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Solving boundary value problems with superabundant data by means of random walk simulation

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1 Introduction

The application of It's formula induces some probabilistic representations of solutions of deterministic linear problems with boundary conditions of Dirichlet, Neumann, Fourier, and mixed types. These representations are used to establish some easily implementable algorithms which compute an approximate solution by means of simulation of reflected random walks. The boundary condition treatment can be reduced to the counting of absorptions and reflections on the boundaries.

We recall first this simulation method and compare numerically some Euler's and Runge-Kutta's schemes used to solve boundary value problems (see, for these schemes, [K92]). Secondly, we consider the following problem with superabundant data on the boundary :

$$\begin{cases} -\frac{1}{2}\Delta u = f & (G) \\ \nabla u \cdot e_i = g_i & (\partial G) \end{cases} \quad (1)$$

where G is an open bounded region in \mathbb{R}^d , with its boundary ∂G , u the unknown function, defined on G , g_i , $i = 1, 2$, some given functions, defined on ∂G , (e_i) an orthonormal basis of \mathbb{R}^d . We then present a plan of my talk.

2 Stochastic representations of solutions

Section 2 is devoted to the stochastic representations of solutions with boundary conditions of different types using It formula. The representations *will be given in the full paper* without demonstration. See, for these representations, [F85] where the data are sufficiently smooth.

3 Resolution algorithms

In Section 3, we present the approaches of these representations by the realizations of random processes, and we establish the corresponding computational algorithms (see, for these simulations and algorithms, [M95]). The algorithms written in pseudo-Pascal *will be given in the full paper*.

4 Numerical experiments

The algorithms have been implemented. The programs, written in Fortran or Pascal, have been run on Sun Spark work stations or compatible PC. Numerical experiments with distributed source in two-dimensional geometries, and computational results with estimation of empirical error, *will be given in the full paper*. For example, we consider problem (1) in the ring G defined by :

$$G = \left\{ (x, y) \in \mathbb{R}^2 \mid 1 < \sqrt{x^2 + y^2} < 3 \right\}$$

with the functions f and g_i defined in G and ∂G respectively such as : $f(x, y) = -2$, $g_1(x, y) = 2x$, and $g_2(x, y) = 2y$. This homogeneous Neumann problem has an exact solution, defined up to an additive constant C : $u(x, y) = x^2 + y^2 + C$.

5 Concluding remarks

The stochastic methods obtained in [M95] do not require selected configurations at the neighborhood of the domain boundary, nor a discretization mesh. The associated simulation methods are obtained and can be applied to problems with superabundant data without specific treatments to domain local geometry.

Programming is short, easy to check step by step. As for the classical Monte Carlo, this stochastic method admit an expected rate of convergence of about $\sqrt{1/NT}$ where NT is the sample size. Relative error and empirical variance can be computed. The essential properties of Monte Carlo methods are maintained.

Références

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