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# Extension of Mori–Tanaka Approach to Hygroelastic Loading of Fiber-reinforced Composites – Comparison with Eshelby–Kröner Self-consistent Model

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**ABSTRACT:** The scale transition model historically proposed by Mori and Tanaka to predict the average and local elastic behavior of heterogeneous structures is extended to hygro-elastic load for the first time. Explicit constitutive laws satisfying the fundamental assumptions of the model are given for the determination of the effective macroscopic coefficients of moisture expansion (CME) in composite structures by considering a jump in moisture content between the fiber and the matrix. Explicit forms are also given for the calculation of local (fiber and matrix scale) internal stress states from the localization of the macroscopic hygro-mechanical states (ply scale).

Comparisons for several compositions of composite structures (volume fraction of the constituents, i.e., the epoxy matrix and the reinforcing fibers) are performed between the numerical predictions given by the Mori–Tanaka model extended to hygro-elastic load and the recently proposed Eshelby–Kröner self-consistent hygro-elastic model. Discrepancies in the calculations appearing with an increase of the reinforcing fiber volume fraction are extensively discussed to conclude on the limitations of the micro-mechanical approach developed in the present work.

**KEY WORDS:** Mori–Tanaka model, Eshelby–Kröner self-consistent model, hygroelastic stresses, fiber-reinforced composites.

## INTRODUCTION

**F**OR THE LAST twenty years, composite laminates have been considered as an interesting substitute to metallic or polymeric materials, in the cases when high strength-to-weight ratio as well as corrosion and fatigue resistance are required in engineering applications. Since carbon/epoxy composites can absorb a significant amount of water and exhibit heterogeneous coefficients of moisture expansion (CME) (i.e., the CME of the epoxy matrix is vastly different from the CME of the carbon fibers), local stresses occur from the hygro-elastic loading of composite structures. Scale transition models providing formalism for the calculation of these local mechanical states are required to estimate a possible damage occurrence in such materials exposed to hygroscopic environments.

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In earlier works, the Eshelby–Kröner self-consistent model [1,2] was extended to hygro-elastic loads [3]. Nevertheless, the programming of this model remains complex, due to the implicit determination of the effective macroscopic properties of the plies, even if closed-form solutions could be expressed for the local mechanical states in very specific cases [4]. Now, in practice, another scale transition model, also based on the mathematical work of Eshelby [1], but providing explicit forms for both the effective macroscopic properties of the plies and the local mechanical states is often preferred to treat the case of composite structures: the Mori and Tanaka approach [5]. This model, first introduced for pure elastic load, has been extended to plastic and thermal loads too: examples of numerical applications of this model to the case of composite structures can be found in [6–8] (elastic-plastic modeling of metal and polymer matrix composites) and [9] (determination of thermal stresses in metal matrix composites). Until now, the available scientific works based on the Mori and Tanaka hypotheses do not provide the possibility to apply this model to treat the cases of composites submitted to hygroscopic loads.

First, a rigorous extension of the Mori and Tanaka model to the case of hygro-elastic load is described in this study. The determination of both the effective macroscopic hygro-elastic properties (the elastic stiffness and the CME) and the local mechanical states is given in detail. The next section shows comparisons of the numerical results obtained with either the Mori–Tanaka approach or the recent Eshelby–Kröner model extended to hygro-elastic loads as a function of the volume fraction of reinforcing fibers constituting the plies. The comparison of the predicted effective macroscopic properties and mechanical states (at both the macroscopic and the local scales) are discussed in the same section: in particular, the limit of validity of the Mori and Tanaka approach to the cases of dilute solutions (small fiber volume fractions) underlined by Benveniste in pure elasticity [10] is confirmed for hygro-elastic loads.

## SCALE TRANSITION MODELING FOR THE HYGRO-ELASTIC BEHAVIOR OF COMPOSITES

### Introduction

Scale transition models are based on a multi-scale representation of materials. In the case of composite materials, for instance, a two-scale model is sufficient:

- (a) The averaged behavior of a ply, defines the macroscopic scale of the model. It is denoted by the superscript ‘I’.
- (b) The properties and mechanical states of either the matrix or fibers are indicated by the superscripts ‘m’ and ‘f’, respectively. These constituents define the so-called ‘pseudo-macroscopic’ scale of the material [11].

In general, scale transition models are assumed to satisfy the following relations on the stresses ( $\sigma$ ) and strains, ( $\varepsilon$ ) demonstrated by Hill [12]:

1. Conditions over the averages of local stresses and strains,

$$\begin{aligned} \langle \sigma^\alpha \rangle_{\alpha=f,m} &= \sigma^I \\ \langle \varepsilon^\alpha \rangle_{\alpha=f,m} &= \varepsilon^I. \end{aligned} \tag{1}$$

In Equation (1), the brackets  $\langle \rangle$  stand for volume average operations. Introducing  $v^f$  and  $v^m$  as the volume fractions of the fibers and matrix (resin) in the composite structure it becomes, as an example, for the stresses:

$$\langle \sigma^\alpha \rangle_{\alpha=f,m} = \sum_{\alpha} v^\alpha \sigma^\alpha = v^m \sigma^m + v^f \sigma^f. \quad (2)$$

2. Hill's theorem:

$$\langle \sigma^\alpha : \varepsilon^\alpha \rangle_{\alpha=f,m} = \langle \sigma^\alpha \rangle_{\alpha=f,m} : \langle \varepsilon^\alpha \rangle_{\alpha=f,m} = \sigma^I : \varepsilon^I. \quad (3)$$

Let us introduce  $\mathbf{B}$  as the elastic stress concentration tensor linking the local stresses to the macroscopic ones in pure elasticity:

$$\sigma^\alpha = \mathbf{B}^\alpha : \sigma^I. \quad (4)$$

Taking into account (4) in (3) enables to demonstrate the following useful relation, after elementary tensorial calculation:

$$\sigma^I : \varepsilon^I = \langle \sigma^\alpha : \varepsilon^\alpha \rangle_{\alpha=f,m} = \langle \mathbf{B}^\alpha : \sigma^I : \varepsilon^\alpha \rangle_{\alpha=f,m} = \left\langle \sigma^I : \mathbf{B}^{\alpha T} : \varepsilon^\alpha \right\rangle_{\alpha=f,m} = \sigma^I : \left\langle \mathbf{B}^{\alpha T} : \varepsilon^\alpha \right\rangle_{\alpha=f,m}. \quad (5)$$

$\sigma^I$  appearing in both the right and the left members of Equation (5), the macroscopic strain generally satisfies:

$$\varepsilon^I = \left\langle \mathbf{B}^{\alpha T} : \varepsilon^\alpha \right\rangle_{\alpha=f,m}. \quad (6)$$

Now, under hygro-elastic loads, macroscopic and local strains are separated into two parts: the elastic strains and the hygroscopic strains, respectively denoted in the following by the subscripts 'el' and 'hy':

$$\begin{aligned} \varepsilon^I &= \varepsilon_{el}^I + \varepsilon_{hy}^I \\ \varepsilon^\alpha &= \varepsilon_{el}^\alpha + \varepsilon_{hy}^\alpha. \end{aligned} \quad (7)$$

The elastic strains are linearly related to the stress states through the stiffness tensor  $\mathbf{L}$ , whereas the hygroscopic dilatation generated by a moisture content increment  $\Delta C$  is treated as a transformation strain, using an approach similar to the one developed in order to take into account thermal strains (see for instance [13]) except that (i) the coefficients of moisture expansion (CME)  $\beta$  replaces the coefficients of thermal expansion (CTE) and (ii) the moisture contents  $\Delta C^I$ ,  $\Delta C^m$ , and  $\Delta C^f$  are strongly different. It finally becomes:

$$\begin{aligned} \varepsilon^I &= \varepsilon_{el}^I + \varepsilon_{hy}^I = \mathbf{L}^{I^{-1}} : \sigma^I + \beta^I \Delta C^I \\ \varepsilon^\alpha &= \varepsilon_{el}^\alpha + \varepsilon_{hy}^\alpha = \mathbf{L}^{\alpha^{-1}} : \sigma^\alpha + \beta^\alpha \Delta C^\alpha. \end{aligned} \quad (8)$$

## General Forms for the Macroscopic Hygro-elastic Properties of Heterogeneous Composite Structures

Within scale transition modeling, the local properties of the  $\alpha$ -superscripted constituents are usually considered to be known (i.e.,  $L^\alpha$  and  $B^\alpha$ ), whereas the effective macroscopic properties of the composite structure are *a priori* unknown and result from computations. Relations (6) and (7) which should be satisfied for any load, provide two general relations enabling the determination of the missing macroscopic stiffness and CME:

1. In order to determine the effective macroscopic elastic stiffness of an heterogeneous structure through scale transition models, the case of a purely elastic load is generally considered [13]. As a consequence, the moisture contents are null:  $\Delta C^I = \Delta C^\alpha = 0$ . As a result, the hygroscopic part of the strains vanishes and the pseudo-macroscopic strains can be written as:  $\varepsilon^\alpha = \varepsilon_{el}^\alpha = L^{\alpha^{-1}} : \sigma^\alpha = L^{\alpha^{-1}} : B^\alpha : \sigma^I$ . Introducing these conditions in (8) yields the following form of Equation (6):

$$\begin{aligned} \varepsilon^I &= \varepsilon_{el}^I = L^{I^{-1}} : \sigma^I \\ &= \left\langle B^{\alpha^T} : \varepsilon^\alpha \right\rangle_{\alpha=f,m} = \left\langle B^{\alpha^T} : \varepsilon_{el}^\alpha \right\rangle_{\alpha=f,m} = \left\langle B^{\alpha^T} : L^{\alpha^{-1}} : \sigma^\alpha \right\rangle_{\alpha=f,m} = \left\langle \sigma^\alpha : L^{\alpha^{-1}} : B^\alpha \right\rangle_{\alpha=f,m}. \end{aligned} \quad (9)$$

Besides, Hill's theorem (3) can be written as:

$$\left\langle \sigma^\alpha : \varepsilon^\alpha \right\rangle_{\alpha=f,m} = \left\langle \sigma^\alpha : L^{\alpha^{-1}} : B^\alpha : \sigma^I \right\rangle_{\alpha=f,m} = \langle \sigma^\alpha \rangle_{\alpha=f,m} : \langle \varepsilon^\alpha \rangle_{\alpha=f,m} = \langle \sigma^\alpha \rangle_{\alpha=f,m} : \left\langle L^{\alpha^{-1}} : B^\alpha : \sigma^I \right\rangle_{\alpha=f,m}. \quad (10)$$

Since  $\sigma^I$  is a constant, the following identity is obtained:

$$\left\langle \sigma^\alpha : L^{\alpha^{-1}} : B^\alpha \right\rangle_{\alpha=f,m} = \langle \sigma^\alpha \rangle_{\alpha=f,m} : \left\langle L^{\alpha^{-1}} : B^\alpha \right\rangle_{\alpha=f,m} = \sigma^I : \left\langle L^{\alpha^{-1}} : B^\alpha \right\rangle_{\alpha=f,m}. \quad (11)$$

Inserting (11) in (9) yields:  $L^{I^{-1}} : \sigma^I = \sigma^I : \left\langle L^{\alpha^{-1}} : B^\alpha \right\rangle_{\alpha=f,m}$ . Consequently, by identification, the elastic stiffness satisfies the relation:

$$L^I = \left\langle L^{\alpha^{-1}} : B^\alpha \right\rangle_{\alpha=f,m}^{-1}. \quad (12)$$

2. In order to determine the effective macroscopic CTE of an heterogeneous structure through scale transition models, the case of a purely thermal load is generally considered [13]. The problem of determining CME instead of CTE is rather similar,  $\beta$  and  $\Delta C$  replacing the CTE and temperature increment  $\Delta T$ , respectively. Thus, by analogy with the thermo-elastic case, finding out the effective macroscopic CME of an heterogeneous structure using the scale transition model implies to consider the case that a purely hygroscopic load is applied to the material. Consequently, the macroscopic stress is null:  $\sigma^I = 0$ . As a result, the elastic part of the macroscopic strains vanishes:  $\varepsilon_{el}^I = 0$ , and  $\varepsilon^I = \varepsilon_{hy}^I = \beta^I \Delta C^I$ . Besides, since  $\varepsilon_{el}^I = 0$ ,

the following average has also to be null in Equation (6):  $\langle \mathbf{B}^{\alpha\top} : \varepsilon_{\text{cl}}^\alpha \rangle_{\alpha=f,m} = 0$ . Introducing these conditions in (8) yields the following form of Equation (6):

$$\begin{aligned} \varepsilon^{\text{I}} &= \beta^{\text{I}} \Delta C^{\text{I}} \\ &= \left\langle \mathbf{B}^{\alpha\top} : \varepsilon^\alpha \right\rangle_{\alpha=f,m} = \left\langle \mathbf{B}^{\alpha\top} : \varepsilon_{\text{hy}}^\alpha \right\rangle_{\alpha=f,m} = \left\langle \mathbf{B}^{\alpha\top} : \beta^\alpha \Delta C^\alpha \right\rangle_{\alpha=f,m}. \end{aligned} \quad (13)$$

From Equation (13), the identification of the general form providing the effective macroscopic CME of an heterogeneous structure is obvious:

$$\beta^{\text{I}} = \frac{1}{\Delta C^{\text{I}}} \left\langle \mathbf{B}^{\alpha\top} : \beta^\alpha \Delta C^\alpha \right\rangle_{\alpha=f,m}. \quad (14)$$

The forms (12) and (14) presented in this section are general: they depend neither on the choice of the scale transition model nor on the hygro-elastic load effectively applied on the structure (in the case that the material properties are assumed to be independent of the hygro-mechanical state). Actually, the constitutive hypotheses of the considered model for the purpose of performing calculations will provide specific (i.e., model dependent) relations for the stress localization tensor  $\mathbf{B}^\alpha$  involved in (12) and (14). The following subsection gives an illustration of the case in which the Mori and Tanaka approximation is taken into account.

### **Mori and Tanaka Model Extended to Hygro-elastic Loading of Composite Structures: Macroscopic Effective Properties and Local Stress States**

#### *INTRODUCTION TO MORI AND TANAKA MODEL: CONSTITUTIVE ASSUMPTIONS*

For simplicity, the case of two-phase composites only is considered here. Nevertheless, the approach detailed in the present subsection could be generalized to any number of phases. Similar to the case of the Kröner and Eshelby self-consistent model, the Mori and Tanaka theory is closely attached to the equivalent inclusion idea of Eshelby. In this framework, the material is divided into a matrix phase (in the present case, the epoxy matrix) and an inclusion phase (the carbon fibers), the last one being constituted of ellipsoidal particles either aligned or randomly oriented. No particular assumption is made on the geometry of the matrix. The two phases are possibly elastically and/or hygroscopically anisotropic.

In this theory, contrary to the case of the Eshelby–Kröner self-consistent model, the inclusions are not considered to be embedded directly in the effective model having the behavior of the composite structure: in the Mori and Tanaka approximation, the particles (fibers) are embedded in the matrix phase, itself being loaded at the infinite by the hygro-mechanical conditions applied on the composite structure. As a consequence, the inclusion phase does not experience any interaction with the macroscopic scale, but with the matrix only. Consequently, the Mori and Tanaka model corresponds to the direct extension of the Eshelby’s single inclusion model [2] to the extent that the volume fraction of inclusions does not remain infinitesimal anymore. Thus, the scale transition relations holding for Eshelby’s dilute model are assumed to be still valid within the Mori and Tanaka procedure. In particular, the stress

localization tensor  $\mathbf{B}^\alpha$  satisfies (an extensive demonstration, yielding an equivalent form, is available in [10]):

$$\mathbf{B}^\alpha = \mathbf{L}^\alpha : \mathbf{T}^\alpha : \langle \mathbf{L}^\alpha : \mathbf{T}^\alpha \rangle^{-1} \quad (15)$$

where the elastic strain localization tensor, usually denoted by  $\mathbf{T}^\alpha$  within the Mori and Tanaka approach, is written as:

$$\mathbf{T}^\alpha = [\mathbf{I} + \mathbf{E}^\alpha : (\mathbf{L}^\alpha - \mathbf{L}^m)]^{-1}. \quad (16)$$

In (16),  $\mathbf{I}$  stands for the fourth-order identity tensor. Hill's tensor  $\mathbf{E}^\alpha$  expresses the dependence of the localization tensor on the morphology assumed for the matrix and its reinforcements [14]. It can be expressed as a function of Eshelby's tensor  $\mathbf{S}_{\text{esh}}^\alpha$ , through  $\mathbf{E}^\alpha = \mathbf{S}_{\text{esh}}^\alpha : \mathbf{L}^m$ . In practice, the calculation of Hill's tensor for the inclusion phase  $\mathbf{E}^f$  only is necessary, since obvious simplifications of (16), leading to  $\mathbf{T}^m = \mathbf{I}$ , occur in the case when the matrix is considered.

### MACROSCOPIC EFFECTIVE PROPERTIES IN MORI AND TANAKA FRAMEWORK

In order to find the effective macroscopic properties of the plies constituting the composite structure, the specific stresses and strains localization tensors (15) and (16) have to be inserted in the general forms (12) and (14) previously obtained for both the macroscopic stiffness and the CME.

$$\mathbf{L}^I = \left\langle \mathbf{L}^{\alpha-1} : \mathbf{B}^\alpha \right\rangle_{\alpha=f,m}^{-1} = \left\langle \mathbf{T}^\alpha : \langle \mathbf{L}^\alpha : \mathbf{T}^\alpha \rangle_{\alpha=f,m}^{-1} \right\rangle_{\alpha=f,m}^{-1} \quad (17)$$

$$\beta^I = \frac{1}{\Delta C^I} \left\langle \mathbf{B}^{\alpha T} : \beta^\alpha \Delta C^\alpha \right\rangle_{\alpha=f,m} = \frac{1}{\Delta C^I} \left\langle (\mathbf{L}^\alpha : \mathbf{T}^\alpha : \langle \mathbf{L}^\alpha : \mathbf{T}^\alpha \rangle^{-1})^T : \beta^\alpha \Delta C^\alpha \right\rangle_{\alpha=f,m}. \quad (18)$$

Since the moisture concentration in the fiber is usually null, the introduction of this additional assumption (i.e.,  $\Delta C^f = 0$ ) provides the following simplifications of (18):

$$\beta^I = \nu^m \frac{\Delta C^m}{\Delta C^I} \mathbf{L}^m : \langle \mathbf{L}^\alpha : \mathbf{T}^\alpha \rangle_{\alpha=f,m}^{-1} : \beta^m. \quad (19)$$

Contrary to the case, previously detailed in [3,4], that the Eshelby–Kröner self-consistent model is used to estimate the macroscopic effective hygro-mechanical properties of composite structures, both the stiffness and the CME are explicit according to the Mori and Tanaka model.

### Local Hygro-elastic Stresses and Strains According to Mori and Tanaka Model

In this subsection, the macroscopic effective properties are assumed to be known from the numerical applications of Equations (18) and (19). Thus Equation (8) can be applied to find either the macroscopic stress or strain state, from the considered hygro-elastic load.

The main question to solve is to express the scale transition relation linking the macroscopic hygro-mechanical states to their corresponding local counterparts.

For reasons of analogies existing between these two kinds of loads, the scale transition relations that have previously been established for both pure elastic and pure thermal loads within the Mori and Tanaka framework will be derived to enable the treatment of the hygro-elastic load. In the case that a thermo-elastic problem is considered (i.e., a macroscopic load  $\{\sigma^I, \Delta T^I = \Delta T^m = \Delta T^f\}$  is applied), the local mechanical states are identical to the sum of the separated contributions to: (i) the pure elastic stress and (ii) the pure thermal load  $\Delta T^I$ :

1. According to the definition of the elastic stress localization tensor, the elastic part of the local stresses satisfies relation (4).
2. For a pure thermal load, the reinforcing phase is assumed to be subjected, at the infinite, to the stress state experienced by the embedding matrix of stiffness  $\mathbf{L}^m$ . The reaction tensor, modulating the stress reaction induced by the strains deviation between the fiber inclusion and the matrix is equal to  $\mathbf{S}_{\text{esh}}^{f^{-1}} - \mathbf{I}$ . The strains deviation in itself corresponds to the sum of both the thermal expansion  $\Delta \varepsilon_T$  and the elastic stiffnesses  $\Delta \varepsilon_S$  mismatches, which respectively reads:

$$\Delta \varepsilon_T = M^f \Delta T^f - M^m \Delta T^m = (M^f - M^m) \Delta T^I \quad (20)$$

$$\Delta \varepsilon_S = \left( L^{f^{-1}} - L^{m^{-1}} \right) : \sigma^f \quad (21)$$

where  $\mathbf{M}$  stands for the CTE.

Consequently, according to the classical formalism of Eshelby [2], the stress in the inclusion (fiber) submitted to a thermal treatment satisfies the following general form:

$$\begin{aligned} \sigma^f - \sigma^m &= -L^m : \left( S_{\text{esh}}^{f^{-1}} - \mathbf{I} \right) : [\Delta \varepsilon_T + \Delta \varepsilon_S] \\ &= -L^m : \left( S_{\text{esh}}^{f^{-1}} - \mathbf{I} \right) : \left[ (M^f - M^m) \Delta T^I + \left( L^{f^{-1}} - L^{m^{-1}} \right) : \sigma^f \right]. \end{aligned} \quad (22)$$

By analogy with the Mori–Tanaka thermo-elastic model, the expression enabling to determination of the local stresses and strains experienced by the matrix and the fiber when a macroscopic load  $\{\sigma^I, \Delta C^I\}$  is applied on a given ply can be provided by the sum of the separated response of the elementary constituents of the ply to: (i) the pure elastic load  $\sigma^I$  and (ii) the pure hygroscopic load  $\Delta C^I$ :

1. Identical to (i) the elastic part of the local stresses has to satisfy relation (4).
2. The treatment of a pure hygroscopic load is analogous to the case of a pure thermal load (ii). The only modification concerns the strains deviations. In the present case, the thermal mismatch has to be replaced by the corresponding hygroscopic mismatch  $\Delta \varepsilon_H$ . It means that Equation (20) transforms as follows (whereas the contribution due to elastic stiffnesses mismatch (21) remains unchanged):

$$\Delta \varepsilon_H = \beta^f \Delta C^f - \beta^m \Delta C^m. \quad (23)$$

As a consequence, the average stress in the inclusion (fiber) of a composite structure submitted to a pure hygroscopic load satisfies the following general form deduced from (22), after replacement of term (20) by (23):

$$\sigma^f - \sigma^m = -L^m : \left( S_{\text{esh}}^{f^{-1}} - \mathbf{I} \right) : \left[ \beta^f \Delta C^f - \beta^m \Delta C^m + \left( L^{f^{-1}} - L^{m^{-1}} \right) : \sigma^f \right]. \quad (24)$$



Besides, for a purely hygroscopic load,  $\langle \sigma^\alpha \rangle_{\alpha=f,m} = 0$ , thus,  $\sigma^f = (-v^m/v^f)\sigma^m$ . Moreover,  $\Delta C^f = 0$ . After simplifying Equation (24), the local hygroscopic stresses are obtained:

$$\sigma^m = v^m B^m : \left[ I - L^m : \left( S_{\text{esh}}^{f^{-1}} - I \right) : \left( L^f - L^m \right) \right]^{-1} : L^m : \left( S_{\text{esh}}^{f^{-1}} - I \right) : \beta^m \Delta C^m \quad (25)$$

$$\sigma^f = (v^f B^f - I) : \left[ I - L^m : \left( S_{\text{esh}}^{f^{-1}} - I \right) : \left( L^f - L^m \right) \right]^{-1} : L^m : \left( S_{\text{esh}}^{f^{-1}} - I \right) : \beta^m \Delta C^m. \quad (26)$$

The hygroscopic stresses are obviously self-equilibrated (this can be easily verified, since the elastic stress concentration tensor has the following main property:  $v^f B^f + v^m B^m = I$ ) and are directly proportional to the moisture content of the matrix.

Finally, the hygro-elastic pseudo-macroscopic stresses in the Mori and Tanaka approach are, as previously explained, expressed by the sum of the elastic (4) and the hygroscopic local stresses ((25) and (26)):

$$\sigma^m = B^m : \sigma^I + v^f B^m : \left[ I - L^m : \left( S_{\text{esh}}^{f^{-1}} - I \right) : \left( L^f - L^m \right) \right]^{-1} : L^m : \left( S_{\text{esh}}^{f^{-1}} - I \right) : \beta^m \Delta C^m \quad (27)$$

$$\sigma^f = B^f : \sigma^I + (v^f B^f - I) : \left[ I - L^m : \left( S_{\text{esh}}^{f^{-1}} - I \right) : \left( L^f - L^m \right) \right]^{-1} : L^m : \left( S_{\text{esh}}^{f^{-1}} - I \right) : \beta^m \Delta C^m. \quad (28)$$

The local strains are evaluated from Equations (27) and (28) through the constitutive relation (8).

### NUMERICAL COMPARISONS BETWEEN MORI–TANAKA AND ESHELBY–KRÖNER SELF-CONSISTENT MODELS EXTENDED TO HYGRO-ELASTIC LOADS

The present section aims to estimate the reliability of the Mori and Tanaka model for the accurate prediction of both the overall effective behavior of composite structures and the calculation of local stress states in its elementary constituents. As explained in the subsection on Mori–Tanaka constitutive assumptions, the Mori–Tanaka approach assumes that the formalism demonstrated by Eshelby for the rigorous treatment of a single inclusion, the volume fraction of which tends toward zero, remains true for any volume fraction of inclusions. In practice, the Mori–Tanaka approach is built on the rough hypothesis that the reinforcement is embedded in a matrix, the thickness of which is relatively large in comparison with the thickness of the reinforcement itself. This condition involves that the embedded fibers can be considered not to directly interact with the effective medium (macroscopic scale), but with the epoxy matrix only. It was demonstrated by Benveniste in [10], for the pure elastic case, that this hypothesis was not always true, for instance, in polycrystals containing cracks. Now, in the case when carbon epoxy composites are considered, the Mori–Tanaka fundamental hypothesis can be highly expected not to be fulfilled, since the thicknesses of the fibers and the surrounding matrix often reach the same order of magnitude: thus, it is not exact to assume that the reinforcing inclusions do interact with the embedding matrix only and not at all with the macroscopic effective medium (ply). Since the Mori and Tanaka procedure is, for the first time, extended to hygro-elastic load of composite structures in the present work, it is necessary

to estimate the domain of validity of this model in terms of volume fraction of the inclusion phase (precisely, the carbon fibers).

To attain this goal, the numerical predictions obtained through the Mori and Tanaka approximation will be compared to the more rigorous Eshelby–Kröner self-consistent approach previously presented in [3,4], which properly extends Eshelby’s fundamental work to the case when the volume fraction of inclusions is not infinitesimal any more.

### Comparison between Macroscopic Effective Hygro-elastic Properties of the Plies

Both the Mori and Tanaka approach and the Eshelby–Kröner self-consistent hygro-elastic models [3,4] have been used to determine both the macroscopic elastic stiffness and the coefficients of moisture expansion of carbon/epoxy composites T300/5208 as a function of T300 carbon fiber volume fraction, starting from  $v^f=0\%$  up to  $v^f=60\%$ . The behavior of each composite is governed by its constituents, i.e., the properties of the fibers, the surrounding matrix, and the assumed morphology for the inclusions. Calculations were performed taking into account the local properties listed in Tables 1 and 2, for elastic constants and CME, respectively. In order to take into account the proper fiber microstructure of the material, the following values were taken for the length of the semi-axis of the inclusions:  $a_2=a_3=1$ , and  $a_1 \rightarrow \infty$ , where the subscripts 1, 2, 3 stand for the longitudinal, transverse, and normal directions of the fibers, respectively. Closed-form solutions for Hill’s tensor corresponding to this specific morphology is provided in [4] for the Eshelby–Kröner model. In the case when the Mori and Tanaka procedure is considered, Hill’s tensor is calculated for an inclusion embedded in the epoxy matrix instead of the effective macroscopic medium, as is the case within the Eshelby–Kröner approach. As a consequence, any  $L_{ij}^I$  involved in the analytical form of Hill’s tensor established for infinite fibers in the Eshelby–Kröner model (Equation (8) of [4]) has to be replaced by the corresponding  $L_{ij}^m$  in order to apply the Mori and Tanaka model. To estimate the macroscopic CME, the ratio between the moisture content of the epoxy matrix and that of the ply, i.e.,  $\Delta C^m/\Delta C^I$  is required. When the equilibrium state is reached, the maximum moisture content of the neat resin  $\Delta C^m$  may actually be estimated from the maximum moisture content of the composite  $\Delta C^I$ . By assuming that the fibers

**Table 1. Mechanical properties of the constituents of the studied composites.**

	$E_1$ (GPa)	$E_2, E_3$ (GPa)	$\nu_{12}, \nu_{13}$	$G_{23}$ (GPa)	$G_{12}$ (GPa)
T300 fibers [15]	230	15	0.2	7	15
N 5208 epoxy matrix [16]	4.5	4.5	0.4	1.6	1.6

**Table 2. CME for the components of T300/5208 composite.**

	$\beta_{11}$	$\beta_{22}, \beta_{33}$
T300 fibers [15]	0	0
N 5208 epoxy matrix [16]	0.6	0.6

do not absorb any moisture,  $\Delta C^I$  and  $\Delta C^m$  are related by the expression given by Loos and Springer [17]:

$$\Delta C^I = \Delta C^m W^m \quad (29)$$

where  $W^m$  is the weight fraction (percent) of the resin in the composite.

The preceding equation develops as follows:

$$\begin{cases} \Delta C^m = \frac{c^m}{\rho^m} \\ \Delta C^I = \frac{c^I}{\rho^I} \\ c^I = (1 - v^f)c^m \end{cases} \Rightarrow \frac{\Delta C^m}{\Delta C^I} = \frac{c^m \rho^I}{c^I \rho^m} = \frac{c^m \rho^I}{(1 - v^f)c^m \rho^m} = \frac{\rho^I}{(1 - v^f)\rho^m} \quad (30)$$

$c$  and  $\rho$  are the moisture concentration and the mass density of the dry material, respectively. The parameters and properties considered during the computations are:  $\rho^m = 1200 \text{ kg/m}^3$ ,  $\rho^f = 1867 \text{ kg/m}^3$ , and  $c^I = 1.5\%$ . Besides, the macroscopic density is obviously given by:

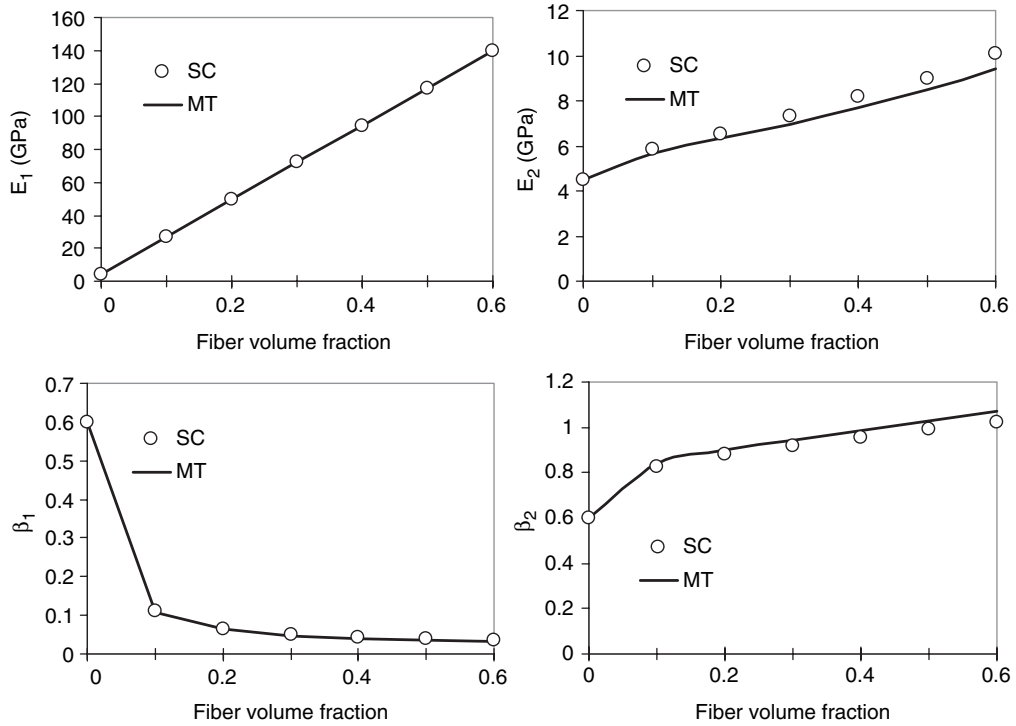
$$\rho^I = v^m \rho^m + v^f \rho^f. \quad (31)$$

The numerical results obtained for both the longitudinal and the transverse macroscopic Young's moduli and the CME have been presented in Figure 1. Graphics displayed in this figure show the following remarkable facts:

1. The Eshelby–Kröner and Mori–Tanaka hygro-elastic models do not predict identical macroscopic stiffnesses and CME for composite structures (in particular, the transverse Young's modulus and CME exhibit significant deviations from one model to the other).
2. The discrepancies existing between the calculated macroscopic hygro-mechanical behavior, according to the Eshelby–Kröner model or the Mori–Tanaka approximation, depend on the fiber volume fraction: they are obviously null for a material containing 0% carbon fibers, and increase with the volume fraction of carbon fibers.
3. Nevertheless, these discrepancies of the hygro-mechanical properties are not critical: the evolution of some constants, like the longitudinal Young's modulus  $E_1$  and the CME  $\beta_1$  as a function of the volume fraction of carbon fiber is almost identical. That is, the effective axial modulus follows, in first approximation, a rule of mixtures (its evolution as a function of fiber volume fraction is more or less linear). As a result, since they yield very similar numerical results, there is no fundamental reason (except the fact that the Mori–Tanaka procedure constitutes only an approximation of the Eshelby–Kröner rigorous mathematical treatment of the inclusion problem) to prefer the Eshelby–Kröner model to the Mori–Tanaka approach in the case that the effective macroscopic hygro-elastic properties of composite structures have to be estimated from the knowledge of the behavior of their elementary constituents.

### Comparison between the Local Mechanical States

In order to check the agreement between the local mechanical states predicted by the Mori–Tanaka and the Eshelby–Kröner models in both the epoxy resin and the carbon



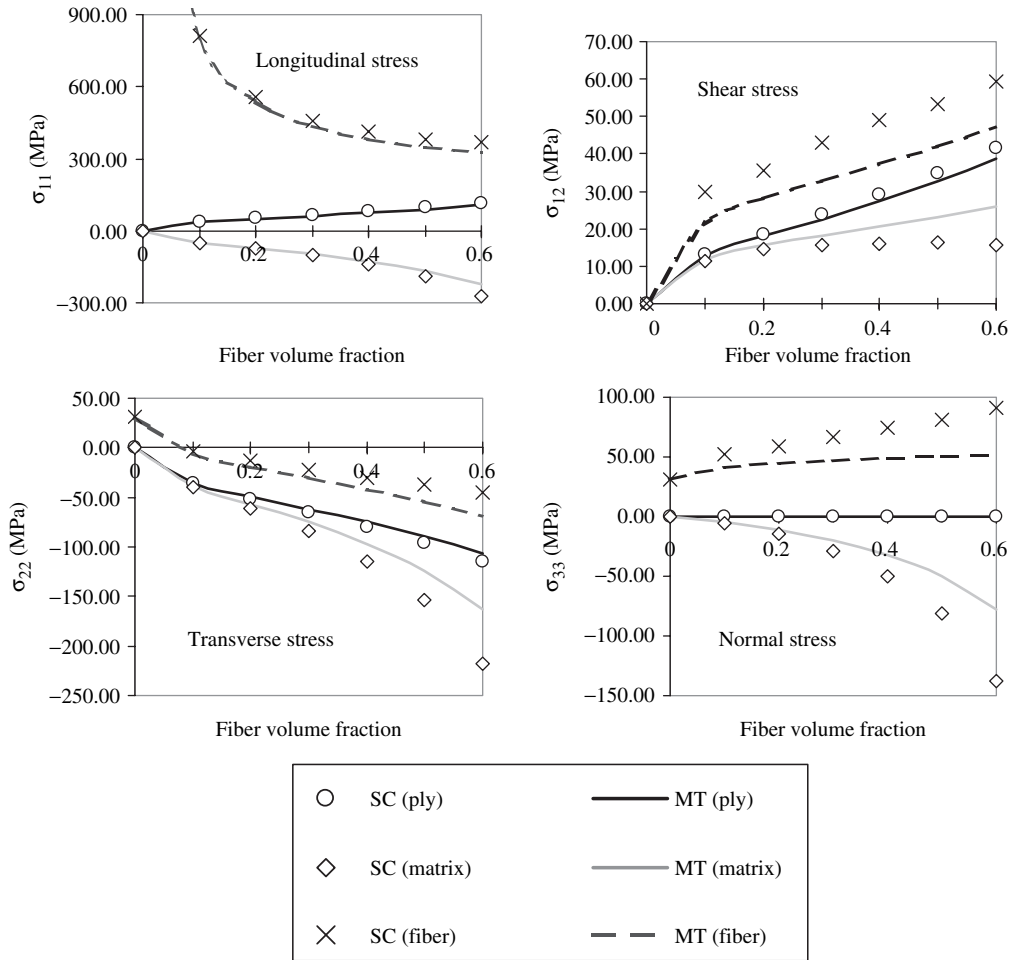
**Figure 1.** Evolution of longitudinal and transverse macroscopic elastic moduli and CME for T300/5208 composites. Comparison between Mori–Tanaka and Eshelby–Kröner models.

fiber of the plies of composite structures submitted to hygroscopic load, further computations were done.

Thin laminated composite pipes, with thickness 4 mm, of T300/5208 carbon-epoxy plies are considered for the determination of the macroscopic stresses. Though initially dry, they are then exposed to an ambient fluid. The closed-form formalism used to determine the mechanical stresses and strains in each ply of the structure is described by Jacquemin and Vautrin [18]. The macroscopic effective hygro-mechanical properties necessary to perform these calculations correspond to the results of the computations presented in the previous subsection.

The macroscopic mechanical states were calculated in a  $[55^\circ/-55^\circ]_S$  laminated cylinder, at the equilibrium of the moisture concentration, i.e., the time when the space dependent macroscopic moisture concentration becomes identical to the moisture concentration corresponding to the boundary conditions:  $c^I(r) = 0.015$ . Thereafter, the local mechanical states were determined, in the central ply of the laminate (in this ply, the macroscopic stresses are almost constant), using for instance Equations (27) and (28) for the Mori–Tanaka model. The equivalent equations satisfied in the Eshelby–Kröner approach can be found in [3,4]. The evolution of the non-zero stress components at the macroscopic (ply) and the local (epoxy resin and carbon fibers) scales, as a function of the proportion of carbon fiber in the structure, is depicted in Figure 2. The main observations are:

1. The macroscopic stresses estimated using continuum mechanics formalism from the effective macroscopic properties determined according to either the Eshelby–Kröner or



**Figure 2.** Macroscopic and local stresses in T300/5208 composites for the central ply of  $[+55^{\circ}/-55^{\circ}]_S$  symmetric laminate, as a function of carbon fiber volume fraction. Comparison between Eshelby–Kröner and Mori–Tanaka predictions.

the Mori–Tanaka models are very close together. The discrepancies occurring in  $\sigma^I$  components increase proportionally to the fiber volume fraction. The explanation of this is directly related to the increasing discrepancies as a function of the fiber concentration between the macroscopic hygro-elastic properties previously observed in the subsection ‘Comparison between Macroscopic Effective Hygro-elastic Properties of the Plies’. Consequently, the same conclusion as before holds: the Mori–Tanaka approximation is in good agreement with the Eshelby–Kröner model for calculating input values involved in macroscopic mechanical states determination in composite laminates submitted to hygroscopic loads, even in the cases when a substantial amount of fibers is present in the considered material.

2. On the contrary, the local stresses in both the fibers and the epoxy resin can, according to Figure 2, strongly depend on the scale transition model used to perform the calculations.

3. As usual, the discrepancies between the two estimations of the local stresses increase with the fiber volume fraction.
4. Besides, in the present case, the deviation existing between the local stresses predicted with the Eshelby–Kröner or Mori–Tanaka approximation is not always close together. Significant deviation can even occur at weak carbon fiber contents: it is especially the case for the shear and the normal stresses (see the evolutions obtained for the local stress in the reinforcing fibers). This result comes from the different scale transition relations assumed to hold within the two models.
5. The Mori and Tanaka approximation generally underestimates the absolute value of the local stresses, in comparison with the corresponding predictions of the Eshelby–Kröner model. In the case when the local stress states are calculated according to the Mori–Tanaka model to estimate a possible damage occurrence of the constituents of the plies, this could lead to a dangerous overestimation of their domain of resistance.
6. The Mori and Tanaka model extended to the hygro-elastic loading of composite structures should only be considered as a rough approximation of the more rigorous Eshelby–Kröner model for the purpose of determining the local mechanical states. A satisfying agreement between the two models fix the domain of reliability of the Mori–Tanaka hygro-elastic model to carbon fiber volume fractions weaker than 10%. This result is compatible with the numerical computations performed in pure elasticity, in earlier works [10].

## CONCLUSIONS AND PERSPECTIVES

The present work deals with in detail an extension of the Mori–Tanaka procedure to hygro-elastic loading, which is often used in the field of the micro-mechanical modeling of composite structures, instead of the more rigorous and more complicated Eshelby–Kröner approach. The hygro-elastic version of the Mori–Tanaka model enables the estimation of both the, possibly anisotropic, effective stiffness and the coefficients of moisture expansion (CME) of the composite structure at the macroscopic scale. The microstructure of the constituents, and, in particular, the morphology of the reinforcing fibers is properly taken into account in the model. Moreover, scale transition relations are provided for the estimation of the heterogeneous local mechanical states existing in the epoxy resin and the carbon fibers, of a given ply of the structure.

Applications of the new extension of the Mori–Tanaka approach to the case of T300/N5208 [55°/–55°]<sub>S</sub> laminated cylinders exposed to an ambient fluid are provided. Comparisons with the numerical predictions of the recently proposed Eshelby–Kröner hygro-elastic model have shown that the two models are in good agreement in the case that the macroscopic behavior and mechanical states of the composite structure only have to be determined. In the case where the local hygro-elastic stresses are considered, the Mori and Tanaka model always underestimates the absolute value of the pseudo-macroscopic mechanical states. As a result, the domain of reliability of the model is limited to small volume fractions of reinforcing fibers: weaker than 10%. In the cases when the composite structures contain a volume fraction of fiber higher than 10%, the Eshelby–Kröner model should be preferred to the Mori–Tanaka estimates. This last result is compatible with previous works performed in pure elasticity.

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