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# A Luenberger like observer for the AM2 model

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## Abstract

In this paper, we deal with the problem of designing a new class of observers for uncertain bioreactor models. The main idea is to construct a nonlinear observer with linear errors, which has an adjustable and robust convergence. Simulation results are presented using a model of chemostat and a model of an anaerobic digestion process for the treatment of wastewater.

*Key words:* Chemostat model, AM2 model, Nonlinear systems, Nonlinear observers.

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## 1 Introduction

The AM2 model was developed under the European project AMOCO. It is a two-step model (corresponding to two biological cascade reactions hence its name), which is represented by the following mathematical system:

$$\begin{cases} \dot{s}_1 = D(t)(s_{1in} - s_1) - k_1\mu_1(s_1)x_1 \\ \dot{x}_1 = [\mu_1(s_1) - D(t)]x_1 \\ \dot{s}_2 = D(t)(s_{2in} - s_2) + k_2\mu_1(s_1)x_1 - k_3\mu_2(s_2)x_2 \\ \dot{x}_2 = [\mu_2(s_2) - D(t)]x_2 \end{cases} \quad (1)$$

In this work we will take as an output

$$y = \begin{pmatrix} s_1 \\ s_2 \end{pmatrix}$$

This model was originally proposed in [4] and it is based on two main reactions, where the substrate  $s_1$  is degraded in the substrate  $s_2$  by the biomass  $x_1$  then the substrate  $s_2$  is degraded by the biomass  $x_2$ , where  $(s_1, s_2) \in R_+^* \times R_+^*$  and  $(x_1, x_2) \in R_+^* \times R_+^*$ ,  $D(t) > 0$  is the dilution rate,  $k$  is the growth yield,  $s_{1in}$  and  $s_{2in}$  are the input substrate concentration of  $s_1$  and  $s_2$  and  $\mu_1(s_1)$  and  $\mu_2(s_2)$  are the specific growth rates.

We will assume that a generic specific growth rate function  $\mu(s)$  satisfies:

- $\mu(s) = 0 \Leftrightarrow s = 0$ ,
- $0 \leq \mu(\cdot) \leq \mu_{max} = cst$ ,
- $\mu(\cdot)$  continue.

In bioprocesses field, the design of nonlinear observers is very challenging, and is an area of intensive research. Till now important papers dealing with the observer of AM2 model are proposed in [12]. The observers are used as software sensors and were applied for the first time in this context by Bastin and Dochain [3].

Several observers have been proposed for anaerobic digestion model, starting with *asymptotic observers* and *interval observers*.

Bastin and Dochain proposed an asymptotic observer that allows (under conditions) to observe the state of the system without any knowledge of the kinetic model. This approach is particularly interesting. Nevertheless, the speed of convergence of this observer, contrary to the Luenberger one, cannot be tuned and depends only of dilution rate. Originally, these observers were designed for relatively simple systems, then they were extended by Chen [5], to include more complex ones. In this context, we note that they are characterized by the simplicity of their design, and preserve some nonlinear aspect of the system, ensuring stability and convergence if the inputs are persistent and bounded. The change of coordinates depends on the stoichiometric coefficients making so these observers not robust, then, to overcome these disadvantages, Gouzé *et al.* [7], [9] generalized them and they made the asymptotic observers robust with a speed

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of convergence partially adjustable.

The interval observers should give a good solutions for systems with large uncertainties see ([1] and [8]). They are the combination of two observers, one observes the lower bound and the other observes the upper bound of the state, under a strong property called *cooperativity*, and it is necessary to know the bounds of uncertainties in the model.

Thanks to the specific form of the output we were leaded to propose a nonlinear Luenberger observer in its general form with the injection of the outputs in the nonlinear function  $\mu(s)$ , we get in turn, a linear system for the errors.

This paper is organized as follows: Section 2 is dedicated to get more insight of the behaviour of our observer by applying it on a chemostat model. Indeed the chemostat model is a subsystem of the AM2 model and the ideas developed here will be used later. Section 3 is devoted to the application of the proposed observer on the AM2 model as a main results with some simulations to show the effectiveness of the proposed method. Finally, in section 4 we present the robustness of the observer and we make a comparison with the asymptotic observer.

## 2 The chemostat model

The chemostat is a laboratory prototype of bioreactors used in waste water treatment, which was introduced by Novick and Szilard [11] and used by Monod [10].

The chemostat is a kind of bioreactor which allows the growth of a population of microorganisms (bacteria, yeast, phytoplankton, zooplankton,...) on some substrates, with suitable environmental conditions (temperature, light, pH and aeration). This device works in continuum mode, i.e, the volume of the bioreactor is kept constant.

The nonlinear model of the chemostat obtained by mass balance is given by:

$$\begin{cases} \dot{s} = D(t)(s_{in} - s) - k\mu(s)x \\ \dot{x} = [\mu(s) - D(t)]x \end{cases} \quad (2)$$

We will take as an output  $y = s$ , where  $s \in R_+^*$  and  $x \in R_+^*$  represent the substrate concentration and the biomass concentration respectively,  $D(t) > 0$  is the dilution rate,  $k$  is the growth yield,  $s_{in}$  is the input substrate concentration and  $\mu(s)$  is the specific growth rate per unit of biomass. To avoid the washout we impose the condition  $D(t) \leq \mu_{max}$ .

**Lemma 1 (H1)** Suppose that  $\lim_{t \rightarrow +\infty} \left( \int_0^t D(\tau) d\tau \right) = +\infty$  (*persistently exciting condition*).

**(H2)**  $\lim_{t \rightarrow +\infty} \left( \int_0^t (D(\tau) - \mu_{max}) d\tau \right) = c$  where  $c$  is a negative constante.

Then there exists a positive constante  $s_{min}$  such that  $s \geq s_{min}$ .

**PROOF.** Let  $\xi = s + kx$ , then

$$\dot{\xi} = D(s_{in} - \xi)$$

and

$$\xi(t) = (\xi(0) - s_{in})e^{-\int_0^t D(\tau) d\tau} + s_{in}$$

It is not hard to see that

$$\min(\xi(0), s_{in}) \leq \xi(t) \leq \max(\xi(0), s_{in})$$

We have

$$\dot{s} = D(t)(s_{in} - s) - k\mu_{max}x + \underbrace{k(\mu_{max} - \mu(s))x}_{\geq 0}$$

so

$$\dot{s} \geq D(t)(s_{in} - s) - k\mu_{max}x$$

as  $kx = \xi - s$ , we have

$$\dot{s} \geq D(t)s_{in} + (\mu_{max} - D)s - \mu_{max} \max(\xi(0), s_{in})$$

then

$$s(t) \geq s_{in} + (s(0) - s_{in})e^{-\int_0^t (D(\tau) - \mu_{max}) d\tau}$$

Let  $\varphi(t) = s_{in} + (s(0) - s_{in})e^{-\int_0^t (D(\tau) - \mu_{max}) d\tau}$ , we have:

$$\varphi(0) = s(0)$$

Under the hypothesis  $\lim_{t \rightarrow +\infty} \left( \int_0^t (D(\tau) - \mu_{max}) d\tau \right) = c$  we have

$$\varphi(+\infty) = s_{in} + (s(0) - s_{in})e^{-c}$$

Now

$$\dot{\varphi}(t) = (s(0) - s_{in})(\mu_{max} - D(t))e^{-\int_0^t (D(\tau) - \mu_{max}) d\tau}$$

for  $s(0) \leq s_{in}$  and as  $D(t) \leq \mu_{max}$  then  $\dot{\varphi}(t) < 0$  so

$$\varphi(+\infty) \leq \varphi(t) \leq \varphi(0)$$

So

$$s(t) \geq s_{min} = s_{in} + (s(0) - s_{in})e^{-c}$$

It can be easily to prove that

$$s(t) \leq s_{in}, \forall t \geq 0$$

**Corollary 2** For a generic  $\mu(\cdot)$ , we have

$$\mu(s) \geq \mu_{min} > 0$$

**Theorem 3** For the system (2), the following system:

$$\begin{cases} \dot{\hat{s}} = D(t)(s_{in} - \hat{s}) - k\mu(s)\hat{x} + a_1(s - \hat{s}) \\ \dot{\hat{x}} = [\mu(s) - D(t)]\hat{x} + a_2(s - \hat{s}) \end{cases} \quad (3)$$

is an observer, where  $a_1$  and  $a_2$  are two tuning parameters.

By putting  $e_1 = s - \hat{s}$  and  $e_2 = x - \hat{x}$ , the error dynamics are given by:

$$\begin{pmatrix} \dot{e}_1 \\ \dot{e}_2 \end{pmatrix} = \begin{pmatrix} -D(t) - a_1 & -k\mu(s) \\ -a_2 & \mu(s) - D(t) \end{pmatrix} \begin{pmatrix} e_1 \\ e_2 \end{pmatrix} \quad (4)$$

Indeed,  $(0,0)$  is an equilibrium point of (4) which is globally uniformly asymptotically stable.

**PROOF.** Let us hide  $D(t)$  from (4) by putting  $p(t) = e^{\int_0^t D(\tau) d\tau}$  and let us multiply  $p(t)$  by the two equations of the system (4) then, the later can be written as follows:

$$\begin{cases} p(t)\dot{e}_1 + D(t)p(t)e_1 = -a_1p(t)e_1 - k\mu(s)p(t)e_2 \\ p(t)\dot{e}_2 + D(t)p(t)e_2 = -a_2p(t)e_1 + \mu(s)p(t)e_2 \end{cases} \quad (5)$$

i.e.

$$\begin{cases} \frac{d}{dt}(p(t)e_1) = -a_1p(t)e_1 - k\mu(s)p(t)e_2 \\ \frac{d}{dt}(p(t)e_2) = -a_2p(t)e_1 + \mu(s)p(t)e_2 \end{cases} \quad (6)$$

Let  $u(t) = p(t)e_1(t)$  and  $v(t) = p(t)e_2(t)$ , then (6) becomes

$$\begin{cases} \dot{u}(t) = -a_1u(t) - k\mu(s)v(t) \\ \dot{v}(t) = -a_2u(t) + \mu(s)v(t) \end{cases} \quad (7)$$

To prove the convergence of the observer we will take the following Lyapunov function candidate:

$$V_\alpha(u, v) = \left\| \begin{pmatrix} \alpha u + v \\ v \end{pmatrix} \right\|^2 = (\alpha u + v)^2 + v^2$$

where  $\alpha \neq 0$ .

Note that this function is also a norm and then is not

radially bounded. Then

$$\dot{V}_\alpha(u, v) = 2(\alpha u + v)(\alpha \dot{u} + \dot{v}) + 2v\dot{v}$$

by substituting  $\dot{u}$  and  $\dot{v}$  in  $\dot{V}_\alpha(u, v)$ , we obtain

$$\begin{aligned} \dot{V}_\alpha(u, v) &= -2\alpha(a_1\alpha + a_2)u^2 + 2\mu(s)(2 - k\alpha)v^2 \\ &\quad + 2[\alpha(1 - k\alpha)\mu(s) - (a_1\alpha + 2a_2)]uv \end{aligned}$$

by putting,  $a_2 = -\frac{\alpha}{2}a_1$ , we have

$$\dot{V}_\alpha(u, v) = -\alpha^2a_1u^2 + 2\mu(s)(2 - k\alpha)v^2 + 2\alpha(1 - k\alpha)\mu(s)uv$$

This function can be written as follows

$$\dot{V}_p(u, v) = \begin{pmatrix} u & v \end{pmatrix} \underbrace{\begin{pmatrix} -\alpha^2a_1 & p(1 - k\alpha)\mu(s) \\ \alpha(1 - k\alpha)\mu(s) & 2\mu(s)(2 - k\alpha) \end{pmatrix}}_A \begin{pmatrix} u \\ v \end{pmatrix}$$

Now, let us prove that the matrix  $A$  is negative definite i.e. let us prove that  $tr(A) < 0$  and  $det(A) > 0$ .

$$tr(A) = -\alpha^2a_1 + 2\mu(s)(2 - k\alpha)$$

and

$$det(A) = -2\alpha^2(2 - k\alpha)\mu a_1 - (1 - k\alpha)^2\mu^2(s)\alpha^2$$

Here  $tr(A)$  will be negative for  $a_1 > 0$  and large enough and  $det(A)$  will be positif with the same condition and  $\alpha > \frac{2}{k}$ .

Note that  $\mu(s) \geq \mu_{min} > 0$  as presented in the corollary which is of great concern for the positivity of  $det(A)$ .

## 2.1 Simulation results

The simulations were carried out using the parameter values given in [2]. They are recalled in table (1) and the chosen initial conditions are given in table (2).

Parameter	Value and Unit
$k$	6.6 KgCOD/Kg x
$\mu_{max}$	1.2 day <sup>-1</sup>
$K$	4.95 KgCOD/m <sup>3</sup>
$s_{in}$	9 Kg/m <sup>3</sup>

Table 1  
Model parameters for simulation runs

The time simulation is taken to be 50 days with variable dilution rate as in the figure (1).

	$s(0)$ (Kg/m <sup>3</sup> )	$x(0)$ (Kg/m <sup>3</sup> )
Model	3	0.5
Observer	3	5

Table 2  
Initial conditions for simulation runs

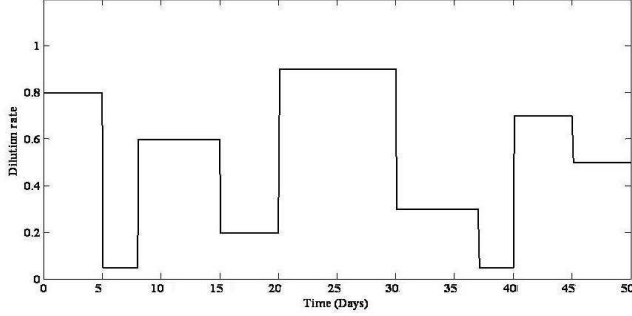


Fig. 1. Dilution rate

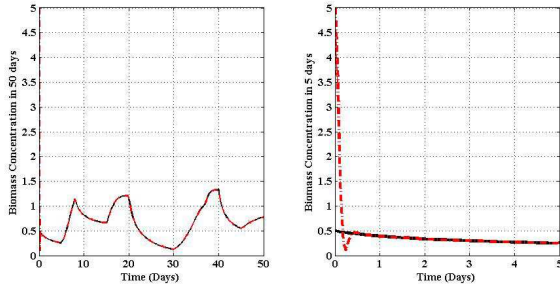


Fig. 2. Biomass Concentration for  $\alpha = 10$  and  $a_1 = 20$

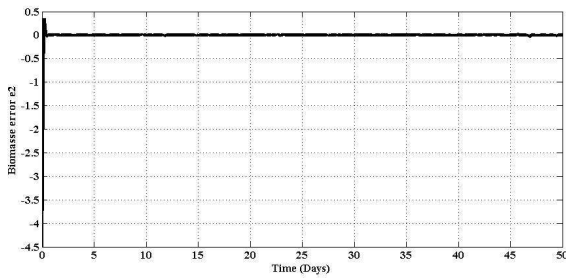


Fig. 3. Biomass error estimation

We can see from these simulations that on the biomass the convergence is obtained in a half a day with an error equal to  $10^{-2}$ . For comparison with the asymptotic observer, it converges only after 5 days and cannot be decreased.

### 3 Main results on AM2 model

**Theorem 4** For the system (1):

$$\begin{cases} \dot{\hat{s}}_1 = D(t)(s_{1in} - \hat{s}_1) - k_1\mu_1(s_1)\hat{x}_1 + a_1(s_1 - \hat{s}_1) \\ \quad + a_2(s_2 - \hat{s}_2) \\ \dot{\hat{x}}_1 = [\mu_1(s_1) - D(t)]\hat{x}_1 + a_3(s_1 - \hat{s}_1) + a_4(s_2 - \hat{s}_2) \\ \dot{\hat{s}}_2 = D(t)(s_{2in} - \hat{s}_2) + k_2\mu_1(s_1)\hat{x}_1 - k_3\mu_2(s_2)\hat{x}_2 \\ \quad + a_5(s_1 - \hat{s}_1) + a_6(s_2 - \hat{s}_2) \\ \dot{\hat{x}}_2 = [\mu_2(s_2) - D(t)]\hat{x}_2 + a_7(s_1 - \hat{s}_1) + a_8(s_2 - \hat{s}_2) \end{cases} \quad (8)$$

is an observer, where  $a_i$ ,  $i = \overline{1,8}$  are tuning parameters.

Let us define  $e_1 = s_1 - \hat{s}_1$ ,  $e_2 = x_1 - \hat{x}_1$ ,  $e_3 = s_2 - \hat{s}_2$  and  $e_4 = x_2 - \hat{x}_2$ .

The error dynamics are given by:

$$\begin{cases} \dot{e}_1 = -(D(t) + a_1)e_1 - k_1\mu_1(s_1)e_2 - a_2e_3 \\ \dot{e}_2 = -a_3e_1 + [\mu_1(s_1) - D(t)]e_2 - a_4e_3 \\ \dot{e}_3 = -a_5e_1 + k_2\mu_1(s_1)e_2 - (D(t) + a_6)e_3 - k_3\mu_2(s_2)e_4 \\ \dot{e}_4 = -a_7e_1 - a_8e_3 + [\mu_2(s_2) - D(t)]e_4 \end{cases} \quad (9)$$

$(0,0,0,0)$  is an equilibrium point of (9) which is globally uniformly asymptotically stable.

**PROOF.** Recall that we have injected the measured variables  $s_1$  and  $s_2$  into  $\mu_1(s_1)$  and  $\mu_2(s_2)$  to write this observer. Note that the system (9) is linear and non autonomous.

Let us write the system (9) as:

$$\dot{e} = B(t)e$$

where  $e = \begin{pmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{pmatrix}$  and  $B(t) =$

$$\begin{pmatrix} -D(t) - a_1 & -k_1\mu_1(s_1) & -a_2 & 0 \\ -a_3 & \mu_1(s_1) - D(t) & -a_4 & 0 \\ -a_5 & k_2\mu_1(s_1) & -D(t) - a_6 & -k_3\mu_2(s_2) \\ -a_7 & 0 & -a_8 & \mu_2(s_2) - D(t) \end{pmatrix}$$

If we take  $a_2 = a_4 = a_5 = a_7 = 0$ , then the matrix  $B(t)$

becomes:

$$\underbrace{\begin{pmatrix} -D(t) - a_1 & -k_1\mu_1(s_1) & \vdots & 0 & 0 \\ -a_3 & \mu_1(s_1) - D(t) & \vdots & 0 & 0 \\ \dots & \dots & \vdots & \dots & \dots \\ 0 & k_2\mu_1(s_1) & \vdots & -D(t) - a_6 & -k_3\mu_2(s_2) \\ 0 & 0 & \vdots & -a_8 & \mu_2(s_2) - D(t) \end{pmatrix}}_{\tilde{B}(t)}$$

We remark that the diagonal blocks are chemostat-like blocks (4).

So, let us hide  $D(t)$  from the system by putting  $p(t) = e^{\int_0^t D(\tau) d\tau}$  and  $u_1(t) = p(t)e_1(t)$ ,  $u_2(t) = p(t)e_2(t)$ ,  $u_3(t) = p(t)e_3(t)$  and  $u_4(t) = p(t)e_4(t)$ .

Then the system  $\dot{e} = \tilde{B}(t)e$  becomes

$$\begin{cases} \dot{u}_1 = -a_1u_1 - k_1\mu_1(s_1)u_2 \\ \dot{u}_2 = -a_3u_1 + \mu_1(s_1)u_2 \\ \dot{u}_3 = k_2\mu_1(s_1)u_2 - a_6u_3 - k_3\mu_2(s_2)u_4 \\ \dot{u}_4 = -a_8u_3 + \mu_2(s_2)u_4 \end{cases} \quad (10)$$

To prove the convergence of this observer we will take the following Lyapunov function candidate:

$$V_{\alpha,\beta}(u_1, u_2, u_3, u_4) = \left\| \begin{pmatrix} \alpha u_1 + u_2 \\ u_2 \\ \beta u_3 + u_4 \\ u_4 \end{pmatrix} \right\|^2$$

where  $\alpha$  and  $\beta$  are positive constants to be chosen later,  $V_{\alpha,\beta}$  is also a norm and then is radially unbounded. So

$$\begin{aligned} \dot{V}_{\alpha,\beta}(u_1, u_2, u_3, u_4) &= 2(\alpha u_1 + u_2)(\alpha \dot{u}_1 + \dot{u}_2) + 2u_2\dot{u}_2 \\ &\quad + 2(\beta u_3 + u_4)(\beta \dot{u}_3 + \dot{u}_4) + 2u_4\dot{u}_4 \end{aligned}$$

by substituting  $\dot{u}_i$ ,  $i = \overline{1,4}$  in  $\dot{V}_{\alpha,\beta}(u_i)$ , we obtain

$$\begin{aligned} \dot{V}_{\alpha,\beta} &= -2\alpha(a_1\alpha + a_3)u_1^2 - 2[a_1\alpha + 2a_3 + \alpha(\alpha k_1 - 1)\mu_1]u_1u_2 \\ &\quad - 2(\alpha k_1 - 2)\mu_1u_2^2 + 2k_2\beta^2\mu_1u_2u_3 + 2k_2\beta\mu_1u_2u_4 \\ &\quad - 2\beta(a_6\beta + a_8)u_3^2 - 2[a_6\beta + 2a_8 + \beta(\beta k_3 - 1)\mu_2]u_3u_4 \\ &\quad - 2(\beta k_3 - 2)\mu_2u_4^2 \end{aligned}$$

This function can be written as follows:

$$\dot{V}_{\alpha,\beta} = U^T C(t) U$$

$$\text{where } U = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{pmatrix} \text{ and } C(t) = \begin{pmatrix} c_{11} & c_{12} & 0 & 0 \\ c_{21} & c_{22} & c_{23} & c_{24} \\ 0 & c_{32} & c_{33} & c_{34} \\ 0 & c_{42} & c_{43} & c_{44} \end{pmatrix}$$

with

$$\begin{aligned} c_{11} &= -2\alpha(a_1\alpha + a_3), \\ c_{12} &= c_{21} = -[a_1\alpha + 2a_3 + \alpha(\alpha k_1 - 1)\mu_1], \\ c_{22} &= -2(\alpha k_1 - 2)\mu_1, c_{23} = c_{32} = k_2\beta^2\mu_1, \\ c_{24} &= c_{42} = k_2\beta\mu_1, c_{33} = -2\beta(a_6\beta + a_8), \\ c_{34} &= c_{43} = -[a_6\beta + 2a_8 + \beta(\beta k_3 - 1)\mu_2] \\ \text{and} \\ c_{44} &= -2(\beta k_3 - 2)\mu_2 \end{aligned}$$

We have to prove that  $C(t)$  is negative definite, i.e.  $-C(t)$  positive definite, so we have to prove that all the principal minors of  $-C(t)$  are positif.

Let  $\delta_1$ ,  $\delta_2$ ,  $\delta_3$  and  $\delta_4$  be these minors.

$$\delta_1 = 2\alpha(a_1\alpha + a_3)$$

$$\delta_2 = -(a_1\alpha + 2a_3)^2 + 2\alpha^2\mu_1(k_1\alpha - 3)a_1 - 4\alpha\mu_1a_3 - \alpha^2(k_1\alpha - 1)^2\mu_1^2$$

Let us put  $a_1\alpha + 2a_3 = 0$  then  $a_3 = -\frac{\alpha}{2}a_1$  so  $\delta_2$  becomes:

$$\delta_2 = 2\alpha^2\mu_1(k_1\alpha - 2)a_1 - \alpha^2(k_1\alpha - 1)^2\mu_1^2$$

For  $a_1 > 0$  and large enough and for  $\alpha > \frac{2}{k_1}$ ,  $\delta_1$  and  $\delta_2$  are both positive.

$$\begin{aligned} \delta_3 &= 2\alpha^2\beta^2\mu_1[2(\alpha k_1 - 2)a_1 - \mu_1(\alpha k_1 - 1)^2]a_6 \\ &\quad + 2\alpha^2\beta\mu_1[2(\alpha k_1 - 2)a_1 - \mu_1(\alpha k_1 - 1)^2]a_8 - \alpha^2\beta^4k_2^2\mu_1^2a_1 \end{aligned}$$

$$\begin{aligned} \delta_4 &= \alpha^2\mu_1[\mu_1(\alpha k_1 - 1)^2 - 2(\alpha k_1 - 2)a_1](\beta a_6 + 2a_8)^2 \\ &\quad + 2\mu_1\mu_2\alpha^2\beta^2(\beta k_3 - 3)[(\alpha k_1 - 2)a_1 - \mu_1(\alpha k_1 - 1)^2]a_6 \\ &\quad + 2\alpha^2\beta\mu_1[(\beta^2k_2^2\mu_1 + 4\mu_2(1 - 2\alpha k_1))a_1 + 2\mu_1\mu_2(\alpha k_1 - 1)^2]a_8 \\ &\quad - 2\mu_1\mu_2\alpha^2\beta^2[\mu_2(\alpha k_1 - 2)(\beta k_3 - 1)^2 - k_2^2]a_1 \\ &\quad + \mu_1^2\mu_2^2\alpha^2\beta^2(\beta k_3 - 1)^2(\alpha k_1 - 1)^2 \end{aligned}$$

Let us put  $a_6\beta + 2a_8 = 0$  then  $a_8 = -\frac{\beta}{2}a_6$  so  $\delta_4$  becomes:

$$\begin{aligned} \delta_4 &= \alpha^2\beta^2\mu_1[2\mu_2((\beta k_3 - 3)(\alpha k_1 - 2) - 2(1 - 2\alpha k_1)) \\ &\quad - \beta^2k_2^2\mu_1]a_1 a_6 - 2\mu_1\mu_2\alpha^2\beta^2[\mu_2(\alpha k_1 - 2)(\beta k_3 - 1)^2 - k_2^2]a_1 \\ &\quad + \mu_1^2\mu_2^2\alpha^2\beta^2(\beta k_3 - 1)^2(\alpha k_1 - 1)^2 \end{aligned}$$

For  $a_6 > 0$  and large enough and for  $\beta > \frac{3}{k_3}$ ,  $\delta_3$  and  $\delta_4$  are both positive.

### 3.1 Simulation results

The simulations were carried out using the parameter values recalled in [2]. They are given in table (1) and the chosen initial conditions are given in table (2).

Parameter	Value and Unit
$k_1$	6.6 KgCOD/Kg $x_1$
$k_2$	7.8 molVFA/Kg $x_1$
$k_3$	611.2 molVFA/Kg $x_2$
$\mu_{1max}$	1.2 day <sup>-1</sup>
$\mu_{2max}$	0.69 day <sup>-1</sup>
$K_1$	4.95 KgCOD/m <sup>3</sup>
$K_2$	9.28 molVFA/m <sup>3</sup>
$K_i$	20 (molVFA/m <sup>3</sup> ) <sup>1/2</sup>
$s_{1in}$	15 Kg/m <sup>3</sup>
$s_{2in}$	80 mol/m <sup>3</sup>

Table 3  
Model parameters for simulation runs

	$s_1(0)$ (Kg/m <sup>3</sup> )	$x_1(0)$ (Kg/m <sup>3</sup> )	$s_2(0)$ (mol/m <sup>3</sup> )	$x_2(0)$ (Kg/m <sup>3</sup> )
Model	3	0.5	15	0.12
Observer	3	3	15	1

Table 4  
Initial conditions for simulation runs

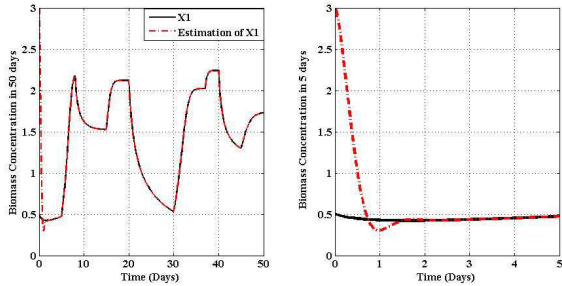


Fig. 4. The first biomass estimation for  $\alpha = 2$ ,  $\beta = 1$ ,  $a_1 = 5$  and  $a_6 = 6$

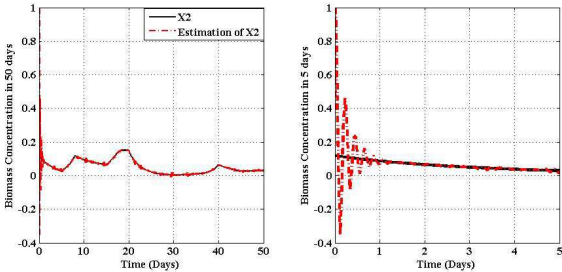


Fig. 5. The second biomass estimation for  $\alpha = 2$ ,  $\beta = 1$ ,  $a_1 = 5$  and  $a_6 = 6$

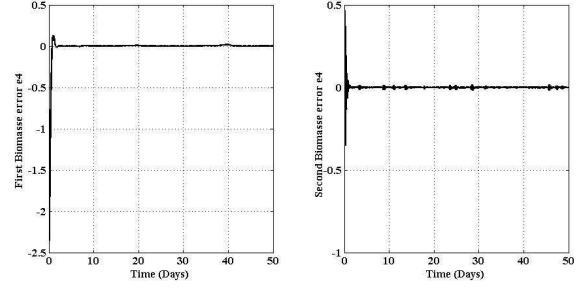


Fig. 6. The biomass errors

We can see from these simulations that on the first biomass  $x_1$  the convergence is obtained in a two days with an error equal to  $10^{-2}$  and on the second biomass  $x_2$  the convergence is obtained in a one days with the same error.

## 4 Robustness

Firstly, we perturbed the first substrate  $s_1$  (see figures (7, 8)), then we injected the same perturbation in the second substrate  $s_2$  (see figures (9, 10)), and we notice that our observer is very robust to measurement noises.

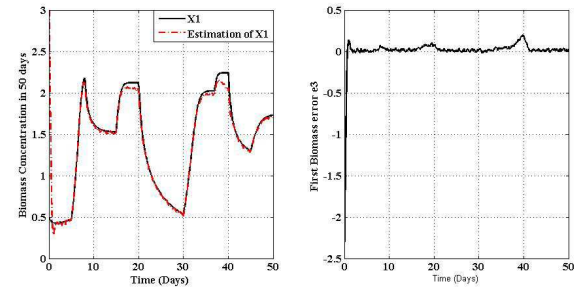


Fig. 7. The first biomass  $x_1$  and its error estimation with a perturbation on  $s_1$

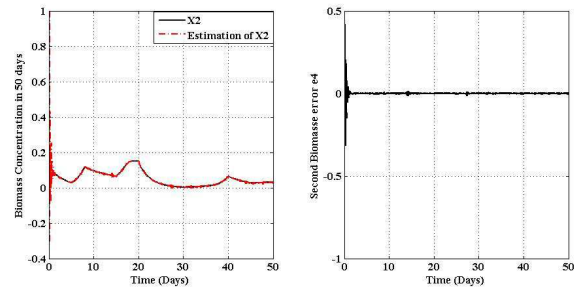


Fig. 8. The second biomass  $x_2$  and its error estimation with a perturbation on  $s_1$

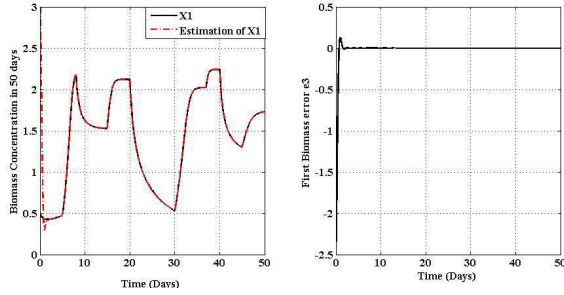


Fig. 9. The first biomass  $x_1$  and its error estimation with a perturbation on  $s_2$

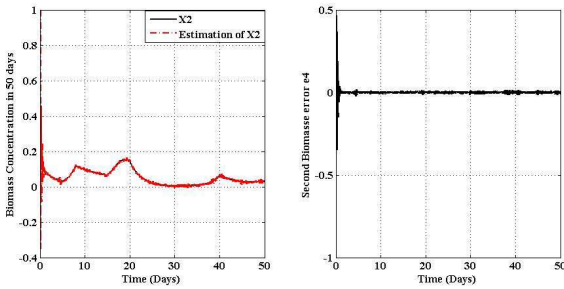


Fig. 10. The second biomass  $x_2$  and its error estimation with a perturbation on  $s_2$

## 5 Conclusion

In this paper, we designed a nonlinear observer for the AM2 model with linear errors, and we proved that this observer is robust with a global convergence and adjustable speed. For large speed of convergence, we noticed that in a small time interval at the beginning, some variables can take erroneous values ( $< 0$ ) this is not harmful if the observer is used only as a software sensor, on the other hand, if the observer is used in a control loop, we have to lower the speed of convergence a little bit to ensure that the variables remains positive. It can be noticed, from an application point of view that the implementation of this observer is very easy.

The quantity of methane produced by the system is given by the following expression

$$q_M = k_6 \mu_2(s_2) x_2$$

and it can be estimated from the estimated variable  $x_1$  and  $x_2$  which qualifies our observer as the software sensor for the methane.

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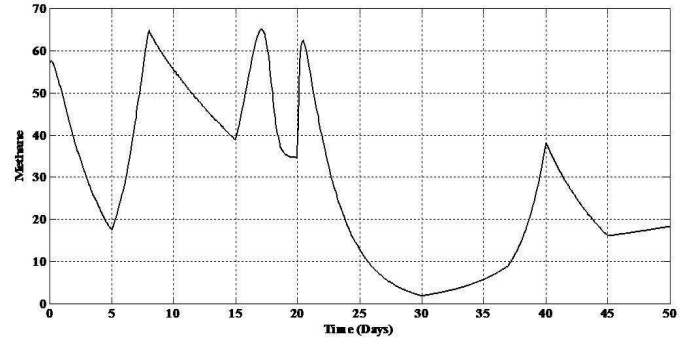


Fig. 11. the output  $q_M$

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