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# State-space modelling of a centerless grinding machine.

Iker Garitaonandia, Xabier Sabalza, M. Helena Fernandes, Jokin Muñoa, Jesús M. Hernández and Joseba Albizuri

**Abstract** — Numerical simulation of the dynamic behaviour of mechanical systems (machine tools), is a field of the mechanical design where it is necessary the use of both experimental techniques and analytical methods to obtain adequate dynamic models. In this work, finite element model updating techniques, using experimental data, and model reduction techniques are studied in order to obtain small size state-space dynamic models. This will allow their utilization for the modelling of mechanical systems in general, including servo-mechanical systems, intelligent structures and mechatronic systems. The finite element model of the machine-tool used as example has about 40000 elements (shell elements, solid and links). The model is improved by using updating techniques and it is further reduced to a few degrees of freedom to be used in the MATLAB/Simulink environment.

**Keywords** — Finite element model reduction, state-space model, updating techniques.

## I. INTRODUCTION

IN lightly damped structures it is possible to represent the behaviour of the system by means of the modal superposition of the modes of the undamped system [1]. In a real (and continuous) system the number of vibration modes is infinite, whereas in a finite element model the number of modes is equal to the number of degrees of freedom of the system. Nevertheless, in order to obtain the response of the system (for instance, the displacement in a given degree of freedom) due to an excitation, it is normally sufficient to use a reduced number of vibration

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I. Garitaonandia is with the University School of Technical Mining Engineering, University of the Basque Country (e-mail: iker.garitaonandia@ehu.es).

X. Sabalza is with IDEKO technology centre. Postal Address: IDEKO Pol. Industrial de Arriaga, 2. 20870 Elgoibar (Spain). (e-mail: xsabalza@ideko.es)

M. H. Fernandes is with University School of Technical Mining Engineering, University of the Basque Country (e-mail: mariahelena.fernandes@ehu.es).

J. Muñoa is with IDEKO technology centre. Postal Address: IDEKO Pol. Industrial de Arriaga, 2. 20870 Elgoibar (Spain). (e-mail: jmunoa@ideko.es)

J. M. Hernández is with the University School of Technical Mining Engineering, University of the Basque Country (e-mail: imphevaj@lg.ehu.es).

J. Albizuri is with the Faculty of Industrial Engineering, University of the Basque Country (e-mail: joseba.albizuri@ehu.es).

modes.

The reduction of the finite element description of a mechanical system to a model with a size more convenient for further control engineering labour, could be performed by means of superposition of the modal contributions, following the next steps:

1. It would be necessary to obtain the modes and vibration frequencies by means of any numerical method for this purpose (subspace iteration, Block Lanczos... ) using or not condensation methods (Guyan, IRS...).
2. Truncation of the modes. Selection of the degrees of freedom where either the forces would be applied or the displacements are wished to be read.
3. Modal reduction. Only the modes that provide a significant contribution to the response would be taken into account.
4. Obtaining the equations of motion in state-space form in modal coordinates.
5. Obtaining the solution in real coordinates.

## II. MODELLING THE SYSTEM

The dynamic characteristics of the centerless grinding machine were studied by means of a finite element model (FE model) carried out in ANSYS. This model has 53200 nodes and 37807 elements. In the Fig. 1 the mesh used is shown and in the table I the vibration frequencies corresponding to the first 10 undamped vibration modes are indicated.

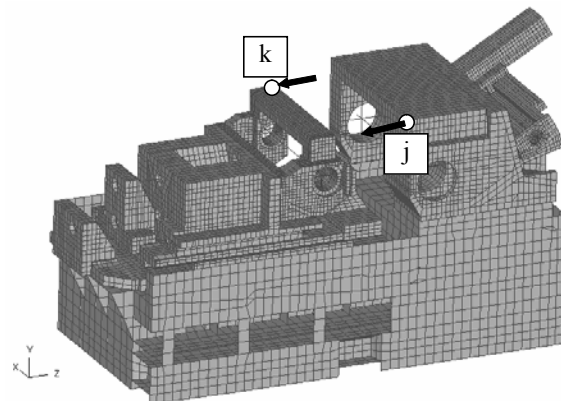


Fig. 1. F.E. mesh of the centerless grinding machine ESTARTA 327 MDA.

TABLE I: VIBRATIONS FREQUENCIES OF THE FEM.

Mode (FEM)	Frequency (Hz)
1	24.205
2	31.520
3	36.297
4	50.313
5	53.847
6	59.310
7	72.039
8	85.717
9	93.070
10	98.796

In order to verify if the results obtained with the finite element model reflect the real behaviour of the grinding machine, an experimental modal analysis was carried out (EMA). The machine was excited by means of a hammer with a force sensor. The responses were obtained in 69 measurement points by means of triaxial accelerometers, and therefore 207 frequency response functions (FRF) were obtained in a frequency range of 0-200 Hz. Fig. 2 shows the geometry of the used measurement points. In this figure, the excitation point is indicated with an arrow (point number 1).

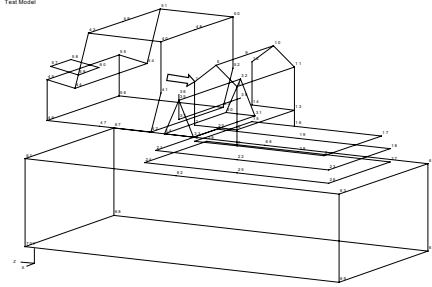


Fig. 2. EMA model.

TABLE II. EMA VIBRATION FREQUENCIES.

Mode (EMA)	Frequency (Hz)	Damping (%)
1	32.99	5.21
2	47.67	3.77
3	59.19	3.79
4	78.59	2.39
5	88.31	4.1
6	108.635	2.73

Based on FRFs, LMS Cada-x software was used to extract the first six natural frequencies, the modal damping ratios and the corresponding real mode shapes. Table II indicates the resulting frequencies and dampings.

Once the analytical and experimental resonance frequencies were obtained, it was noticed that important differences existed between them. Therefore, it is not possible to establish a clear correspondence between the theoretical and the experimental modes. A widely used technique for the mode shape pairing is based on the calculation of the MAC [2], [3] (equation 1). This expression is a measurement of the existing degree of correlation between the  $j$  mode and the  $k$  mode. Fig. 3 shows the MAC matrix obtained between the analytical and experimental mode shapes.

$$MAC_{jk} = \frac{|\varphi_j^T \varphi_k|^2}{(\varphi_j^T \varphi_j)(\varphi_k^T \varphi_k)} \quad (1)$$

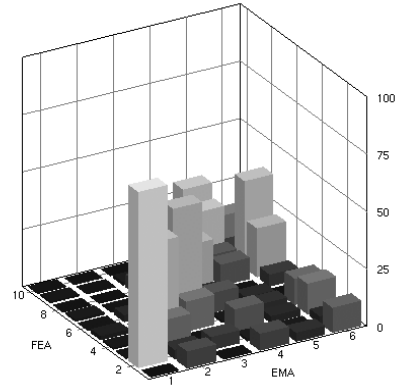


Fig. 3. FEM-EMA initial correlation.

Fig. 3 shows that it is not straightforward to establish a valid correlation between the mode shapes from initial finite element model and the experimental modes. Practically any form of automatic model updating technique will not give good results. In this particular case, the initial finite element model included a not well modelled zone, which was providing an excessive flexibility between different components of the grinding machine. The problem was solved stiffening the flexible zone. Fig. 4 shows the new obtained MAC.

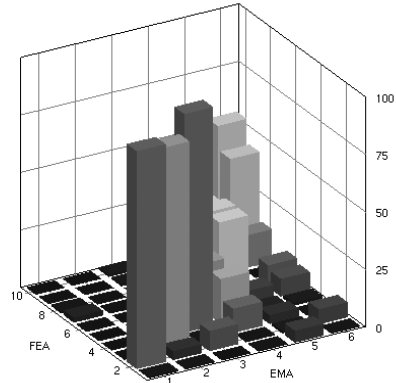


Fig. 4. FEM-EMA correlation after the correction.

The correlation for the lowest modes is satisfactory. Specifically, three analytical modes could be updated as the corresponding resonance frequencies match the experimental ones. A sensitivity analysis was carried out to locate the worse modelled zones in the finite element mesh. It was concluded that the parameters with major influence in the variation of the vibration frequencies were the stiffness values of the springs that joint some components of the grinding machine. During the updating phase, these stiffness values were modified following an iterative process, obtaining the degree of correlation indicated in the table III.

TABLE III. CORRELATION AFTER UPDATING.

	FEM	Hz	EMA	Hz	Diff (%)	MAC
1	2	33.48	1	33.41	0.20	96.3
2	3	48.43	2	48.43	0.00	86.4
3	4	58.59	3	58.91	-0.55	94.1

This table shows that the updated finite element model simulates adequately the dynamics of the grinding machine up to the frequency of 59 Hz, which is the excited frequency when chatter (self-excited vibrations) appears.

### III. MODEL REDUCTION.

The improved finite element model has a very high number of degrees of freedom, what can be problematic for the design and simulation of a vibration control system. Because of this disadvantage, it is necessary to reduce the order of the model in such a way that the small model and the complete original model have the same frequency response characteristic in the range of interest.

First, the degrees of freedom of the finite element model where either the forces will be applied or the response will be measured were selected. These degrees of freedom consisted in the translations in the three directions corresponding to the nodes paired with the 69 points used in the experimental modal analysis.

The following step consisted in the selection of the most significant modes in the frequency range of interest. The range taken in account was the same used for the experimental measurements (0-200 Hz.). Here, 20 vibration frequencies were calculated together with their (undamped) mode shapes for the 207 degrees of freedom selected.

The transfer function between the input  $k$  and the output  $j$  is represented by the equation (2), where it can be observed that the function is a linear combination of  $m$  systems of one degree of freedom.

$$\frac{z_j}{F_k} = \sum_{i=1}^m \frac{\varphi_{nji}\varphi_{nki}}{s^2 + 2\zeta_i\omega_i s + \omega_i^2} \quad (2)$$

where  $\varphi_{nji}$  is the component  $j$  of the mode  $i$  normalized with respect to the mass matrix,  $\omega_i$  is the vibration frequency and  $\zeta_i$  is the modal damping.

A reduction strategy consists of truncating the least significant modes, including their static contribution in the response. The criteria followed to eliminate the least important modes can be either eliminate the modes that present a minor static contribution in the response (dc gain criterion), or eliminate the modes that present a minor contribution in the resonance (peak gain criterion) [4].

The previous formulation based on transfer functions is adapted for the use of classic control techniques (PID control, pole placement...). Nevertheless, the objective of this work is essentially to get a reduced model for further use in the design of modern control systems like LQR,  $H_\infty$ ..., hence the most suitable formulation is the based in the state-space model [5].

In order to obtain the state-space model that simulates the dynamic behaviour of the grinding machine in the frequency range of interest, the obtained first 20 vibration frequencies and mode shapes were grouped in the natural frequencies matrix ( $[\Omega]=\text{diag}(\omega_i)$ ) and in the normalized modal matrix ( $[\Phi]=[\phi_1, \phi_2, \dots, \phi_{20}]$ ). These matrices were exported to the MATLAB environment, where a state-space model defined by the equation (3) was created.

$$\begin{cases} x' = Ax + Bu \\ y = Cx + Du \end{cases} \quad (3)$$

The matrices were defined in modal coordinates so that the equations of equilibrium are uncoupled. By means of a suitable definition of the state variables [5], [6], it is possible to obtain the  $A$  matrix as it is indicated in the expression (4). The data necessary to define it are both the natural frequencies matrix previously obtained and the damping coefficients of each mode, which were obtained from the experimental modal analysis (table II). With 20 considered modes, this matrix  $A$  has dimensions 40x40.

$$A = \begin{bmatrix} 0 & \Omega \\ -\Omega & -2\zeta\Omega \end{bmatrix} \quad (4)$$

For the matrix  $B$ , it is necessary to take into account that the transformation from the natural coordinates to the modal coordinates of any vector of components in natural coordinates is carried out by means of the normalized modal matrix according to the expression (5).

$$B = \begin{bmatrix} 0 \\ [L_u^T \Phi]^T r \end{bmatrix} \quad (5)$$

where  $L_u$  is the influence matrix of the input forces.

The matrix  $C$  depends on the desired output. If these outputs consist of displacements, it adopts the form:

$$C = [L_y^T \Phi \Omega^{-1} \quad 0] \quad (6)$$

where  $L_y$  is the influence matrix of the output displacements. In this case,  $D$  is a null matrix of suitable dimensions.

In Fig. 5 the transfer function between the degrees of freedom  $j$  and  $k$  of the grinding machine (see Fig. 1) are represented. This function was calculated in MATLAB using the state-space model of 40 states.

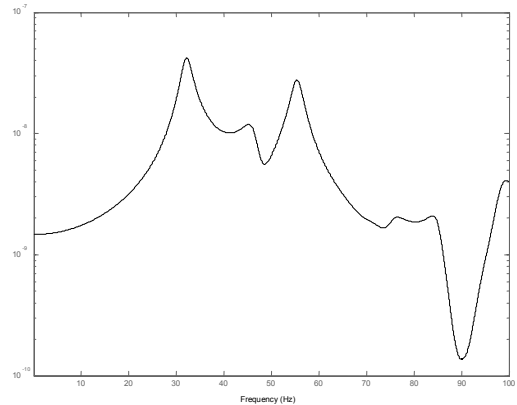


Fig. 5. Transfer function between  $j$ - $k$  dof. (40 states).

To obtain a reduced model in state-space, a reasonable approach is to make use of the controllability and the observability criteria [4], [7]. In control engineering, the controllability measures the capacity or faculty of a particular configuration of the actuators in order to control all the states of the system, whereas the observability measures the capacity of a particular configuration of the sensors in order to provide all the necessary information to estimate all the states of the system.

The reduction of the state-space model is based on the calculation of the controllability and observability

gramians, equations (7) and (8).

$$W_c = \int_0^{\infty} e^{A\tau} B B^T e^{A^T\tau} d\tau \quad (7)$$

$$W_o = \int_0^{\infty} e^{A^T\tau} C^T C e^{A\tau} d\tau \quad (8)$$

These gramians are, respectively, the solutions of Lyapunov's equations (9) and (10):

$$A W_c + W_c A^T + B B^T = 0 \quad (9)$$

$$A^T W_o + W_o A + C^T C = 0 \quad (10)$$

The gramians are square matrices in which the principal diagonal terms indicate the relative controllability and observability of the different states. The objective is to eliminate the less controllable and less observable states to obtain a reduced model. In order to carry out this operation, the model must be previously transformed into a balanced realization, where the different states are equally controllable and observable. It is possible to obtain it by means of an appropriate transformation of the state coordinates [8].

$$\bar{x} = T x \quad (11)$$

where  $T$  is the transformation matrix. So, the equation (3) turns into (12).

$$\begin{cases} \dot{\bar{x}} = T A T^{-1} \bar{x} + T B u \\ y = C T^{-1} \bar{x} + D u \end{cases} \quad (12)$$

In this balanced realization, the controllability and observability gramians are identical and strictly diagonal. The diagonal terms are the Hankel singular values of the system, and they provide information about the relative controllability and observability of the different states.

The Hankel singular values obtained from the balanced realization of the grinding machine are shown in Fig. 6. They are ranked from large to small.

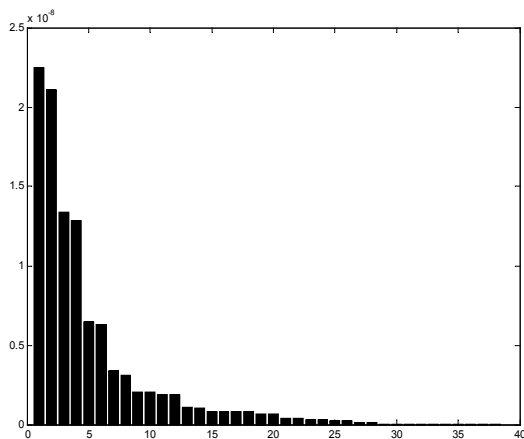


Fig. 6. Hankel singular values of the balanced realization (40 states).

Fig. 6 shows that some of the 40 states of the model are more controllable and more observable than others. It was considered that the first 8 states were the most representative ones, so that the remaining states were eliminated from the model. Fig. 7 displays the results obtained from the simulation of the transfer function between the input dof  $k$  and output dof  $j$  (see Fig. 1).

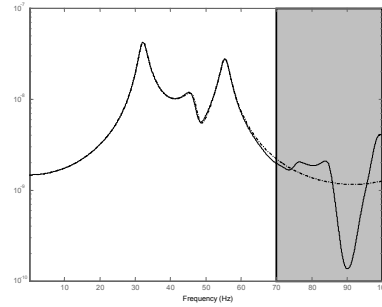


Fig. 7. Transfer function between  $i-j$  dof.

Fig. 7 shows the FRFs corresponding to the model with 40 states (continuous curve) and 8 states (dotted curve). It is convenient to notice that when the grinding machine works in chatter conditions (self excited vibrations) its dynamic behaviour is dominated by the third vibration mode, close to 59 Hz. Fig. 7 shows a shaded area corresponding to the contribution of high frequency modes to illustrate that there is a good fit of the two FRFs in the lower frequency band (no shaded area).

#### IV. CONCLUSION

In this work, a low order state space model of a centerless grinding machine is developed starting from a large scale finite element model. The success in obtaining the above mentioned model lies in the suitable selection of the representative modes (for example for the simulation of chatter). On the other hand, it is necessary that the original finite element model represents correctly the real behaviour of the system, being thus essential to make use of experimental data and finite element model updating techniques.

The reduced model and the original full model have the same frequency response characteristics in the frequency range of interest.

The proposed research is the first stage in the simulation of a control algorithm based on the reduced model. In the next phase the objective is to obtain a practical implementation of this methodology. Being a first approximation, it is expected that once the first steps in the practical development will be done, corrections should be made in the methodology.

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