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Cut-off Frequencies Induced by the Length of Strain Gauges Measuring Impact Events

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ABSTRACT: The finite length of strain gauges may induce filtering effects when measuring impact events. In this study, we are interested in quantifying these effects. Precisely, we determined the cut-off frequencies of strain gauges cemented on visco-elastic bars and measuring impact-induced strain waves. This study shows that the cut-off frequencies increase with the bar's wave velocity and decrease with the bar's diameter. The asymptotic value, corresponding to an infinite bar diameter, is reached rapidly (bar diameter ≈ 15 mm). Moreover, we showed that the mode cut-off frequencies are more severe (lower) than the gauge length cut-off frequencies for bar diameters greater than 8 mm.

KEY WORDS: cut-off frequency, complex Young modulus, Hopkinson bar, strain gauge length, wave dispersion

Introduction

The use of strain gauges is widespread in engineering applications. One of these fields is impact engineering [1]. For instance, they are used to indirectly measure the force and velocity [2–10], to measure the wave dispersion relation [11–14] and to identify complex Young's modulus [15–19]. Strain gauges are also used to measure ballistic impact events [20, 21]. They furnish valuable data to validate numerical models. Generally, strain gauges are preferred to piezoelectric sensors in shock events. This is motivated by the weightless nature of the strain gauges. Indeed, the inertia of the piezoelectric sensors can interfere with the real signals.

In impact engineering, the non-zero length of strain gauges will obviously induce filtration of high frequencies. This effect was studied previously, in the case of slender elastic bars [22, 23]. In this study, we aim at studying the gauges length cut-off frequencies in the more general cases of three-dimensional elastic and visco-elastic bars. For comparison purpose, the case of slender elastic bars is reinvestigated.

Cut-off Frequency

Let us consider a long straight (visco)-elastic rod. The strain at a point X_0 , $\varepsilon(X_0, t)$, is measured with a strain gauge. However, this gauge has a length $l_g \neq 0$. Therefore, the gauge signal rather corresponds to the mean value of the strain on the interval $[X_0 - \frac{l_g}{2}, X_0 + \frac{l_g}{2}]$ and not the strain at the point X_0 . In other terms, the strain measured by the gauge is:

$$\hat{\varepsilon}(X_0, t) = \frac{1}{l_g} \int_{X_0 - \frac{l_g}{2}}^{X_0 + \frac{l_g}{2}} \varepsilon(X, t) \, dX. \quad (1)$$

If only the first mode is considered, the Fourier transform of the strain at a point X , is given by Equation (3):

$$\hat{\varepsilon}(X, \omega) = F(\omega)e^{-i\xi(\omega)X} + D(\omega)e^{i\xi(\omega)X}, \quad (2)$$

where $F(\omega)$, $D(\omega)$ and $\xi(\omega)$ are the forward-going wave, the downward-going wave and the complex wave number. Thus, the Fourier transform of the strain gauge signal is obtained by combining Equations (1) and (2):

$$\tilde{\varepsilon}(X_0, \omega) = \frac{1}{l_g} \int_{X_0 - \frac{l_g}{2}}^{X_0 + \frac{l_g}{2}} \left(F(\omega)e^{-i\xi(\omega)X} + D(\omega)e^{i\xi(\omega)X} \right) dX. \quad (3)$$

After calculating the integral, Equation (3) yields:

$$\tilde{\varepsilon}(X_0, \omega) = \frac{\sin\left(\frac{\xi(\omega)l_g}{2}\right)}{\frac{\xi(\omega)l_g}{2}} \left(F(\omega)e^{-i\xi(\omega)X_0} + D(\omega)e^{i\xi(\omega)X_0} \right), \quad (4)$$

or equivalently,

$$\tilde{\varepsilon}(X_0, \omega) = \frac{\sin\left(\frac{\xi(\omega)l_g}{2}\right)}{\frac{\xi(\omega)l_g}{2}} \tilde{\varepsilon}(X_0, \omega), \quad (5)$$

Then, the filtering ratio can be defined as:

$$R(\omega) = \frac{\sin\left(\frac{\xi(\omega)l_g}{2}\right)}{\frac{\xi(\omega)l_g}{2}}. \quad (6)$$

As expected, this ratio is independent of the gauge position. Furthermore, at low frequencies, $\lim_{\omega \rightarrow 0} \xi(\omega) = 0$. Therefore,

$$\lim_{\omega \rightarrow 0} R(\omega) = 1. \quad (7)$$

Equation (7) means that there is almost no-filtering at low frequencies. Indeed, the strain gauge length is insignificant compared with the wave lengths at low frequencies. Therefore, the strain is almost constant along the strain gauge and the average strain given by the strain gauge is almost equal to this constant value.

Let φ_c be the cut-off frequency at -3 db, that is, the cut-off frequency such as $20 \log_{10}(|R(2\pi\varphi_c)|) = -3$. φ_c can then be defined as the solution of the following equation:

$$|R(2\pi\varphi_c)|^2 = \frac{1}{2}. \quad (8)$$

To solve Equation (8), we introduce the following mathematical map:

$$m: \begin{matrix} IR \rightarrow IR \\ x \mapsto \frac{\sin^2 x}{x^2} \end{matrix}. \quad (9)$$

From the definition of the filtering ratio R in Equation (6), we have $|R(\omega)|^2 = \left| m\left(\frac{\xi(\omega)l_g}{2}\right) \right|^2$. By considering Equation (8), the cut-off frequency corresponds to $\left| m\left(\frac{\xi(\omega)l_g}{2}\right) \right| = \frac{1}{2}$.

The equation $|m(x)| = 1/2$ has only one solution x_0 for positive x ; precisely, $x_0 \approx 1.3916$. Therefore, the cut-off frequency is given by:

$$\xi_c = \xi(2\pi\varphi_c) = \frac{2x_0}{l_g}, \quad (10)$$

where ξ_c is the wave number corresponding to the cut-off frequency. From Equation (10), we can also conclude that the cut-off frequency correspond to a critical wave length λ_c which is equal to the gauge's length multiplied by the ratio π/x_0 , that is, $\lambda_c = \pi l_g/x_0$. The ratio is slightly greater than 2; precisely $\pi/x_0 \approx 2.256$. The cut-off frequency corresponds to a wave length which is 2.256 times as long as the strain gauge.

From the properties of the mathematical map m , we have $|m(x)| < 1/2$ for any x greater than x_0 . Hence,

$$\forall \omega > 2\pi\varphi_c, |R(\omega)|^2 < \frac{1}{2}. \quad (11)$$

This means, that the length of gauges induces a low-pass filter. Precisely, the strain gauge cuts all wave lengths which are 2.256 times the gauge's length or lower.

Typically, $|\Im(\xi(\omega))| \ll |\Re(\xi(\omega))|$. Therefore, we can assume that $\xi(2\pi\varphi_c)$ is real. Consequently, Equation (10) has only one solution φ_c . In the following, the cut-off frequency will be calculated for three cases: slender elastic, three-dimensional elastic and three-dimensional visco-elastic bars.

Case of Slender Bars

In the case of slender bars, that is, $|\xi(\omega)r| \ll 1$ (r the bar radius) [24], the wave number is reduced to:

$$\xi(\omega) = \frac{\omega}{c_0}, \quad (12)$$

where c_0 is the wave velocity. Therefore, Equation (10) yields:

$$\varphi_c = \frac{c_0 x_0}{\pi l_g} = \frac{x_0}{\pi l_g} \sqrt{\frac{E}{\rho}} \equiv \varphi_0, \quad (13)$$

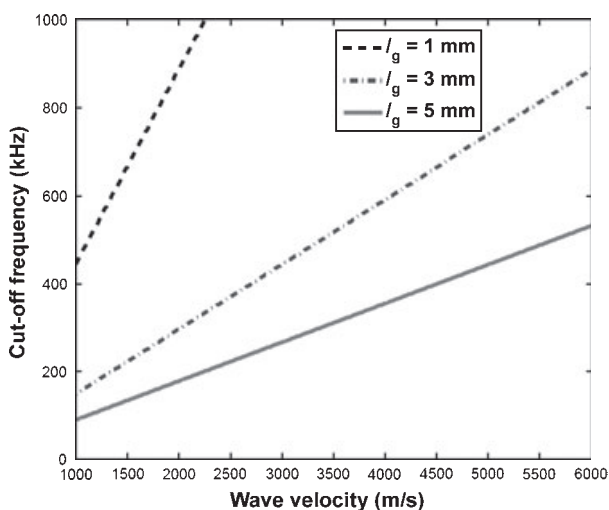


Figure 1: Cut-off frequencies of slender elastic bars

where φ_0 denotes the cut-off frequency of a slender bar, and E and ρ are the material Young's modulus and density, respectively.

In Figure 1, the cut-off frequency φ_0 is depicted in terms of the wave velocity for three different values of strain gauge length: $l_g = 1, 3$ and 5 mm. For the all considered cases, the cut-off frequency is higher than 100 kHz. For an aluminium or steel bar (wave velocity $c_0 \approx 5000$ m s⁻¹), which are the most used bars in impact events, the cut-off frequency is 2216, 738 and 443 kHz, for a strain gauge length of 1, 3 and 5 mm, respectively. The typical length of a strain gauge is about 3 mm. In this case, the lowest cut-off frequency is 147 kHz for a bar wave velocity of 1000 m s⁻¹.

Case of Three-Dimensional Bars

Elastic bars

The wave velocity in three-dimensional elastic bars is no more constant with (angular) frequency. The wave number $\xi(\omega)$ is solution of the so called Pochhammer-Chree equation [25, 26]:

$$\begin{aligned} \varphi(\xi, \omega, E, \nu, r, \rho) \\ = \frac{2\alpha}{r} (\beta^2 + \xi^2) J_1(\alpha r) J_1(\beta r) - (\beta^2 - \xi^2)^2 J_0(\alpha r) J_1(\beta r) \\ - 4\alpha\beta\xi^2 J_1(\alpha r) J_0(\beta r) = 0. \end{aligned} \quad (14)$$

In Equation (14), ν and $J_n()$ hold for the Poisson ratio and the first kind Bessel function of order n . Furthermore, $\alpha^2 = \frac{\rho\omega^2}{\lambda+2\mu} - \xi^2$ and $\beta^2 = \frac{\rho\omega^2}{\mu} - \xi^2$, where $\lambda = \frac{E\nu}{(1+\nu)(1-2\nu)}$ and $\mu = \frac{E}{2(1+\nu)}$ are the Lamé coefficients.

Equation (14) is a non-linear implicit equation and can be solved, for example, by the iterative formula of Newton [27]. It has multiple solutions; each solution gives a mode of propagation. For frequencies lower than a given frequency Φ_2 , only the first solution is real, the other solutions are pure complex. The first mode, corresponding to the first solution, yields a wave propagating through the bar. However, the other modes have no propagating term (the real part of the wave number) and they are damped. Their presence is limited to the neighbourhood of the bar extremities. For frequencies between Φ_2 and a second given frequency Φ_3 only the first and second mode solutions are real. Φ_2 and Φ_3 are called the cut-off frequencies of the second and third modes.

Now, we consider an aluminium bar ($E = 70$ GPa, $\rho = 2800$ kg m⁻³, $\nu = 0.34$). Six values of the diameter are also considered: 5, 10, 15, 20, 40 and 80 mm. The first solution of the Pochhammer-Chree equation is computed for the different values of the diameter. Only the case of $l_g = 3$ mm will be considered so forth. In Figure 2, we plot ratio $\zeta = \frac{\xi l_g}{2x_0}$ in terms of the frequency. The cut-off frequencies ε_c correspond to the frequencies where $\zeta = 1$.

Subsequently, we show in Figure 3 the cut-off frequencies for the different values of the bar diameter (black circles filled with grey). The zero diameter point corresponds to the case of a slender bar. The cut-off frequency decreases sharply from that point to reach an asymptotic value: 419 kHz. This is explained by the wave velocity–frequency profile. Indeed, for three-dimensional elastic bars, the wave velocity decreases with increasing frequency and reach an

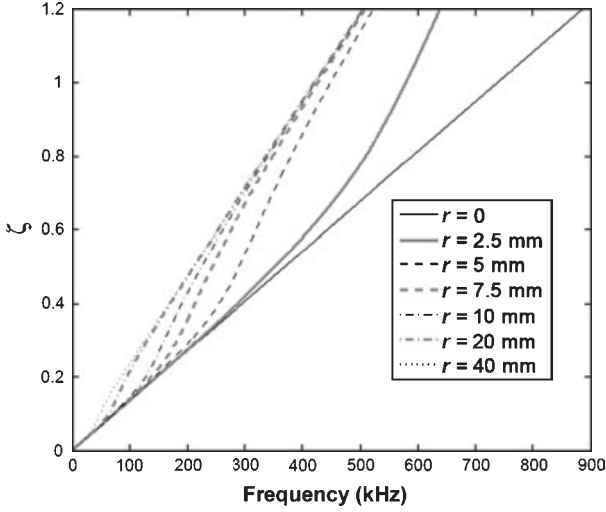


Figure 2: Variation of the ratio ζ in terms of the frequency

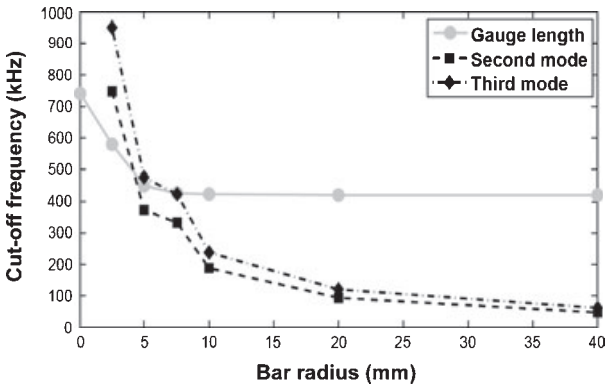


Figure 3: Strain-gauge length, second mode, third mode cut-off frequencies for a three-dimensional aluminium bar

asymptotic velocity which can be approximated by $c_\infty = \sqrt{\mu/\rho}$. The rate of velocity decreasing is higher for important bar diameter. For bar diameters greater than 15 mm, the cut-off frequency is located in the asymptotic part of the wave velocity–frequency profile. Therefore, the angular cut-off frequency can be approximated by the gauge length cut-off frequency of an infinite-diameter bar. This cut-off frequency can be obtained by considering in Equation (10) that wave velocity is equal to c_∞ :

$$\varphi_c \approx \frac{x_0 c_\infty}{\pi l_g} = \frac{x_0}{\pi l_g} \sqrt{\frac{\mu}{\rho}} \equiv \varphi_\infty. \quad (15)$$

Considering Equation (15), the cut-off frequency, for important-diameter bars (>15 mm), can be calculated and is equal to 451 kHz for the aluminium bar, which is not far from the numerical value (419 kHz) obtained above with the Pochhammer-Chree equation. The error is less than 7%. Equation (15) is established for an aluminium bar but holds also for any elastic bar, as the velocity–frequency profile is the same.

To assess the importance of the gauge-length cut-off frequencies, they are compared with the cut-off frequencies of the second and third modes, Φ_2 and Φ_3 . To obtain last two frequencies, Equation (14) is resolved for $\xi = 0$,

$$\varphi'(\omega, E, \nu, r, \rho) = \frac{2\alpha_0}{r} \beta_0^2 J_1(\alpha_0 r) J_1(\beta_0 r) - \beta_0^4 J_0(\alpha_0 r) J_1(\beta_0 r) = 0, \quad (16)$$

where $\alpha_0^2 = \frac{\rho\omega^2}{\kappa+2\mu}$ and $\beta_0^2 = \frac{\rho\omega^2}{\mu}$. Let $\omega_1 = 0 < \omega_2 < \omega_3 < \dots < \omega_n < \dots$ be the solutions of Equation (16). The cut-off frequency of the second and third modes can be approximated by $\Phi_2 \approx \omega_2/2\pi$ and $\Phi_3 \approx \omega_3/2\pi$. The variations of these frequencies in terms of the bar diameter are also depicted in Figure 3. For very low diameters (<8 mm), the gauge length cut-off frequency is lower than the mode frequencies. Besides, for important diameters (>15 mm), the mode cut-off frequencies are lower than the filtering cut-off frequency induced by the gauge-length.

Visco-elastic bars

In a visco-elastic bar, waves not only change shape but also attenuate while propagating. To consider these two effects, the wave number is considered to be complex number:

$$\xi(\omega) = \frac{\omega}{c(\omega)} + i\delta(\omega), \quad (17)$$

where $c(\omega)$ and $\delta(\omega)$ are the variation of the wave velocity and wave damping coefficient in terms of angular frequency. This complex wave number is obtained by solving the generalised Pochhammer-Chree equation [25], which is similar to Equation (14), except that the Young's modulus is also complex: $E(\omega) = E'(\omega) + iE''(\omega)$. Following Zhao and Gary [27], the material visco-elastic behaviour is described by a simple linear visco-elastic rheological model in which a set of a spring (Young's modulus E_2) and a dashpot (η) that are parallel, is in series with a second spring (Young's modulus E_1). The complex Young's modulus then reads:

$$E(\omega) = E_1 \frac{E_2 - i\eta\omega}{E_1 + E_2 - i\eta\omega}. \quad (18)$$

Knowing the evolution of the complex Young's modulus in terms of angular frequency, the complex wave number is determined by solving Equation (14) with the generalised iterative formula of Newton as explained [25]. Subsequently, we calculate the square of the absolute value of the filtering ratio R , which is defined in Equation (6). The cut-off frequencies correspond to $|R|^2 = 1/2$, as stated in Equation (8).

In this study, a polymer-like material is considered ($E_1 = 4$ GPa, $E_2 = 5$ GPa, $\eta = 1$ MPa·s, $\nu = 0.35$ and $\rho = 1200$ kg m⁻³ [25]). The variation of $|R|^2$ in terms of the frequency is depicted in Figure 4, for different values of the bar diameter. Subsequently, the cut-off frequencies are shown in Figure 5 in terms of the bar diameter. We obtain the same tendency as in the case of three-dimensional elastic bars. The cut-off frequency is decreasing with increasing bar diameter. It decreases sharply for low diameters and reaches rapidly an asymptotic value. This asymptotic value can also be approximated by:

$$\varphi_\infty \approx \frac{x_0 c_\infty}{\pi l_g} \approx \frac{x_0}{\pi l_g} \sqrt{\frac{\mu_\infty}{\rho}} = \frac{x_0}{\pi l_g} \sqrt{\frac{E_\infty}{2\rho(1+\nu)}} = \frac{x_0}{\pi l_g} \sqrt{\frac{E_1}{2\rho(1+\nu)}}, \quad (19)$$

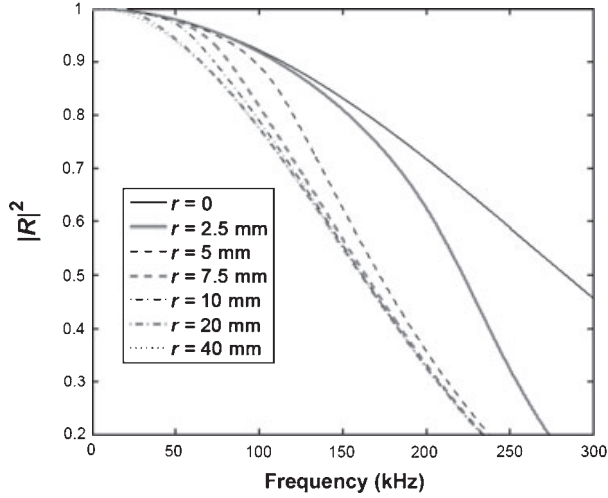


Figure 4: Variation of the $|R|^2$ in terms of the frequency

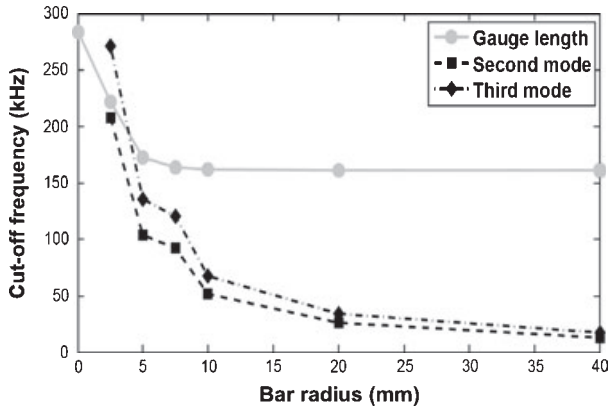


Figure 5: Strain-gauge length, second mode, third mode cut-off frequencies for a three-dimensional polymer-like bar

where $E_\infty \equiv \lim_{\omega \rightarrow +\infty} (E(\omega)) = E_1$. In the case of our polymer-like material, Equation (19) gives a cut-off frequency of an infinite-diameter bar of $\phi_\infty \approx 164$ kHz which is not far from 161 kHz obtained for the 80-mm diameter bar by solving the generalised Pochhammer-chree equation.

The cut-off frequency of a slender visco-elastic bar can be naturally approximated by:

$$\phi_0 \approx \frac{x_0 c_\infty^{r=0}}{\pi l_g}, \quad (20)$$

where $c_\infty^{r=0}$ is the wave velocity when the angular frequency and bar radius (diameter) tend to infinity and zero, respectively. This velocity is given by:

$$c_\infty^{r=0} = \sqrt{\frac{E_\infty}{\rho}} = \sqrt{\frac{E_1}{\rho}}, \quad (21)$$

Equations (20) and (21) give $\phi_0 \approx 270$ kHz which is not far from the value obtained in Figure 5 (i.e. 284 kHz).

In Figure 5, the strain gauge length cut-off frequencies are compared with the second and third mode cut-off frequencies. Here also, we obtain similar results as for the elastic case. Indeed, the imaginary part of the complex

wave number is very low compared with the real part ($|\Im(\xi(\omega))| \ll |\Re(\xi(\omega))|$) even in the case of visco-elastic bars. Thus, the real part influences solely the results.

Discussion

In the previous sections, the strain gauge length cut-off frequencies are calculated for the slender elastic, three-dimensional elastic and three-dimensional visco-elastic bars. For the last two cases, the influence of the bars diameter is investigated. We establish that the cut-off frequencies are mainly influenced by the bar wave propagation velocity. The higher the wave velocity, the higher are the cut-off frequencies. Besides, the cut-off frequencies decrease with increasing bar diameters. The rate of decrease is very important for low-bar diameters as the cut-off frequencies reach rapidly an asymptotic value.

Results of these sections can be synthesised as follows. The cut-off frequency of an elastic or visco-elastic slender bar is given by:

$$\phi_0 \approx \frac{x_0}{\pi l_g} \sqrt{\frac{E_\infty}{\rho}}, \quad (22)$$

We recall that $E_\infty = \lim_{\omega \rightarrow +\infty} (E(\omega))$, $x_0 \approx 1.3916$, l_g is the strain gauge length and ρ is the material density. For bar diameters lower than 15 mm, the cut-off frequency decreases sharply with increasing diameter. It almost takes a constant value thereafter. This asymptotic value corresponds to the cut-off frequency of an infinite diameter bar and reads:

$$\phi_\infty \approx \frac{x_0}{\pi l_g} \sqrt{\frac{E_\infty}{2\rho(1+\nu)}}. \quad (23)$$

where ν is the Poisson ratio. The ratio $\phi_\infty/\phi_0 = \sqrt{1/2(1+\nu)}$ depends only on the Poisson's ratio, which is an intuitive result. It decreases with increasing Poisson's ratio. It is equal to 0.707, 0.62 and 0.577 for $\nu = 0, 0.3$ and 0.5 , respectively.

Compared with the second and third mode cut-off frequencies, the gauge length cut-off frequency is more severe (lower than) for bars with low diameters (< 8 mm). For the opposite case (bar diameters higher than 8 mm), the care should be paid to mode cut-off frequency rather than that induced by the strain gauge length. Let Δt be the wave pulse duration, the above cut-off frequencies can be ignored if:

$$\frac{1}{\Delta t} \ll \min(\phi_c, \Phi_2), \quad (24)$$

where ϕ_c and Φ_2 are the gauge length and second mode cut-off frequency.

In Kolsky-Hopkinson bar machine applications, $\Delta t \approx \epsilon_{\max}/\dot{\epsilon}$, where ϵ_{\max} is the desired maximum strain in the specimen and $\dot{\epsilon}$ is the average strain rate. Consequently, the gauge length and second mode cut-off frequency are insignificant when

$$\dot{\epsilon} \ll \epsilon_{\max} \cdot \min(\phi_c, \Phi_2). \quad (25)$$

Conclusions

In several impact engineering applications, impact events are measured by strain gauges. However, these gauges have

a non-zero length that induces a well-known, but non-quantified filtering effect. In this study, we were interested in the strain induced by an impact wave in elastic or viscoelastic bars. We demonstrated that the strain gauge length induces a low-pass filter and that the cut-off frequency corresponds to wave lengths 2.256 times as long as the gauge. As wave lengths depend upon the bar material and diameter, we investigated the cases of three-dimensional elastic and viscoelastic bars. We showed that the gauge cut-off frequencies decrease with decreasing wave velocity and increasing bar diameter. Besides, we compared these frequencies with the second and third mode cut-off frequencies. It comes that the gauge cut-off frequencies are more restrictive for bar diameters lower than 8 mm. For bar diameters higher than 8 mm, the mode cut-off frequencies are more severe than the cut-off frequencies induced by the gauge length.

REFERENCES

- Knapp, J., Altmann, E., Niemann, J. and Werner, K.-D. (1998) Measurement of shock events by means of strain gauges and accelerometers. *Measurement* **24**, 87–96.
- Othman, R., Bussac, M. N., Collet, P. and Gary, G. (2001) Séparation et reconstruction des ondes dans les barres élastiques et viscoélastiques à partir de mesures Redondantes. *C. R. Acad. Sci. Sér. Iib* **329**, 369–376.
- Bussac, M. N., Collet, P., Gary, G. and Othman, R. (2002) An optimisation method for separating and rebuilding one-dimensional dispersive waves from multi-point measurements – application to elastic or viscoelastic bars. *J. Mech. Phys. Solids* **50**, 321–349.
- Jacquelin, E. and Hamelin, P. (2003) Force recovered from three recorded strains. *Int. J. Solids Struct.* **40**, 73–88.
- Casem, D. T., Fourney, W. and Chang, P. (2003) Wave separation in viscoelastic pressure bar using single point measurements of strain and velocity. *Polym. Test.* **22**, 155–164.
- Zhao, H., Abdennadher, S. and Othman, R. (2006) An experimental study of square tube crushing under impact loading using a modified large scale SHPB. *Int. J. Impact Eng.* **32**, 1174–1189.
- Wang, L., Xu, M., Zhu, J. and Shi, S. (2006) A method of combined SHPB technique and BP neural network to study impact response of materials. *Strain* **42**, 149–158.
- Othman, R. and Gary, G. (2007) Testing aluminum alloy from quasi-static to dynamic strain-rates with a modified Split Hopkinson Bar method. *Exp. Mech.* **47**, 295–299.
- Wang, L., Zhu, J. and Lai, H. (2008) A new method combining Lagrangian analysis with Hopkinson pressure bar technique. *Strain*. doi: 10.1111/j.1475-1305.2008.00533.x.
- Othman, R., Guégan, P., Challita, G., LeBreton, D. and Pasco, F. (2009) A modified servo-hydraulic machine for testing at intermediate strain-rates. *Int. J. Impact Eng.* **36**, 460–467.
- Yew, E. H. and Chen, C. S. (1978) Experimental study of dispersive waves in beam and rod using FFT. *J. Appl. Mech.* **45**, 940–942.
- Bacon, C. (1998) An experimental method for considering dispersion and attenuation in a viscoelastic Hopkinson Bar. *Exp. Mech.* **38**, 242–249.
- Othman, R., Blanc, R. H., Bussac, M. N., Collet, P. and Gary, G. (2002) Identification of the dispersion relation in rods. *C. R. Mécanique* **320**, 849–855.
- Pelker, R. R. and Saha, S. (1983) Stress wave propagation in bone. *J. Biomech.* **16**, 481–489.
- Lundberg, B. and Blanc, R. H. (1988) Determination of mechanical material properties from the two-point response of an impacted linearly viscoelastic rod specimen. *J. Sound Vib.* **137**, 483–493.
- Humen, V. and Potesnil, A. (1993) Pulse method used to identify material properties in linear viscoelastic media. *Int. J. Impact Eng.* **13**, 85–98.
- Lundberg, B. and Ödeen, S. (1993) In situ determination of the complex modulus from strain measurements on an impacted structure. *J. Sound Vib.* **167**, 413–419.
- Blanc, R. H. (1993) Transient wave propagation methods for determining the viscoelastic properties of solids. *J. Appl. Mech.* **60**, 763–768.
- Hillström, L., Mossberg, M. and Lundberg, B. (2000) Identification of complex modulus from measured strains on an axially impacted bar using least squares. *J. Sound Vib.* **230**, 689–707.
- Guégan, P., Othman, R., LeBreton, D., Pasco, F., Swiergiel, N. and Thevenet, P. (2010) Experimental investigation of rubber ball impacts on aluminium plates. *Int. J. Crashworthiness* **15**, 391–399.
- Galpin, B., Grolleau, V., Umiastowski, S., Rio, G. and Maheo, L. (2008) Design and application of an instrumented projectile for load measurements during impact. *Int. J. Crashworthiness* **13**, 139–148.
- Kaiser, M. A. (1998) Advancements in the split Hopkinson bar test. MSc thesis. Virginia Polytechnic Institute and State University.
- Wicks, A. L., Kaiser, M. A., Wilson, L. T. and Swantek, S. D. (2000) Experimentally derived bar dispersion and transducer selection for split Hopkinson bars. Shock compression of condensed matter-1999. *AIP Conf. Proc.* **505**, 1099–1102.
- Davies, R. M. (1948) A critical study of Hopkinson pressure bar. *Philos. Trans. R. Soc. London Ser. A* **240**, 375–457.
- Pochhammer, L. (1876) Über die fortpflanzungsgeschwindigkeiten kleiner schwingungen in einem unbergrenzten isotropen kreiszylinder. *J. für die Reine und Angewandte Mathematic* **81**, 324–336.
- Chree, C. (1889) The equations of an isotropic elastic solid in polar and cylindrical co-ords, their solutions and applications. *Cambridge Phil. Soc. Trans.* **14**, 250–369.
- Zhao, H. and Gary, G. (1995) A three dimensional analytical solution of the longitudinal wave propagation in an infinite linear viscoelastic bar, application to experimental techniques. *J. Mech. Phys. Solids* **43**, 1335–1348.