A non symmetric extension of Amblard-Girard copulas

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Dragan European University of Lugoj

Abstract

A copula is a function that completely describes the dependence structure between the marginal distributions. One of the most important parametric family of copulas is the Farlie-Gumbel-Morgenstern (FGM) family. In practical applications this copula has been shown to be somewhat limited and a symmetric extension of this family, known as the Amblard-Girard copula, has been introduced. We propose a new non symmetric extension of this family.

A copula is a function that completely describes the dependence structure. It contains all the information to link the marginal distributions to their joint distribution. To obtain a valid multivariate distribution function, it suffices to combine several marginal distribution functions with any copula function. Thus, for the purposes of statistical modeling, it is desirable to have a large collection of copulas at one's disposal. Many examples of copulas can be found in the literature, most are members of families with one or more real parameters.

One of the most important parametric family of copulas is the Farlie-Gumbel-Morgenstern (FGM) family defined as

$$C_\theta^{FGM}(u,v) = uv + \theta uv(1-u)(1-v), \quad u,v \in [0,1], \tag{1}$$

where $\theta \in [-1,1]$. The density function of FGM copulas is given by

$$\frac{\partial^2 C_\theta^{FGM}(u,v)}{\partial u \partial v} = \theta\,(2u-1)\,(2v-1) + 1,$$

for any $u, v \in [0,1]$.

Members of the FGM family are symmetric, i.e., $C_\theta^{FGM}(u,v) = C_\theta^{FGM}(v,u)$ for all (u,v) in $[0,1]^2$ and have the lower and upper tail dependence coefficients equal to 0.

The copula given in (1) is PQD for $\theta \in (0,1]$ and NQD for $\theta \in [-1,0)$. In practical applications this copula has been shown to be somewhat limited, for copula dependence parameter $\theta \in [-1,1]$, Spearman's correlation $\rho \in [-1/3, 1/3]$ and Kendall's $\tau \in [-2/9, 2/9]$, for more details on copulas see, for example, [1].

To overcome this limited dependence, several authors proposed extensions of this family.

- Theoretical issues: [2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24]
- Applications: [25, 26, 27, 28, 29, 30]

Let C_1, C_2 be two copulas proposed by [9]:

$$C_i(u,v) = uv + \theta_i \phi_i(u)\phi_i(v)$$

with $i \in \{1,2\}$. The parameter θ_i is called the dependence parameter while the function ϕ_i is called the generator. Let $C = C_1 \star C_2$ where $\star$ is the copula-product, see for instance [1]. The density associated to C can be computed as:

$$\begin{aligned} c(u,v) &= \int_0^1 c_1(u,t)c_2(v,t)dt \\ &= \int_0^1 [1+\theta_1\phi_1'(u)\phi_1'(t)][1+\theta_2\phi_2'(v)\phi_2'(t)]dt \\ &= 1+\theta_1\theta_2\int_0^1 \phi_1'(t)\phi_2'(t)dt\phi_1'(u)\phi_2'(v), \end{aligned}$$

the cross-product terms being null in view of the constraint $\phi_i(0) = \phi_i(1) = 0$, for all $i \in \{1,2\}$. Letting $\theta = \theta_1\theta_2 \int_0^1 \phi_1'(t)\phi_2'(t)dt$, routine calculations show that

$$C(u,v) = uv + \theta\phi_1(u)\phi_2(v).$$

Therefore, C can be considered as a non-symmetric extension of [9].

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