Dzyaloshinskii-Moriya interactions effects on the entanglement dynamics of a two qubit xxz spin system in non-Markovian environment

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Georges Collince Fouokeng, Martin Tchoffo, Edwine Tendong, Lukong Cornelius Fai
Mesoscopic and Multilayer Structure Laboratory, Department of Physics, Faculty of Science, University of Dschang, Cameroon.

We investigate the exact entanglement dynamics of a two-qubit Heisenberg XXZ chain with Dzyaloshinskii-Moriya (DM) interactions, interacting with an anisotropic spin bath in thermal equilibrium at a temperature T, driven by an external magnetic field B along the z-axis. We establish that, for an initially entangled qubit pair, the DM interactions generate entanglement and enhance the entanglement in the revival region. The effects of the DM interaction are also seen to be very important at high temperatures and for weak coupling between the two qubits where it is seen to preserve entanglement. These effects are weakened when the magnetic field B and the Heisenberg coupling are switched on. If the two-qubits are prepared in an initially separable state, the DM interaction instead has a negative effect on their entanglement. On a whole entanglement can better be preserve in the spin chain even at high temperatures by increases the external magnetic field B and the Heisenberg couplings, and by tuning the strength of the DM interaction.

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I. INTRODUCTION

Entanglement also called the quantum non-local connection [1] has been studied intensely in recent years due to its potential applications in quantum communication and information processing tasks [2] such as quantum teleportation [3], super-dense coding [4] quantum key distribution, and telecoloning. It has also been recognized as an essential resource in quantum information processing [5]. Quantum entanglement also plays a fundamental role in the quantum phase transitions that occur in interacting lattice systems at zero temperature [6]. These Potential applications of entanglement have stimulated research on ways to quantify and control it. Different physical systems have been proposed as reliable candidates for the underlying technology of quantum computing and quantum information processing [7] [8]. The basic idea in each one of these systems is to choose a certain quantum degree of freedom to serve as a qubit, such as the charge, orbital, or spin angular momentum. This is usually followed by finding a controllable mechanism to form entanglement between a two-qubit system in such a way as to produce a fundamental quantum computing gate such as an exclusive OR (XOR) gate. In addition, one should be able to manipulate coherently such an entangled state to provide an efficient computational process. Such coherent manipulation of entangled states has been realized in different systems such as isolated trapped ions [9] and superconducting junctions [9]. The coherent control of a two-electron spin state in a coupled quantum dot has been achieved experimentally, in which the coupling mechanism is the Heisenberg exchange interaction between the electron spins [11] [12]. Heisenberg spin chains are among one of the major quantum systems, which have been proposed for the physical realization of good qubits needed in the implementation of the quantum computer [13] [14]. [15]. However spin systems suffer from decoherence effects due to the influences of the environmental degrees of freedom on the dynamics of the system. In such spin systems the unavoidable interactions between the qubits and their environment causes the decay of qubit superposition states (entangled states) into a classical, statistical mixture of states; [16] a phenomenon known as decoherence, and this can seriously hinder the various quantum information processing tasks. Decoherence effects have therefore in recent years attracted extensive research attention. In the dynamics of a quantum spin system, one of the major spin interactions inducing decoherence, which play an important role in the entanglement dynamics of spin qubits is the Dzyaloshinskii-Moriya (DM) interaction [17] [18]. It is an anisotropic anti-symmetric interaction between spins, which arises from the consideration of spin orbit coupling effects in Anderson's super-exchange interaction theory. The effects of the DM interactions have been widely considered in research [19] [20] [21] . Recently, the influence of the anisotropic Dzialoshinski-Moriya (DM) interaction on the entanglement of two qubits in various magnetic spin models, including the XX, XY, XXX, XXZ and the most general XYZ Heisenberg models have been studied [22].The XXZ model encompasses the XX model, XY model, the isotropic Ising model and the XXX model which are all relevant for QIP. Understanding, quantifying and exploring entanglement dynamics may provide an answer for many questions regarding the decoherence behaviour of quantum spin systems. In the Heinsenberg spin chain, the interaction with the spin bath system often leads to strong non-Markovian behavior. That is, to study the dynamics of such a problem, the usual Markovian quantum master equations, which are widely used in the area of atomic physics and quantum optics, may fail for many spin bath models. Therefore, it becomes more and more important to develop methods that are capable of going beyond the Markovian approximation. Recently [22], the exact dynamics of a two qubit chain in an XY environment have been studied using a simple mathematical technique based on a unitary linear transformation. The authors have shown the behavior of the system was extremely non Markovian. H. P. Breuer, D. Burgarth et al [24] have studied the dynamics of a single spin in a spin star environment, using exact methods and various approximation techniques, and they show that the Markovian approximations perform poorly. In this paper, our objective is to study analytically and numerically the exact entanglement dynamics of a two qubit Heisenberg XXZ spin chain with DM interactions interacting with a spin bath in the presence of an external magnetic field using a simple mathematical technique based on a unitary linear transformation [22] [25] [26]. We examine the effects of the external magnetic field strength, temperature, intra-bath coupling strength, system bath coupling and anisotropy of the two qubit spin chain, on the entanglement dynamics, considering the interactions of the qubit systems with the environment, and we calculates the concurrence of a qubit pair, for an initially disentangled state and for an initially maximally entangled state. The organization of the paper is as follows: in section 2 we present a brief description of the theoretical approach used and the model for simulation the two qubit XXZ spin chain with Dyaloshinski-Moriya interactions interacting with a spin bath. In section 3, to study the entanglement dynamics of the model system presented in section 2, we evaluate the concurrence that quantifies the degree of the pair-wise entanglement between the two central qubits and then conclude with discussions our findings in Section 4.

II. THEORETICAL APPROACH AND THE MODEL HAMILTONIAN

The model used here describes two coupled spin qubit interacting with a spin bath in the presence of an external magnetic field in the z-direction via interactions of Heisenberg XXZ type alongside DM interactions, which are considered both within the spin chain and also in the spin bath. The environment is modeled here as a one dimensional Heisenberg XY chain with nearest neighbor spin couplings [22]. The total Hamiltonian of the system described above together with the DM interactions can be written in the form

$$H = H_S + H_{SB} + H_B \tag{1}$$

with

$$\begin{aligned} H_S =& \mu_0(S_{01}^z + S_{02}^z) + \Omega(S_{01}^+ S_{02}^- \\ &+ S_{01}^- S_{02}^+) + J_z S_{01}^z S_{O2}^z + i d_z(S_{01}^+ S_{02}^- - S_{01}^- S_{02}^+) \end{aligned} \tag{2}$$

$$H_{SB} = \frac{g}{\sqrt{N}}\{(S_{01}^+ + S_{02}^+)\sum_{a=1}^{N} S_a^- + (S_{01}^- + S_{02}^-)\sum_{a=1}^{N} S_a^+\} \tag{3}$$

$$\begin{aligned} H_B =& \sum_{a\neq b}^{N}\{\frac{g}{N}(S_a^+ S_b^- + S_a^- S_b^+) + i\frac{D_z}{N}(S_a^+ S_b^- - S_a^- S_b^+)\} \\ &+ \sum_{a=1}^{N}\frac{2\gamma}{N} S_a^z \end{aligned} \tag{4}$$

Here μ_0 represents the strength of the coupling of the two spin qubits to the external magnetic field, Ω is the coupling strength between the two spin qubits while J_z represents the coupling strength in the z-direction of the XXZ chain. $S_{0i}^{\pm}(i = 1, 2)$, represent the spin creation and annihilation operators for the two qubit spin chain while $S_i^{\pm}(i = a, b)$ represent the spin creation and annihilation operators for the bath spins. γ, represents the strength of coupling of the bath spins with the external magnetic field, g_0 and g are respectively the spin system-bath coupling strength and the and the intra-bath coupling strength. D_z and d_z represent the z-component of the DM coupling vector between the bath spins and between the two spin qubits. Finally N represents the number of spins in the bath. All the coupling strengths are rescaled so that the free energy of the system remains finite when $N \longrightarrow \infty$. We choose the above Hamiltonian due to its relevance for various QIP tasks and it models the environment as closely as possible so that the effects of the environment on the dynamics of the central spin can fully be taken into account. The XXZ model encompasses the XX model, XY model, the isotropic Ising model and the XXX model which are all relevant for QIP. A similar Hamiltonian has been examined recently in [27] . By introducing the collective angular momentum operators

$$J^{\pm} = \sum_{a=1}^{N} S_a^{\pm}; J^z = \sum_{a=1}^{N} S_a^z \tag{5}$$

we rewrite the Hamiltonians (3) and (4) as

$$H_{SB} = \frac{g}{\sqrt{N}}\{(S_{01}^+ + S_{02}^+)J^- + (S_{01}^- + S_{02}^-)J^+\} \tag{6}$$

$$\begin{aligned} H_B =& \frac{g}{N}\{(J^+J^- + J^-J^+) + i\frac{D_z}{N}(J^+J^- + J^-J^+)\} \\ &+ 2\frac{\gamma}{N}J^z - g - i\frac{D_z}{N}J^z \end{aligned} \tag{7}$$

The low temperature excitation spectrum of the system can be obtained by introducing the following Holstein-Primakoff transformation

$$\begin{aligned} &J^+ = b^+\sqrt{(2\Delta - b^+b)}; J^- = \sqrt{(2\Delta - b^+b)}b \\ &J^z = \Delta - b^+b \end{aligned} \tag{8}$$

Where Δ denotes the length of the collective environment pseudo-spin $\frac{N}{2}$. Thus;

$$N = 2\Delta \tag{9}$$

The above transformation transforms the spin operators J^+, J^- and J^z into bosonic creation and annihilation operators b^+ and b^- obeying the commutation relation $[b^+b] = 1$. After the transformation and in the thermodynamic limit ($N \longrightarrow \infty$) the Hamiltonians (6) and (7) become

$$H_{SB} = g_0[(S_{01}^+ + S_{02}^+)b + (S_{01}^- + S_{02}^-)b^+] \tag{10}$$

$$H_B = 2gb^+b - 2iD_z + \gamma \tag{11}$$

The equations (2); (10) and (11) are effectively the Hamiltonian of a two coupled spin qubits system interacting with a single-mode thermal bosonic bath with Dzyaloshinski-Moriya interactions both in the bath and in the two qubit chain. We note here that due to the high symmetry of the model, the coupling to the environment is actually represented by a coupling to a single collective environment spin. The effect of this single-mode environment on the dynamics of the two coupled qubits is extremely non-Markovian hence the traditional master equations used in describing the Markovian dynamics of open quantum systems, cannot be used in this case. We assume that the initial state of the system-bath is a separable state so its initial density matrix can be written in the form

$$\rho(0) = |\varphi(0)\rangle\langle\varphi(0)| \otimes \rho_B \tag{12}$$

The density matrix of the spin bath ρ_B satisfies the Boltzmann distribution, i.e ;

$$\rho_B = \frac{1}{Z} e^{-\frac{H_B}{T}} \tag{13}$$

where $Z = Tr(e^{-\frac{H_B}{T}})$ is the partition function. Here Tr denotes the trace and $T = K_B\tau$ where $\tau$is the temperature and K_B is the Boltzmann constant (subsequently we simply refer to T as the temperature). The partition function Z is given by;

$$Z = e^{2iD_z - \gamma}\Big(\frac{1}{1 - e^{-\frac{2g}{T}}}\Big) \tag{14}$$

At absolute zero temperature, no excitation will exist. The bath is in a thoroughly polarized state with all spins down. With the increase of temperatures, the number of spin-up atoms increases and the bath is no longer in a polarized state. The most general form of an initial pure state of the two-qubit system can be written as:

$$|\varphi(0)\rangle = \alpha|00\rangle + \varepsilon|01\rangle + \delta|10\rangle + \beta|11\rangle \tag{15}$$

With the normalization condition yielding

$$|\alpha|^2 + |\varepsilon|^2 + |\delta|^2 + |\beta|^2 = 1 \tag{16}$$

For analytic simplicity, we set $\varepsilon = \delta = 0$. So the initial state can be written as

$$|\varphi(0)\rangle = \alpha|00\rangle + \beta|11\rangle \tag{17}$$

and the initial density matrix of the system plus bath takes the form:

$$\rho(0) = (\alpha|00\rangle + \beta|11\rangle)(\langle 00|\alpha^* + \langle 11|\beta^*) \otimes \frac{1}{Z}(e^{-\frac{H_B}{T}}) \tag{18}$$

We note that the time dependent density matrix of the system coupled to the bath obeys to the folowing relation

$$\rho_s(t) = Tr_B(\rho(t)) \tag{19}$$

where $U(t) = e^{iHt}$, is the unitary time evolution operator. The qubit system alone does not evolve in a unitary manner. We can obtain the dynamics of the qubit system alone by tracing over the bath modes in order to obtain the reduced density matrix of the qubit system $\rho_s(t)$

$$\rho(t) = U^*(t)\rho(0)U(t) \tag{20}$$

where Tr_B denotes the partial trace of the density matrix taken over the bath modes. We obtain the reduced density matrix of the form:

$$\begin{aligned}\rho_s(t) =& Tr_B\Big\{(\frac{1}{Z})\Big(|\alpha|^2 e^{-iHt}|00\rangle e^{-\frac{H_B}{T}}\langle 00|e^{iHt} \\ &+ \alpha\beta^* e^{-iHt}|00\rangle e^{-\frac{H_B}{T}}\langle 11|e^{iHt} \\ &+ \alpha^*\beta e^{-iHt}|11\rangle e^{-\frac{H_B}{T}}\langle 00|e^{iHt} \\ &+ |\beta|^2 e^{-iHt}|11\rangle e^{-\frac{H_B}{T}}\langle 11|e^{iHt}\Big)\Big\}\end{aligned} \tag{21}$$

In order to obtain the full form of the reduced density matrix, we need to evaluate the time evolution of the initial qubit state: $e^{-iHt}|00\rangle$, and $e^{-iHt}|11\rangle$. We observe that by applying the time dependent Schrodinger equation,

$$i\frac{d}{dt}|\varphi(t)\rangle = H|\varphi(t)\rangle \tag{22}$$

where

$$|\varphi(t)\rangle = U(t)(\alpha|00\rangle + \beta|11\rangle) \tag{23}$$

From the total Hamiltonian H, we can see that it consists of operators of the form S_0i^+ , S_0i^- (i=1,2) which change the state of the i^th spin from $|0\rangle$ to $|1\rangle$and from $|1\rangle$, to $|0\rangle$ respectively. Thus, the qubit will evolve from the initial pure state into the most general mixed state as follows

$$e^{-iHt}|11\rangle = A|00\rangle + B|01\rangle + C|10\rangle + D|11\rangle \tag{24}$$

$$e^{-iHt}|00\rangle = E|00\rangle + F|01\rangle + G|10\rangle + Q|11\rangle \tag{25}$$

where A, B, C, D, E, F, G, Q are functions of b^+, b and t . Thus:

$$\begin{aligned}|\varphi(t)\rangle =& \alpha e^{-iHt}|00\rangle + \beta e^{-iHt}|11\rangle \\ =& \alpha(E|00\rangle + F|01\rangle + G|10\rangle + Q|11\rangle) \\ &+ \beta(A|00\rangle + B|01\rangle + C|10\rangle + D|11\rangle)\end{aligned} \tag{26}$$

To obtain the exact form of the reduced density matrix $\rho_s(t)$ we need to evaluate the form of the expressions A, B, C, D, E, F, G, Q . From the time dependent Schrdinger equation it follows that:

$$i\frac{d}{dt}(e^{-iHt}|11\rangle) = H(e^{-iHt}|11\rangle) \tag{27}$$

and

$$i\frac{d}{dt}(e^{-iHt}|00\rangle) = H(e^{-iHt}|00\rangle) \tag{28}$$

With the initial conditions from being $A0) = B(0) = C(0) = 0$ and $D(0) = 1$, and with the following set of transformations

$$A = -b^+ b e^{-i(2g(b^+b+1)+u)t} A_1 \tag{29}$$

$$B = -b^+ e^{-i(2g(b^+b+1)+u)t} B_1 \tag{30}$$

$$C = -b^+ e^{-i(2g(b^+b+1)+u)t} C_1 \tag{31}$$

$$D = e^{-i(2g(b^+b+1)+u)t} D_1 \tag{32}$$

the following first order four differentials equations derives

$$i\frac{d}{dt}A_1 = i(\mu_0 - 2g)A_1 - ig_0(B_1 + C_1) \tag{33}$$

$$i\frac{d}{dt}B_1 = 2iJ_zB_1 - i(\Omega - id_z)C_1 - ig_0(\hat{n}+2)A_1 - ig_0D_1 \tag{34}$$

$$i\frac{d}{dt}C_1 = 2iJ_zC_1 - i(\Omega - id_z)B_1 - ig_0(\hat{n}+2)A_1 - ig_0D_1 \tag{35}$$

$$i\frac{d}{dt}D_1 = i(\mu_0 - 2g)D_1 - ig_0(\hat{n}+1)(B_1 + C_1) \tag{36}$$

The solution of the coupled differential equations (27) is obtained analytically through the initial conditions (29) to (32) for the case $d_z = 0$ with the resonant condition $\mu_0 = 2g$; the external magnetic field can easily be tuned to satisfy this condition. However the numerical results are present for the case where $d_z \neq 0$. Thus, the following analytically solutions are obtained

$$A_1(t) = \frac{-1}{3+2\hat{n}} + \frac{2g_o^2}{\sqrt{(2J_z-\Omega)^2 + 8g_o^2(3+2\hat{n})}}\Big\{\frac{e^{i\lambda_1 t}}{\lambda_1} - \frac{e^{i\lambda_2 t}}{\lambda_2}\Big\} \tag{37}$$

$$B_1(t) = C_1(t) = -\frac{g_o^2}{\sqrt{(2J_z-\Omega)^2 + 8g_o^2(3+2\hat{n})}}\{e^{i\lambda_1 t} - e^{i\lambda_2 t}\} \tag{38}$$

$$D_1(t) = \frac{2+\hat{n}}{3+2\hat{n}} + \frac{2g_o^2(1+\hat{n})}{\sqrt{(2J_z-\Omega)^2 + 8g_o^2(3+2\hat{n})}}\Big\{\frac{e^{i\lambda_1 t}}{\lambda_1} - \frac{e^{i\lambda_2 t}}{\lambda_2}\Big\} \tag{39}$$

where $U = \gamma - 2iD_z + J_z$, $b^+b = \hat{n}$ and

$$\lambda_{1,2} = \frac{(2J_z - \Omega) \pm \sqrt{(2J_z-\Omega)^2 + 8g_o^2(3+2\hat{n})}}{2} \tag{40}$$

From Eq.(28), we find also that with the transformations

$$E = e^{-i(2g(b^+b-1)+u)t} E_1 \tag{41}$$

$$F = b e^{-i(2g(b^+b-1)+u)t} F_1 \tag{42}$$

$$G = b e^{-i(2g(b^+b-1)+u)t} G_1 \tag{43}$$

$$Q = bb^+ e^{-i(2g(b^+b-1)+u)t} Q_1 \tag{44}$$

we obtain

$$E_1(t) = \frac{\hat{n}-2}{2\hat{n}-1} + \frac{2g_o^2\hat{n}}{\sqrt{(2J_z-\Omega)^2 + 8g_o^2(3+2\hat{n})}}\Big\{\frac{e^{i\lambda'_1 t}}{\lambda'_1} - \frac{e^{i\lambda'_2 t}}{\lambda'_2}\Big\} \tag{45}$$

$$Q_1(t) = \frac{-1}{(2\hat{n}-1)} + \frac{2g_o^2\hat{n}}{\sqrt{(2J_z-\Omega)^2 + 8g_o^2(3+2\hat{n})}}\Big\{\frac{e^{i\lambda'_1 t}}{\lambda'_1} - \frac{e^{i\lambda'_2 t}}{\lambda'_2}\Big\} \tag{46}$$

$$F_1(t) = G_1(t) = -\frac{g_o^2}{\sqrt{(2J_z-\Omega)^2 + 8g_o^2(3+2\hat{n})}}\{e^{i\lambda'_1 t} - e^{i\lambda'_2 t}\} \tag{47}$$

where

$$\lambda'_{1,2} = \frac{(2J_z - \Omega) \pm \sqrt{(2J_z-\Omega)^2 + 8g_o^2(3+2\hat{n})}}{2} \tag{48}$$

The exact form of the reduced density matrix is then obtained by tracing over the bath modes and replacing the operator $\hat{n}$ by its Eigen value n as:

$$\rho_s(t) = \begin{pmatrix} \rho_{11} & 0 & 0 & \rho_{14} \\ 0 & \rho_{22} & \rho_{23} & 0 \\ 0 & \rho_{32} & \rho_{33} & 0 \\ \rho_{14}^* & 0 & 0 & \rho_{44} \end{pmatrix} \tag{49}$$

where

$$\rho_{11} = \Big(\frac{1}{Z}\Big)\Big(|\alpha|^2 \sum_{n=0}^{\infty} E_1E_1^* e^{-\frac{2gn-2iD_z+\gamma}{T}} + |\beta|^2 \sum_{n=0}^{\infty}(n+1)(n+2)A_1A_1^* e^{-\frac{2gn-2iD_z+\gamma}{T}}\Big) \tag{50}$$

$$\rho_{14} = (\frac{1}{Z})\Big(\alpha\beta^* \sum_{n=0}^{\infty} E_1 D_1^* e^{-\frac{2gn-2iD_z+\gamma}{T}} e^{4igt} \qquad (51)$$

$$\rho_{22} = \rho_{23} = \rho_{32} = \rho_{33} = (\frac{1}{Z})\Big(|\alpha|^2 \sum_{n=0}^{\infty} F_1 F_1^* n e^{-\frac{2gn-2iD_z+\gamma}{T}} + |\beta|^2 \sum_{n=0}^{\infty} (n+1) B_1 B_1^* e^{-\frac{2gn-2iD_z+\gamma}{T}}\Big) \qquad (52)$$

$$\rho_{44} = (\frac{1}{Z})\Big(|\alpha|^2 \sum_{n=0}^{\infty} Q_1 Q_1^* n(n-1) e^{-\frac{2gn-2iD_z+\gamma}{T}} + |\beta|^2 \sum_{n=0}^{\infty} (n+1) D_1 D_1^* e^{-\frac{2gn-2iD_z+\gamma}{T}}\Big) \qquad (53)$$

III. ENTANGLEMENT DYNAMICS

For the reduce density matrix given in equation (49) the concurrence quantifies the degree of the pair-wise entanglement between the two central qubits and is defined as [28]:

$$C_{12} = max\{\lambda_1 - \lambda_2 - \lambda_3 - \lambda_4, 0\} \qquad (54)$$

Where λ_1, λ_2 , λ_3 , λ_4 are the squaroots of the Eigen values in order of decreasing magnitude of the operator:

$$R_{12} = \rho_s(\sigma^y \otimes \sigma^y)\rho_s^*(\sigma^y \otimes \sigma^y) \qquad (55)$$

As a measure of the degree of entanglement the concurrence varies from 0 to a maximum value 1. If the concurrence is equal to zero then the two states are said to be completely disentangled or separable while a concurrence $C_{12} = 1$ means the two states are maximally entangled. The Eigen values of R_{12} in order of reducing magnitude are found to be:

$$\begin{aligned} \lambda_1 &= \sqrt{\rho_{11}\rho_{44}} + |\rho_{14}| \\ \lambda_2 &= \sqrt{\rho_{11}\rho_{44}} - |\rho_{14}| \\ \lambda_3 &= 2\rho_{22} \\ \lambda_4 &= 0 \end{aligned} \qquad (56)$$

We shall present our results here for $J_z \geq 0$ and $\Omega \geq 0$ which corresponds to the antiferromagnetic XXZ chain. The generation of entanglement, is a competition between the effects of the environment and the coupling between the two qubits. On the one hand we have the case in which there exist no couplings between the two qubits. Here the entanglement is generated via the interaction of the two qubits with a common environment. Such environment induced entanglement has been reported in [29]. Hence the environment which is known to cause decoherence can never the less generate some entanglement between the two qubits. A similar conclusion has been made by the authors of [28]. Such environment induced entanglement is very fragile and increases with increasing coupling strength between the system and the bath. On the other hand we also have the case of entanglement generated through the coupling between the two qubits. The coupling generated entanglement is stronger and dominates the preceding case. For coupling induced entanglement, increasing the coupling strength between the two qubit systems and the environment will rather cause faster decay of entanglement. To study the effects of the DM interaction on the entanglement dynamics, we consider an initially disentangled state . For this case of two initially separable qubits which become entangled in time through the effects of the environment, their entanglement is destroyed by increasing the strength of the DM interactions as seen in figure 1. Thus the DM interactions destroy environment induced entanglement but enhance coupling induced entanglement.

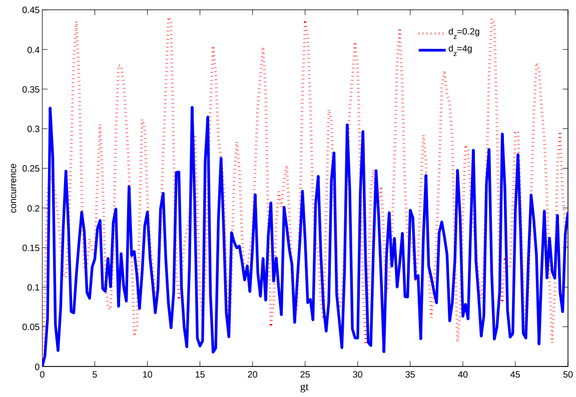

FIG. 1: Concurrence VS time for an initially disentangled qubit (i.e, $|\psi\rangle = |00\rangle$),for different values of d_z, with the corresponding values of the parameters $\Omega = 0$, $J_z = 0$, $g = g_0 = 2$, $\mu_0 = 2g$ et $T = 2g$.

The behavior of an initially entangled spin chain is very different from that of a spin chain with no initial entanglement. We study the behavior of the concurrence for a maximally entangled initial qubit state $|\psi\rangle = \frac{1}{2}(|00\rangle + |11\rangle)$. We find that the effects of the DM interaction depend largely on the temperature, and on the Heisenberg couplings J_z and Ω . In the absence of the couplings i.e. $J_z = \Omega = 0$ as seen in figure.2 the DM interactions preserve the entanglement and also to greatly enhance the revival of entanglement. However when the Heisenberg couplings set in this enhancement effect is reduced and the concurrence shows a sinusoidal oscillation in space as presented in figure.3. This can be seen as being due to competing effects between the antisymmetric DM interaction and the symmetric Heisenberg interactions. This because in contrast to the Heisenberg interactions which tend to render neighbor spins parallel, the DM interaction has the effect of turning them perpendicular to one another.

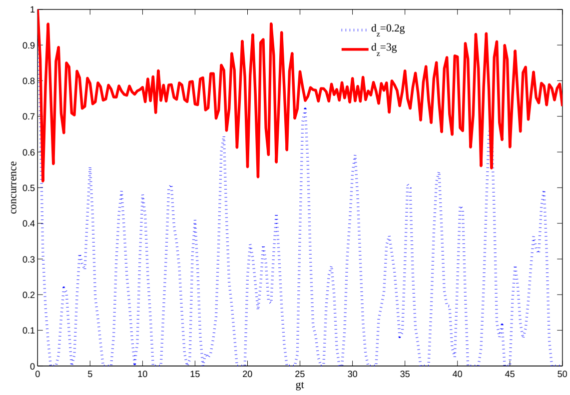

FIG. 2: Concurrence VS time for an initially maximally entangled state $|\psi\rangle = \frac{1}{2}(|00\rangle + |11\rangle)$, for different values of d_z, with the corresponding values of the parameters $\Omega = 0$, $J_z = 0$, $g = g_0 = 2$, $\mu_0 = 2g$ et $T = 3g$.

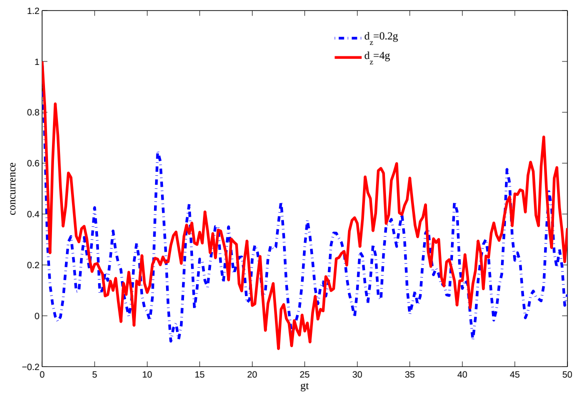

FIG. 3: Concurrence VS time for an initially maximally entangled state $|\psi\rangle = \frac{1}{2}(|00\rangle + |11\rangle)$,for different values of d_z, with the corresponding values of the parameters $\Omega = 1$, $J_z = 0.5$, $g = g_0 = 2$, $\mu_0 = 2g$ et $T = 2g$.

It is observed in figure4 that, the entanglement decays more rapidly as the temperature increases. Increasing the temperature introduces thermal fluctuations which destroys quantum correlations. At low temperatures the entanglement exhibits periodic oscillations. At high temperatures we also observe the interesting phenomenon of entanglement sudden death (ESD) i.e. when the entanglement of the system is observed to suddenly disappear without any exponential decay. In [30] Yu and Eberly have shown that noisy environments may cause entanglement to vanish completely in finite time and they call the phenomenon entanglement sudden death. The sudden death of entanglement is a very undesirable effect since major quantum protocols for quantum computing; depend on the preservation of entanglement in the system. Here the non-Markovian environment is seen to cause the revival or rebirth of entanglement after ESD.

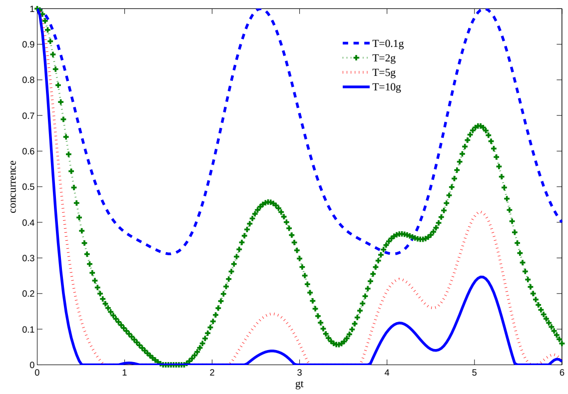

FIG. 4: Concurrence VS time for an initially maximally entangled state $|\psi\rangle = \frac{1}{2}(|00\rangle + |11\rangle)$, for different temperatures with the corresponding values of the parameters $\Omega = 0$, $J_z = 0$, $g = g_0 = 2$, $\mu_0 = 2g$ and $d_z = 0.2g$.

The oscillatory collapse and revival behavior of the entanglement due to the influence of the non-Markovian environment can be understood by analogy to the collapse and revival of atomic population inversion of a two-level atom interacting with a single mode field in quantum optics. It is known that for a two level system coupled to an oscillating driving field the probability of being in the ground or excited states, exhibit oscillatory behavior (Rabi oscillations). Similarly our qubits are coupled to a single mode bath thus the quantum fluctuations of the system due to the bath may become uncorrelated in time leading to the collapse of entanglement. As time goes on these quantum fluctuations may again become correlated leading to the revival of entanglement. The DM interaction increases the frequency of the quantum fluctuations [31] thus enhancing the entanglement. Furthermore this sudden death of entanglement can be avoided by increasing the strength of DM interaction as observed in figure 5.

When coupling between the two qubits is switched on, the entanglement is observed to be preserved for a longer time. The dependence of the concurrence on the coupling strengths J_z and Ω is closely linked. For example, we note that if $J_z > 0$ then increasing Ω will cause the concurrence to reduce while if $J_z < 0$ then increasing Ω will improve the concurrence. The same holds for the variation of J_z with a fixed value of Ω. In our numerical analysis, we note that the effective Heisenberg coupling between the two qubits can be written as $\chi = |J_z - \Omega|$. The entanglement is enhanced by increasing the factor χ

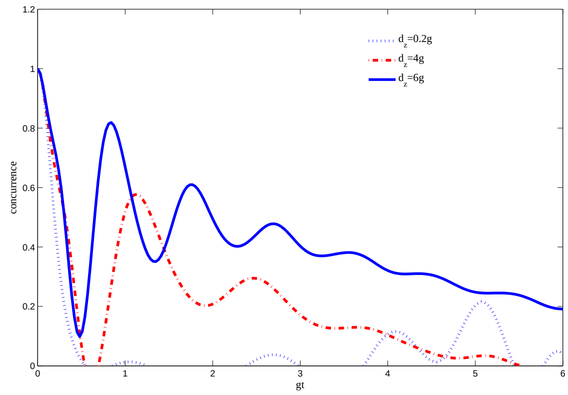

FIG. 5: Concurrence VS time for an initially maximally entangled state $|\psi\rangle = \frac{1}{2}(|00\rangle + |11\rangle)$ at high temperature (i.e, $T = 10g$) for different values of d_z, with the corresponding values of the parameters $\Omega = 0$, $J_z = 0$, $g = g_0 = 2$, $\mu_0 = 2g$.

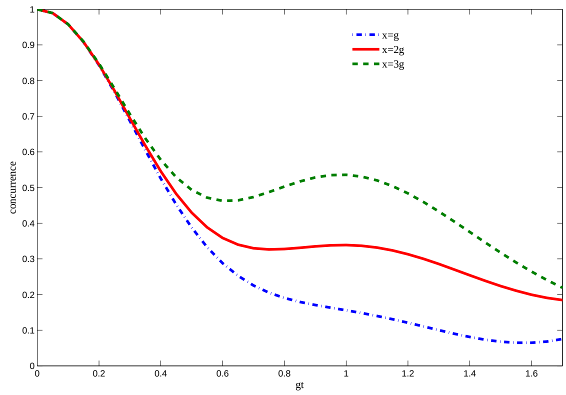

FIG. 6: Concurrence VS time for an initially maximally entangled state $|\psi\rangle = \frac{1}{2}(|00\rangle + |11\rangle)$ for different values of χ considering $\Omega = 0$,with the corresponding values of the parameters $d_z = 0$, $g = g_0 = 2$, $\mu_0 = 2g$, $T = 3g$.

as seen in figure 6. The entanglement does not depend on how large J_z and Ω are but rather depends on their difference. We find that for $J_z = \Omega$, i.e. $\chi = 0$ the concurrence is low and it increases as the value of χ increases. From this we can conclude that the anisotropic XXZ chain will be better than the Isotropic XXX chain for preserving entanglement and hence for various quantum information processing tasks. We also find that the entanglement decays very slowly when the system bath coupling is small and faster for a strong coupling between the system and the bath (see figure 7). This is so because in the case where the system, is strongly coupled to the bath, decoherence from the bath is more prominent and leads to faster decay of the quantum correlations. When the coupling amongst the bath spins is strong we observe that the entanglement decays more slowly (such presented in figure 8).This is an indication that strong coupling amongst the bath spins effectively decouples the bath from the system thus preserving entanglement . As expected strong coupling between the two qubits also allows them to keep their entanglement for longer times.

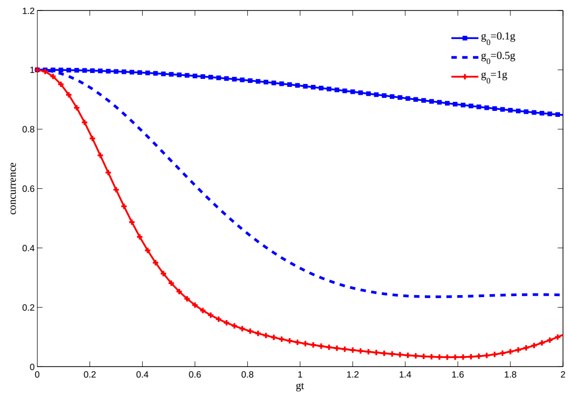

FIG. 7: Concurrence VS time for an initially maximally entangled state $|\psi\rangle = \frac{1}{2}(|00\rangle + |11\rangle)$ for different values of g_0, with the corresponding values of the parameters $\Omega = 2$, $d_z = 1$, $J_z = 1$,$g = 2$, $\mu_0 = 2g$, $T = 2g$.

IV. DISCUSSION AND CONCLUDING REMARKS

The entanglement dynamics for a system of two qubits XXZ spin chian coupled to antiferromagnetic spin bath with DM interactions have been studied under the influnce of an external magnetic field, by employing the simple mathematical technique based on a unitary linear transformation. Exact results on the entanglement dynamics have been obtained and shows consequently the strong dependence on the nature of the bath. Numerical analysis of the behavior of the concurence vis-a-viz the various system parameters revealed the effects of the DM interaction depend on the initial state of the sytem and on how the entanglement is generated. For an initially disentangled qubit pair, and in the absence of any coupling between the two qubits, the common bath can generate some effective entanglement between the two qubits. DM interaction destroy such environment generated entanglement; This is contrary to the case of an initially entangled qubit where the DM interactions rather enhance the entanglement. For the long time behaviour of the entanglement of an initially entangled state it was also observed to initially decay exponentially with time and then to undergo continous cycles of collapse and revival. This collapse and revival behaviour is attributed to the non-Markovian nature of the bath in which memory effects of the bath can reconstuct the entanglement of the

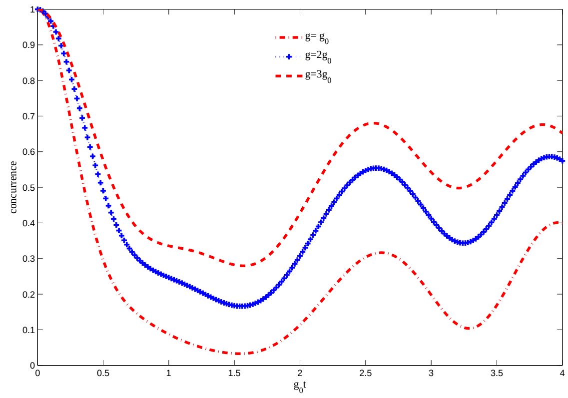

FIG. 8: Concurrence VS time for an initially maximally entangled state $|\psi\rangle = \frac{1}{2}(|00\rangle+|11\rangle)$ for different values of g, with the corresponding values of the parameters $\Omega = 2$, $d_z = 0.2$, $J_z = 2$,$g_0 = 2$, $\mu_0 = 2g$, $T = 3g_0$.

system, with time. It is seen that the DM interactions play an important role in the weak inter-qubit coupling limit, and for high temperatures where they delay the decay of entanglement and enhance revival oscillations in the entanglement. Increasing the temperature can lead to appearance of the Entanglement sudden death effect. However this sudden death of entanglement can be avoided by increasing the strength of the DM interaction. The effects of J_z and Ω on the entanglement are closely linked and we find that the effective Heisenberg coupling in the XXZ chain is given by $\chi = |J_z - \Omega|$. Increasing χ enhances and preserves the entanglement while we see that the entanglement decays faster for small χ even if J_z and Ω are both large. This suggests that the anisotropic XXZ chain is better than the isotropic XXX (where $\chi = 0$) chain for QIP tasks. Further more strong intrabath coupling is seen to effectively decouple the bath from the system thus delaying the loss of entanglement in the system. Our result reveals that entanglement can better preserved for large finite temperatures and for longer times, by tunning the strenght of the external magnetic field, the DM interaction and the system parameters J_z , Ω , g_0, g.We expect that our analysis will shed some light on the study of the dynamics of a multipartite entangled state under local noise. An interesting feature of this model is that it can be used as better quantum channel when entanglement transfer is considered. Therefore, in principle, it can be exploited as a quantum channel for teleportation with nonclassical fidelity at finite temperature, both very low and moderately low. Considering future research along these lines of investigation, it will be interesting to consider practical schemes for the realization of this kind of spin chains in highly controllable situations.

[1] R Horodecki, P Horodecki, M Horodecki and K Horodecki: Quantum entanglement arXiv:quant-ph/0702225v2 20 Apr 2007
[2] Special issue on quantum information, Phys. World, 11, 33-57 (1998)
[3] C. H. Bennett, G. Brassard, C. Crepeau, R. Jozsa, A.Peres and W. K. Wootters, Phys. Rev. Lett. 70, 1895(1993)
[4] C. H. Bennett and S.J. Wiesner, Phys. Rev. Lett. 69,2881 (1992)
[5] Nielsen M A, Chuand I L. Quantum Computation and Quantum Information.
[6] G. Vidal, J. I. Latorre,E. Rico,and A. Kitaev, Entanglement in quantum critical phenomena. arXiv:quant-ph/0211074v1 13 Nov (2002)
[7] L. Vandersypen, M. Ste?en, G. Breyta, C. Yannoni, M. Sherwood, and I. Chuang, Nature 414, 883 (2001).
[8] J. Jones, M. Mosca, and R. Hansen, Nature 393, 344 (1998).
[9] J. e. a. Chiaverini, Science 308, 997 (2005).
[10] D. Vion, A. Aassime, A. Cottet, H. Joyez, P.and Pothier, C. Urbina, D. Esteve, and M. H. Devoret, Science 296, 886 (2002).
[11] A. C. Johnson, J. R. Petta, J. M. Taylor, A. Yacoby, M. D. Lukin, C. M. Marcus, M. P. Hanson, and A. C. Gossard, Nature 435, 925 (2005).
[12] F. H. L. Koppens, J. A. Folk, J. M. Elzerman, R. Hanson, L. H. Willems van Beveren,I. T. Vink, H. P. Tranitz, W. Wegscheider, L. P. Kouwenhoven, and L. M. K. Vandersypen,Science 309, 1346 (2005).
[13] D. A. Lidar, D. Bacon, and K. B. Whaley, Phys. Rev.Lett. 82, 4556 (1999)
[14] D. Loss and D. P. Divincenzo, Phys. Rev. A 57, 120 (1998).
[15] G. Burkard, D. Loss, and D. P. DiVincenzo, Phys. Rev. B 59, 2070 (1999).
[16] Jan Fischer, Daniel Loss: Dealing with Decoherence, Science , 324: 1277(2009).
[17] I Dzialoshinski A thermodynamic theory of weak ferromagnetics, J. Phys. Chem. Solids 4 (1958), pp.241.
[18] T. Moriya,Anistropic superexchange interaction and weak ferromagnetism, Phys. Rev. Lett. 120 (1960), pp.91.
[19] M Tursun, A. Abliz, R. Mamtimin, A Abliz, and Q. Pan-Pan; Various Correlations in Anisotropic Heisenberg XYZ Model with Dzyaloshinskii-Moriya Interaction, arXiv:1207.0277v4 [quant-ph] (2013)
[20] Yi-Ying Yan , Li-Guo Qin and Li-Jun Tian; Dynamics of quantum correlations for two-qubit coupled to a spin chain with Dzyaloshinskii-Moriya interaction. arXiv:1109.5458v1 [quant-ph] (2011)
[21] Ming-Liang Hu; Disentanglement dynamics of interacting two qubits and two qutrits in an XYspin Chain environment with the Dzyaloshinsky-Moriya interaction; Phy Lett A Vol 374, Issue 34, (July 2010), Pages 3520-3528
[22] Z.N. Gurkan and O.K. Pashaev ;Two Qubit Entanglement in Magnetic Chains with DM Antisymmet-

ric Anisotropic Exchange Interaction; arXiv:0804.0710v2 [quant-ph] (2008).

[23] Xiao-Zhong Yuan, Hsi-Sheng Goan, and Ka-Di Zhu; Non-Markovian reduced dynamics and entanglement evolution of two coupled spins in a quantum spin environment, Phy. Rev, B75, 045331(2007)

[24] H. P. Breuer, D.lBurgarth ,and F Petruione :NonMarkovian dynamics in a spin star system ;Exact solution and approximation techniques; arXiv:quant-ph/0401051v2 10 Jan 2004

[25] Martin Tchoffo1, Georges Collince Fouokeng, et al, World Journal of Condensed Matter Physics, 2012, 2, 246-256

[26] Martin Tchoffo, Georges Collince Fouokeng, Lukong Cornelius Fai, Mathurin Esouague Ateuafack, Journal of Quantum Information Science, 2013, 3, 10-15.

[27] Physics Letters A Volume 374, Issue 34, 26 July 2010, Pages 3520-3528

[28] William K. Wootters : Entanglement of Formation of an Arbitrary State of Two Qubits; arXiv:quant-ph/9709029v2 (1997)

[29] X. X. Yi, H. T. Cui, and L. C. Wang, Entanglement induced in spin-1/2 particles by a spin chain near its critical points Phys. Rev. A 74, 054102 (2006)

[30] T .Yu and J. H. Eberly ; Sudden Death of Entanglement : Classical Noise Effects; Optics communications Volume 264, Issue 2, (200), pp 393-397

[31] Wen-Long You and Yu-Li Dong ;The entanglement dynamics of interacting qubits embedded in a spin environment with Dzyaloshinsky-Moriya term; Eur Phys J D (2010), vol 57, Issue 3, pp 439-44