation:

$$p_c = p_{co} \exp(\beta \varepsilon_v^p). \quad (34)$$

Let us reconsider the correlations in *figure 4*. We can observe that the grain ruptures, by provoking an increase of d_{60}/d_{10} , will cause a shift of the initial relation of perfect plasticity towards the smaller void ratios.

Under these conditions, the critical pressure p_{co} corresponding to a reference void ratio e_0 diminishes while the slopes of the lines are almost equal. We therefore propose the hypothesis that the modulus of plastic compressibility β remains constant during the whole loading even if the physical and mechanical properties evolve as shown in *figure 4*.

The calculation of the value of p_{co} obtained from experimental tests made on quartz sand as well as on carbonate sand shows that its diminution is correlated with that of calculated plastic work. The value of p_{co} carried up to its maximum value varies between 1 and 0.

We can then write:

$$p_{co} = p_{coi}(1 - \chi(S)). \quad (35)$$

The most appropriate form for the relation between p_{co} and W^{*p} is the hyperbolic form (*figure 9*). The variable $\chi(S)$ has the following analytical form (Hicher et al., 1995) and (Daouadji, 1999):

$$\chi(S) = 1 - \frac{p_{co}}{p_{coi}} = \frac{W^{*p}}{B + W^{*p}}. \quad (36)$$

Hardening in density will then be written as follows:

$$p_c = p_{coi} \cdot \left(1 - \frac{W^{*p}}{B + W^{*p}}\right) \cdot \exp(\beta \cdot \varepsilon_v^p) \quad (37)$$

with

p_{co} : value of the critical reference pressure after loading (grain crushing may occur),

p_{coi} : value of the critical reference pressure before loading (without grain crushing),

B : parameter taking into account the effect of grain ruptures which depend on the initial properties (shape, size, distribution. . .) of the grains.

Parameter B allows us to characterise different materials while taking into account the individual characteristics of grains (resistance, form. . .) and of their grain size distribution.

We have also taken into account the influence of the loading path as it has been stated above. The use of a multimechanism model that distinguishes between deviatoric and isotropic paths allows us to take into account this influence and support the choice of this model. We have therefore taken into consideration the plastic work corresponding to each of these mechanisms. Thus, if one mechanism is more activated than another, the plastic work corresponding to this mechanism is higher than that of other mechanisms. This is the case notably for the simulation of a triaxial test where the deviatoric mechanisms intervene in a more important way than does the isotropic mechanism.

In re-writing equation (37) for each mechanism, we obtain:

$$p_{ck} = p_{coi} \cdot \left(1 - \frac{W_k^{*p}}{B_k + W_k^{*p}} \right) \cdot \exp(\beta \cdot \varepsilon_v^p). \quad (38)$$

Parameter B_k takes on different values for deviatoric mechanisms and for isotropic mechanism.

We note that the coupling between different mechanisms is always provided by the plastic volume change. The plastic multipliers are obtained by resolving for each mechanism the equation (27).

8. Validation of the model

In order to validate our approach, we have simulated the behaviour of different kinds of sands (Colliat-Dangus, 1986). The first one is a quartz sand coming from the Hostun quarry in France. This is a fine sand of angular grains and of uniform grain size distribution. The second sand is of marine origin with a very high percentage of calcite (98%), and containing a great amount of shell debris. The grains are rounded and have an intra-granular porosity of 9%. The grain size distribution for the second sand is more spread out than it is for the first sand. *Table II* recapitulates the information concerning these two sands.

The individual resistance of the sand grains varies very much. It is calculated in running a simple compression test on a grain, in assimilating it to a sphere of diameter d . The resistance to rupture by indirect traction is given by Jaeger (1967), Billam (1971), Colliat-Dangus (1986) and Mc Dowell and Bolton (1998):

$$R_t = \gamma^* \frac{P_r}{d^2} \quad (39)$$

with

P_r : load provoking rupture,

Table II. Characteristics concerning the Hostun's sand and the calcareous sand.

Sand	Mineralogy	e_{\min}	e_{\max}	D_r (%)	C_u	d_{50} (mm)	Shape	R_t (MPa)
Hostun	Quartzic	0.656	1.00	90	1.7	0.32	angular	200
Calcareous	Calcareous	1.014	1.670	71	2.8	0.17	rounded	20

Table III. Values of the parameters of the model for the calcareous sand and the quartzic Hostun's sand.

Value of the parameters	Calcareous sand	Hostun's sand
Elastic parameters		
K	215	354
G	165	265
N	0.5	0.5
Parameters of the critical state		
ϕ	39	31
β	30	10
p_{co}	1.5 ^(*)	2.2 ^(**)
d	8	8
Hardening parameters		
– deviatoric parameters		
a	$8 \cdot 10^{-3}$	$5.5 \cdot 10^{-3}$
$a_{cyc} = a/2$	$4 \cdot 10^{-3}$	$2.75 \cdot 10^{-3}$
b	0.1	0.16
α	2	1.75
– isotropic parameters		
c	0.01	0.01
Parameter r		
$r_k^{el} = r_4^{el}$	10^{-2}	$1.5 \cdot 10^{-2}$

^(*) For an effective mean stress $p'_0 = 0.05$ MPa.

^(**) For an effective mean stress $p'_0 = 2.2$ MPa.

d : diameter of the sphere,

γ : coefficient of proportionality dependent on the grain.

These tests were made on ten grains of the same size and on all grain sizes. The values of resistance to indirect traction are presented in *figure 10* as are the dispersions obtained for each grain size. The tests performed by Billam (1971) as well as those by Ramamurthy et al. (1974) are also presented. For example, the value of resistance to indirect traction for a diameter corresponding to d_{50} is 20 MPa for the calcareous sand, whereas for the quartz sand, it is 200 MPa.

We have simulated laboratory tests on isotropic and triaxial paths in a large range of stresses. We have tried to reproduce the influence of grain ruptures which were produced during the isotropic phase as well as during the shearing phase. The determination of the parameters of the model was made from tests performed under small mean stresses. We have used the correlations proposed by Hicher and Rahma (1994) in order to determine a set of initial parameters which has been adjusted on triaxial test results at small stresses. All the parameters, without exception, were maintained constant for all the tests and are given in *table III*.

We can therefore take into account the effect of the grain rupture with the help of two parameters B_k (deviatoric mechanism) and B_4 (isotropic mechanism). *Figure 11* illustrates the influence of parameter B_k on a drained triaxial path at lateral stresses equal to 1 MPa on the calcareous sand. We can note first of all that the set of parameters defined for the small stresses (50 KPa) and which characterises the initial material can no longer propose a good simulation for this level of mean stress. The evolution of the grain size distribution curve (*figure 12*) shows effectively a significant change in the grain sizes. We also notice that an appropriate

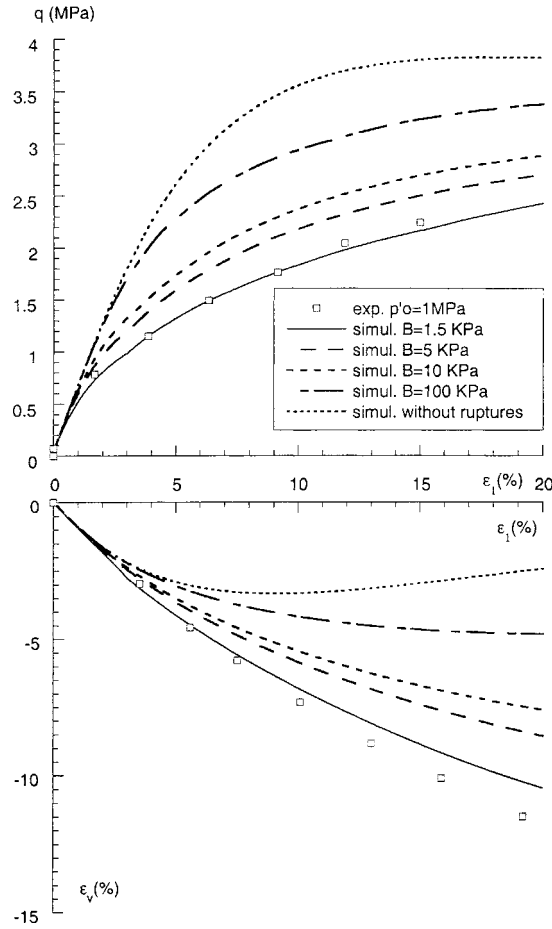


Figure 11. Influence of grain breakage amplitude in a numerical simulation of a triaxial test on calcareous sand at $p'_0 = 1$ MPa.

value of parameter B_k (obtained by curve fitting) allows us to simulate well the experimental results for both the stress–strain relationship (q – ε_1 plane) and the volume change (ε_1 – ε_v plane).

When we increase the amount of grain breakage, the peak in the stress–strain curve given by the model without grain breakage vanishes, the material becomes more ductile and the axial strain corresponding to the maximum value of the stress deviator increases. We can note in *figure 11* that the dilatancy phenomenon vanishes with the increase of breakage. The volumetric strain reaches the value noted during the test. Let us point out that at the end of the simulation the volumetric strain increases from 2% (without breakage) to 10% (with breakage).

The same can be said for the isotropic stress paths thanks to the choice of the parameter B_4 . In *figures 13* and *14* we present the isotropic consolidation curves up to a stress value of 15 MPa. The value of B_4 obtained for the quartzic sand and for the calcareous sand are respectively 1.5 MPa and 2 KPa. We notice that in the case of the Hostun sand very little breakage occurs during this phase whereas a great number of grains are crushed in the case of the calcareous sand (see the grain size distribution curves in *figure 12*). In the first case, the volumetric strain obtained is 8% while for the calcareous sand, this value rises to 30%.

This increase in the consolidation slope is here directly tied to the quantity of grain breakage obtained during loading. For the Hostun sand, its value is close to the values usually obtained for granular materials

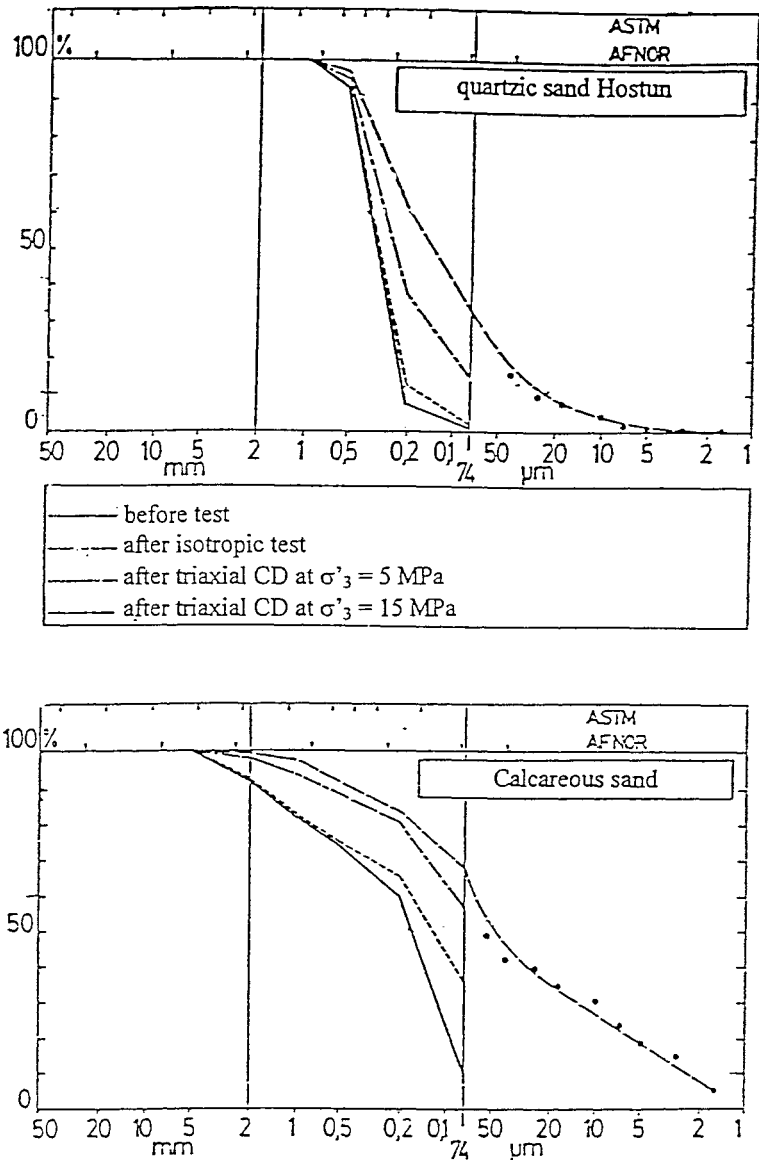


Figure 12. Grain size distribution curves before and after tests on Hostun and calcareous sands.

$C_c \approx 0.1-0.2$ (Biarez and Hicher, 1994). On the other hand, for the calcareous sand, it is clearly higher $C_c = 0.48$. This high value is a consequence of very great grain breakage (Biarez and Hicher, 1994) and (Daouadji and Hicher, 1997). We can state that given a well-chosen value B_4 , the modified model is capable of reproducing the experimental results.

It is therefore necessary to simulate the isotropic consolidation phase preceding shearing since there may be the presence of significant grain breakage and the triaxial tests cannot be simulated in a satisfactory manner without taking into account this phase.

We present in *figure 15* the experimental results and the simulations of four triaxial tests performed on calcareous sand with lateral stresses between 50 KPa and 1 MPa ($B_k = 1.5$ KPa). We can state that the modified

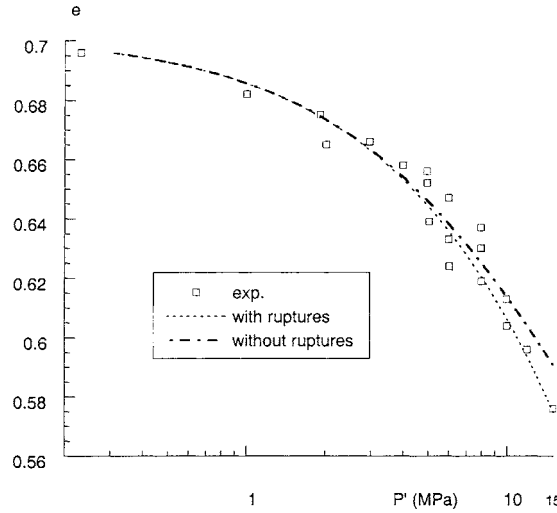


Figure 13. Isotropic test on Hostun sand. Comparison between numerical and experimental results (p' up to 15 MPa).

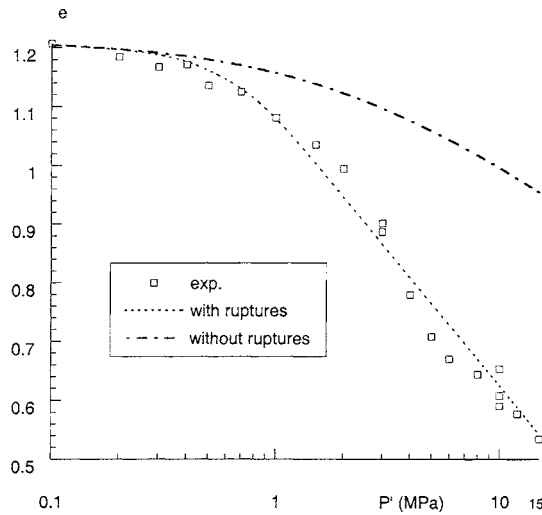


Figure 14. Isotropic test on calcareous sand. Comparison between numerical and experimental results (p' up to 15 MPa).

model is capable of taking into account the behaviour of granular materials at both small stresses where it is strongly dilatant as well as at higher stresses under which the materials become very contractant due to the effect of numerous grain ruptures provoking a significant variation of the grain size distribution.

Simulations of fine Hostun sand are shown in *figure 16*. Simulations without breakage lead to results in the $q-\varepsilon_1$ plane which approximate the experimental curves. On the other hand, values obtained in the $\varepsilon_v-\varepsilon_1$ plane for simulations under confining pressure greater than 3 MPa are much smaller than values obtained during testing. For a confining pressure under 2 MPa, little grain breakage occurs as we can see from the evolution of the percentage of fine material (*figure 12*). With grain breakage, we can simulate more correctly triaxial tests up to 15 MPa. This test is presented in *figure 17* with ($B_k = 0.7$ MPa) and without grain breakage. In the case of grain breakage, we observe that for 20% of axial strain, we can reduce the deviatoric stress for more than 5 MPa and increase significantly the volumetric strain.

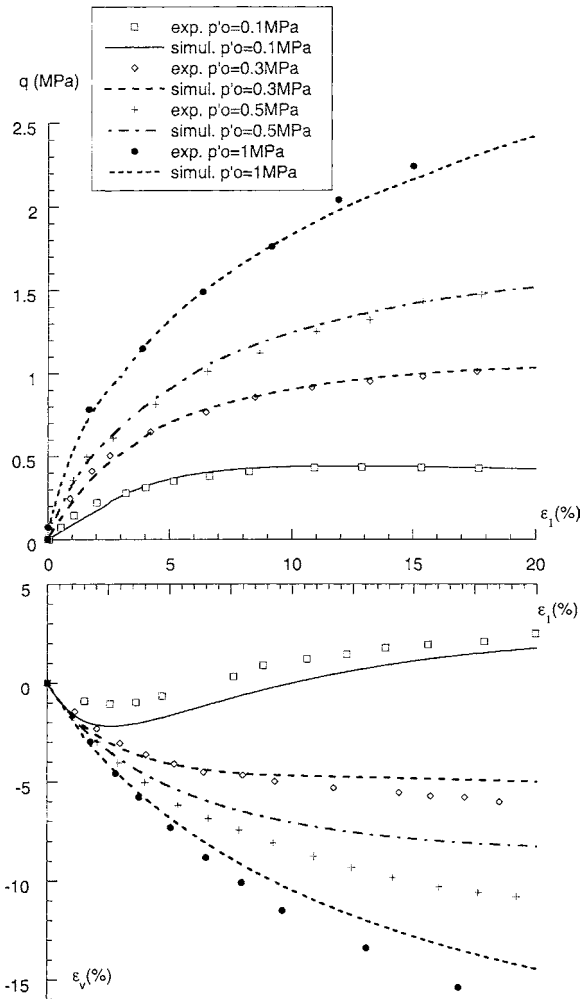


Figure 15. Triaxial test on calcareous sand. Comparison between numerical and experimental results ($0.05 \leq p'_0 \leq 1$ MPa).

Contrary to tests on calcareous sand which require little energy in order to induce grain breakage, tests on fine Hostun sand show that several MPa in confinement are needed in order to induce the noticeable influence of grain breakage.

9. Conclusion

In establishing the correlations between the characteristics of the discontinuous medium and the mechanical behaviour of the equivalent continuous medium, we find it possible to study the role of grain breakage, particularly of an elasto-fragile type breakage.

The parameter which best explains this phenomenon is the evolution of the grain size distribution represented by the value of the coefficient of uniformity d_{60}/d_{10} , which exerts a strong influence on the position of the isotropic consolidation line and of the perfect plastic one in the e - $\text{Log } p$ plane.

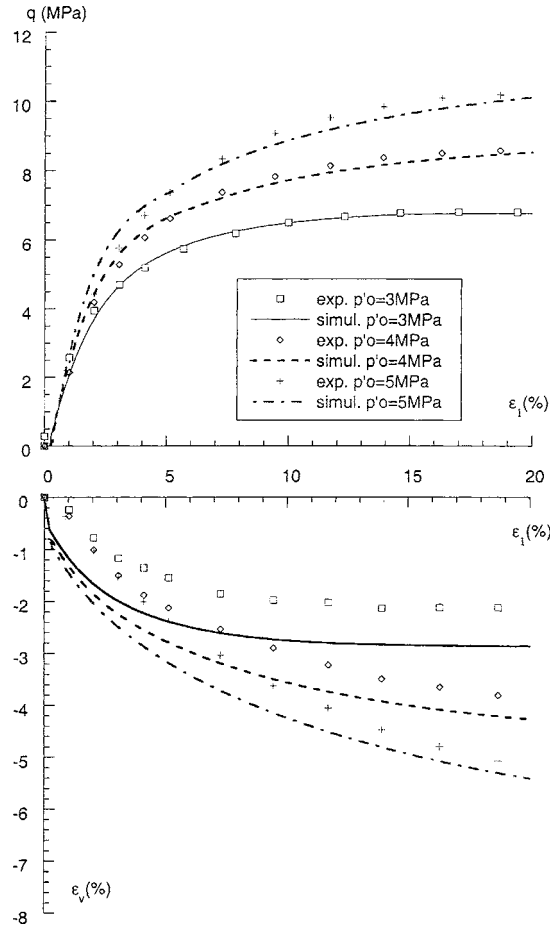


Figure 16. Triaxial test on Hostun sand. Comparison between numerical and experimental results ($3 \leq p'_0 \leq 5$ MPa).

In particular, an increase of the coefficient of uniformity corresponds to a slipping of these relations towards the small void ratios. Grain breakage, in provoking an increase of d_{60}/d_{10} , consequently increases, on the one hand, the compressibility of granular material on a consolidation path (isotropic or one-dimensional compression), and on the other hand, the contractancy on a deviatoric path. This increase in contractancy is accompanied by an increase in ductility.

The grain breakage influence has been introduced in an elastoplastic model by making the critical state line dependent on the evolution of the grain size distribution. This evolution is computed by means of a function of the plastic work, which depends on the initial nature parameters of the granular material.

In modifying the model in this manner, we may simulate the behaviour of granular materials under a large range of stresses when there is an evolution of the physical and mechanical characteristics of grains.

A primary validation of this approach has been done on two different types of sands, a quartzic and calcareous one and on various loading paths with different stress intensities. The results we obtained reproduced with a sufficient accuracy the results of the laboratory tests.

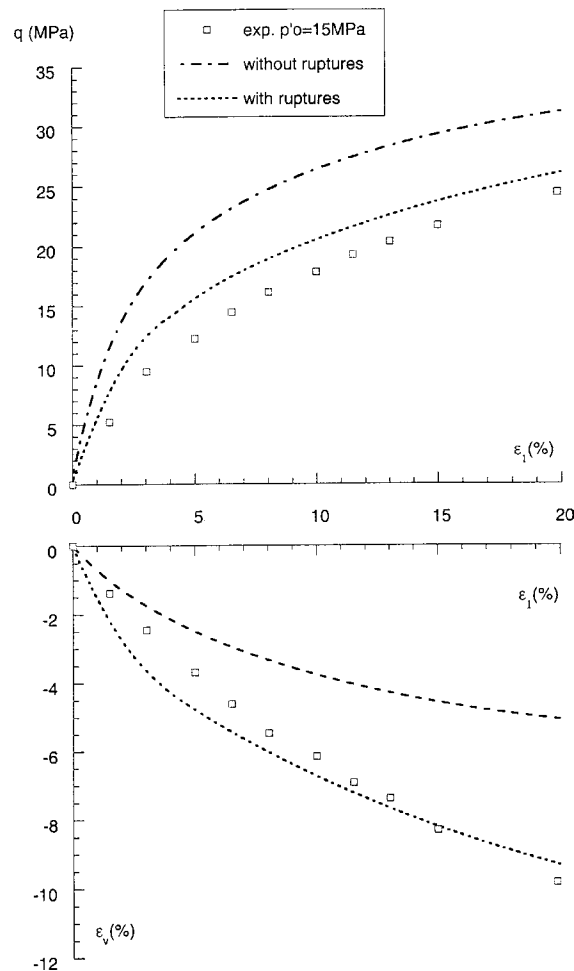


Figure 17. Triaxial test on Hostun sand. Comparison between numerical and experimental results ($p'_0 = 15$ MPa).

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