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Fault detection of event based control system

M.A Sid, S. Aberkane, D. Maquin, D. Sauter

Abstract— A framework for the joint design of on-line sensor scheduling and fault detection is proposed. First, the synthesis of fault detection filter under any event triggering mechanism is given. The proposed filter can be viewed as a special structure of the Bayesian filter. To demonstrate its performance, this filter is tested under mixed event triggering mechanism designed to satisfy control requirements. A numerical example illustrates the effectiveness of the proposed approach.

I. INTRODUCTION

Most of the research in control theory and engineering considers periodic or time-driven control systems where continuous-time signals are represented by their sampled values at a fixed sample frequency [20]. To economize energy and bandwidth capacity in networked control systems, measurements can be taken and transmitted only when an event occurs rather than each periodic sampling time [19], [18]. However, this fact raises the complexity of the FD problems considerably, mainly in the situation when no measurement is received any more. In control problem some event-triggering mechanism (ETM) invokes transmission of the outputs in a node when the difference between the current values of the outputs in the node and their previously transmitted values (innovation) becomes greater than a given constant. In the sensor/control networking community the magnitude-driven/level-crossing sampling is known as the send-on-delta or Lebesgue sampling [10], [9]. Using the same terminology logic, the classical periodic scheme is called Riemann sampling. For dealing with abrupt changes in the output signal due to perturbations, the integral of the innovation signal can be used for event generation [1], this technique is named $\dot{\gamma}$ integral sampling \dot{z} . In a recent work [16] the transmission occurs when the innovated difference is 'large' compared with the current value of the sensor. An additional threshold is added in [2] to avoid Zeno phenomenon when the sensor node performs an infinite number of sampling operations in a finite time. Another solution to this problem is the periodic event triggered (ET) control as given in [3]. In [5] robust ET control strategy is designed to minimize the noise effect on the system performance. There are few works that deal with event triggered state estimation, the interested reader is referred to the recent works [15], [17], [4].

It is clear that the new developed sampling techniques require specific FDI algorithm design, but according to our best knowledge the only works that had considered

this issue are [7], [12], [8]. In this paper we will consider FD filter design problem.

First, we present an event triggered fault detection algorithm (ETFD) that is compatible with any given ETM. Then, this algorithm is used for fault detection of networked control system operating under a predefined ETM, more precisely the mixed ETM. We find similar design problem when the FD algorithm is co-implemented with the control algorithm at the same processor. In this case the ETM is specifically designed to insure the control performance.

The rest of the paper is organized as follow: section II gives the problem formulation and introduce the concept of the ETM algorithm. In section III, the general event based fault detection (ETFD) filter is given under any ETM scheme. To illustrate the effectiveness of the proposed method, we apply ETFD algorithm under a specific ETM namely the mixed ETM in section IV. A numerical example is given in section V. Finally, we provide some conclusions and some future research directions in section VI.

II. PROBLEM FORMULATION

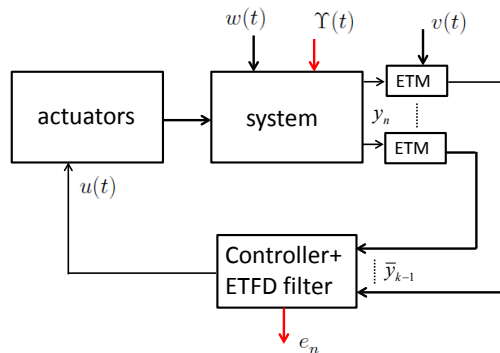


Fig. 1. Event triggered FD

Consider a faulty networked control system illustrated as in **Fig 1.**, where the linear continuous-time dynamics is described by

$$\begin{cases} \dot{x}(t) &= Ax(t) + Bu(t) + F\Upsilon(t) + w(t) \\ y(t) &= Cx(t) + v(t) \end{cases} \quad (1)$$

$x(t) \in \mathbb{R}^n$ is the state vector, $u(t)$ is the control input, $F = [f_1, f_2, \dots, f_q] \in \mathbb{R}^{n \times q}$ is the fault distribution matrix, $\Upsilon(t) = [\Upsilon^1, \Upsilon^2, \dots, \Upsilon^q]^T \in \mathbb{R}^q$ is the fault

vector and $y(t) \in \mathfrak{R}^m$ is the measurement signals vector. We assume that each component of the output vector $y^i \in \mathfrak{R}^{m_i}$ with $i \in \{1, 2, \dots, n_s\}$ and $\sum_1^{n_s} m_i = m$ represent an intelligent sensor node equipped with event triggered mechanism (ETM). The initial state vector x_0 , process noise $w(t)$ and measurement noise $v(t)$ are uncorrelated, white Gaussian random processes with $x_0 \sim \mathcal{N}(x_0, \bar{P}_0)$, $w(t) \sim \mathcal{N}(w, 0, W)$ and $v(t) \sim \mathcal{N}(v, 0, V)$ respectively, where \bar{P}_0, W and R are symmetric, positive definite matrices. The Gaussian function (shortly noted as Gaussian) of vectors $x \in \mathfrak{R}^n$, $\mu \in \mathfrak{R}^n$ and $P \in \mathfrak{R}^{n \times n}$ is defined as $\mathcal{N}(x, \mu, P) : \mathfrak{R}^n \times \mathfrak{R}^n \times \mathfrak{R}^{n \times n} \rightarrow \mathfrak{R}$, i.e

$$\mathcal{N}(x, \mu, P) = \frac{1}{\sqrt{(2\pi)^n |P|}} e^{-0.5(x-\mu)^T P^{-1}(x-\mu)}$$

If $p(x) = \mathcal{N}(x, \mu, P)$ then by definition it holds that $E(x) = \mu$ and $cov(x) = P$.

In our case we will consider two problems: the first one is a system with a predefined ETM this case is coherent to the system architecture where the fault diagnosis scheme and the controller are implemented in the same computer. In this case, the ETM is dedicated only for improving control performances. In the second design problem, the ETM is designed for improving the FD performance. For simplicity, we ignore transmission delay and other communication constraints.

Event triggered mechanism (ETM)

The ETM is an algorithm that allows the sensor node to transmit recent measurement to the estimator node only if its value verifies the event triggering condition. This method has been explored and applied by researchers because of its efficiency in the network bandwidth improvement. The sensor measurements are processed and the event triggering conditions are checked in discrete instants $t_n = nh$ where $h > 0$ is the sampling period. At each synchronous time t_n and for each sensor 'i' we define the ET condition as follows

$$y^i(t_n) \notin \mathbb{S}_n^i \subset \mathfrak{R}^{m_i} \quad (2)$$

where \mathbb{S}_n^i is the ET set which is a specific designed bounded set that relies on the used event triggering technique. The transmission instances for the sensor i are given by the recursion

$$\bar{t}_k^i = \inf\{t_n > \bar{t}_{k-1}^i \mid \text{the condition (2) is verified}\} \quad (3)$$

If the ETM is verified at $t_n = \bar{t}_k^i$ then the sensor measurement is directly transmitted to the controller

$$t_n = \bar{t}_k^i \Leftrightarrow \bar{y}^i(\bar{t}_k^i) = y^i(t_n)$$

In this case we say that sensor node 'i' applies an event sampling method for obtaining the k^{th} event sampled measurement $\bar{y}^i(\bar{t}_k^i)$. After being transmitted, this measurement is then used for estimating $x(t_n)$ according

to a Gaussian probability density function (PDF). This estimated state is characterized by some mean $\hat{x}_{n|n}$ and error-covariance $P_{n|n}$. For monitoring the system, an estimate of the state is required synchronously in time rather than at event occurrence instants. To that extent, let us define \mathbb{T}_e^i to be the set of event instants for the i^{th} sensor node and \mathbb{T} the set of synchronous instants. Then we can write

$$\mathbb{T}_e^i = \{\bar{t}_k^i \mid k \in \mathbb{N}\}; \text{ and } \mathbb{T} = \{ch \mid c \in \mathbb{N}\}$$

As illustrated in **Fig 2**, it is clear that $\mathbb{T}_e^i \subset \mathbb{T}$.

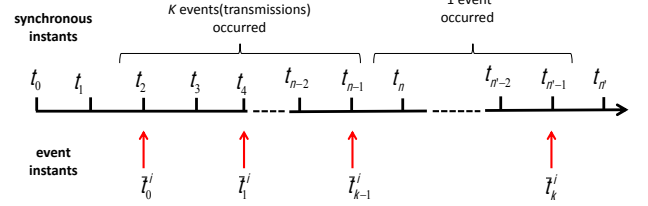


Fig. 2. Synchronous and event instants

At each synchronous time t_n and from ET condition (2) we can define the Borel set $\mathbb{Y}_n^i \in \mathfrak{R}^{m_i}$ for each $i \in \{1, \dots, n_s\}$ as follows

$$\mathbb{Y}_n^i = \begin{cases} \bar{y}^i(\bar{t}_k^i) = y^i(t_n) & \text{if } t_n = \bar{t}_k^i \\ \mathbb{S}_n^i & \text{otherwise} \end{cases} \quad (4)$$

The Borel set that corresponds to all measurement vector $y(t_n)$ is denoted \mathbb{Y}_n . This set is available to the controller at each synchronous time t_n and can be used for fault detection.

Remark 1. For scalar measurement sensors ($m_i = 1$) the Borel set \mathbb{Y}_n^i can be considered as bounded interval with uniform probability distribution over its values.

III. EVENT TRIGGERED FAULT DETECTION ALGORITHM

As illustrated in **Figure 1**, a fault detection filter is used for generating the residual signal e_n . More specifically, this filter uses a bunch of set-measurements $\mathbb{Y}_{0:n}$ for the state vector estimation based on Bayesian approach, developed to deal with such type of measurements.

The ETFD algorithm is described by the following steps:

- 1) Prediction of the state based on the system model(1) and the last updated state $\hat{x}_{n-1|n-1}$;
- 2) Generation and evaluation of the output residual e_n ;
- 3) Formulation of the likelihood $p(\mathbb{Y}_n|x_n)$ as a summation of N Gaussians;

- 4) Calculation of the updated estimation result as a summation of N Gaussians;
- 5) Approximation of the resulting $p(x_n|\mathbb{Y}_{0:n})$ as a single Gaussian;

More details about these steps are given in the sequel.

A. Recursive Bayes filter

The estimation of the state vector $x(t_n)$ given the set-measurement is equivalent to finding the following PDF function

$$p(x(t_n)|y(0) \in \mathbb{Y}_0, y(1) \in \mathbb{Y}_1, \dots, y(t_n) \in \mathbb{Y}_n) \quad (5)$$

For brevity of notation we denote $\star(t_n)$ as \star_n where \star can represent x, y, \bar{y} . We denote also (5) as $p(x_n|\mathbb{Y}_{0:n})$.

According to Bayes formula we can write the following

$$p(x_n|\mathbb{Y}_{0:n}) = \frac{p(x_n|\mathbb{Y}_{0:n-1})p(\mathbb{Y}_n|x_n)}{\int_{\mathbb{R}^n} p(x_n|\mathbb{Y}_{0:n-1})p(\mathbb{Y}_n|x_n)dx_n} \quad (6)$$

where the prediction PDF $p(x_n|\mathbb{Y}_{0:n-1})$ is given by

$$p(x_n|\mathbb{Y}_{0:n-1}) = \int_{\mathbb{R}^n} p(x_n|x_{n-1})p(x_{n-1}|\mathbb{Y}_{0:n-1})dx_{n-1} \quad (7)$$

Evaluating $p(x_n|\mathbb{Y}_{0:n})$ is not always straightforward [21]. The developed (ETFD) algorithm uses the results given in [15] to evaluate this integrals at each discrete instant t_n . A Bayesian Gaussian sum filter [6], [11] is used for limiting the complexity of computations by approximating the filtering and predictive distributions by weighted Gaussian mixtures. The result of the estimation are used for the residual generation as shown in the sequel.

B. Prediction step

In this step, we calculate the PDF of the state $p(x_n|\mathbb{Y}_{0:n-1})$, at each synchronous time t_n using the previous updated state $\hat{x}_{n-1|n-1}$ and the system mathematical model. Because the state is Gaussian process, one can write

$$p(x_n|\mathbb{Y}_{0:n-1}) = \mathcal{N}(x_n, \hat{x}_{n|n-1}, P_{n|n-1}) \quad (8)$$

with

$$\hat{x}_{n|n-1} = A_h \hat{x}_{n-1|n-1} + B_h u_{n-1} \quad (9)$$

$$P_{n|n-1} = A_h P_{n-1|n-1} A_h^T + E_h W E_h^T \quad (10)$$

where

$$A_h = e^{A h}, E_h = \int_0^h e^{A \eta} d\eta, B_h = \int_0^h e^{A \eta} B d\eta$$

Remark 2. For the computation of B_h we suppose that the actuators are provided with a zero-order-hold circuit that maintains the continuous input constant until the arrival of the new control value.

C. Residual signal generation

In model based FD, the purpose of residual generation is to find a fault indicating signal using available input and output information from the monitored system. The signal $e^i(t_n)$ denotes the residual for the rest of this paper. When the measurement is available the residual can be generated by the usual method *i.e* the difference between the available measurement and the estimated one. For the non-transmitting period, the fault effect can be detectable or non-detectable as illustrated in the **Fig 3**. In the case of detectable fault the estimated output \hat{y}_n^i based on the healthy system doesn't belong to set of actual possible output value of the sensor node namely \mathbb{S}_n^i which indicates a high possibility of fault occurrence. However, in the second case the only information that we possess is that both the real output y_n^i and estimated output \hat{y}_n^i belongs to same set \mathbb{S}_n^i and no information about the distance between y_n^i and \hat{y}_n^i is provided. In this case no decision about the occurrence of the fault can be given.

During the non-transmitting period $\bar{t}_{k-1}^i < t_n \leq \bar{t}_k^i$ the residual signal can be inspired from the ET condition. In other words, we can define a vector membership function $\xi^{\mathbb{S}_n^i}$ (see **Appendix**) such that $\xi^{\mathbb{S}_n^i}(y_n^i) > 0$ if $y_n^i \in \mathbb{S}_n^i$ and $\xi^{\mathbb{S}_n^i}(y_n^i) < 0$ otherwise. The residual signal can be given by

$$e_n^i = \begin{cases} \xi^{\mathbb{S}_n^i}(\hat{y}_n^i) & \text{if } t_n \neq \bar{t}_k^i \\ \bar{y}_k^i - \hat{y}_n^i & \text{if } t_n = \bar{t}_k^i \end{cases} \quad (11)$$

- real output
- estimated healthy output

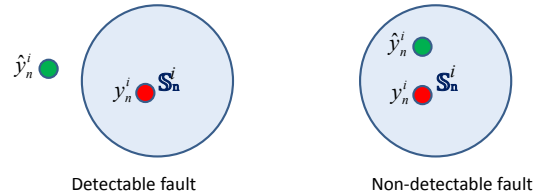


Fig. 3. Possible fault effects in the non-transmitting period

where $\hat{y}_n^i = C^i \hat{x}_{n|n-1}$ and $C^i \in \mathbb{R}^{m_i \times m_i}$ is the m_i row of the matrix C corresponding to the i^{th} sensor node. In general, the residual should be normally zero or close to zero when no fault is present, but is distinguishably different from zero when a fault occurs. For our choice of residual, we note that, between two sampling instances, the residual signal is positive in the faulty case and negative otherwise. The evaluation of $\xi^{\mathbb{S}_n^i}(\hat{y}_n^i)$ is a way to detect the occurrence of the fault. For instance, while transmitting, we use the classical way for residual generation and the faulty behaviour corresponds to a positive and distinguishably different from zero value of the residual signal given by $\bar{y}_k^i - \hat{y}_n^i$.

D. Likelihood formulation

The probability $p(y_n \in \mathbb{Y}_n | x_n)$, named also the likelihood, can be seen as a quantized measurement characterized by the set \mathbb{Y}_n . This section gives a unified formula of the PDF $p(y_n \in \mathbb{Y}_n | x_n)$ as a uniform distribution for all $y_n \in \mathbb{Y}_n$ such that

$$p(y_n \in \mathbb{Y}_n | x_n) = \int_{\mathbb{R}^m} p(y_n | x_n) p(y_n \in \mathbb{Y}_n) dy \quad (12)$$

where

$$p(y_n | x_n) = \mathcal{N}(y_n, C x_n, V)$$

To find an expression for the second PDF, i.e. $p(y_n \in \mathbb{Y}_n)$, let us define $\wedge_C(y_n)$ as a uniform distribution of $y_n \in \mathbb{R}^m$, which is constant within the set $\mathcal{C} \subset \mathbb{R}^m$ and zero outside \mathcal{C} . Therefore, $\wedge_{\mathbb{Y}_n}(y)$ is the desired uniform distribution of $p(y_n \in \mathbb{Y}_n)$, which can further be rewritten for each sensor node 'i' and for $\bar{t}_{k-1}^i < t_n \leq \bar{t}_k^i$ as follows

$$p(y_n^i \in \mathbb{Y}_n^i) = \begin{cases} \delta(y_n^i - \bar{y}_k^i) & \text{if } t_n = \bar{t}_k^i \in \mathbb{T}_e \\ \wedge_{\mathbb{Y}_n^i}(y_n^i) & \text{otherwise} \end{cases} \quad (13)$$

where δ is the Dirac delta function.

This distribution can be approximated by a sum of N Gaussian distributions as follows

if $t_n = \bar{t}_k^i \in \mathbb{T}_e$ then

$$p(y_n^i \in \mathbb{Y}_n^i) = \lim_{\tilde{V}_n^i \rightarrow 0} \frac{1}{N} \sum_{j=1}^N \mathcal{N}(y_n^i, \bar{y}_k^i, \tilde{V}_n^i)$$

if $t_n \neq \bar{t}_k^i$ then

$$p(y_n^i \in \mathbb{Y}_n^i) = \wedge_{\mathbb{Y}_n^i}(y_n^i) \approx \frac{1}{N} \sum_{j=1}^N \mathcal{N}(y_n^i, \bar{\mu}_{n,j}^i, \bar{V}_n^i)$$

by substitution into (12) we get

$$p(y_n \in \mathbb{Y}_n | x_n) \approx \frac{1}{N} \sum_{j=1}^N \int_{\mathbb{R}^m} \mathcal{N}(y_n, C x_n, V) \mathcal{N}(y_n, \mu_{n,j}, V_n) dy_n \quad (14)$$

where

$$\mu_{n,j} \in \mathbb{R}^m \text{ such that } \mu_{n,j}^i = \begin{cases} \bar{y}_k^i & \text{if } t_n = \bar{t}_k^i \\ \bar{\mu}_{n,j}^i & \text{otherwise} \end{cases}$$

$$V_n = \text{diag}(V_n^1, \dots, V_n^{n_s}) \in \mathbb{R}^{m \times m}$$

$$V_n^i = \begin{cases} 0_{m^i \times m^i} & \text{if } t_n = \bar{t}_k^i \\ \tilde{V}_n^i & \text{otherwise} \end{cases}$$

Proposition 1. [6] An explicit solution of the integral (14) is given by

$$p(y_n \in \mathbb{Y}_n | x_n) \approx \frac{1}{N} \sum_{j=1}^N \mathcal{N}(\mu_{n,j}, C x_n, R_n) \quad (15)$$

where

$$R_n = V + V_n \quad (16)$$

Proof. The proof of this proposition can be found in [13]. \square

E. State update

An approximation of the product $p(x_n | \mathbb{Y}_{0:n-1}) p(\mathbb{Y}_n | x_n)$ can be given by

$$p(x_n | \mathbb{Y}_{0:n-1}) p(\mathbb{Y}_n | x_n) \approx \frac{1}{N} \sum_{j=1}^N \mathcal{N}(x_n, \hat{x}_{n|n-1}, P_{n|n-1}) \times \mathcal{N}(\mu_{n,j}, C x_n, R_n)$$

Using again the approximation of a Gaussian product, the above summation of Gaussian products becomes

$$p(x_n | \mathbb{Y}_{0:n-1}) p(\mathbb{Y}_n | x_n) \approx \frac{1}{N} \sum_{j=1}^N \omega_{n,j} \mathcal{N}(x_n, \hat{\theta}_{n,j}, \Theta_n) \quad (17)$$

with

$$\Theta_n = (P_{n|n-1}^{-1} + C^T R_n^{-1} C)^{-1} \quad (18)$$

$$\hat{\theta}_{n,j} = \Theta_n (P_{n|n-1}^{-1} \hat{x}_{n|n-1} + C^T R_n^{-1} \mu_{n,j}) \quad (19)$$

$$\omega_{n,j} = \mathcal{N}(\mu_{n,j}, C \hat{x}_{n|n-1}, C P_{n|n-1} C^T + R_n) \quad (20)$$

Now, the updated PDF $p(x_n | \mathbb{Y}_{0:n})$ of (6) can be established. Note that the characterization presented in (6) has the expression of (17) in its nominator, while its denominator is the integral of (17) over x_n . As this integral equals $\frac{1}{N} \sum_{j=1}^N \omega_{n,j}$, the explicit formula of $p(x_n | \mathbb{Y}_{0:n})$ is given by

$$p(x_n | \mathbb{Y}_{0:n}) \approx \sum_{j=1}^N \frac{\omega_{n,j}}{\sum_{j=1}^N \omega_{n,j}} \mathcal{N}(x_n, \hat{\theta}_{n,j}, \Theta_n) \quad (21)$$

F. State approximation

Proposition 2. In this step the sum given in (21) is approximated by a single Gaussian that has the same expectation and covariance matrix, hence:

$$p(x_n | \mathbb{Y}_{0:n}) \approx \mathcal{N}(x_n, \hat{x}_{n|n}, P_{n|n}) \quad (22)$$

where

$$\hat{x}_{n|n} = \sum_{j=1}^N \frac{\omega_{n,j}}{\sum_{j=1}^N \omega_{n,j}} \hat{\theta}_{n,j} \quad (23)$$

$$P_{n|n} = \sum_{j=1}^N \frac{\omega_{n,j}}{\sum_{j=1}^N \omega_{n,j}} \left(\Theta_n + (\hat{x}_{n|n} - \hat{\theta}_{n,j})(\hat{x}_{n|n} - \hat{\theta}_{n,j})^T \right) \quad (24)$$

Proof. The proof of this proposition can be found in [13]. \square

In the next section, we will examine the efficiency of the proposed algorithm by applying it to an event based control system in which the ETM is designed to ensure high performances with less number of measurements transmission;

Before applying of the ETFD, we need to define the used ETM and the corresponding sum of Gaussian approximation as proceeded in the sequel.

IV. FD UNDER PREDEFINED ETM

The mixed output based ETM is one of the most alternative methods in event triggered control literature. The transmission is only possible if the following ET condition is verified.

$$\|y_n^i - \bar{y}_{k-1}^i\|^2 > \sigma^i \|y_n^i\|^2 + \epsilon^i \quad (25)$$

The parameters σ^i and ϵ^i can be designed to insure the system stability with very less number of measurement transmissions as given in [3].

We can also define the ETM by the next membership function by

$$\xi^{\mathbb{S}_n^i}(y_n^i) = \xi^i(y_n^i, \bar{y}_{k-1}^i) = \|y_n^i - \bar{y}_{k-1}^i\|^2 - \sigma^i \|y_n^i\|^2 - \epsilon^i$$

The scalars $\bar{y}^i \in \mathfrak{R}$ ($m_i = 1$) represents the transmitted measurements and \bar{t}_k^i is the k^{th} event instant.

The ETFD estimates x_n given the bunch of received observations until time t_n . Notice that due to the definition of event sampling, we can extract information of all the measurement sets \mathbb{Y}_n . If ETM condition is not verified at the time interval $\bar{t}_{k-1}^i < t_n \leq \bar{t}_k^i$ then the output y_n^i remains in a finite subset given by

$$\bar{m}^i(\bar{t}_k^i) - \Delta^i(\bar{t}_k^i) < y_n^i < \bar{m}^i(\bar{t}_k^i) + \Delta^i(\bar{t}_k^i)$$

where $\bar{m}^i(\bar{t}_k^i) = \frac{\bar{y}_k^i}{1-\sigma^i}$ and $\Delta^i(\bar{t}_k^i) = \frac{\sqrt{\sigma^i (\bar{y}_k^i)^2 + \epsilon^i - \epsilon^i \sigma^i}}{1-\sigma^i}$.

It is clear that

$$\mathbb{S}_n^i = [\bar{m}^i(\bar{t}_k^i) - \Delta^i(\bar{t}_k^i), \bar{m}^i(\bar{t}_k^i) + \Delta^i(\bar{t}_k^i)] \quad (26)$$

While no-transmitting, the values $\bar{m}^i(\bar{t}_k^i) - \Delta^i(\bar{t}_k^i)$ and $\bar{m}^i(\bar{t}_k^i) + \Delta^i(\bar{t}_k^i)$ represent the upper and the lower bound of the measurement y_n^i , respectively.

For the Gaussian approximation one can write:

$$p(y_n \in \mathbb{Y}_n) = \frac{1}{N} \sum_{j=1}^N \mathcal{N}(y_n, \bar{\mu}_{n,j}, \bar{V}_n) \quad (27)$$

such that

$$\mu_{n,j} \in \mathfrak{R}^m$$

$$\mu_{n,j}^i = \begin{cases} \bar{y}_k^i & \text{if } t_n = \bar{t}_k^i \\ \bar{\mu}_{n,j}^i & \text{otherwise} \end{cases}$$

$$V_n = \text{diag}[V_n^1, \dots, V_n^m] \in \mathfrak{R}^{m \times m}$$

$$V_n^i = \begin{cases} 0 & \text{if } t_n = \bar{t}_k^i \\ \bar{V}_n^i & \text{otherwise} \end{cases}$$

where $\bar{\mu}_{n,j}^i$ and \bar{V}_n^i are given by :

$$\bar{\mu}_{n,j}^i = \bar{m}^i(\bar{t}_k^i) - (N - 2(j - 1) - 1) N^{-1} \Delta^i(\bar{t}_k^i)$$

$$\bar{V}_n^i = (\Delta^i(\bar{t}_k^i))^2 N^{-2} \left(1 - 0.2e^{-\frac{4(N-1)}{15}} - 0.32e^{-\frac{4(N-1)}{180}} \right)$$

More details of this approximation are presented in the **Appendix**.

In model based FD the purpose of residual generation is to generate a fault indicating signal, using available input and output information from the monitored system. In the sequel, the signal $e^i(t_n)$ is used as fault indicator. This signal inspired from the mixed ET condition is given by

$$e_n^i = \begin{cases} \xi^i(\hat{y}_n^i, \bar{y}_{k-1}^i) = \|\hat{y}_n^i - \bar{y}_{k-1}^i\|^2 - \sigma^i \|\hat{y}_n^i\|^2 - \epsilon^i & \text{if } t_n \neq \bar{t}_k^i \\ \bar{y}_k^i - \hat{y}_n^i & \text{if } t_n = \bar{t}_k^i \end{cases}$$

with

$$\hat{y}_n^i = C^i \hat{x}_{n|n-1} \text{ and } C^i \text{ is the } i^{\text{th}} \text{ row of the matrix } C.$$

V. ILLUSTRATIVE EXAMPLE

Consider the following continuous time system

$$A = \begin{bmatrix} -10 & 0 & 1 & 22 \\ 0 & -35 & 1 & -0.4 \\ 1 & 0 & -8 & 1 \\ 0 & 1 & 0.4 & -3 \end{bmatrix}, B = \begin{bmatrix} 0 & -1 \\ 0 & 0.25 \\ 1 & 0 \\ -1 & 0.5 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0.6 & 0.1 & 0 \\ 1 & 0.4 & 0 & 0 \\ 0.5 & 0 & 1 & 0.1 \\ 0 & 0 & 0 & -1 \end{bmatrix}, F = \begin{bmatrix} -1 & 1 \\ 1 & 0 \\ 0 & -1 \\ 1 & 1 \end{bmatrix},$$

$$W = \begin{bmatrix} 0.95 & 0 & 0 & 0 \\ 0 & 0.88 & 0 & 0 \\ 0 & 0 & 0.59 & 0 \\ 0 & 0 & 0 & 0.87 \end{bmatrix}; V = I_4$$

The fault associated to the first column of the matrix F occurs at time intervals $[20, 40]$ and $[70, 100]$ with $\Upsilon^1 = 12$; while the second fault (associated to the

second column of F) occurs at time interval $[20, 40]$ with $\Upsilon^1 = 12$. Figures **Fig .4** and **Fig .5** depict the residual signal e_n^1 with its corresponding ETM state (1 activated, 0 not activated) in both healthy and faulty case. We omit the plot of the rest of residual signals e_n^2, e_n^3, e_n^4 because they show similar behaviour as e_n^1 .

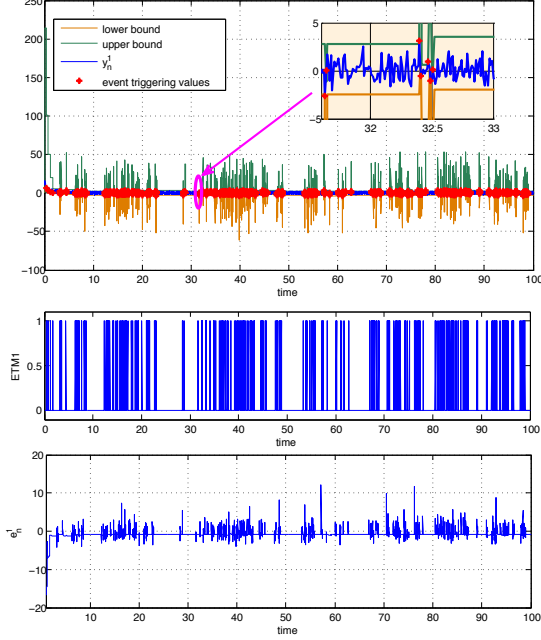


Fig. 4. The residual e_n^1 in the healthy case

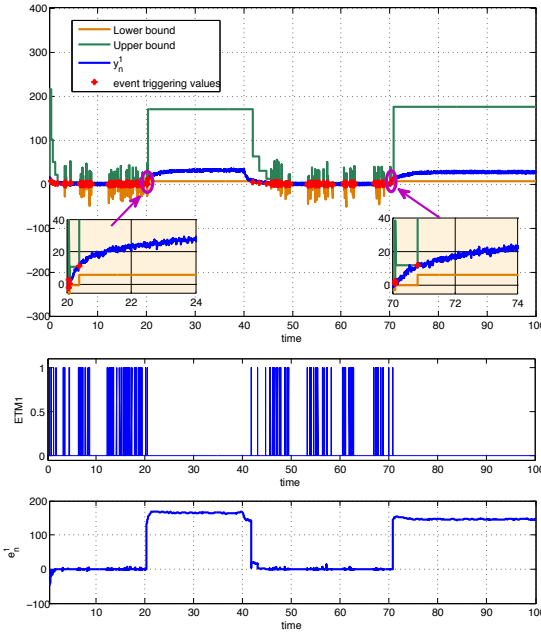


Fig. 5. The residual e_n^1 in the faulty case

First, we see that residual signals e^1 , can detect the fault even in the non transmission time interval $t \in [20, 40]$. For $t \in [70, 100]$ the fault is detected using mainly set

type measurement. The ETFD filter uses the ET condition and the predicted state based on the healthy system dynamics for the detection of the faulty behaviour. One can see that based on the proposed method, one has the possibility to detect fault even for non-transmission case.

VI. CONCLUSION

In this chapter, an FD framework for networked control systems has been proposed. First, a general structure of the ETFD algorithm is proposed. Then, to demonstrate its performance, this algorithm is used for fault detection in NCS under predefined ETM. Future directions of research will include the design of ETM dedicated for improving fault diagnosis performance. Self triggering algorithm based on updating the ETM to reduce the number of measurement transmissions and hence the power consumption is also a possible perspective for the on-line sensor scheduling and fault detection co-design field.

APPENDIX

Membership function

For a given set \mathbb{S} we say that ξ is a membership function on \mathbb{S} only if it satisfies the following:

For any element s the next statement holds true

$$\begin{cases} \xi^{\mathbb{S}}(s) > 0 & \text{if } s \in \mathbb{S} \\ \xi^{\mathbb{S}}(s) < 0 & \text{otherwise} \end{cases} \quad (28)$$

Sum of Gaussian approximation

The uniform distribution $\wedge^i_{\mathbb{Y}_n}(y_n^i)$ is defined as following:

$$\text{if } \bar{m}^i(t_{n-1}) - \Delta^i(t_{n-1}) < y^i(t_n) < \bar{m}^i(t_{n-1}) + \Delta^i(t_{n-1})$$

then

$$\wedge^i_{\mathbb{Y}_n}(y_n^i) = \frac{1}{2 \Delta^i(t_{n-1})}$$

$$\text{if } y^i(t_n) < \bar{m}^i(t_{n-1}) - \Delta^i(t_{n-1}) \text{ or } y^i(t_n) > \bar{m}^i(t_{n-1}) + \Delta^i(t_{n-1})$$

then

$$\wedge^i_{\mathbb{Y}_n}(y_n^i) = 0$$

Our aim is to find an approximation of $\wedge^i_{\mathbb{Y}_n}(y_n^i)$ as a sum of Gaussian distributions given by

$$\wedge^i_{\mathbb{Y}_n}(y_n^i) \approx \frac{1}{N} \sum_{j=1}^N \mathcal{N}(y_n^i, \bar{\mu}_{n,j}^i, \bar{V}_n^i)$$

V_n^i and $\mu_{n,j}^i$ are calculated in order to minimize the next cost function

$$\begin{aligned} & \mathcal{F}(N, \bar{\mu}_{n,1}^i, \dots, \bar{\mu}_{n,N}^i, \bar{V}_n^i) \\ &= \int_{-\infty}^{+\infty} \left(\wedge_{\mathbb{Y}_n}^i(y_n^i) - \frac{1}{N} \sum_{j=1}^N \mathcal{N}(y_n^i, \bar{\mu}_{n,j}^i, \bar{V}_n^i) \right)^2 dy_n^i \end{aligned}$$

For that the rectangle of $\wedge_{\mathbb{Y}_n}^i(y_n^i)$ is divided into N new sub-rectangles. Meaning that a large quantization of 2Δ is divided into N smaller quantization of $\frac{2\Delta}{N}$. Each quantization-section is approximated by a single Gaussian function with a different expectation $\bar{\mu}_{n,j}^i$ and equal covariances \bar{V}_n^i . The expectation $\bar{\mu}_{n,j}^i$ is the center of each quantization and is given by

$$\bar{\mu}_{n,j}^i = \bar{m}^i(t_{n-1}) - (N - 2(j - 1) - 1) N^{-1} \Delta^i(t_{n-1})$$

The covariance \bar{V}_n^i is estimated as a function of Δ and N and due to the fact that width of a Gaussian function is linear to the square-root of its covariance, two aspects of this function are assumed: \bar{V}_n^i is linear with Δ^2 and when $N \rightarrow \infty$, \bar{V}_n^i increases linearly with $\frac{1}{N^2}$. With these two assumptions \bar{V}_n^i is further estimated using trial and error for different values of N and Δ .

Finally, the resulting function is given by [14]

$$\bar{V}_n^i = (\Delta^i(\bar{t}_k^i))^2 N^{-2} \left(1 - 0.2e^{-\frac{4(N-1)}{15}} - 0.32e^{-\frac{4(N-1)}{180}} \right)$$

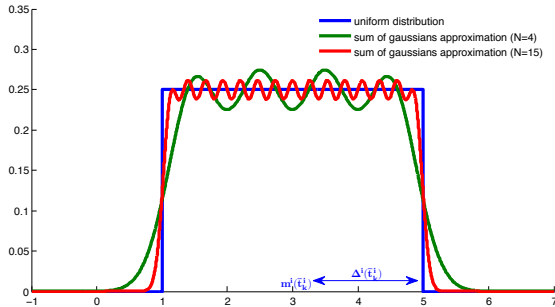


Fig. 6. Sum of Gaussians approximation

It is clear that better approximation needs more number of Gaussians, or in other words if $M > N$ then $\mathcal{F}(N, \bar{\mu}_{n,1}^i, \dots, \bar{\mu}_{n,N}^i, \bar{V}_n^i) \geq \mathcal{F}(M, \bar{\mu}_{n,1}^i, \dots, \bar{\mu}_{n,M}^i, \bar{V}_n^i)$. Figure 6 shows an example of the proposed approximation for two cases ($N = 4$ and $N = 15$) where $\Delta^i(t_{n-1}) = 2$ and $\bar{m}^i(t_{n-1}) = 3$.

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