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# Design of reset strategy for consensus in networks with cluster pattern

Marcos-Cesar Bragagnolo, Irinel-Constantin Morărescu,  
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## Abstract

This work focuses on the problem of consensus in network of agents with linear reset dynamics. This continues a preliminary work in which we have characterized the consensus value and its stability for networks of linear reset systems. The contributions of this work are the characterization of the convergence speed and the design of the interactions allowing to reach a prescribed consensus value. One numerical example illustrates the results.

## 1 Introduction

The problem of agreement of dynamic agents belonging to a network has been studied in many frameworks depending on the application (see [14] and the references therein). All existing works agree that the connectivity of the network is a main ingredient for consensus of agents with linear [8, 12, 13, 10] or nonlinear dynamics [9, 4]. Therefore, some works have been oriented toward decentralized control design able to preserve the network connectivity [16, 6]. Whenever the connectivity is lost, clustering behavior was emphasized [11, 15].

In [3] we considered networks that are partitioned in several clusters. The agents are able to continuously interact only with neighbors belonging to the same cluster. Moreover, each cluster contains an agent with powerful communication capacity called leader. Each leader can interact with some

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other leaders via a network with communication constraints. Thus, the leaders will interact only at specific isolated instants that will be defined in the next section as a nearly periodic sequence.

This model can be interpreted in terms of opinion dynamics - decision making process in social networks. The network is partitioned in a number of communities. The opinion of each agent continuously evolves taking into account the opinion of its neighbors. At some discrete times the leaders interact in a dedicated network and reset their opinion. We show that, if no external perturbation occurs and the process continues for a long period of time, all the opinions approach a common one. This global agreement opinion depends on the centrality ([2]) of each node inside each community and in the leaders' network. Roughly speaking the centrality of an agent expresses the importance of its opinion in the network. Thus, it characterizes somehow the level of "democracy" inside the network.

All the clusters as well as the network of leaders are represented by fixed, strongly connected directed graphs. The agents have a continuous dynamics but the leaders reset their state when they are communicating between them. In other words, we address the problem of consensus for agents subject to both continuous and discrete dynamics. Precisely, we are focusing on agreement in networks of reset systems, which are a particular class of hybrid systems (see [1, 5]). The existing literature on reset systems treats mostly linear dynamics.

In [3] we evaluated and studied the stability of the consensus value when the leaders interact and reset their state in a near-periodic manner. We shown that the consensus value depends only on the initial conditions and the topologies of the involved networks (i.e the networks associated with the clusters and that associated with the leaders). It is noteworthy that the consensus value does not depend on the reset period used for the leaders' opinions. A first objective of this paper is to study the convergence speed of the algorithm proposed in [3]. A second objective is the design of the network of leaders allowing to reach an a priori specified consensus value. In order to reach this goal, we consider that each cluster has a fixed and known interconnection topology. Therefore, we modify the consensus value of the whole network by changing the weights in the network of leaders. The set of consensus values that can be reached is contained in the convex hull of the initial local agreement values, which, for scalar states, is just the interval defined by the minimum and maximum initial local agreements.

**Notation.** The following standard notation will be used throughout the paper. The sets of nonnegative integers, real and nonnegative real numbers are denoted by  $\mathbb{N}$ ,  $\mathbb{R}$  and  $\mathbb{R}_+$ , respectively. For a vector  $x$  we denote by  $\|x\|$

its Euclidian norm. The transpose of a matrix  $A$  is denoted by  $A^\top$ . Given a symmetric matrix  $A \in \mathbb{R}^{n \times n}$ , notation  $A > 0$  ( $A \geq 0$ ) means that  $A$  is positive (semi-)definite. By  $I_k$  we denote the  $k \times k$  identity matrix.  $\mathbb{1}_k$  and  $\mathbf{0}_k$  are the column vectors of size  $k$  having all the components equal 1 and 0, respectively. We also use  $x(t_k^-) = \lim_{t \rightarrow t_k, t \leq t_k} x(t)$ .

## 2 Problem formulation

We consider a network of  $n$  agents described by the digraph (i.e. directed graph)  $\mathcal{G} = (\mathcal{V}, \mathcal{E})$  where the vertex set  $\mathcal{V}$  represents the set of agents and the edge set  $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$  represents the interactions.

In the sequel, we consider that the agent set  $\mathcal{V}$  is partitioned in  $m$  strongly connected clusters/communities  $\mathcal{C}_1, \dots, \mathcal{C}_m$ . The state of each agent continuously evolves by taking into account the states of its neighbors belonging to the same community. On top of that, each community possesses one particular agent called leader and denoted in the following by  $l_i \in \mathcal{C}_i$ ,  $\forall i \in \{1, \dots, m\}$ . The set of leaders will be referred to as  $\mathcal{L} = \{l_1, \dots, l_m\}$ . At specific time instants  $t_k$ ,  $k \geq 1$ , called reset times, the leaders interact between them following a predefined interaction map  $\mathcal{E}_l \subset \mathcal{L} \times \mathcal{L}$ . We also suppose that  $\mathcal{G}_l = (\mathcal{L}, \mathcal{E}_l)$  is strongly connected. The rest of the agents will be called followers and denoted by  $f_j$ . For the sake of clarity we consider that the leader is the first element of its community:

$$\mathcal{C}_i = \{l_i, f_{m_{i-1}+2}, \dots, f_{m_i}\}, \forall i \in \{1, \dots, m\} \quad (1)$$

where  $m_0 = 0$ ,  $m_m = n$  and the cardinality of  $\mathcal{C}_i$  is given by  $|\mathcal{C}_i| \triangleq n_i = m_i - m_{i-1}$ ,  $\forall i \geq 1$ . In order to keep the presentation simple, each agent will have a scalar state denoted also by  $l_i$  for the leader  $l_i$  and  $f_j$  for the follower  $f_j$ . We also introduce the vectors  $x = (l_1, f_2, \dots, f_{m_1}, \dots, l_m, \dots, f_{m_m})^\top \in \mathbb{R}^n$  and  $x_l = (l_1, l_2, \dots, l_m)^\top \in \mathbb{R}^m$  collecting all the states of the agents and all the leaders' states, respectively.

We are ready now to introduce the linear reset system describing the overall network dynamics:

$$\begin{cases} \dot{x}(t) = -Lx(t), & \forall t \in \mathbb{R}_+ \setminus \mathcal{T} \\ x_l(t_k) = P_l x_l(t_k^-) & \forall t_k \in \mathcal{T} \\ x(0) = x_0 \end{cases} \quad (2)$$

where  $\mathcal{T} = \{t_k \in \mathbb{R}_+ \mid t_k < t_{k+1}, \forall k \in \mathbb{N}, t_k \text{ reset time}\}$ ,  $L \in \mathbb{R}^{n \times n}$  is a generalized Laplacian matrix associated to the graph  $\mathcal{G}$  and  $P_l \in \mathbb{R}^{m \times m}$

is a Perron matrix associated to the graph  $\mathcal{G}_l = (\mathcal{L}, \mathcal{E}_l)$ . The components  $L_{(i,j)}$  and  $P_{l(i,j)}$  of  $L$  and  $P$ , represent the weight of the state of the agent  $j$  in the updating process of the state of agent  $i$  when using the continuous and discrete dynamics, respectively. In particular,  $L$  has the following block diagonal structure  $L = \text{diag}(L_1, \dots, L_m)$ ,  $L_i \in \mathbb{R}^{n_i}$  with  $L_i \mathbb{1}_{n_i} = \mathbf{0}_{n_i}$  and  $P_l \mathbb{1}_m = \mathbb{1}_m$ . Due to strong connectivity of  $\mathcal{C}_i$ ,  $i = 1, m$  and  $\mathcal{G}_l$ , 0 is a simple eigenvalue of each  $L_i$  and 1 is a simple eigenvalue of  $P_l$ .

In the sequel, we denote by  $w_i$  the left eigenvector of  $L_i$  associated with the the eigenvalue 0 such that  $w_i^\top \mathbb{1}_{n_i} = 1$ . Similarly, let  $v = (v_1, \dots, v_m)^\top$  be the left eigenvector of  $P_l$  associated with the eigenvalue 1 such that  $v^\top \mathbb{1}_m = 1$ . Due to the structure (1) of the communities, we emphasize that each vector  $w_i$  can be decomposed in its first component  $w_{i,l}$  and the rest of its components grouped in the vector  $w_{i,f}$ . Let us introduce the matrix of the left eigenvectors of the communities:

$$W = \begin{bmatrix} w_1^\top & 0 & \cdots & 0 \\ 0 & w_2^\top & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & w_m^\top \end{bmatrix} \in \mathbb{R}^{m \times n}. \quad (3)$$

We also introduce the following vectors:

$$x^*(t) = (x_1^*(t), x_2^*(t), \dots, x_m^*(t))^\top \in \mathbb{R}^m, \quad u = (v_1/w_{1,l}, v_2/w_{2,l}, \dots, v_m/w_{m,l})^\top \in \mathbb{R}^m \quad (4)$$

where  $x_i^*(t)$  is the local agreement value of the cluster  $i$  at time  $t$ . It is noteworthy that  $x^*(t)$  is time-varying but piecewise constant:  $x^*(t) = x^*(k) \forall t \in (t_k, t_{k+1})$ .

### 3 Consensus value and its stability analysis

In this section we briefly recall the results that we obtained in [3]. The reset sequence is defined such that  $t_{k+1} - t_k = \delta + \delta'$  where  $\delta \in \mathbb{R}_+$  is fixed and  $\delta' \in \Delta$  with  $\Delta \subset \mathbb{R}_+$  a compact set. Thus the set of reset times  $\mathcal{T}$  belongs to the set  $\Phi(\Delta) \triangleq \left\{ \{t_k\}_{k \in \mathbb{N}}, t_{k+1} - t_k = \delta + \delta'_k, \delta'_k \in \Delta, \forall k \in \mathbb{N} \right\}$  of all admissible reset sequences associated with  $\Delta$ .

**Proposition 1** *When the collective state  $x(\cdot)$  follows the dynamics (2), one has  $u^\top x^*(t) = u^\top x^*(0)$ ,  $\forall t \in \mathbb{R}_+$ . Moreover, assuming the agents of this system reach a consensus, the consensus value is*

$$x^* = \frac{u^\top W x(0)}{\sum_{i=1}^m u_i}. \quad (5)$$

Let us define the disagreement vector  $y = x - x^* \mathbb{1}_n$ . We also introduce an extended stochastic matrix  $P_{ex} = T^\top \begin{bmatrix} P_l & 0 \\ 0 & I_{n-m} \end{bmatrix} T$  where  $T$  is a permutation matrix allowing to recover the cluster structure of  $L$ .

In the sequel, we define a quasi-quadratic Lyapunov function satisfying Theorem 1 in [7] by means of LMI conditions.. Therefore, the following result gives sufficient conditions for the stability of the equilibrium point of  $x^* \mathbb{1}_n$  for the system (2).

**Theorem 2** *Consider the system (2) with  $\mathcal{T}$  in the admissible reset sequences  $\Phi(\Delta)$ . If there exist matrices  $S(\delta'), S(\cdot) : \Delta \mapsto \mathbb{R}^{n \times n}$  continuous with respect to  $\delta'$ ,  $S(\delta') = S^\top(\delta') > 0$ ,  $\delta' \in \Delta$  such that the LMI*

$$\begin{aligned} & \left( I_n - \mathbb{1}_n u^\top W \right)^\top S(\delta_a) \left( I_n - \mathbb{1}_n u^\top W \right) - \left( Y(\delta_a) - \mathbb{1}_n u^\top W \right)^\top S(\delta_b) \left( Y(\delta_a) - \mathbb{1}_n u^\top W \right) > 0, \\ & Y(\delta_a) \triangleq P_{ex} e^{-L(\delta + \delta_a)} \end{aligned} \quad (6)$$

*is satisfied on  $\text{span}\{\mathbb{1}_n\}^\perp$  for all  $\delta_a, \delta_b \in \Delta$ , then  $x^*$  is globally uniformly exponentially stable for (2). Moreover, the stability is characterized by the quasi-quadratic Lyapunov function  $V(t) = V(x(t)) \triangleq \max_{\delta' \in \Delta} (x(t) - x^* \mathbb{1}_n)^\top S(\delta') (x(t) - x^* \mathbb{1}_n)$  satisfying  $V(t_k) > V(t_{k+1})$ .*

Due to the space limitations we do not provide the proofs of the results above. However, we point out that Theorem 2 requires to solve a parametric LMI which can be approximated by a finite number of LMIs using polytopic embeddings. The set  $\{X \in \mathbb{R}^{n \times n} \mid X = e^{-L\delta_a}, \delta_a \in \Delta\}$  can be embedded into the polytopic set defined by the vertices  $Z_1, \dots, Z_{h+1}$  where

$$Z_1 = I_n, \quad Z_i = \sum_{l=0}^{i-1} \frac{(-L)^l}{l!} \delta_{max}^l, \quad \forall i \in \{2, \dots, h+1\}$$

with  $\delta_{max} = \max_{\delta' \in \Delta} \delta'$ ,  $(-L)^0 = I_n$  and  $0! = 1$ . Then, Theorem 2 can be replaced by the following result.

**Theorem 3** *Consider the system (2) with  $\mathcal{T}$  in the admissible reset sequences  $\Phi(\Delta)$ . If there exist symmetric positive definite matrices  $S_i$ ,  $1 \leq$*

$i \leq h + 1$  such that the LMI

$$\begin{aligned} & \left( I_n - \mathbb{1}_n u^\top W \right)^\top S_i \left( I_n - \mathbb{1}_n u^\top W \right) - \\ & \left( Y(\delta) Z_i - \mathbb{1}_n u^\top W \right)^\top S_j \left( Y(\delta) Z_i - \mathbb{1}_n u^\top W \right) > 0, \\ & Y(\delta) \triangleq P_{ex} e^{-L(\delta)} \end{aligned} \quad (7)$$

is satisfied on  $\text{span}\{\mathbb{1}_n\}^\perp$  for all  $i, j \in \{1, \dots, h + 1\}$ , then  $x^*$  is globally uniformly exponentially stable for (2).

## 4 Main results

### 4.1 Decay rate analysis

Once the global uniform exponential stability of  $x^*$  ensured by Theorem 2 we can compute the convergence speed of system (2). Denoting it by  $\lambda$ , one has  $\lambda = \frac{\ln \lambda_d}{\delta + \delta_{max}}$  (see [7] for details) where  $\delta_{max} = \max_{\delta' \in \Delta} \delta'$  and  $\lambda_d$  is the decay rate of the linear difference inclusion (LDI)

$$x(t_{k+1}) \in \mathcal{F}(x(t_k)), \quad k \in \mathbb{N} \quad (8)$$

where

$$\mathcal{F}(x) = \left\{ P_{ex} e^{-L(\delta + \delta')}, \delta' \in \Delta \right\}.$$

Precisely, for the LDI (8) there exist  $M > 0$  and  $\xi \in [0, 1]$  such that

$$\|x(t_k) - x^* \mathbb{1}_n\| \leq M \xi^k \|x(0) - x^* \mathbb{1}_n\|, \quad \forall k \in \mathbb{N} \quad (9)$$

and  $\lambda_d$  is defined as the smaller  $\xi$  satisfying (9).

Thus, in order to quantify the convergence speed of system (2), we only have to evaluate  $\lambda_d$ . Let us denote again  $y = x - x^* \mathbb{1}_n$  and note that  $V(y)$  defined by Theorem 2 is a norm. That implies there exist  $\alpha, \beta > 0$  such that

$$\alpha \|y\|^2 \leq V(y) \leq \beta \|y\|^2.$$

Consequently, one obtains that the decay rate  $\lambda_d$  coincides with the decay rate of  $V$ . Thus, the following result can be derived directly from Theorem 2.

**Proposition 4** Assume there exist  $\alpha > 0$ ,  $\beta > 0$ ,  $\xi \in (0, 1]$  and the matrices  $S(\delta'), S(\cdot) : \Delta \mapsto \mathbb{R}^{n \times n}$  continuous with respect to  $\delta'$ ,  $S(\delta') = S^\top(\delta') > 0$ ,  $\delta' \in \Delta$  fulfilling the following constraints

$$\begin{cases} \alpha I_n \leq S(\delta') \leq \beta I_n, & \forall \delta' \in \Delta \\ \xi^2 \left( I_n - \mathbb{1}_n u^\top W \right)^\top S(\delta_a) \left( I_n - \mathbb{1}_n u^\top W \right) - \\ \left( Y(\delta_a) - \mathbb{1}_n u^\top W \right)^\top S(\delta_b) \left( Y(\delta_a) - \mathbb{1}_n u^\top W \right) > 0, \\ Y(\delta_a) \triangleq P_{ex} e^{-L(\delta + \delta_a)} \end{cases} \quad (10)$$

on  $\text{span}\{\mathbb{1}_n\}^\perp$  for all  $\delta_a, \delta_b \in \Delta$ . Then, the decay rate is defined as

$$\lambda_d = \min_{\xi \text{ satisfies (10)}} \xi$$

and

$$\|x(t_k) - x^* \mathbb{1}_n\| \leq \frac{\beta}{\alpha} (\lambda_d)^k \|x(0) - x^* \mathbb{1}_n\|, \quad \forall k \in \mathbb{N}.$$

**Remark 1** It is noteworthy that  $0 < \lambda_d \leq 1$  and for a priori fixed values of  $\alpha$ ,  $\beta$  we can use the bisection algorithm to approach as close as we want the value of  $\lambda_d$ .

**Remark 2** To complete the decay rate analysis, we can consider that  $P_{ex}$ ,  $L$  and  $\lambda_d$  are fixed and perform a line search to find the nominal reset period  $\delta$  that ensures the convergence speed constraint. In other words, we check if (10) has solutions for  $\xi = \lambda_d$  and  $\delta$  heuristically sweeping the positive real axis. Moreover, we can progressively decrease  $\lambda_d$  and re-iterate the line search in order to find the smaller reachable decay rate.

## 4.2 Convergence toward a prescribed value

In what follows we assume that the value  $x^*$  is a priori fixed and at least a vector  $u$  satisfying (5) exists. Under this assumption we are wondering if there exists a matrix  $P_l$  that allows system (2) to reach the consensus value  $x^*$ . It is worth noting that the network topology is still considered fixed and known for each cluster. Under these assumptions, a consensus value is imposed by a certain choice of  $v$  such that  $v^\top \mathbb{1}_m = 1$  and  $v$  left eigenvector of  $P_l$  associated with the eigenvalue 1. In other words we arrive to a joint design of  $P_l$  and the Lyapunov function  $V$  guaranteeing that the trajectory of (2) ends up on  $x^*$ .



**Theorem 5** *Let us consider the system (2) with  $\mathcal{T}$  in the admissible reset sequences  $\Phi(\Delta)$  and let  $x^*$  be a priori fixed by a certain choice of  $v$ . If there exist matrices  $R(\delta')$ ,  $R(\cdot) : \Delta \mapsto \mathbb{R}^{n \times n}$  continuous with respect to  $\delta'$ ,  $R(\delta') = R^\top(\delta') > 0$ ,  $\delta' \in \Delta$  and  $P_l \in \mathbb{R}^{m \times m}$  such that the LMI*

$$\begin{bmatrix} Z(\delta_a) & \left(Y(\delta_a) - \mathbb{1}_n u^\top W\right)^\top \\ \left(Y(\delta_a) - \mathbb{1}_n u^\top W\right) & R(\delta_b) \end{bmatrix} > 0, \quad (11)$$

$$Y(\delta_a) \triangleq P_{ex} e^{-L(\delta + \delta_a)}$$

$$Z(\delta_a) \triangleq \left(I_n - \mathbb{1}_n u^\top W\right)^\top + \left(I_n - \mathbb{1}_n u^\top W\right) - R(\delta_a)$$

with the constraints

$$v^\top P_l = v^\top, \quad P_l \mathbb{1}_m = \mathbb{1}_m, \quad P_{l(i,j)} \geq 0$$

is satisfied on  $\text{span}\{\mathbb{1}_n\}^\perp$  for all  $\delta_a, \delta_b \in \Delta$ , then  $x^*$  is GUES for (2). Moreover, the stability is characterized by the quasi-quadratic Lyapunov function  $V(t) = \max_{\delta' \in \Delta} (x(t) - x^* \mathbb{1}_n)^\top R(\delta')^{-1} (x(t) - x^* \mathbb{1}_n)$  satisfying  $V(t_k) > V(t_{k+1})$ .

**Proof:** First notice that

$$\left((I_n - \mathbb{1}_n u^\top W)^\top S(\delta_a) - I_n\right) S(\delta_a)^{-1} \times \left(S(\delta_a)(I_n - \mathbb{1}_n u^\top W) - I_n\right) \geq 0$$

leads to

$$(I_n - \mathbb{1}_n u^\top W)^\top S(\delta_a)(I_n - \mathbb{1}_n u^\top W) \geq (I_n - \mathbb{1}_n u^\top W)^\top + (I_n - \mathbb{1}_n u^\top W) - S(\delta_a)^{-1}$$

Thus, once the solution to the LMI problem (11) is obtained we can define  $S(\delta_a) = R(\delta_a)^{-1}$  and  $S(\delta_b) = R(\delta_b)^{-1}$ . Then:

$$\begin{bmatrix} Z(\delta_a) & \left(Y(\delta_a) - \mathbb{1}_n u^\top W\right)^\top \\ \left(Y(\delta_a) - \mathbb{1}_n u^\top W\right) & S(\delta_b)^{-1} \end{bmatrix} > 0$$

where

$$Z(\delta_a) = (I_n - \mathbb{1}_n u^\top W)^\top + (I_n - \mathbb{1}_n u^\top W) - S(\delta_a)^{-1}$$

and hence

$$\begin{bmatrix} \bar{Z}(\delta_a) & \left(Y(\delta_a) - \mathbb{1}_n u^\top W\right)^\top \\ \left(Y(\delta_a) - \mathbb{1}_n u^\top W\right) & S(\delta_b)^{-1} \end{bmatrix} > 0$$

where

$$\bar{Z}(\delta_a) = (I_n - \mathbb{1}_n u^\top W)^\top S(\delta_a) (I_n - \mathbb{1}_n u^\top W).$$

By Schur complement, the last LMI is nothing than (6) in Theorem 2. Moreover, the constraints  $v^\top P_l = v^\top$ ,  $P_l \mathbb{1}_m = \mathbb{1}_m$  and the coefficients of  $P_l$  positive ensure the matrix  $P_l$  is row stochastic and the consensus value is exactly  $x^*$ .

## 5 Illustrative examples

### 5.1 Academic example

An academic example consisting in a network of 5 agents partitioned in 2 clusters ( $n_1 = 3, n_2 = 2$ ) is used in the sequel to illustrate the theoretical results. We consider the dynamics (2) with

$$L = \begin{bmatrix} 4 & -2 & -2 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 \\ 0 & -2 & 2 & 0 & 0 \\ 0 & 0 & 0 & 3 & -3 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix}, \quad P_l = \begin{bmatrix} 0.45 & 0.55 \\ 0.25 & 0.75 \end{bmatrix} \quad (12)$$

and the reset sequence given by  $\delta = 0.5$  and  $\delta'_k$  randomly chosen in  $\Delta = [0, 0.2]$ . The initial condition of the system is  $x(0) = (8, 7, 9, 2, 3)$  and the corresponding consensus value computed by (5) is  $x^* = 4.6757$ . The convergence of the 5 agents towards  $x^*$  is illustrated in Figure 1 emphasizing that the leaders trajectories are non-smooth while the followers trajectories are.

The table below collects the first 10 time intervals between consecutive reset instants. As expected, these time intervals have random lengths within  $[0.5, 0.7]$  and no monotony occurs.

$t_1 - t_0$	0.6189 s	$t_6 - t_5$	0.5979 s
$t_2 - t_1$	0.6131 s	$t_7 - t_6$	0.5372 s
$t_3 - t_2$	0.6433 s	$t_8 - t_7$	0.6401 s
$t_4 - t_3$	0.6023 s	$t_9 - t_8$	0.6965 s
$t_5 - t_4$	0.6553 s	$t_{10} - t_9$	0.6613 s

The jumps and decreasing of the Lyapunov function defined by Theorem 2 are pointed out in Figure 2.

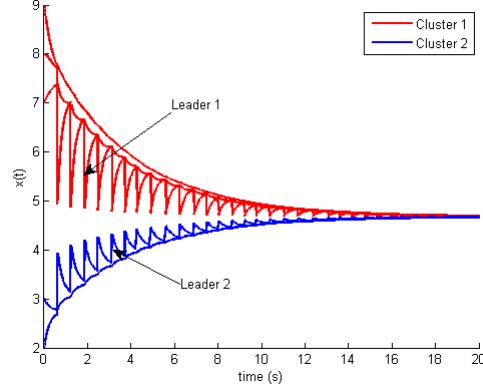


Figure 1: The state-trajectories of the agents converging to the calculated consensus value.

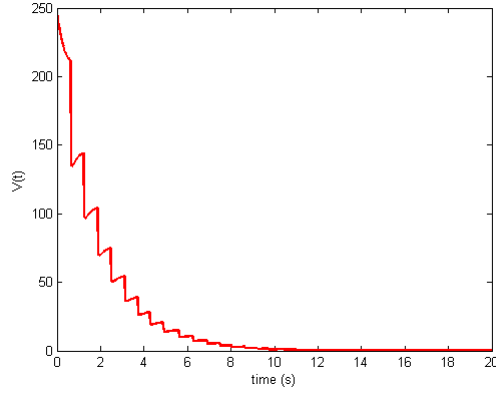


Figure 2: The behavior of the Lyapunov function given by Theorem 2.

To find the decay rate  $\lambda_d$  we use the bisection algorithm as stated in Remark 1. The value of  $\lambda_d$  obtained was  $\lambda_d = 0.855$  and the number of iterations of the bisection algorithm is  $k = 30$ .

The consensus value is always a convex combination of the initial agreement values of the clusters. In the present case, one has two clusters and the two initial agreements are 2.75 and 7.5. Thus, we can try to reach only consensus values belonging to  $[2.75, 7.5]$ . In Figure 3 the consensus value was fixed at  $x^* = 6.5$ .

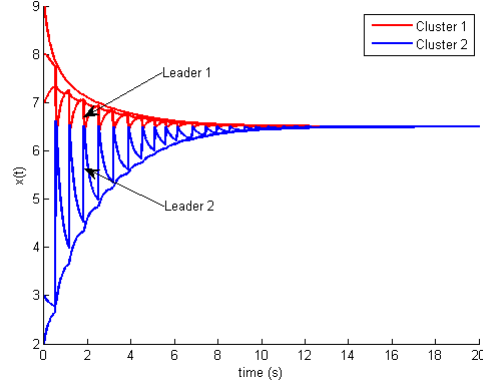


Figure 3: The states of a system ( $x^* = 6.5$ ).

The obtained  $P_l$  matrix was

$$P_l = \begin{bmatrix} 0.6870 & 0.3130 \\ 0.7825 & 0.2175 \end{bmatrix}.$$

In Figure 4 the consensus value was fixed as  $x^* = 3.5$ . The obtained  $P_l$  matrix was

$$P_l = \begin{bmatrix} 0.3010 & 0.6990 \\ 0.0874 & 0.9126 \end{bmatrix}. \quad (13)$$

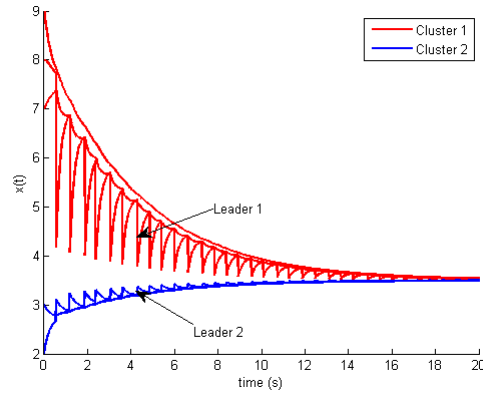


Figure 4: The states of a system ( $x^* = 3.5$ ).

## 5.2 Larger network analysis

In order to prove that the algorithms are implementable in real networks we consider in the following a larger system. Precisely, we present an example consisting of a network of 100 agents partitioned in 3 clusters. The size of the clusters as well as the connections between agents are randomized, resulting in non-symmetric matrices  $L$  and  $P_l$ . The random initialization leads at  $n_1 = 59, n_2 = 20, n_3 = 21$  and

$$P_l = \begin{bmatrix} 0.1538 & 0.8080 & 0.0382 \\ 0.4886 & 0.3876 & 0.1238 \\ 0.1266 & 0.2805 & 0.5929 \end{bmatrix}. \quad (14)$$

The initial condition is also randomized but, in order to guarantee a relatively large interval for the possible consensus value, for the first cluster the initial states of the agents are randomly chosen within  $[0, 3]$ , for the second one within  $[3, 7]$  and for the third one within  $[7, 10]$ . The corresponding initial local agreement values are 1.2970, 5.2578 and 8.7556, respectively. We illustrate the theoretical results by using the dynamics (2) with the reset sequence given by  $\delta = 0.5$ . The corresponding consensus value computed by (5) is  $x^* = 4.2562$ . The convergence of the 100 agents towards  $x^*$  is shown in Figure 5.

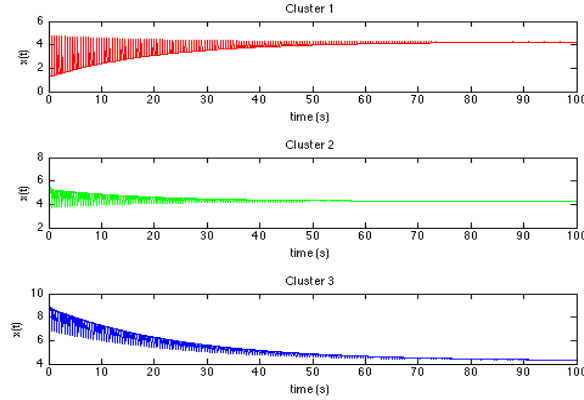


Figure 5: The state-trajectories of the agents converging to the calculated consensus value.

To find the decay rate  $\lambda_d$  we use the bisection algorithm as stated in Remark 1. In Figure 6 we demonstrate that the initial conditions of Lyapunov

function may vary due to the conditioning of the matrix  $S(\delta)$ , but the decay rate remains the same. The value of  $\lambda_d$  obtained was  $\lambda_d = 0.9712$  and the number of iterations of the bisection algorithm is  $k = 10$ .

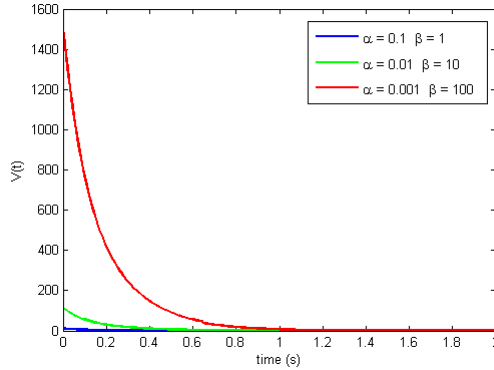


Figure 6: Lyapunov function for different conditionings

The consensus value is always a convex combination of the initial agreement values of the clusters. Therefore, for the initialization above, any consensus value can be imposed between 1.2970 and 8.7556. In Figure 7 the consensus value was fixed at  $x^* = 6.5$  and one obtained  $P_l$  matrix is

$$P_l = \begin{bmatrix} 0.0643 & 0.3720 & 0.5637 \\ 0.3064 & 0.0358 & 0.6578 \\ 0.0360 & 0.1917 & 0.7723 \end{bmatrix}.$$

## 6 Conclusions and perspectives

In this work we have considered networks of linear agents partitioned in several clusters disconnected one of each other. Each cluster has a linear impulsive leader that resets its state nearly periodically by taking into account the state of some neighboring leaders. In [3] we have characterized the consensus value for this type of networks and we have performed its stability analysis. In this work we provided the convergence speed towards consensus and we designed the interconnection network between the leaders allowing to reach a prescribed consensus value.

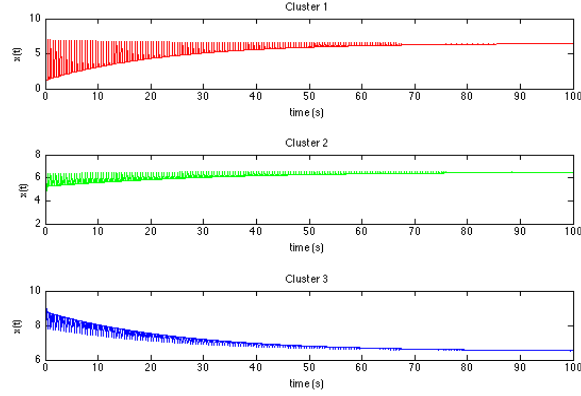


Figure 7: The states of a system ( $x^* = 6.5$ ).

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