Derivative based global sensitivity analysis for screening and robustness studies of electromagnetic devices

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***Abstract*— In this paper we apply a recent index [1] based on gradient information [2] (local information), but giving a global sensitivity measure. Moreover, this index gives a superior limit to classical Sobol's total index, and converges faster. It is applied on PMSM (Permanent Magnet Synchronous Machine) design. A first stage is to reduce parameters for optimization procedure; the second stage is to perform robustness analysis on the optimal solution.**

***Index Terms*— Derivative, Robustness, Screening, Sensitivity.**

I. Sensitivity Analysis

Sensitivity analysis aims at sorting parameters regarding their influence on performances. It helps designer understanding its model behavior regarding their variations on defined ranges. These ranges can be small (local sensitivity) or big (global sensitivity) depending on different goals. A first goal can be to perform a parameter screening in order to reduce dimension of a sizing search procedure (such as optimization). Another goal can be to study robustness of a design solution regarding uncertainties.

II. DGSM

Derivative based global sensitivity (DGSM) can be positioned between Morris screening method [3] and Sobol indices. It is fast as Morris method, but closer to Sobol indices since it takes probability distribution into account.

Consider a model ***f (x)***, where $x = (x_1, x_2, \cdots, x_n) \in \Re^n$. DGSM method is averaging all squared derivatives inside the space according to variables distribution. Under the conditions of variables independency, the DGSM index can be defined as equation (1):

$$\vartheta_i = \int_{\Re^n} \left(\frac{\partial f}{\partial x_i} \right)^2 \rho(x).dx \qquad (1)$$

with $\rho(\boldsymbol{x})$, the probability density functions.

The major strength of this index is the inequality that binds to S_i^T, the Sobol total index. In the case of uniformly distributed variables over the interval [a,b], Sobol and Kucherenko [1] have defined the index Y_i such as:

$$Y_i = \frac{(b-a)^2}{\pi^2.D} \vartheta_i \geq S_i^T \qquad (2)$$

with $D = \int_{\Re^n} f^2(x)dx - f_0^2$, the total variance of ***f(x)***.

In the case of normal distribution of variance σ^2, the index is:

$$Y_i = \frac{\sigma^2}{D} \vartheta_i \geq S_i^T \qquad (3)$$

III. Application to PMSM Design

PMSM (Permanent Magnet Synchronous Machine) design requires simulation model in order to achieve good performances at a low cost. A Reluctance Network (RN) method which furnishes the harmonics of output parameters (i.e. torque and back-emf) has been made. Air-gap reluctances widths have been calculated for each angular pitch using Fourier series method introduced in [4]. The following figure gives sensitivity indices Y_i for electromagnetic torque regarding 23 parameters. It has been computed using model's Jacobian which is available in our modeling framework using automatic and symbolic differentiation [2].

There are no doubts about main influent parameters, essentially related to magnet effects. But in this machine, in which we try to constrain torque ripple, as well as back-emf harmonics, such a sensitivity measure is helps modeling and design. A full analysis (including screening and robustness analysis) will be presented during the conference as well as convergence comparison with Sobol index.

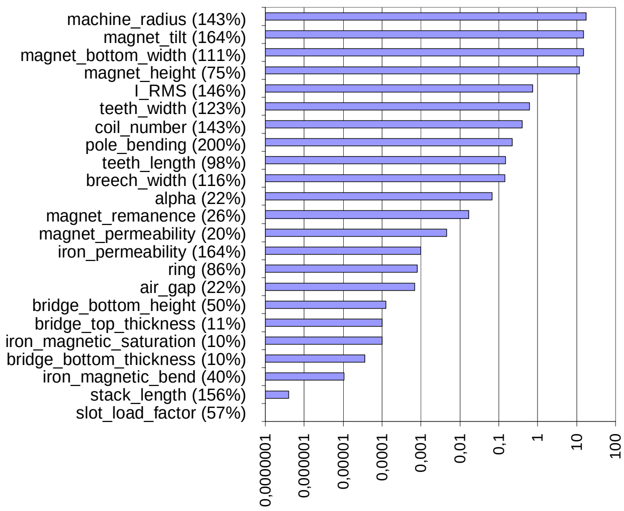

Fig. 2. Y_i indices for PMSM Torque sensitivity analysis.
Number inside parenthesis is variable range of variation.

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