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# Cracking cohesive law thermodynamically equivalent to a non-local damage model

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**Abstract.** This paper deals with the transition from a localized damage state to crack formation. Several attempts have already been made in this field. Our approach is in the continuity of studies where thermodynamic considerations lead to the definition of an equivalent crack concept. The main idea consists in replacing a damaged localized zone by a crack in order to recover the same amount of dissipated energy. On the one hand, a nonlocal model is used to modelize accurately localized damage. On the other hand, an elastic model which authorizes the formation of a crack described by a cohesive zone model is used. This cohesive zone model is defined thermodynamically in order to be in concordance with the damage model. The method allows obtaining the cohesive zone model traction curve from the knowledge of the nonlocal damage model solution. The numerical implementation is done using a Lagrangian multiplier that ensures the energetic equivalence between both models.

## Introduction

Localized damage and fracture mechanics are two ways of modelling the same phenomena. Both methods introduce a characteristic length in the behaviour and can be considered as localization limiters. Our study aims at bridging those two theories in a well defined thermodynamic framework. In this view, we use the equivalent crack concept defined by Mazars [1] where energy dissipation is the value which has to be preserved in order to bridge models. This concept was used by Mazars and Pijaudier-Cabot in [2] with a non-local damage model. Another important ingredient of the method is the thermodynamical definition of the cohesive zone model, within the framework defined by Gurtin [3]. It is assumed that unload occurs linearly, as for a damage model. After presenting in detail the method, we will show an application with the Mazars non-local damage model where the corresponding cohesive zone model is incrementally constructed. We will then discuss the validity of the method. Finally we will present our current work which deals with the generalization of the method when plastic strains are taken into account.

## The bridging method.

**Energetic criterion for bridging models.** We assume isothermal processes. The energetic criterion concerns the energy dissipation of the structure noted  $\Phi$ . If we note  $\check{\Phi}$  the non-local damage model dissipated energy and  $\hat{\Phi}$  the cohesive zone model dissipated energy, the criterion writes:

$$\hat{\Phi} = \check{\Phi} .$$

For the non-local damage model,  $\check{\Phi}$  is calculated with the formula:

$$d\check{\Phi} = \int_{\Omega} -Y dD dV ,$$

with  $D$  the scalar damage variable, and  $Y$  its conjugate variable.

**Additional constraints on the cohesive zone model.** In addition to the energetic criterion, we impose that the cohesive model unloads in the same way as a damage model, which means that we observe no energy dissipation and a linear behaviour. In order to take into account these assumptions for the cohesive zone model, we use the thermodynamic framework defined by Gurtin in [3] for interfaces. With this construction of the cohesive zone model, it can be shown that the expression of the dissipated energy increment writes:

$$d\hat{\Phi} = \frac{1}{2} \int_{\Gamma} (\sigma d\delta - \delta d\sigma),$$

with  $\sigma$  the stress and  $\delta$  the displacement jump across the cohesive crack.

### Application to the incremental construction of a cohesive law.

For this study, we assume that the nonlocal damage model is given, whereas the cohesive law is unknown and has to be determined. The dissipated energy is calculated from the nonlocal damage model and then used with the energetic criterion to construct incrementally the cohesive law of the equivalent crack. For the cohesive zone model, we assume that there is only one crack and that it appears in the middle of the beam.

**The nonlocal model.** We here present the results obtained with a nonlocal damage model. The evolution law chosen for damage follows Mazar's model. For this model, damage is linked to an equivalent strain  $\varepsilon_{eq}$  which writes for a one-dimensional problem:

$$\varepsilon_{eq} = |\varepsilon|.$$

The maximum of  $\varepsilon_{eq}$  during time is noted  $\kappa$ . Damage is then computed with the formula:

$$D = 1 - \frac{\varepsilon_0(1 - A_t)}{\kappa} - \frac{A_t}{\exp(B_t(\kappa - \varepsilon_0))} \quad \text{if } \kappa > \varepsilon_0,$$

where  $\varepsilon_0$ ,  $A_t$  and  $B_t$  are material constants. The nonlocal features of the model are obtained by using a smoothed strain, calculated with the help of a Gaussian function. An analytical solution of the nonlocal problem serves of reference for the calculation of the cohesive law. The analytical solution is obtained with the use of the nonlocal smoothing function on a discontinuous strain field. This method was introduced in [4] and can be considered in itself as a way of bridging the gap between nonlocal models and fracture mechanics. The calculation of the nonlocal strain field is schematized below:

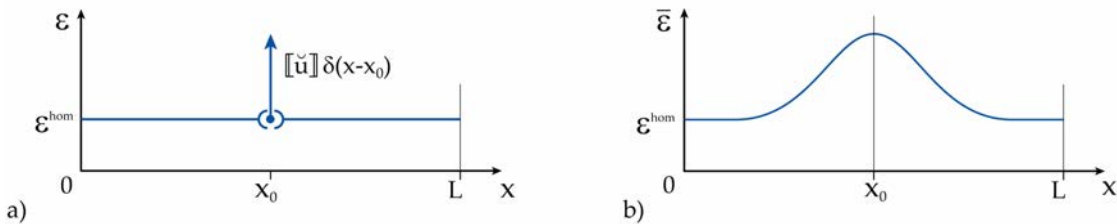


Fig 1

**The cohesive zone model.** The knowledge of the solution of the nonlocal problem allows us to calculate dissipated energy increments that will be transferred in the cohesive zone model. This information is sufficient to compute incrementally the solution of the problem with a cohesive crack. This calculation requires a special numerical implementation that will be presented now.

**Numerical calculation of the cohesive law.** The resolution of the problem with a cohesive crack is treated by isolating the elastic part of the structure. This means that cohesive stresses are considered as external loads. We choose to consider the lips of the crack as a prescribed-displacement zone. A Lagrangian multiplier imposes the wanted dissipated energy increment on the crack between two time steps. The incremental-calculation process of the cohesive law and the obtained cohesive law are represented on the following graph:

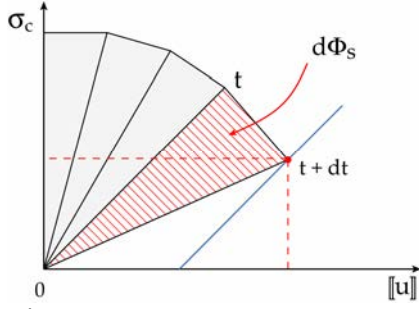


Fig. 2

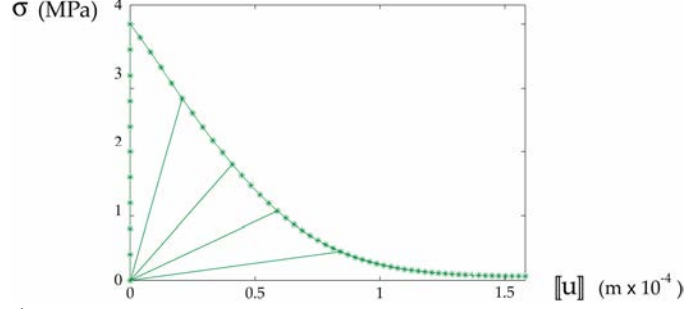


Fig. 3

### Validity of the method.

We define the total energy  $E$  as the energetic quantity that remains stable during time. If we neglect kinetic energy and temperature variations, the First and Second Principles of thermodynamics give the following expression of  $E$ :

$$E = \Psi - W_{ext} + \Phi,$$

where  $\Phi$ ,  $W_{ext}$  and  $\Psi$  are respectively the dissipated energy, the external-loads work and the free energy of the whole structure. The proposed energetic criterion imposes that dissipated energy is the same for both models. An interesting property of the method proposed is that  $\Psi$  and  $W_{ext}$  are also kept identical for both models. The key feature that allows concluding on this is the fact that both models unload in a linear manner, which allows us to write:

$$d\Psi = \int_{\Gamma} \underline{F} \underline{u} dS,$$

with  $\underline{F}$  and  $\underline{u}$  the external loads and displacements applied to the structure. This property is obviously true for the damage model and was also preserved for the cohesive zone model during its thermodynamic construction.

### Generalization to plasticity.

Our current work deals with the generalization of the method to more sophisticated models. We will next present our first results in the use of the method when plastic strains are taken into account.

**Difficulties arisen by the presence of plastic strains.** Plastic strains complicate the study of the energetic behaviour of the structure because, even if the unloading is done inside the elastic domain, the free energy of the unloaded state isn't equal to zero. There are two reasons for this: the first one is the fact there is some free energy stored in the microstructure of the material and the other one is the presence of residual stresses in the unloaded state. The first problem can be tackled by building a fictitious free energy potential from the knowledge of the mechanical behaviour of the material. This potential is defined so that dissipated energy is not directly linked to heat production, but merely to unrecoverable energy.

**New bridging criterion.** The cohesive model is defined in the same way as for damage, with the only difference that we now allow the presence of a plastic displacement jump across the cohesive zone. This new parameter in the cohesive zone requires refining the energetic criterion, which now writes:

$$\begin{aligned}d\hat{\Phi}^p &= d\check{\Phi}^p, \\d\hat{\Phi}^e &= d\check{\Phi}^e,\end{aligned}$$

with  $d\Phi^p$  the dissipated energy increment associated with the variation of the plastic internal variables and  $d\Phi^e$  the increment of dissipated energy associated with the damage variation.

### Conclusion

The method proposed here allows building incrementally a cohesive zone model from the knowledge of the solution of a nonlocal damage problem. This method is generalized so that it can be used with plastic-damageable models. Further work has to be done for the three dimensional generalization of the method.

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