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Langevin's twin paradox and the forwards and backwards movement of a rotating cylinder experiment

Jean Stratonovitch

1 - LANGEVIN'S TWIN PARADOX

Langevin slightly theatricalised this thought experiment resulting from the Lorentz transformation. A pair of twins, representing two clocks, stand at a Galilean place O. Then, while one stays 'motionless' at O, the other one moves away from him at a uniform speed v , passes some time, turns back and returns to O at a uniform speed $-v$. When the twins meet again, the 'motionless' twin is now older than the one who travelled.

This very famous paradox has had no easy life. Though it does not lead to any contradiction, it appeared to many people as being in itself evidence of the logical inconsistency of special relativity.

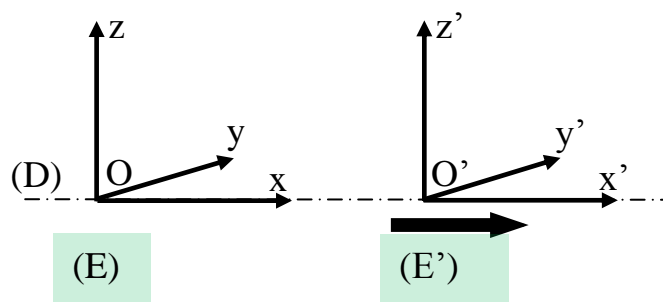
Bergson, in *Duration and Simultaneity*, published in 1922, explains in substance that the Lorentz formulae just describe a 4-dimensionnal effect of perspective: « "Supposing, as has been said, a traveller inside a rocket shot from the Earth at a speed one twenty-thousandth lower than the speed of light, would reach a star and be shot back to Earth at the same speed. When the traveller comes out of his rocket, he will be two years older, but he will discover our globe has aged two hundred years". – Are we really sure ? Let us have a closer look at this. We shall see the mirage fade, as it is nothing but a mirage ».

If Bergson were still alive, he would certainly not persist in this opinion, for nowadays various experiments have made obvious that fast-moving particles increase their life span in accordance with predictions of special relativity.

Let us first do the calculation. Let (E) be the Galilean space in which the first twin stays 'motionless' at a place O, (E') the space in which his brother, riding away from him, stays 'motionless' at a place O'.

A **Galilean space** is the set of the "events" (in the sense the word has in special relativity) the three spatial coordinates of the event are eternally invariable relative to a Galilean system of reference.

Let us fit these two spaces with Galilean systems of reference RG and RG', placed according to the standard way: the x and x' axes are on the same straight line (D), the other axes are one-to-one parallels, the spatial origins O and O' coincide at the instant 0 of each system.



The change of coordinates between (E') and (E) is given by the Lorentz transformation.

Lorentz transformation from (E') to (E):

$$\begin{cases} x = \gamma(x' + v t') \\ y = y' \\ z = z' \\ t = \gamma\left(t' + \frac{v x'}{c^2}\right) \end{cases}$$

In these formulae, γ is the relativistic factor $\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$, which is higher than 1.

The second twin, when going at a uniform speed, and such is the case except for a negligible part of his time, is a clock, whose **intrinsic** period, i.e. the one observed in the Galilean system of reference RG' in which it stays unmoving, has a duration T . Because the change of coordinates is an affine transformation, the period, when observed from (E), is constant. Thus, in order to know its duration, we can consider any cycle of the clock. The simplest way is to chose the one beginning at the instant $t' = 0$.

With relation to RG' , the spatiotemporal coordinates of the beginning and the end of this cycle are $(0, 0, 0, 0)$ and $(0, 0, 0, T)$. Their images under the coordinate change are $(0, 0, 0, 0)$ and $(\gamma v T, 0, 0, \gamma T)$, thus the period of the clock, considered from (E), is γT : it is now longer, and so the clock turns more slowly.

If we substitute $-v$ for v , the formula remains unchanged: on the return journey, the same phenomenon occurs.

Let i_1, i_2, \dots, i_{n+1} be the local instants (also called events) when-and-where the successive cycles of the moving clock begin. At i_1 , it goes away from O and at i_{n+1} it has just returned to this spot. For the outwards journey, the duration of the first $\frac{n}{2}$ cycles is $\frac{n}{2} T$ relative to RG' and $\frac{n}{2} \gamma T$ relative to RG ; for the return journey, they have the same respective values if we substitute RG'' for RG' . Thus, when the two clocks join up, if the one who made the journey has counted a duration τ , the one who stayed motionless has counted a duration $\gamma \tau$. Pursuing the metaphor, if the clocks were the twins' cardiac muscles, the muscle of the twin who made the journey has beaten γ times less than his brother's. So the twin who stayed at O is now older than the other one.

There is no difficulty in constructing a model of the relativistic kinematics within the frame of set theory. So, if we consider as absurd the result of this thought experiment, it is the whole of mathematics we ought to consider as absurd – however this experiment, strictly speaking, does not lead to any contradiction. The two clocks are physical mechanisms with different stories, there is nothing that could force

We construct this model from a model of Galilean kinematics in which we take out all “material points” whose speed relative to a given Galilean space (E) is at one or another instant higher than the speed C of light, and then modify by affine coordinate changes the spatiotemporal coordinates relative to the spaces other than (E) in such a way that the changes of coordinates between them and (E) become C-transformations of Lorentz. Because Lorentz transformations are a group, the chains of coordinate changes are still C-transformations of Lorentz, and the universe has become Lorentzien. The model thus obtained has the solidity of Galilean kinematics, which is that of the Cartesian product $\mathbb{R}^3 \times \mathbb{R}$, whose solidity is immediately that of mathematics in general.

them to have at the common ending run the same number of cycles – nothing but, clearly, some old habits of thinking. Langevin’s twin paradox therefore conceals no paradox. Moreover, it properly describes physical reality.

Still inside the frame of set theory, this model can be fitted with the laws of mechanics of material points. So, special relativity, concerning kinematics as well as mechanics of material points, is a theory free from any logical flaw, unless we admit the non-solidity of mathematics.



2 - PRESENTATION OF THE FORWARDS AND BACKWARDS MOVEMENT OF A ROTATING CYLINDER EXPERIMENT

In Langevin’s twin experiment, the clocks are undefined. But it can be proved by an epistemological argument that the prime clock of physics, the one who generates a time directly such that the momentum and the angular momentum obey conservational laws, is the inertial clock. For instance a cylinder globally motionless with relation to a certain Galilean space, freely spinning on its axis and not subjected to any action. Counting the number of its turns provides a ‘perfectly regular’ time.

This clock, this rotating cylinder, can be driven into an axial forwards and backwards experiment similar to Langevin’s twins’ one. The crucial difference is that *the space between the ‘motionless’ part and the one making the journey is always bridged by some continuous portion of the cylinder*

We shall study the case of a ‘thin’ cylinder, that is, when its thickness is infinitesimal.

Therefore, it is a limit case we examine. The hypothesis of thinness not only simplifies analysis, but also makes the experiment use an arbitrarily small quantity of material, and so legitimates using the “flat” frame of special relativity.

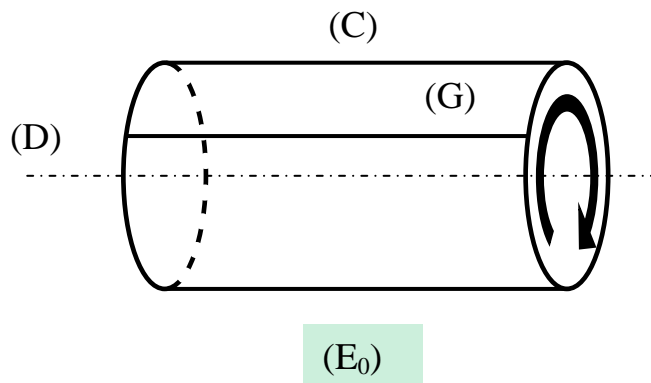
The material it is made of is not supposed rigid (in the sense of infinitely rigid): its shape changes when actions are exerted on it. We suppose it is *elastic*, in other words, that its mechanical properties remain invariable throughout the whole experiment.

It has long been known, through various thought experiments, that the rigid body is incompatible with special relativity, because physical actions should be instantaneously propagated through it.

No harm is done to special relativity, because rigid bodies just do not exist.

The first part of the experiment goes on in a ‘totally uniform’ frame. The cylinder (C) is globally motionless relative to the Galilean space (E_0), and freely spinning around its axis. So, at this moment of the experiment, and only at this moment, it has an invariable shape.

(G) is a generatrix of (C), which is once and for all drawn on (C) – we shall say ‘engraved’ on (C).

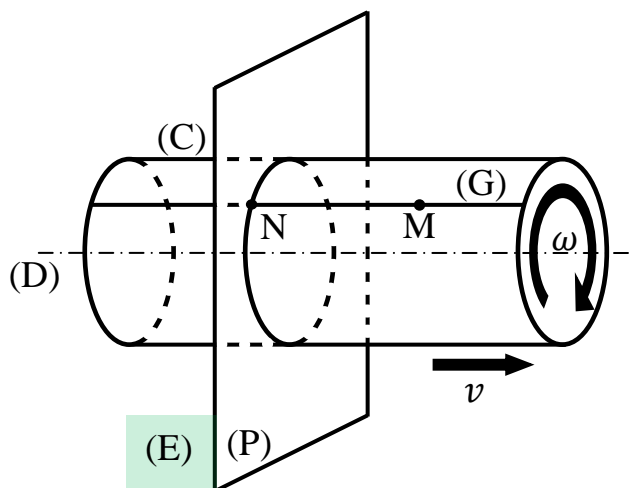


We call **generatrix** of (C) a set of points on its surface which, in this early stage of the experiment, when the movement of (C) is totally uniform in all its components, and when (C) is observed from the Galilean space (E_0) in which it is globally motionless, make a straight line parallel to the axis. The generatrix is “engraved” or “painted”: in a second stage of the experiment, when (C) will be subjected to an action, it will lose its prime shape, for no rigid body exists, and therefore (G) will also lose its prime shape at the same time (C) does.

(E) is another Galilean space, relative to which (C) is in globally uniform translatory movement along its axis (D). So this axis is motionless both in (E) and (E_0), as is the common support of the x and x' axes in the usual presentation of the Lorentz transformation.

(G) intersects a plane (P) motionless in (E) and perpendicular to (D) at a point N which plays the role of the first twin. The second twin's role is played by the point M, which is fixed – “engraved”, also – on (G). The global translatory speed of (C) relative to (E) is v , and its **intrinsic** angular velocity, that is the one observed in (E), is ω .

When an object can be considered as motionless (or globally motionless if it is spinning) relative to a given Galilean space, we call its characteristics relative to this space **intrinsic**; relative to another Galilean space, we call them **extrinsic**. Thus the intrinsic angular velocity of (C) is that observed from (E₀).



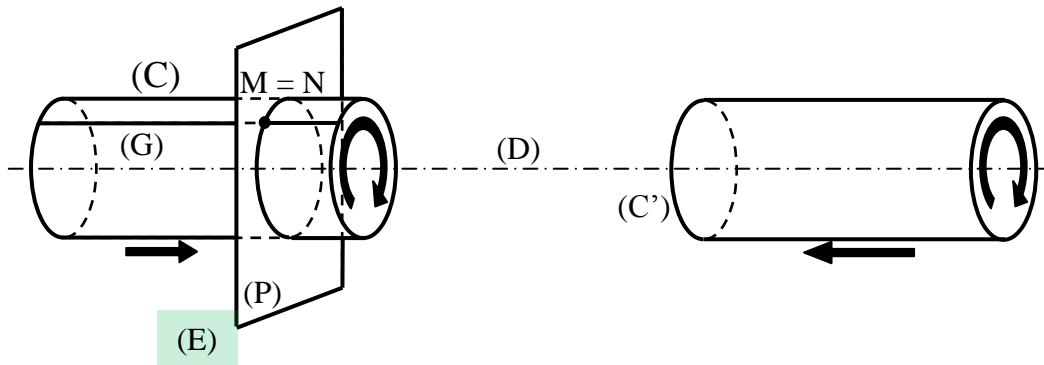
Contrarily to what this figure shows, when (C) is in uniform movement relative to (E), one of its generatrix (G), described in (E), is at every instant not a straight line, but a regular helix (see further).

The points M and N are thus perfectly defined, and their definition will not change.

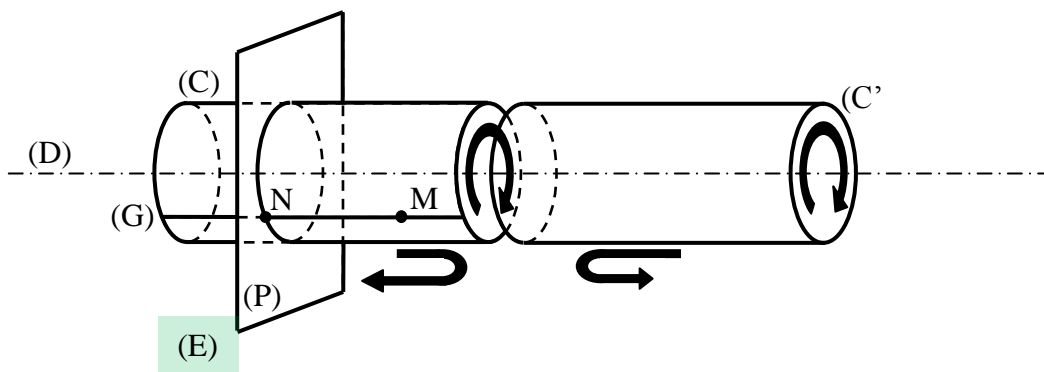
One should be wary of confusing N with the orthogonal projection, in the Galilean space (E₀) or in another one, of M onto (P). This erroneous definition refers to the simultaneity of this space, instead of the one which has been given. Moreover, it cuts out the continuous material link between M and N, which is crucial.

A cylinder (C') identical to (C), the movement of which being uniform in all its components, and also spinning around (D) at the intrinsic angular velocity ω , will strike (C).

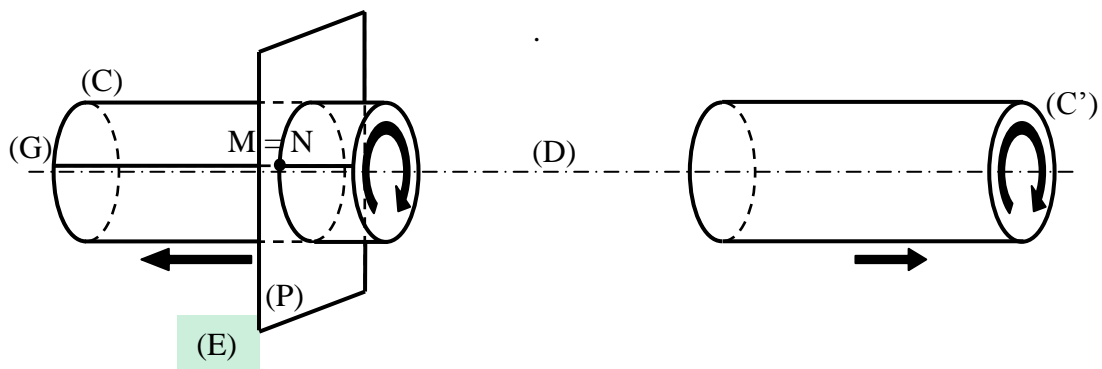
The experiment, described relative to (E), begins at the instant i of (E) when the point M goes through the plane (P) forwards and coincides instantaneously with the point N.



The second part of the experiment begins when (C') strikes (C). The movements of the cylinders then stop being uniform in all their components. A complex process starts, that will make (C) go backwards relative to the Galilean space of reference (E).



M goes through (P) backwards at the (E)-instant j , at which the experiment finishes. This point then coincides again with N.

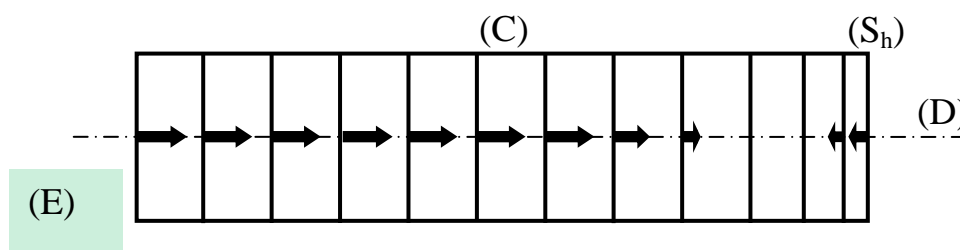


Because (C) does not entirely goes through (P), the point N exists at each instant of the interval $[i, j]$ of the duration of the experiment, at the end of which M has made m turns around (D) and N n turns.

Since actions are propagated at a finite speed, the reversal of the movement cannot be immediate. The shape of (C) changes and its sections no longer go at the same speed in relation to (E). For instance, immediately after the impact, because the head section (S_h) has just been struck by (C'), whose speed relative to (E) is superior to that of (C), the sections close to (S_h) are already going backwards, while those at a greater distance are still going forwards.

We call **section** of (C) its cutting by a plane perpendicular to its axis. Sections, like generatrices, are supposed "engraved" on (C).

A body whose parts are in the same uniform movement loses its shape if and only if its parts cease to be in that same uniform movement.



The result of the experiment depends on how the bodies lose their shape, and this has to be analysed. The changes of shape depend in particular on the material of which the cylinders are made, whose possibilities of variation are unlimited. To simplify, we shall suppose it is homogenous, isotropic and elastic.

The movement of (C) is no longer uniform; but we shall continue to analyze it relative to a Galilean system of reference, as is necessary. The assertion that special relativity must be confined to

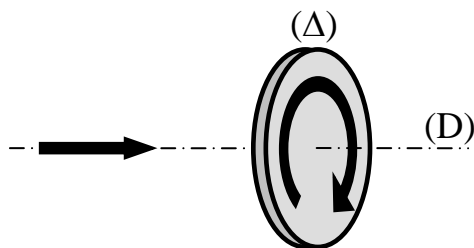
We find in manuals of special relativity the formulae giving how acceleration changes when the Galilean system of reference changes. Moreover, *no physical experiment* can be run in a universe where all the movements are uniform.

uniform movements is erroneous: we are only obliged to use Galilean systems of reference. For instance, when Langevin's traveller twin turns back, he experiences an acceleration.



3 – ANGULAR MOMENTUM IS 'ABSOLUTE'

Let us consider an elastic solid disk (Δ) spinning around its axis (D), and globally motionless relative to a Galilean space (E_0). Let us call ω its intrinsic angular velocity. At some instant, (Δ) receives an impulse along (D), exerted for instance by a particle moving along (D). This makes (Δ), once it has recovered its prime shape, be globally motionless in the Galilean space (E) moving at speed v relative to (E_0). Its new intrinsic angular velocity is ω' .



For symmetry of revolution, the particle still goes along (D): its angular momentum about (D) is therefore zero after the impact as it was before. Because the angular momentum is conserved, the angular momentum of (C) about (D) has not changed.

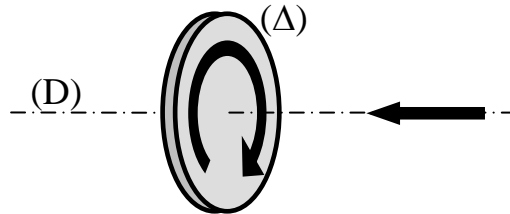
The value of ω determines the angular momentum of (Δ); and the values of the angular momentum and of v determine ω' , so that ω' is a function of v and ω : $\omega' = f(v, \omega) = f_v(\omega)$.

An identical disk (Δ_1), globally motionless in (E_0), but which turns at the angular speed $\omega_1 > \omega$, would have a higher angular momentum; this remains true with conservation of the angular momentum, if (Δ_1) is pushed as above at the speed v . Its final angular speed would thus be $\omega'_1 > \omega'$: for any given v , the

Because mass depends on speed, and because (Δ_1) turns faster than (Δ), the axial impulse exerted on (Δ_1) to make it go at the speed v is not the same as the one exerted on (Δ).

function $\left\{ \begin{array}{l} \mathbb{R}_+ \rightarrow \mathbb{R}_+ \\ \omega \mapsto \omega' = f_v(\omega) \end{array} \right.$ is increasing.

Let us now apply on (Δ) an axial impulse from the opposite direction, chosen to bring it back to a position of global immobility relative to (E_0) .



In consequence of the principle of relativity, and since the speed of (E_0) relative to (E) is the same as the speed of (E) relative to (E_0) , that is to say v , the new angular speed obeys the same law: $\omega'' = f_v(\omega')$.

Thus $\omega'' = f_v(f_v(\omega))$.

The angular momentum of (Δ) relative to (E) , however, has remained unchanged in each interaction. Since (Δ) is once more globally motionless in (E_0) , with the same angular momentum, $\omega'' = \omega = f_v(f_v(\omega))$.

Composing the increasing function f with itself, we obtain the identity. That shows f_v is the identity: $f_v(\omega) = \omega$.

Thus, after the first interaction, the angular speed of (Δ) is unchanged.

Its intrinsic angular momentum, the one relative to the space (E) in which it is then globally motionless, is therefore the same as it was (or will be) relative to (E_0) when (Δ) was (or will be) globally motionless relative to this space.

It is also the same, in consequence of the law of conservation, as the extrinsic angular momentum, the one relative to (E_0) .

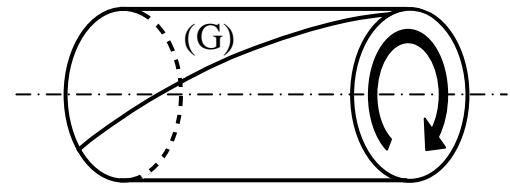
If we take away the centre of the disk, this result will extend using subtraction to rings spinning around (D) , which are the elementary bodies occurring in the problem of the rotating cylinder.

Intrinsic and extrinsic angular momentum are equal: the angular momentum, instead of durations, lengths, masses, is 'absolute', at least in relation to Galilean systems of reference gliding along (D) .

The angular momentum of a ring rotating around its axis (D), considered at an instant t of its existence, is the same relative to all the Galileans spaces having a translational movement along (D).

4 - TWIST PHENOMENA

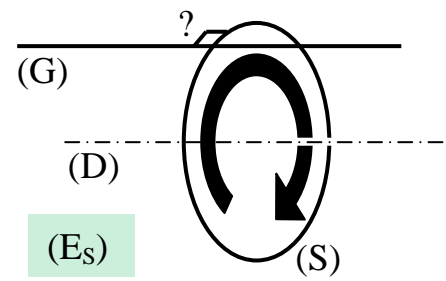
During the forwards and backwards experiment, the system maintains its symmetry of revolution. Nevertheless, some **twist** phenomena may or may not happen, which do not alter in any manner this symmetry of revolution.



Three types of twist can be distinguished, which are linked to each other.

Intrinsic Twist

In order to define it, the generatrices must have been previously engraved on (C) when it spins freely on its axis, globally motionless in a certain Galilean space. This cylinder has an **intrinsic twist** at the level of a section (S) if, when observing this tube from the **Galilean space tangent to the movement of translation** of one of its points M, we state – for instance with a set square, or by any process equivalent in theory – that the angle between (S) and the generatrix passing through M is not a right angle. The choice of M does not matter, because of the symmetry of revolution. The absence



When a material system can at a certain instant be considered as made of points all going at the same speed, we call **Galilean space tangent to its movement of translation** the unique Galilean space in which its speed is zero at this instant.

or existence of any intrinsic twist is a local characteristic: the set square is theoretically infinitesimal, and nevertheless of an infinite accuracy.

The Galilean spaces used in that study are all moving one relative to another in a direction parallel to (D) – allowing for one exception, the one we just used, the Galilean space tangent to the movement of M. As changes of coordinates in several different directions generate difficult calculations, that is a situation we ought to avoid.

(S) is considered here as a curve and not as a plane. For intrinsic twist can also exist when the changes of shape of (C) make its radius vary, so that this cylinder may have, in the neighbourhood of (S), a form equivalent to that of a portion of a cone. The criterion for (S) and (G) being perpendicular is then the one for two curves, and not the one for a curve and a plane.

Let (E_S) be the Galilean space tangent to the translatory movement of (S). The movement of (E_M) relative to (E_S) is collinear to the tangent at M to (S), and the Lorentz transformation changes a straight line perpendicular to the direction of the movement into a straight line perpendicular to the direction of the movement, so the inexistence of an intrinsic twist can be checked from (E_S) :

We call **Galilean space tangent to the movement of translation** of a section (or an infinitesimal slice) the unique Galilean space in which its translatory speed is zero at the instant it is considered. The section (or the infinitesimal slice) is then globally motionless relative to this space, although spinning on its axis.

A **slice** of (C) is the set of the points that are within range made by two sections.

The intrinsic twist at the level of one section is zero if and only if this section is, in the Galilean space tangent to its movement of translation, perpendicular to the generatrices.

Mechanical Twist

We say that (C) shows at a certain instant a **mechanical twist** at one of its sections (S) if the infinitesimal slices on each part of (S) exert torque on each other.

The torque exerted by one slice on the other is the quantity (actual or virtual) of angular momentum they exchange per unit of time, that is to say its time derivative. It depends on the system of reference gliding along (D) that we chose, for the angular momentum does not depend on this choice, while the time depends on it. But its being zero do not depend on it.

Relationship Between Mechanical Twist and Intrinsic

Twist

In order to know whether mechanical twist is present at the level of a section (S) at the instant t of the system of reference, we have a simple test: we can consider a situation *exactly identical, but* in which (S) is replaced by a zero thickness cutting-line, which divides (C) into two consecutive parts (C_1) and (C_2). The operation replaces (S) by two adjacent faces (S_1) and (S_2), that we suppose to be perfectly slippery. Since the strike is compressive, they remain adjacent in the instants immediately after t .

Two possibilities may occur in these instants:

a) (S_1) and (S_2) begin spinning at different angular speeds. Because they turn at the same speed when (C) has not been cut, we conclude the existence of mechanical twist at (S), in a direction given by the sign of the difference of the angular speeds.

b) (S_1) and (S_2) still turn at the same speed. We conclude there is no mechanical twist at (S).

Mechanical twist at (S) will not be present at the instant t if and only if a perfectly slippery cut of (C) at (S) is such that the adjacent faces still spin at the same velocity during the instants immediately after t .

However, we can also know if there is any mechanical twist at (S) by studying how the shape of (C) is changed in the neighbourhood of (S).

By the symmetry of revolution of the situation, the manner the shape is changed is the same at every point of (S). So it is sufficient to study it at any of its points.

The specific shape of (C) in the neighbourhood of a point M of (S), considered in the Galilean space tangent to the movement of this point, is indicative of the existence or non-existence of mechanical twist at (S).

Under the principle of relativity, the local intrinsic properties of the elastic material of which (C) is made do not depend on its translatory speed.

Thus, for a neighbourhood of M, being such that an instantaneous slippery cut along (S) may or may not make the two parts immediately glide one on the other, this fact depends only on the intrinsic shape of this neighbourhood, and not on its translatory movement.

So we can reduce the case we are studying to the one where angular speed is zero. In any possible interaction with a cylinder (C') absolutely identical to (C), and therefore the angular speed of which is also zero, the initial situation shows a symmetry about the plane containing (D) and the generatrix (G) passing through M. This symmetry will remain for the length of the interaction. So the changes of shape of (G) will in every case be such that (G) remains included in that plane, and therefore perpendicular to (S). On the other hand, as the angular speeds of all the sections are zero, the exchanges of angular momentum are constantly zero, and so is the mechanical twist.

This reasoning seems to ignore centrifugal forces, but actually it does not, because they take part at their own exact rate to the generation of the local shape of (C).

Because the local intrinsic shape is an indicator for local mechanical twist:

There is mechanical twist at a section (S) if and only if there is intrinsic twist at (S).

Extrinsic Twist

Let us examine, for the general case where the different parts of (C) do not go at the same speed, the local extrinsic characteristics of its movement.

Let (S) be a section of (C) going, at the instant t of (E), at the speed v relative to (E), and such that its intrinsic angular speed be ω and the intrinsic twist at its level be τ .

Let (E_S) be the Galilean space tangent to the movement of translation of (S) at this instant.

Without loss of generality, we can chose two systems of reference RG for (E) and RG' for (E_S), placed according to the standard manner, such that (S)

crosses the plane $x = 0$ at the instant 0 of RG, which is the one at which we intend to study its behaviour, and locally corresponds to the instant 0 of RG'.

**Lorentz transformation
from (E) to (E_S):**

$$\begin{cases} x' = \gamma(x - v t) \\ y' = y \\ z' = z \\ t' = \gamma\left(t - \frac{v x}{c^2}\right) \end{cases}$$

The local intrinsic twist of the generatrix (G) is τ at the instant 0 of (E_S), but this local twist is not constant. The spatiotemporal coordinates of a generic point M of (G) are thus, in (E_S), at this instant, without loss of generality:

$$(x', R \cos(\tau x') + o(x'), R \sin(\tau x') + o(x'), 0).$$

The translatory speeds of the sections of (C) are not supposed to be constant in the neighbourhood of M, nor are their angular speeds. But they have to be supposed continuous. The point M, being infinitely little, has no physical reality. Only the infinitesimal neighbourhoods of M do 'exist', whose acknowledgement as pertinent elementary physical entities having a certain speed presupposes the continuity of the speeds. So, at the instant t' , the spatiotemporal coordinates of M are, taking into account the continuity of the functions cosine and sine:

$$(x', R \cos(\tau x' + \omega t') + o(x', t'),$$

$$R \sin(\tau x' + \omega t') + o(x', t'), t'),$$

this formula in which the $o(x', t')$ are, in accordance with Landau's notation, negligible compared with $\|(x', t')\|$.

A point of coordinates relative to (E) (x, y, z, t) is on the generatrix if and only if there exist x' and t' such that the image of this point under the Lorentz transformation has the form shown above.

As the $o(x', t')$ are some of $o(x, t)$, this is equivalent to:

$$\begin{cases} y = R \cos \left[\tau \gamma (x - v t) + \omega \gamma \left(t - \frac{v x}{c^2} \right) \right] + o(x, t) \\ z = R \sin \left[\tau \gamma (x - v t) + \omega \gamma \left(t - \frac{v x}{c^2} \right) \right] + o(x, t) \end{cases}$$

In this way, we obtain the spatiotemporal equation of (G) relative to (E) in the neighbourhoods of the origin and the instant 0.

If we ‘stop’ t at the instant 0, we obtain the equation of (G) in (E) at the instant 0 in the neighbourhood of the origin of RG:

$$\begin{cases} y = R \cos \left[\gamma \left(\tau - \frac{v \omega}{c^2} \right) x \right] + o(x) \\ z = R \sin \left[\gamma \left(\tau - \frac{v \omega}{c^2} \right) x \right] + o(x) \end{cases}$$

(G) is thus, at the instant 0 and in the neighbourhood of (S), tangent to a regular helix of twist $\gamma \left(\tau - \frac{v \omega}{c^2} \right)$.

If $\tau = 0$, the local intrinsic twist of (C) is $-\frac{\gamma v \omega}{c^2}$. It is not associated with mechanical twist, and is simply the consequence that two sections of (C) neighbouring (S), when considered at the same instant relative to (E), are considered in (E_S) under a temporal gap which, since (C) spins on its axis, generates an angular lag. Calling x the difference of the abscises of these sections in (E) at the considered instant, the temporal gap in (E_S) is $\gamma \left(\frac{-v x}{c^2} \right)$.

Since the intrinsic angular speed is ω , the angular lag is $-\frac{\gamma v x \omega}{c^2}$; and then the extrinsic twist is

This « twist » is not torsion in its usual mathematical sense, but the “axial parameter” of the helix: the angle through which one of its points turns when its projection on the axis moves forward by a unit of length.

If we make $x = 0$ in the above formula, we obtain the angular speed of N in (E): $\gamma(\omega - v \tau)$.
This result will be used later.

$\frac{\frac{\gamma v x \omega}{c^2}}{x} = -\frac{\gamma v \omega}{c^2}$. So we return to and explain the value established by the former reasoning.

As the Lorentz transformation implies a simultaneity depending on the Galilean space of reference, it generates a phenomenon of extrinsic twist of the rotating cylinder, that is not linked to any physical change of shape.

We did not determine the extrinsic twist when the changes of shape of (C) at (S) make its radius vary, and then make (C) tangent to a cone. There is a trick that can help us. As the extrinsic twist is not linked to a physical change of shape, it is the same on a thick tube as on a thin one, and the same again on all the thin surfaces we can cut out of that thick tube, in particular this portion of a cone.

When the changes of shape to which the cylinder is subjected make its radius vary, the formulae providing the extrinsic twist and the angular speed of N remain unchanged.

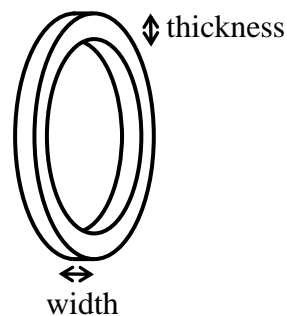


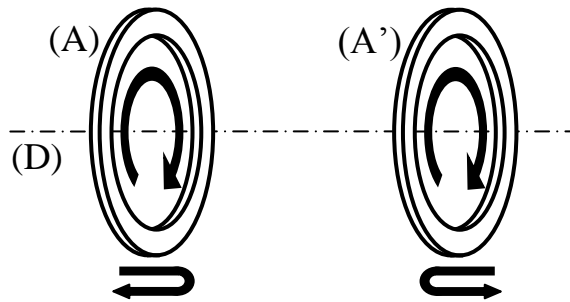
5 - INTERACTIONS BETWEEN TWO THIN RINGS

A Sufficient Condition So That Two Interacting Rings Do Not Exert Torque On Each Other

Let (A) and (A') be two identical elastic rings, which have the same axis (D), and of which the width and thickness are infinitesimal. Both of them are free from mechanical twist.

They move towards each other, strike and go backwards.





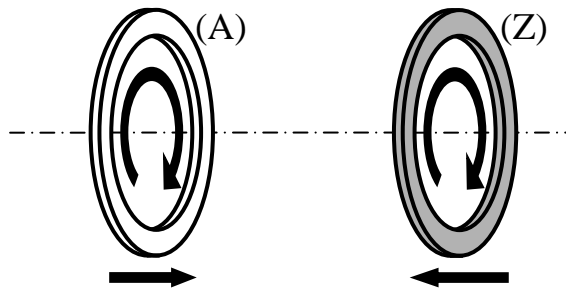
As the situation between (A) and (A') is symmetrical, they do not exchange any angular momentum during the interaction. So the torque one exerts on the other is always zero.

This result does not depend upon the speed of one ring relative to the other.

By reason of symmetry, their angular speeds are identical during the interaction, and so the points of them that coincide at the very first instant of the impact still coincide during the whole interaction: the rings do not slip on each other. Their adjacent surfaces may be either perfectly slippery or rough, that makes no difference.

The common intrinsic angular speed of the two rings is ω at the beginning of the impact; but this value might vary during the interaction; because their angular momentum μ remains unchanged, though their temporary changes of shape could make their intrinsic moment of inertia vary, and thus make their intrinsic angular speed $\frac{\mu}{J}$ vary.

Let us substitute for the ring (A') a thin elastic ring (Z) of the same size, moving like (A') at a uniform translatory speed along (D), and spinning at angular speed ω' . Like (A), it is made of a homogeneous and isotropic material, of which other characteristics may be different. It has no intrinsic twist before being struck.



During the interaction, the sides of (A) and (Z) that face each other are at every instant adjacent. So their contact area is an infinitesimally thin annulus.

Let us study how the situation depends on the value of ω' , beginning with the case $\omega' < \omega$.

First, let us suppose that the adjacent sides are perfectly slippery. The exchange of angular momentum is thus zero throughout the impact.

An observer carried along the movement of (A), staying close to the contact area, watches the situation.

If the intrinsic angular speed of (Z) were ω , he would, according to what we have just established, see the points of (Z) turning at the same speed as those of (A). But the intrinsic angular speed of (Z) is strictly inferior to ω , so the observer sees the points of (Z) sliding along (A) in the opposite direction to the rotation of (A). He concludes that if there were friction, (Z) would act against the rotation of (A), that is to say it would exert a torque contrary to this movement.

Similarly, if $\omega' > \omega$ and the friction was not zero, (Z) would exert on (A) a torque acting in the direction of its movement.

As the torque exerted by (Z) on (A) continuously depends on ω' , the intermediate value theorem states that it is zero when $\omega' = \omega$ and friction is not zero.

According to the same reasoning, this result remains true at every instant of the interaction; but the values of ω and ω' could have varied because the momenta of inertia of the rings have changed, in which case the hypotheses are no longer valid.

Let us consider two thin rings spinning around the same axis, and in translation along this axis. If, at a certain instant, these rings are interacting, and if, at that instant,

- they have no intrinsic twist,*
- they turn at the angular speed ω ,*

then, at this instant, these rings do not exert torque on each other.

So the instantaneous exchange of angular momentum between them is zero.

If moreover the interaction is such that the changes of shape induce no variation of the moments of inertia, the rings have a very simple behaviour:

If we add to the above hypotheses that throughout the interaction the intrinsic momenta of inertia of the rings remain invariable, then, throughout the interaction,

- they exchange no angular momentum,*
- they do not exert torque on each other,*
- they are not subjected to any phenomenon of intrinsic twist,*
- their intrinsic angular speed is constantly equal to ω .*

Possibility of an Interaction With Invariable Intrinsic Moments of Inertia

The variations of the intrinsic moments of inertia of the rings depend on how they are made.

If we use composite materials such that compression phenomena generated by the impact projects more material inwards than outwards, that decreases the intrinsic moment of inertia – possibly enough to overcome the relativistic increase of mass due to the conversion of energy in the compressed material, that has the opposite action.

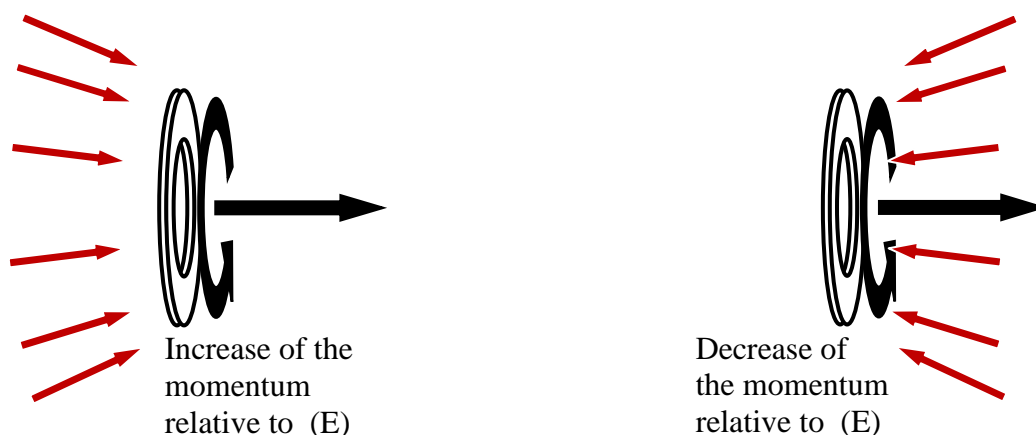
Inversely, if we imagine more material being projected outwards than inwards, then the moment of inertia increases during the impact.

Between the two, it seems that there exists the intermediate possibility of a neutral ring, whose moment of inertia remains unchanged during the impact. But it is only a limited and approximate possibility. A ring that would be perfectly neutral for a certain intensity of impact would very probably not be so for another. And if by chance it were, we would be unable to prove it with strict logic.

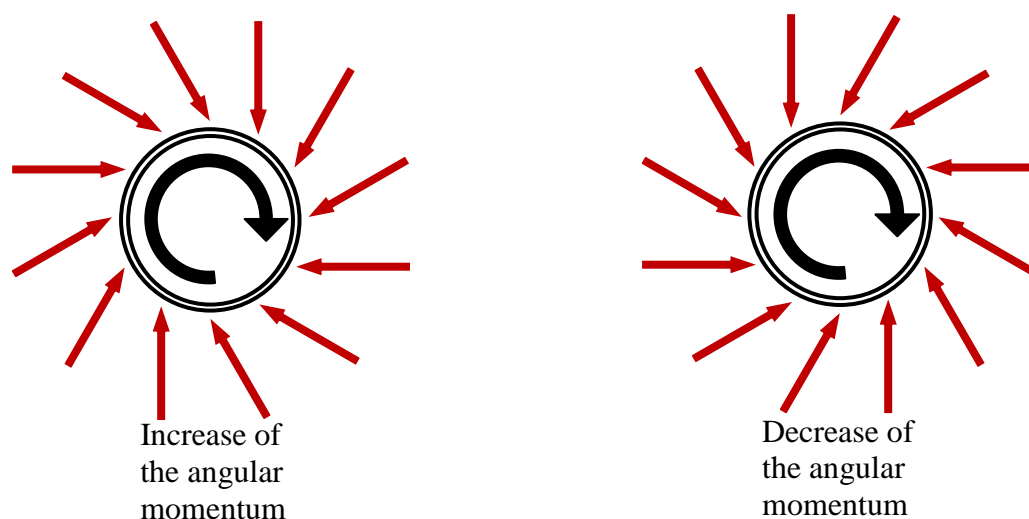
However, there is another way to make the rings neutral. It suffices to exert on (A) a lateral pressure that cancels out the variations of its moment of inertia. In order to do that, let us bombard (A) during the interaction with a continuous flow of particles that all strike it all at the same angle and at the same speed, respecting its

symmetry of revolution. If we want to cancel out an increase of the intrinsic moment of inertia, the particles will strike the outside cylindrical surface; if a decrease, the inside cylindrical surface. Let us without loss of generality consider the first case.

According to the way the flow arrives on (A), it can make its momentum relative to (E) increase or decrease.

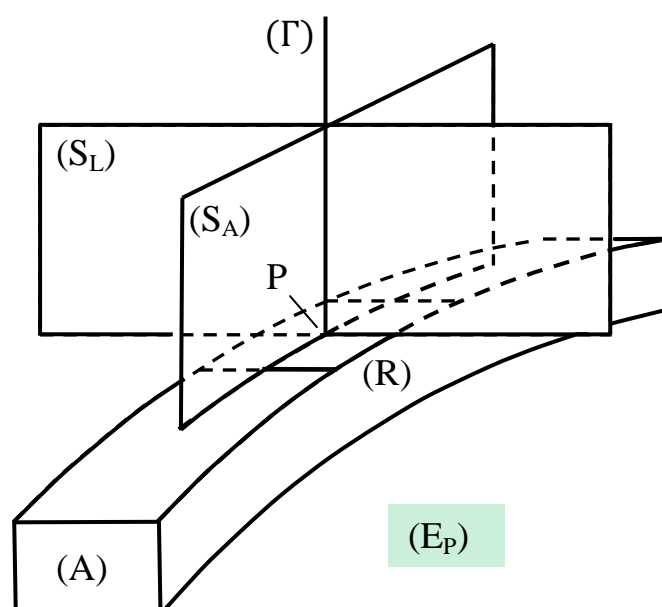


Similarly, it can also make the angular momentum increase or decrease.



Let (R) be an infinitesimal rectangle on the outside cylindrical surface of (A), drawn from one edge to the other. Let us observe the situation from the Galilean space tangent to its movement.

The action exerted on (R) can be represented one-to-one by a vector the terminal point of which is the centre P of (R), parallel to the flow, and the length of which is proportional to its intensity.



The original points of these vectors can be chosen everywhere in the half-space above (R). The flows making the momentum of (A) relative to (E) increase are characterised without loss of generality by a vector pushing (R) rightwards; and those making the momentum decrease, by a vector pushing (R) leftwards. This discrimination splits the original points of vectors into two regions separated by a revolution surface (S_A) transversal to the axis of the ring.

In a similar way, the flows that make the angular momentum increase and those that made it decrease are on opposite sides of a surface (S_L) set along the axis. The two surfaces intersect on a curve (Γ), which characterizes the flows that modify neither the momentum of (A) relative to (E) nor its angular momentum.

Among these flows, some have an intensity I too weak to cancel out the increase of the moment of inertia: for instance, the flow zero.

Let us decide from now on to run the experiment at ‘reasonable’ speeds of approach and rotation of the cylinders, that is to say in such a way that changes of shape and relativistic effects be ‘tiny’, or even ‘negligible’. Without intending to neglect anything, on the contrary we want to reason with the absolute accuracy of geometry, which infinitely exceeds that of our actual experiments.

Special relativity is a geometrical theory, meaning that it is ruled by axioms the accuracy of which is not supposed to be approximate but perfect. So one can, as with geometry, produce reasoning whose precision goes much further than our actual ability to observe and verify; and in particular concerning bodies the velocity of which can be as low as we want, much slower than that of a snail. In strict logic, the ‘relativistic effects’, though invisible, continue to exist.

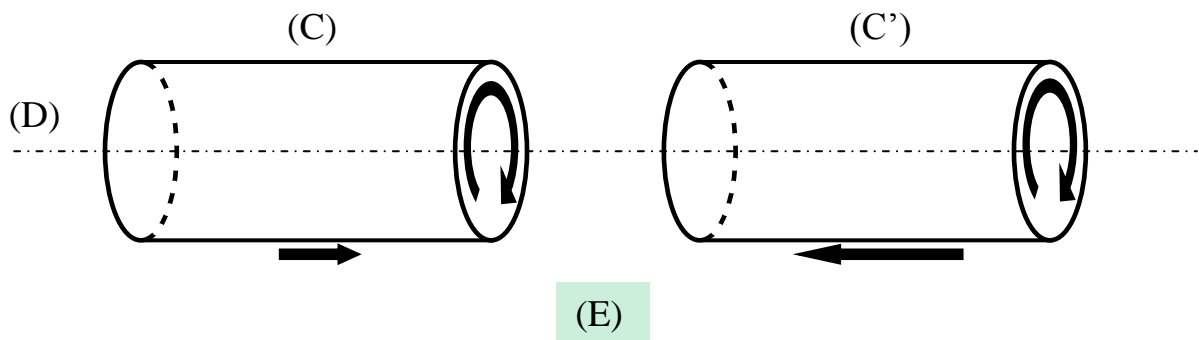
But, since these phenomena are ‘tiny’ or ‘negligible’, we can, by acting on elastic material, overpower them. A sufficiently high intensity I of the flow will have the effect of surpassing this “tiny” and will make the momentum of inertia decrease. As it is continuously dependant on I , at every instant of the interaction, there exists an intermediate value of I that cancels out its variation.

Thus we can exert on (A) throughout the interaction a time-varying homogeneous pressure which, without transmitting to it momentum nor angular momentum, is such that its moment of inertia remains unchanged.



6 – POSSIBILITY OF RUNNING A FORWARDS AND BACKWARDS EXPERIMENT AT CONSTANT INTRINSIC ANGULAR SPEED AND WITH ZERO INTRINSIC TWIST

Lets us resume our analysis of the interaction of two thin elastic cylinders coming towards each other. The situation is studied from the Galilean space (E).



Let us assume the mathematical induction hypothesis that, at a certain instant t relative to (E), we know the whole set of positions and speeds of the points of the two cylinders, and that

- each section turns at the angular speed ω ;
- there is no intrinsic twist anywhere;
- the angular momentum of each infinitesimal slice is the same as it was the first time of the experiment, when the movement of (C) was uniform in all its components.

As we know the whole set of positions and speeds of the points of the system, and because this data, added to the complete knowledge of the mechanical characteristics, determines its evolution at this instant t , we are able, at least in theory, to calculate this evolution, and so to obtain the knowledge of the whole set of positions and speeds at the instant $t + dt$.

But there is no reason why the induction hypothesis would still be valid at this new instant, because the changes of shape make the moments of inertia of infinitesimal slices vary. As a consequence, intrinsic angular speeds do not remain unchanged, thus phenomena of exchange of angular momentum occur, and thus so do phenomena of mechanical twist, that is to say, of intrinsic twist.

In order to counteract this inconvenience, we have to interfere a little: exert on each infinitesimal slice of (C) and (C') the action we have just studied, which exactly cancels out the variations of its intrinsic moment of inertia.

The successive infinitesimal slices making (C) are joined together, but that makes no change to the fact they then do not exert torque on each other.

Indeed, let us suppose (C), at the instant t when we study its behaviour at the level of any section (S), is suddenly cut in two parts along (S). The two adjacent cylinders thus obtained (that

compression phenomena prevent from separating), considered at this instant in a neighbourhood of the cutting-line,

- are turning at the same intrinsic angular speed everywhere,
- have no intrinsic twist anywhere.

So they do not exert torque on each other at this instant.

The flow cancelling the variations of the moments of inertia also does not exert torque, thus no mechanical twist will appear in the instants immediately after t , and thus no intrinsic twist.

As the intrinsic moments of inertia and the angular momenta remain unchanged, these neighbourhoods will in the instants immediately after t continue turning everywhere at the unchanged intrinsic angular speed ω and will not exert torque on each other. The adjacent sides can be rough or perfectly slippery, it does not matter.

Two adjacent points on each side of the cutting-line will continue coinciding as they did when there was no cut, and that shows that torque is not exerted at (S) when there is no cut.

As this is true wherever the cut is made, there is no exchange of angular momentum occurring through any section of (C). All the elementary slices of the now uncut cylinder keep unchanged their angular momenta, and because their moments of inertia also remain unchanged, they continue turning at the same unchanged angular speed ω . Moreover, since there is no exchange of angular momentum anywhere, no torque appears, and thus no intrinsic twist.

Whether (C) be cut or not, whether the sides made by the cutting line are perfectly slippery or not, the behaviour of this cylinder through the interaction will be the same concerning the absence of intrinsic twist as the invariability of the intrinsic angular speed of its sections. It can even be sliced into an arbitrary large number of rings, its generatrix will remain a continuous curve free from intrinsic twist, and each ring will still turn at intrinsic angular speed ω .

So, the induction hypothesis (no intrinsic twist, unchanged angular momenta of slices and angular speeds of sections) is true again at the instant $t + dt$: the inductive step is performed. Because the basis is obviously true, in the first part of the

Our reasoning, using mathematical induction concerning an arithmetic progression with infinitesimal increments dt , is legitimate in non-standard analysis – the one we implicitly use when reasoning ‘like physicists’, in particular employing infinitesimals.
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experiment, we can state:

It is possible to construct a forwards and backwards movement of a rotating cylinder experiment throughout which the intrinsic angular speed of all sections remains what it was before the impact, and the intrinsic twist remains zero everywhere.

From now on, we shall consider the experiment is done thus.



7 – AN INTEGER WHICH IS BOTH ZERO AND NON-ZERO

Thus, during the round trip, no intrinsic twist appears on (C), and its sections constantly turn at the intrinsic angular speed ω .

According to what we settled when we studied extrinsic twist, and because intrinsic twist is zero, the angular speed of N relative to (E) is $\omega_N(t) = \gamma_N(t) \omega$, in which formula $\gamma_N(t)$ is the Lorentz factor associated with the translatory speed of the section crossing (P) at the instant t relative to (E). As for M, since the section it belongs to it is a clock turning at the intrinsic angular speed ω , the angular speed of this section relative to (E) is $\omega_M(t) = \frac{\omega}{\gamma_M(t)}$, in which $\gamma_M(t)$ is the Lorentz factor – in general different from $\gamma_N(t)$ – associated with the translatory speed of the section. We have already done the calculation in our study of Langevin's twin paradox, we need not do it again.

Because $\gamma_N(t)$ and $\gamma_M(t)$, except for isolated instants, are strictly higher than 1, $\omega_N(t) > \omega_M(t)$.

The numbers of turns made by M and N around (D) between the instants i and j of the beginning and the end of the experiment are respectively $m = \int_i^j \omega_M(t) dt$ and $n = \int_i^j \omega_N(t) dt$. The functions $\omega_M(t)$ and $\omega_N(t)$ are continuous and the second one is almost everywhere strictly higher than the first one, therefore $n > m$.

Moreover, M and N coincide at the instants i and j . The difference between the numbers of turns they make is thus an integer.

$n - m$ is a positive integer.

All other things remaining unchanged, let us now vary a parameter: the position of (P). This plane is still motionless in (E) throughout the experiment, and still perpendicular to (D), but the point at which it intersects with (D) is no longer the same in one as it is in another.

Each position of (P) is characterized by the abscise x of the point at which it intersects (D), and the set of numbers x such that (C) – whose movement relative to (E) is strictly unchanged – crosses (P) but not entirely, that is to say it generates a forwards and backwards experiment, is a \mathbb{R} -interval $[x_1, x_2]$.

The longest possible experiment, in which the almost entire cylinder temporarily crosses (P), except for its last section, is run when $x = x_1$. The shortest one, in which the point M reaches (P) only at a single instant, at the furthest point of its movement, is run when $x = x_2$.

For a given x in $[x_1, x_2]$, the experiment characterized by x gives the two numbers of turns $m(x)$ and $n(x)$. An infinitesimal variation of x can only induce a infinitesimal variation of $m(x)$ and $n(x)$, thus $n(x) - m(x)$ continuously depends on x belonging to the interval $[x_1, x_2]$. As moreover $n(x) - m(x)$ is an integer, it is constant on that interval. This constant is the value obtained in the particular experiment studied throughout this article, the positive integer $n - m$. It is also the value obtained when $x = x_2$. Since the experiment has then a duration of zero, $m(x_2) = n(x_2) = 0$. So:

$n - m = 0$

Contrary to the former result, this one shows that the experiment, when analyzed in accordance with special relativity, leads to contradiction.

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8 – TEMPORARY CONCLUSION

Concerning mathematics, the emergence of a contradiction is a catastrophe. The theory, because asserting the existence of an integer which is both zero and non-zero, allows, by multiplying it by an arbitrary real number, to state that all the real numbers are zero, and thus are equal – which does not prevent us from stating, at the same time, that they are different from zero: when a theory is contradictory, one can prove both anything and its opposite.

This situation looks similar to that which Greek mathematics seems to have known when their prime belief that all numbers be rational suddenly collapsed with the discovery that the square root of 2 is irrational. Because if we suppose that $\sqrt{2} = \frac{p}{q}$, with p and q as mutually prime integers, we can show that q is both even and odd, and thus that $0 = 1$. As a result, the theory collapses, which is unfair as well as incomprehensible considering the great many proofs it has already shown of its value. It is precisely due to this great number of proofs that the theory could not be scrapped. It was too efficient, too powerful to be ruined by a simple question of parity, completely outside the range of the experiment; since rational numbers are everywhere dense among real numbers, no experimental protocol could ever make any difference between one and the other.

There was necessarily a solution, which the Greeks finally discovered: to admit that not all numbers are rational. Once they had admitted this unconceivable solution, everything returned to normal. The old theorems recovered their prime solidity, and geometry, which had been ruined for a time, was reinforced by that temporary disaster. Moreover, mathematics were from now on enriched by an essential and fruitful knowledge, the existence of irrational numbers.

Like Euclidian geometry at that time, special relativity has given enough evidence of its adequacy to reality to survive the catastrophe, even if it has to be modified in at least one point. As the modification is necessarily minor, we can consider it as being only a ‘detail’. But, since this ‘detail’ is necessarily related to an erroneous way of thinking, as was the naïve belief that all real numbers are rational, it is more than a simple ‘detail’ and must teach us something of interest.

In a later paper,
 – we shall determine what is the smallest modification applicable to special relativity to make it compatible with the forwards and backwards experiment,
 – verify, as happened with ‘pre-irrational’ and ‘post-irrational’ geometry, that this modification preserves the whole pragmatic part of the theory,
 – and examine its main consequences.



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