Inelastic behavior modelling of concrete in dynamics

Frédéric Ragueneau, Fabrice Gatuingt

Inelastic behavior modelling of concrete in low and high strain rate dynamics

Frédéric Ragueneau and Fabrice Gatuingt

Laboratoire de Mécanique et Technologie

Ecole Normale Supérieure de Cachan, 61 avenue du Président Wilson,

F-94235 Cachan cedex, France

Abstract

This work deals with response modeling of concrete for dynamic loading. As in statics one has to account for substantial difference of inelastic response in tension and compression, the anisotropy of the response induced by complex cracking patterns and the need of irreversible deformation due to frictional sliding or non-closing cracks. On the top of that, in dynamics, we also have to handle the hardening or softening phenomena which explain a particular hysteretic response for a given cyclic loading as well as the strain rate effects. The latter should further be addressed separately for high as opposed to low strain rates. The main goal of this work is to develop the concrete constitutive model capable of reproducing the salient features experimentally observed. We present one theoretical development for the constitutive model of concrete at low strain rates. The same kind of developments are then carried out for high strain dynamic behavior. Both chosen models belong to the class of coupled plasticity–damage models briefly presented.

Key words: concrete, dynamics, damage, plasticity

1 Introduction

The concrete like all geo-materials and ceramics, is perceived like being brittle in traction and more ductile under compressive loading. Moreover, direct tensile tests allow to observe, under displacement control in the testing machine, a steep softening beyond the peak resistance in traction [1]. Other experimental procedures to obtain an estimate of the concrete traction strength such as the splitting tensile tests (Brazilian test) or by 3 points bend tests on notched samples [2] and [3] are simple to carry out (e.g. the compressed part of the beam ensures in this case a better stability of the test) but more difficult to interpret [4].

The behaviour in compression is more easily measured thanks to the greater ductility of material. During tests on cylindrical specimens, a network of microscopic cracks nucleates parallel to the axis of loading coalescing at the complete rupture. For measurement quality, it is important to ensure friction free boundary condition between the plates of the machine and the two ends of the sample generating a not-uniaxial state of stress, which can be achieved by anti-hooping devices [5], [6].

Cracking, as the main source of the damage of brittle materials, is perceived as the stiffness decrease. The differences in stiffness decrease in longitudinal and transverse direction, which is typical of concrete, induces anisotropic response which is very difficult to handle (e.g. see [7]). Moreover, the imperfections of the cracks precludes their complete closure upon unloading, thus generating irreversible strains which can be represented by a plasticity model. The same model can be used to capture friction and sliding along the cracks.

In short, the mechanical model which is capable of representing the salient features of described response of concrete is the coupled model of damage and plasticity. The model ought to take into account: different type of behavior for concrete under tension and compression, anisotropic decrease in material stiffness as the microcracks open, stiffness recovery as crack closure occurs and irreversible strains concomitant to damage.

The same type of concrete model can be extended to the case of dynamic loading, which is of main interest for this work, both for the case of low strain rate [8] and high strain rate type of response [9]. To that end, the main modification concerns the capability of the constitutive model to account for the strain rate effect.

Many issues remain ambiguous in the dynamic experiment as opposed to a static one and in particular the influence of time. For more frequent structural testing (rather than material one) the effects of time typically appear in two ways: by inertial forces induced by nonzero acceleration to which is subjected the structure and by a dependence of the mechanical characteristics of each elementary volume of material with respect to the time evolutions which is described in terms of viscosity.

This distinction is strictly related to the traditional concept of representative volume element used to define the material behavior and it is indeed not excluded that the effects of viscosity are due to microscopic inertial phenomena. This remark is particularly pertinent to the case of concrete like materials, where the size of the representative volume element is of the order of centimeter. We will consider herein, that the dynamic aspect of the behavior is only due to the influence of time on the mechanical characteristics and exclude the

inertial effects.

A number of works has been published in order to get better insight into the concrete behavior in dynamic compression and to study the influence of the loading rate. For example, Bischoff and Perry [10] show, as corroborated by a significant bibliographical synthesis in figure 1, that there is an increase of the concrete compressive strength at higher deformation rate. Moreover they point out that we can distinguish two different modes (see figure 1) producing the relative increase of the resistance in compression: the first mode corresponds to a range of variations of deformation rate ranging between the quasi-static $\dot{\varepsilon} = 10^{-6}$ s^{-1} and $\dot{\varepsilon} < 10$ s^{-1} leading to $1,5$ times increase of the resistance in compression. The majority of the authors agree on explanation for such a strength increase pertinent to a material viscosity related to the presence of water in the pores of the concrete [11], [12]. The second mode for which the deformation rate $\dot{\varepsilon} > 10$ s $^{-1}$ all the way to $\dot{\varepsilon} = 10^{3}$ s $^{-1}$ allows to multiply by four the resistance of the concrete, according to various studies, is considered to be of structural origin [13], [14], [15]. This origin seems to be a state of nonhomogeneous stress in the specimen brought about by the inertia which generates considerable radial constraints acting like a confining pressure [16]. One can also consider that this increase in resistance is due to a delayed appearance of the microscopic cracks [17] for increasing deformation rate. Moreover, recent studies [18] carried out on ceramics, show that the higher the rate of deformation, the more cracking remains diffuse (no localization of cracking), which also increases resistance.

Just like compression test, the simple tension test must also be adapted to dynamic. In particular, by using a modified version of the Hopkinson bar test, Brara and Klepaczko [19] obtained experimental results in which the rate of

deformation is about 100 s $^{-1}$ (see figure 2). They also collected the results presented in the literature as shown in figure 2. We can see from this figure that just like in compression, two distinct modes appear in the increase of the tensile strength due to the high rate of deformation. Moreover, for both modes, the increase seems to be higher than in compression: with a rate of deformation of about 1 s $^{-1}$ the tensile strength is multiplied by two, while if the rate of deformation is about 100 s $^{-1}$ one multiplies resistance by 10. Contrary to compression test, we cannot explain the phenomena which induce resistance increase. If the first mode dynamic resistance increase, seems again to be due to the presence of water in material, the second mode is still not explained. Numerical simulations [20] of this test obtained with the discrete elements method without accounting for viscous effects, showed that we were not able to reproduce increase in tension resistance, successfully achieved by the same method in compression. It thus seems that the " inertial " effects do not have the same influence in traction (see figure 3) and that it might be necessary to account for other phenomena which intervene (e.g. crack speed). The "inertial " effects of traction produce a stress state close to a multiaxial state of traction and thus, in principle, we can not really talk about an increase in resistance.

While the salient features of the static behavior of the concrete under multiaxial stress state requests is nowadays, rather well known (see figure 4), it is much less so in dynamics. The main reason for that pertains in the difficulty to synchronize evolutions of the different loadings components in order to produce a multiaxial stress state in a concrete specimen. For that reason in most experimental techniques only one component of the loading is dynamic while the others remain static. In order to have a touchy dynamic triaxial

compression test, we carried out a test with the Hopkinson bar, in which the concrete sample was confined by a metal ring [9]. In this test, the state of deformation is close to an œdometric state what makes it possible to consider very high pressure and volume change of the concrete. The curve in figure 7 makes it possible to compare static with dynamic results and to observe a strong influence of rate of loading which cannot be explained by an inertial effect in measurement or in experiments with the restrained radial deformation blocked.

The main goal of this work is to develop the concrete constitutive model capable of reproducing one salient features observed in these experimental results. The outline of the paper is as follows. In section 3 we present one theoretical development for the constitutive model of concrete at low strain rates. The same kind of developments are then carried out in section 4 for a high strain dynamic behavior of concrete. Although both chosen models belong to the class of coupled plasticity -damage models briefly presented in section 2, the actual choice for either is quite specify. Concluding remarks are given in section 5.

2 Thermodynamic framework for coupled plasticity damage model for concrete

In order to perform 3D nonlinear transient analyses for concrete structures, the constitutive equations must provide sufficiently robust and reliable representations for any different loading path. The thermodynamics of irreversible processes provides one such framework for material modelling which allows to ensure continuity of the stress-strain response under 3D state of loading and

to obtain the consistent tangent modulus, the essential ingredients of efficient numerical implementation. The expression of constitutive equations based on thermodynamics is based on several steps. The first one is to define the state variables, including the internal variables as macroscopic representation of microscopic mechanism and phenomena (dislocation sliding leading to plasticity and hardening, cracking inducing damage, etc...). The local state method postulates that the thermo-mechanic knowledge of a continuum at time t is completely defined by the value of these state variables. Any process leading to evolution of state variables is thermodynamically admissible, if, the Clausius-Duhem inequality (the second principle) is satisfied. Assuming a convex state potential, from which the evolution laws are derived, allows to verify a priori the thermodynamic stability condition.

Next ingredient of the model which is required is the specific Helmholtz free energy as a function of the state variables. From the stand point of subsequent numerical implementation, it is advantageous to include the total strain ϵ along with a set of internal variables V_k and write the free energy as:

$$\psi = \psi(\epsilon, V_k) \tag{1}$$

We further appeal to the Clausius-Duhem inequality postulating the positiveness of the dissipated energy, which in an isothermal case can be written as:

$$\sigma\dot{\epsilon} - \rho\dot{\psi} \geq 0 \tag{2}$$

where σ is the Cauchy stress tensor, ρ the material density and superposed dot denotes the time derivative.

With respect of the choice of state variable in 1 we can further write:

$$\dot{\psi} = \frac{\partial \psi}{\partial \epsilon}\dot{\epsilon} + \frac{\partial \psi}{\partial V_k}\dot{V}_k \tag{3}$$

And by substituting the last result in (2) we obtain:

$$(\sigma - \rho\frac{\partial \psi}{\partial \epsilon})\dot{\epsilon} - \rho\frac{\partial \psi}{\partial V_k}\dot{V}_k \geq 0 \tag{4}$$

From this expression we further deduce the state laws as:

$$\sigma_{ij} = \rho\frac{\partial \psi}{\partial \epsilon_{ij}} \tag{5}$$

$$-A_k\dot{V}_k \geq 0; \quad A_k = \rho\frac{\partial \psi}{\partial V_k} \tag{6}$$

where A_k is the thermodynamic force associated to the state variable V_k. In order to complete this development, we have to define particular yield and damage functions which define the elastic domain:

$$f^p(\sigma, A_k^p) \leq 0, \quad f^d(\sigma, A_k^d) \leq 0 \tag{7}$$

For a very large number of physical mechanisms as described in the previous section for dynamics of concrete, we in general need a coupled plasticity damage model, as indicated in 7 above. The main goal in the next section is to elaborate on two possible approaches to construct the constitutive model on continuum damage mechanics and of plasticity coupled to damage, one for low strain rate and another for high strain rate dynamics.

For completeness of this section we briefly review the main features of the existing damage and plasticity models. The formers can be separated into two main categories: the first one exploits micromechanic considerations (e.g. see

Kachanov [21] , Andrieux *et al.* [22]) to define the internal variables which are introduced in a free energy potential [23], [24]. The second kind of models are based on phenomenological interpretations of experimental results and the expression of the elastic free energy does not contain any microscale variables. The models of the first category allow for interpretation of internal variables, but they ought to pass through homogenization procedure, which leads to a very costly numerical implementation. On other hand, the phenomenological plasticity coupled to damage models render the choice of the internal variables somewhat more delicate, but they remain placed within the standard thermodynamics framework and thus may benefit from advanced numerical algorithms for plasticity [25], [39].

From a macroscopic point of view, initially introduced by Kachanov [26] for creep failure problems, the damage mechanics formulation requires the addition of a phenomenological internal variable which represents the loss of stiffness (Lemaitre and Chaboche [27]). This can be achieved in many ways. The classical one is to relate the damaged material's and the intact material's elastic properties, which in the most general case would require the damage variable as an fourth-order tensor. But since its identification would be practically impossible, we further reduce the rank of the damage variable while trying to maintain the most reliable representation of the experimentally observed properties of the material. In order to achieve this objective, the effective stress concept is introduced (e.g. see [28], [27], [29]) through a strain equivalence, such that the effective stress $\tilde{\sigma}$, when applied to the undamaged material, will produce the same strain as the true stress acting upon the damaged variable.

Using this concept of effective stress, the coupling between damage mechanics

and plasticity can be introducing directly by considering the plasticity yield surface in terms of the effective stress $f^p(\tilde{\sigma}, A_k) \leq 0$.

Another approach, considering damage as a fourth order tensor, consists in using directly the material compliance tensor D as the internal damage variable related to crack growth and nucleation [30], [31]. With this kind of approach, we also introduce the damage criterion $f^d(\sigma, A_k) \leq 0$ which is formally equivalent to the plasticity criterion. The final group of models using the damage as second-order tensors, although frequently used, till poses a problem that the symmetry of the elasticity operator is not ensured [32] and can be recovered as shown by Cordebois and Sidoroff [33] by postulating a state potential in terms of only the effective stress and computing the elasticity operator through the energy equivalence. This energetic approach will be further developed herein. Other strategies allowing to account for anisotropy in damage modeling for concrete are based on the microplan approach [34].

The plasticity models can also be developed by following two kinds of approaches with respect to the choice of internal variables; on one side, we can take the micromechanics approach, with the clearly defined internal variables discribing the sliding of the plastic crystals (e.g. Hill [35] or Lubliner [36]) and, on the other side, we can rather choose the phenomenological plasticity models. The latter approach is favorised herein mostly for the reason of computational efficiency. The strain rate effect which are of interest for dynamics can easily be incorporated within such model, with either classical Perzyna [37] or Duvaut and Lions [38] models, which can be implemented in a unified manner as shown by Ibrahimbegovic *et al.* [39]. Further refinement of the plasticity model of this kind which concern the nonlinear kinematic hardening of Armstrong and Fredericks [44] can also be handled on efficient manner as

shown in Ibrahimbegovic and Chorfi [40].

3 Damage coupled to plasticity for low strain rate dynamics

In order to obtain more reliable results for transient non-linear computations of structures, physical and numerical energy dissipation have to be accounted for . The major difficulty relates to select the corresponding form of the damping matrix, such as viscous or hysteretic [41]. A more realistic damping model includes inelastic behavior and these possibilities ought to be examined carefully since related to internal dissipation. As shown by recent experiments on reinforced concrete mock-up subjected to seismic loading, there exist strong interactions between state of failure and resulting global damping [42]. Most of the coupled damage-plasticity models, which are capable to reproduce realistically the behavior of concrete in the non-linear static case, do not provide the same accuracy for cyclic loading. Namely, the influence of heterogeneities and roughness of the crack surfaces imply that at a fixed level of damage, concrete still exhibits dissipation due to the frictional sliding between crack surfaces. This property can be observed experimentally through typical cyclic loading producing hysteresis loops.

A new constitutive model is proposed for concrete, capable of including residual hysteretic loops at a fixed level of damage. The model is developed within the thermodynamics framework and it allows to couple the state of cracking with the hysteretic dissipation induced by the crack surfaces sliding. The model employs a particular Helmholtz's free energy which allows to introduce a coupling of the level of damage in one direction to a frictional stress, associated with a plasticity-like behavior and a non-linear kinematic harden-

ing. The introduced dissipative potential allows the description of dilatancy, fundamental feature of materials like concrete, sand or rocks.

A physically realistic description of the oriented crack growth in concrete can be obtained by using a second-order damage tensor formulation. Helmholtz's strain-based free energy is chosen as a function of an effective strain tensor which is defined in the principal axis of the damage tensor according to (no summation on repeated indices):

$$\tilde{\epsilon}_{ij} = (1 - d_i)^{1/4} \epsilon_{ij} (1 - d_j)^{1/4} \tag{8}$$

As in Cordebois and Sidorof [33], this effective strain, introduced directly into the state potential, allows for the description of an elasto-damage material exhibiting orthotropic cracks:

$$\rho\psi_d = \frac{1}{2}\{2\mu\tilde{\epsilon}_{ij}\tilde{\epsilon}_{ij} + \lambda\tilde{\epsilon}_{kk}^2\} \tag{9}$$

where λ and μ are the Lame coefficients defined for the undamaged material. The main assumption postulated to describe the hysteretic behavior of a partially open crack created after fracture is based on the concept of frictional sliding. It induces the occurrence of a sliding stress which prevents the crack from opening easily. In other words, the energy consumed during cracking is not entirely dissipated but partially stored in the sliding potential. It is through the damage variable that this energy shift can be obtained. Since each dissipative nonlinear phenomenon needs its own internal variable, measure of the frictional sliding is defined as a particular second order tensor, the sliding strain ϵ^s. Since they result from the same microscopic mechanism (rugosity of the cracked surfaces), inelastic strains and hysteresis loops are represented by

the same internal variable. Sliding is incorporated into the behavior through an equivalent strain which couples damage with elastic strain component of sliding.

$$\hat{\epsilon}_{ij} = d_i^{1/4}(\epsilon_{ij} - \epsilon_{ij}^s)d_j^{1/4} \tag{10}$$

The complete thermodynamic potential will now have the following form:

$$\rho\psi = \rho\psi_d(\tilde{\epsilon}_{ij}) + \rho\psi_s(\hat{\epsilon}_{ij}) \tag{11}$$

with

$$\rho\psi_s = \frac{1}{2}\{2\mu\hat{\epsilon}_{ij}\hat{\epsilon}_{ij} + \lambda\hat{\epsilon}_{kk}^2\} + \frac{1}{2}b\alpha_{ij}\alpha_{ij} \tag{12}$$

where α_{ij} is the internal variable associated with the kinematic hardening phenomenon and b is a material parameter. The constitutive equations may now be derived as:

$$\sigma_{ij} = \rho\frac{\partial\psi}{\partial\epsilon_{ij}} = 2\mu(1-d_i)^{1/2}\epsilon_{ij}(1-d_j)^{1/2} + \lambda(1-d_i)^{1/2}\sum_{k=1}^{3}\epsilon_{kk}(1-d_k)^{1/2}\delta_{ij} + \sigma_{ij}^s \tag{13}$$

and

$$\sigma_{ij}^s = -\rho\frac{\partial\psi}{\partial\epsilon_{ij}^s} = 2\mu d_i^{1/2}(\epsilon_{ij} - \epsilon_{ij}^s)d_j^{1/2} + \lambda d_i^{1/2}\sum_{k=1}^{3}(\epsilon_{kk} - \epsilon_{kk}^s)d_k^{1/2}\delta_{ij} \tag{14}$$

Finally, the back stress id defined as

$$X_{ij} = \rho\frac{\partial\psi}{\partial\alpha_{ij}} = b\alpha_{ij} \tag{15}$$

We can observe that the total stress is divided into two parts : a classical elasto-damage one and a sliding stress component. One can easily recognize a

classical elasto-damage coupling and a new term allowing the energy to shift from the elasto-damageable part to the frictional sliding part. The coupling between sliding and cracking is made possible thanks to the presence of the damage variable as a multiplier in the right-hand side of (14). Damage is classically controlled by the elasto-damage stress and the sliding strain is linked only to the sliding part of the stress. This kind of partitioning, in conjunction with the two yield functions, allows the description of a hysteretic behavior at a fixed level of damage. Such an approach could be compared to multi-surface modelling [43], except for the fact that the surfaces are not expressed in the same space (strain space for damage and stress space for sliding). The sliding and plastic strains being different, the thermodynamic forces associated with the total strain and the sliding strain are also different. Such a formulation differs greatly from the classical plasticity-damage coupling. This choice of introducing damage into the sliding stress is guided by the idea that all inelastic phenomena in concrete result from the cracks' growth. Such an assumption implies a limitation of this model in the case of hydrostatic compression, the response remains elastic

According to experimental investigations, the damage in brittle materials such as concrete is governed principally by their tensile behavior and much less pronounced in compression. To take into account this asymmetry, two different damage tensors must be introduced, one in tension and another in compression. The splitting between the tensile and the compressive damage tensors is achieved through the sign of the sliding strains expressed in their respective principal directions:

$$d_i = d_i^+ H^+(\epsilon_{ij}^s) + d_i^- H^-(\epsilon_{ij}^s) \tag{16}$$

where $H^+(\epsilon^s_{ij}) = \mathbf{P}^{-1}H(\lambda_{ij})\mathbf{P}$ and $H^-(\epsilon^s_{ij}) = \mathbf{P}^{-1}(\delta_{ij} - H(\lambda_{ij}))\mathbf{P} = \delta_{ij} - H^+(\epsilon^s_{ij})$, λ_{ij} is the tensor of sliding strain eigenvalues, $\mathbf{P}$ is the transformation matrix and $H(x)$ is the Heaviside function. The associated damage criterion is also expressed in the principal axis of the strain tensor. choosing for each direction i :

$$f_i = \epsilon_i^+ - \epsilon_{d0} - \kappa_i(\epsilon_i^+) \leq 0 \tag{17}$$

where $\kappa_i(\epsilon_i^+)$ is the hardening variable (historic) and ϵ_{d0} is the initial threshold. The evolution of damage variable is governed by the positive value of strains:

$$\dot{d}_i^+ = \left[\frac{\epsilon_{d0}}{\epsilon_i^{+2}}(1 + \epsilon_i^+ Bt)exp(Bt(\epsilon_{d0} - \epsilon_i^+))\right]\dot{\epsilon}_i^+ \tag{18}$$

where Bt is a material parameter driving the slope of the softening branch. This last equation was derived from empirical considerations. Such a formulation can not handle the behavior of material under strongly non-proportional loading. Compressive damage in a particular direction is considered proportional to the state of tensile cracking along the orthogonal directions and it is therefore taken equal to a function as follows:

$$d_i^- = \left(\frac{d_j^+ + d_k^+}{2}\right)^\beta \tag{19}$$

where β is a material parameter connecting the damaged Young's moduli for two orthogonal directions. Typically we obtain it by comparison of apparent Young's moduli in the longitudinal and radial directions.

The sliding part of the constitutive relation is assumed to be represented by a plasticity-like model. In order to better reproduce the observed hysteresis loops, nonlinear kinematic hardening is considered as the model initially in-

troduced by Armstrong and Frederick [44] and further developed by [45]. Such a model allows to overcome the major drawback of Prager's kinematic hardening law, i.e. the linearity of the state law defining the forces associated with kinematic hardening, by adding the nonlinear terms in the dissipative potential. In order to retain simplicity and adequacy of classical return mapping numerical algorithm for stress computation, the sliding criterion is given as:

$$f = J_2(\sigma^s_{ij} - X_{ij}) - \sigma_y \leq 0 \tag{20}$$

where $J_2(\sigma^s_{ij} - X_{ij})$ represents Von Mises' equivalent stress. The specific aspects of non-linear behavior of concrete have been introduced in the plastic flow potential. Namely, nonlinear kinematic hardening imposes the use of a non-associated flow rule:

$$\phi = J_2(\sigma^s_{ij} - X_{ij}) + \frac{3}{4} a X_{ij} X_{ij} + c I_1 - \sigma_y \tag{21}$$

a and c are material parameters. $I_1 = 1/3\sigma^s_{kk}$ is the first invariant of the sliding stress tensor and this choice enables us to take into account the dilatancy phenomenon observed experimentally on geomaterials such as concrete or rocks. The evolution laws of the internal variables are thus expressed as:

$$\dot{\epsilon^s}_{ij} = \dot{\lambda} \frac{\partial \phi}{\partial \sigma^s_{ij}} = \dot{\lambda} [\frac{3}{2J_2} (\sigma^s_{ij} - \frac{1}{3} \sigma^s_{kk} \delta_{ij} - X_{ij}) + \frac{1}{3} c \delta_{ij}] \tag{22}$$

and

$$\dot{\alpha}_{ij} = -\dot{\lambda} \frac{\partial \phi}{\partial X_{ij}} = -\dot{\lambda} [-\frac{3}{2J_2} (\sigma^s_{ij} - \frac{1}{3} \sigma^s_{kk} \delta_{ij} - X_{ij}) + \frac{3}{2} a X_{ij}] \tag{23}$$

where $\dot{\lambda}$ is the plastic multiplier.

The first analysis is performed at the material level and details concerning the numerical scheme allowing to solve the constitutive equations may be found in [46]. The Figure 5 represents the results of the numerical simulation of a uniaxial compression test. We can observe that the model is able to describe the volumetric response of the material satisfactorily.

The material parameters used for the analysis are : $E = 36000MPa$, $\nu = 0.24$, $\epsilon_{d0} = 1.10^{-4}$, $Bt_{induced} = 300$,$Bt_{direct} = 9000$, $a_{tension} = 5.10^{-6}Pa^{-1}$, $b_{tension} = 1.10^{10}Pa$, $a_{compression} = 39.10^{-8}Pa^{-1}$, $b_{compression} = 5.10^{10}Pa$, $c = 0.09$ and $\beta = 12$.

The two curves in Figure 6 show the ability of the model to describe the hysteresis loops under compression loading paths. The level of inelastic strains and hysteresis is directly linked to state of damage. The hysteretic dissipation capability of the model can be illustrated by plotting the absorbed energy of an unloading loop against the value of damage in Figure 6. We can easily appreciate the effect of the coupling between the state of damage and the sliding stress

4 Damage coupled to plasticity for high strain rate dynamics

Modelling of concrete structures subjected to dynamic loading such as explosions, impacts and perforations is very demanding, both from the point of view of computer codes and from the point of view of material modelling. It seems that constitutive modelling of concrete has not reached the same level

of sophistication as computer methods. Most approaches are inspired by existing constitutive relations for alloys and ceramics, where a split between the spherical and deviatoric parts of the stress-strain relations is assumed (see e.g. [47,48]). The deviatoric part is modelled with a plasticity-based, or viscoplastic model while the spherical part, also called "equation of state", is fitted with a cap model [49]. In the present paper, which is a sequel of recent research efforts carried out at LMT-Cachan [50], we present a constitutive relation for concrete in dynamics based on viscoplasticity combined with rate dependent continuum damage. This relation is restricted to cases with moderate strain rates in concrete, corresponding typically to explosions and impacts of projectiles at a velocity less than 350 m/s, inducing a hydrostatic pressure in the material which is less than 1 GPa.

4.1 Constitutive equations

Let us consider the case of an impact on a concrete slab and look at the various modes of deformation and failure in concrete. Near the striker, inertial forces in concrete are such that the material is confined. Concrete experiences a very high state of triaxial compression. In statics, the response of concrete for such a state of stress has been characterized experimentally [51,52]. The equation of state, (*e.g.* the curve relating the volumetric deformation to the hydrostatic stress), exhibits an increase of the tangent and decreasing volumetric moduli with residual plastic strains upon unloading. Recently Gary *et al.* [53] obtained similar results in dynamics and observed a substantial rate effect. The hydrostatic stress at a given volumetric strain increases with the strain rate. Although the physical mechanisms responsible for the rate effect are not

clearly explained at the moment (it might be due to intersticial water trapped in the porous microstructure of the material), it is generally accepted that the observed hardening phenomenon is due to irreversible material compaction at a given strain rate.

Farther away from the impactor, concrete is subjected to compression with a moderate amount of confinement. Its response is essentially controlled by microcracking in compression and internal friction. This state of stress can be captured by constitutive relations based on plasticity or plasticity coupled to damage [54,55]. Two characteristics of the material response ought to be recalled:

(1) The volumetric and deviatoric responses of the material cannot be separated.The coupling between the volumetric and deviatoric responses of concrete has been experimentally demonstrated by Burlion *et al.* [52] by comparing hydrostatic and uniaxial tests on confined specimens.
(2) In uniaxial compression, it has been demonstrated with three dimensional computations that rate effects are very small compared to the confinement generated by inertial forces which are opposite to the lateral expansion of the specimen during loading [56,9]. The increase of the material strength, which is classically observed (see e.g. Ref. [10]) can be almost totally explained (in the range of strain rates considered in this paper) by such a transient confinement.

The third failure mechanism is scabbing and occurs on the back of the impacted structure. It is due to the interaction between the compressive wave generated by the impactor and the free surface. The compression wave is transformed into a tensile wave which might produce tensile cracking if the

material strength is reached. This failure mechanism is practically rate dependent. Contrary to compression, inertial forces in uniaxial tension induce a three dimensional state of tensile stresses in the material and cannot contribute to an increase of apparent strength. The response of the material in tension can be captured with a rate dependent damage model [28,57] or a rate dependent smeared crack model [59].

An accurate description of the response of concrete subjected to impacts and explosions requires the combination of at least the three features above mentioned. Most constitutive relations implemented in existing computer codes are not entirely satisfactory because they do not account for the interaction between the deviatoric response of the material and its volumetric response and they do not combine the above three mechanisms. Bazant and co-workers have used the microplane approach for such a combination [60], but the rate effect on the equation of state is not considered in that model. Burlion *et al.* [50] proposed a coupled damage–plasticity model in which compaction and tensile cracking are combined into a single damage variable. Although the model remains quite simple, it is very delicate to obtain a good description of uniaxial compression especially since strain rate effects are not included.

The constitutive relations presented herein are an extension of this model, aimed at providing a more accurate description of the material response, at still a moderate degree of complexity. The proposed relations are based on three main characteristics:

- The variation of porosity of concrete, which remains lower than 30 %, is taken into account with the help of a homogenisation technique and, in particular, the method of Mori-Tanaka which provides explicit expressions

of the shear and bulk moduli of (uncracked) concrete. Thus, the elastic properties of material are functions of the variation of porosity, especially during hydrostatic loadings.

- Microcracking is captured with a rate dependent damage model [58], which uses two damage variables in order to provide a realistic response of the material in uniaxial compression while preserving a fairly good description of what occurs in tension, which is a characteristic of damage models. Rate effects are necessary in order to reproduce the results of dynamic experiments (mostly dynamic tensile tests). In addition, rate dependency preserves well posedness of the equations of motion when strain softening occurs [61].
- In order to describe the material response in triaxial compression, a viscoplastic model is implemented, based on Perzyna's approach associated with a modified Gurson's yield function [62]. Rate effects modification of plasticity model is needed to capture the increase of stress with strain rate in hydrostatic compression observed experimentally in dynamics [9].

These mechanical effects are combined in the relationships which relate the stresses to the elastic strains:

$$\sigma_{ij} = (1 - D)[K\varepsilon^e_{kk}\delta_{ij} + 2G(\varepsilon^e_{ij} - \frac{1}{3}\varepsilon^e_{kk}\delta_{ij})] \tag{24}$$

where the shear G and bulk moduli K are defined by Mori-Tanaka's expressions:

$$K = \frac{4K_M G_M (1-f^*)}{4G_M + 3K_M f^*} \tag{25}$$

$$G = \frac{G_M (1-f^*)}{1 + \frac{6K_M + 12G_M}{9K_M + 8G_M} f^*}$$

where K_M and G_M are the bulk and shear moduli of the material without pores respectively.

In this model, the damage is isotropic and represent by a scalar variable D. The damage growth is defined by:

$$D = \alpha_c D_c + \alpha_t D_t \tag{26}$$

where D_c and D_t represent the compressive and tensile damage respectively, while α_c and α_t are parameters defined as:

$$\begin{cases} \text{uniaxial tension} & \alpha_c = 0 \\ \text{uniaxial compression} & \alpha_t = 0 \\ \text{general loading} & \alpha_t + \alpha_c = 1 \end{cases} \tag{27}$$

The definition of these parameters is given in the original work of Mazars [57,63]. The growth of the two damage variables is governed by the elastic equivalent strain[63]:

$$\tilde{\varepsilon}^e = \sqrt{\sum_i \left(\langle \varepsilon_i^e \rangle^+\right)^2} \tag{28}$$

where ε_i^e is the ith component of the tensor of the principal strains and $\langle x \rangle^+$ is the positive part of x.

The growth of D_c and D_t is defined by the following equations which are similar to the equations used by Dube *et al.*[58]:

$$\dot{D}_c = \left(\frac{< \tilde{\varepsilon}^e - \varepsilon_{D0} - \frac{1}{a_c}\left(\frac{D_c}{1-D_c}\right)^{(1/b_c)} >}{m_{D_c}} \right)^{n_{D_c}} \tag{29}$$

and

$$\dot{D}_t = \left(\frac{< \tilde{\varepsilon}^e - \varepsilon_{D0} - \frac{1}{a_t}\left(\frac{D_t}{1-D_t}\right)^{(1/b_t)} >}{m_{D_t}} \right)^{n_{D_t}} \tag{30}$$

where m_{D_t}, n_{D_t}, m_{D_c} and n_{D_c} are material parameters which control the rate effect. a_c, a_t, b_c, b_t are material parameters which govern the growth of damage in quasistatic tension and compression and ε_{D0} is the initial threshold of damage.

The experiments carried out within the French research network GEO showed that there was a dependence of the loading rate on the curve relating the volumetric strain to the hydrostatic stress [9], in addition to permanent plastic strains and to the material compaction which induces an increase of the bulk and shear moduli of the material. It is thus pertinent to implement a viscoplastic model. Within the classical framework of (small strain) elasto-plasticity, we use the basic assumption of additive strain decomposition:

$$\dot{\varepsilon}_{ij} = \dot{\varepsilon}^e_{ij} + \dot{\varepsilon}^{vp}_{ij} \tag{31}$$

where $\dot{\varepsilon}_{ij}$ is the total strain rate, $\dot{\varepsilon}^e_{ij}$, the elastic one and $\dot{\varepsilon}^{vp}_{ij}$ the viscoplastic one.

The viscoplastic strains are obtained following Perzyna's approach:

$$\dot{\varepsilon}^{vp}_{ij} = \dot{\lambda}\frac{\partial F_{NT}}{\partial \sigma_{ij}} \tag{32}$$

F_{NT} is the modified Gurson's yield function proposed by Needleman and Tvergaard [62]:

$$F_{NT}(\sigma_{ij}, \sigma_M, f^*) = \frac{3J_2}{\sigma_M^2} + 2q_1 f^* \cosh\left(q_2 \frac{I_1}{2\sigma_M}\right) - (1 + (q_3 f^*)^2) = 0 \tag{33}$$

where σ_M is the stress in concrete without voids, f^* the porosity and q_1, q_2, q_3 are scalars parameters.

Colantonio and Stainier [64] proposed a similar model in which the definition of the plastic multiplier accounts for the variation of porosity of the material. We follow here the same approach and define the viscoplastic multiplier as :

$$\dot{\lambda} = \frac{f^*}{(1-f^*)} \left\langle \frac{F_{NT}}{m_{vp}} \right\rangle^{n_{vp}} \tag{34}$$

where m_{vp} and n_{vp} are material parameters. Just like in the model by Burlion [50], we assume that the evolution of the porosity is controlled by the irreversible volumetric strain:

$$\mathrm{d}f^* = k(1-f^*)f^* \mathrm{d}\varepsilon^p_{kk} \tag{35}$$

where a parameter k is introduced in equation 35 in order to be able to calibrate the velocity with which porosity is closed. Figure 7 shows a comparison of the hydrostatic responses between static and dynamic simulations and the corresponding experiments taken from Ref. [65].

Figure 8a shows the response of the damage model in uniaxial tension tests carried out at various strain rates. The response is strongly dependent on the strain rate, which is in agreement with experiments [65]. Figure 8b shows the response of the damage model in uniaxial compression. We can see that we chose to have a behaviour in compression which is only slightly dependent on the strain rate. This is again quite consistent with test data. It should be pointed out, however that the model response will be substantially modified in compression, due to the coupling with a viscoplastic model.

Figure 9a shows the response of the model for hydrostatic compression followed by hydrostatic tension. We observe a hardening behaviour when the porosity of the material decreases. We see also that during unloading, the modulus of elasticity of material is higher than at the beginning of the loading (this is due to the decrease of porosity). Finally, when tension is reached, the rate dependent damage model is recovered, with the elastic constants K and G which have increased due to compaction. A static loading and a dynamic loading were carried out in order to show the influence the strain rate on the model response.

With regard to the material behaviour in uniaxial compression, the values of the model parameters provide a behavior which is either "coupled" or not, in the sense that the viscoplastic criterion and the damage criterion can be activated at the same time or not. Figure 9b shows, in the case of a dynamic compression test (loading–unloading–reloading), a curve where the model parameters are such as in one case the damage criterion is activated only (no irreversible strains) and in the second case the two criteria are activated simultaneously. In the second case, a decrease of stiffness occurs at the same time that incremental viscoplastic strains are non zero. Obviously, the first case is much easier to handle as far as calibration is concerned because viscoplasticity is restricted to cases where the material is subjected to compression with a significant amount of confinement.

Overall, the constitutive relations contains two parameters which define the elastic behaviour (the shear and bulk moduli of concrete without voids), five parameters which control the material response in tension (including the damage threshold), four parameters which control damage growth in compression. Four parameters enter in the viscoplastic model and two enter in the equation

which governs the variation of porosity, including the initial material porosity (the quantities q_1, q_2, q_3 assume fixed values usually). The initial porosity depends on the concrete mix. It is usually in the range of 0.3. The total number of model parameters might be considered to be quite high, but in view of the three mechanisms described by the model and the various rate effects, it seems difficult to arrive at a significantly smaller number of parameters. Quasistatic experiments in tension, uniaxial compression and triaxial compression (such as uniaxially confined tests [52,50]) provide all the coefficients, except the viscoplastic parameters (see Burlion *et al.* [50]). The determination of these parameters is more intricate because it requires test data obtained for different strain rates. Such test data have been obtained for a single concrete mix, including scabbing tests, split Hopkinson tests on confined and unconfined specimens [65].

Given the complexity of the constitutive relations and their aim which is to model concrete in fast transient dynamics, the model has been implemented in the explicit finite element code LS-DYNA3D using the Euler forward integration scheme. The nonlinear response of the material is obtained by an explicit correction of the elastic prediction at each time steps [50]. It is well known that explicit time integration is not accurate if the time increment is too large (*i.e.* if the viscoplastic incremental strains, damage or incremental porosity are too high). It was observed that in practical cases where the three dimensional finite element mesh is sufficiently fine in order to achieve an accurate description of the irreversible phenomena in the structure, the critical time step due to the explicit integration of the equations of motion is so small that the explicit integration of the constitutive relations is accurate enough. Given the fact that both the constitutive relations and the equations of motion

are integrated explicitly, error accumulation might occur at these two levels. The balance between the internal, kinetic and dissipated energy is monitored in order to detect such situations.

5 Conclusions

As shown in this work, modeling mechanical response of concrete for dynamic loading inherits all the difficulties we have to deal with in statics, like substantial difference of inelastic response in tension and compression, the anisotropy of the response induced by complex cracking patterns and the need to account for irreversible deformation due to frictional sliding or non-closing cracks. On the top of that, in dynamics, we also have to take into account the eventual hardening or softening phenomena which explain a particular hysteretic response for a given cyclic loading as well as the strain rate effects as also shown herein. A great number of mechanism can be identified explaining different features of concrete behavior in dynamics and most of sufficiently versatile models will have a fair number of parameters to be identified. Although the mechanism interpretation and parameters identifications can be carried out at the micro level, the computation is in principle performed at the macro one. Nowadays it is still hard to imagine a complete analysis of civil engineering structures only based on a micro-scale description [66]. Such an approach of computational mechanics in nonlinear transient dynamics remains the most pragmatic one allowing to keep, as far as possible, the physical features of the micro-scale without neglecting the need of an efficient and robust numerical tool at the structural level.

6 Acknowledgment

The authors wish to thank Pr. A. Ibrahimbegovic, J. Mazars and G. Pijaudier-Cabot for their useful help in this work.

References

[1] Terrien M., "Emission acoustique et comportement mécanique post-critique d'un béton sollicité en traction", *Bull. Liais. du L.C.P.C.*, Vol. 105, réf. 23983, 1980.

[2] Hillerborg A., "The theoretical basis of a method to determine the fracture energy Gf of concrete", *Matériaux et Structures*, RILEM, Vol. 18, No 106, pp. 292-296, 1986.

[3] Hillerborg A., "Results of three comparative test series for determining the fracture energy Gf of concrete", *Matériaux et Structures*, RILEM, Vol. 18, No 107, pp. 407-413, 1986.

[4] Saouridis C., "Identification et numérisation objectives des comportements adoucissants : une approche multi-échelle de l'endommagement du béton", *Thèse de doctorat*, Univ. Paris VI, 1988.

[5] Van Mier J.G.M., "Strain softening of concrete under multiaxial loading conditions", *PhD Dissertation*, Eindhoven University of Technology, The Netherlands, 1984

[6] Torrenti J.M., "Comportement multiaxial du béton - aspects expérimentaux et modélisation", *Thèse de doctorat*, Ecole National des Ponts et Chaussées, 1987.

[7] Ladevèze P., "On an anisotropic damage theory", *Proc. CNRS Int. Coll.*, 351

Villars-de-Lans, Failure criteria of structured media, Ed. by J.P. Boehler, pp. 355-363, 1983.

[8] Ragueneau F., "Fonctionnement dynamique des structures en béton - Influence des comportements hystérétiques locaux", *Thèse de doctorat*, ENS de Cachan, France, 1999.

[9] Gatuingt F., "Prévision de la rupture des ouvrages en béton sollicités en dynamique rapide", *Thèse de doctorat*, ENS de Cachan, France, 1999.

[10] Bischoff P.H. and Perry S.H., "Compressive Behaviour of Concrete at High Strain Rates", *Materials and Structures*, **24**, pp. 425–450 ,1991.

[11] Rossi P., "Influence of cracking in the presence of free water on the mechanical behaviour of concrete", *Magazine of concrete research*, vol 43, 154, pp. 53–57, 1991.

[12] Toutlemonde F., "Résistance au choc des structures en béton", *Thèse de doctorat*, École National des Ponts et Chaussées", France, 1994.

[13] Bailly P., "Une modélisation d'un matériau fragile avec prise en compte d'effets dynamiques", *Compte Rendu Académie des Sciences, Série II*, pp. 727–732, 1994.

[14] Le Nard H. and Bailly, P., "Simulation de la compression dynamique de cylindre en béton", *Communication–Congrès GEO*, Aussois–France, 1996.

[15] Dubé J.F., Kanji Nanji A. and Wielgosz, "Comportement du béton en dynamique rapide", *J. Phys. IV, supplément au journal de physique III d'aot 1997*, France, 1997.

[16] Kotsovos M.D., "Effect on testing techniques on the post-ultimate behaviour of concrete in compression", *Materiaux et construction*, vol. 16, 91, 1983.

[17] Sercombe J., "Modélisation du comportement du béton en dynamique rapide", *Thèse de doctorat*, École National des Ponts et Chaussées, 1997.

[18] Denoual C., "Approche probabiliste du comportement l'impact du carbure de silicium: application aux blindages moyens", *Thèse de doctorat*, ENS de Cachan, 1998.

[19] Klepaczko J.R. and Brara A., "An experimental method for dynamic tensile testing of concrete by spalling", *International Journal of Impact Engineering*, Vol. 25, pp. 387-409, 2001.

[20] Camborde F. and Mariotti C., "Étude du comportement dynamique des matériaux fragiles par les éléments discrets", *14ème Congrès Franais de Mécanique–Toulouse'99*, CD-ROM, 1999.

[21] Kachanov M.A., " A microcrack model of rock inelasticity - Part I ; frictional sliding on microcracks", *Mech. Materials*, 1, pp 19-27, 1982.

[22] Andrieux S., Bamberger Y. and Marigo J.J., "Un modèle de matériau microfissuré pour les bétons et les roches", *Journal de Mécanique théorique et appliquée*, Vol. 5, No. 3, pp. 471-513, 1986.

[23] Halm D. and Dragon A., "An anisotropic model of damage and frictional sliding for brittle materials", *Eur. J. Mech. A/Solids*, No. 3, pp. 439-460, 1998.

[24] Pensée V. and Kondo D., "Une analyse micromécanique 3-D de l'endommagement par mésofissuration", *C.R. Acad. Sci. Paris*, t. 329, Série II b, pp. 271-276, 2001.

[25] Ortiz M. and Simo J.C., "An Analysis of a new class of integration algorithns for elastoplastic constitutive relations", *Int. J. Num. Meth. Eng.*, Vol 23, pp 353–366, 1986.

[26] Kachanov L.M., "Time of the rupture process under creep conditions", *Izv. Akad. Nauk. S.S.R.*, Otd. Tekh. Nauk., no. 8: 26-31, 1958.

[27] Lemaitre J. and Chaboche J.L., "Mechanics of solids material", Cambridge University Press, 1990.

[28] Simo J.C. and Ju J.W., "Strain and Stress Based Continuum Damage Models –I Formulation", *Int. J. Solids and Structures*, **23**, 821–840 , 1987.

[29] Ju J.W., "On energy-based coupled elastoplastic damage theories: constitutive modelling and computational aspects", *Int. J. Solids and Structures*, 25 (7), pp. 803-833, 1989.

[30] Meschke G., Lackner R. and Mang H.A., 'An anisotropic elastoplastic-damage model for plain concrete', Int. J. Num. Meth. Engng., 42, pp 703-727, 1998.

[31] Govindjee S., Kay G.J. and Simo J.C., 'Anisotropic modelling and numerical simulations of brittle damage in concrete', Int. J. Num. Meth. Engng., 38, pp 3611-3633, 1995.

[32] Murakami S. and Ohno N., "A constitutive equation of creep damage in polycrystalline metals", I.U.T.A.M. colloquium, Euromech 111, Marienbad, 1978.

[33] Cordebois J.P. and Sidoroff F., "Damage Induced Elastic Anisotropy", Colloque Euromech, 115, Villars de Lans: pp.761-774, 1979.

[34] Carol I., Prat P.C. and Bazant Z.P. , "New explicit microplane model for concrete: theoretical aspects and numerical implementation", *Int. J. Solids Structures*, **29**, No. 9, pp. 1173–1191 ,1992.

[35] Hill R., "Aspects of Invariance in Solids Mechanics", *Advances in Applied Mechanics*, 18, pp 1-75, 1978.

[36] Lubliner J., "A simple model of generalized plasticity.", *International journal of solids and structures*, vol. 28, no. 6, pp. 769, 1991.

[37] Perzyna P., "Fundamental problems in viscoplasticity", *Advances in applied mechanics*, vol. 9, pp. 935–950, 1966.

[38] Duvaut G. and Lions J.L., "Les Inéquations en Mécanique et en Physique", Dunod–Paris, 1972.

[39] Ibrahimbegovic A., Gharzeddine F. and Chorfi L., "Classical Plasticity and Viscoplasticity Models Reformulated: Theoretical Basis and Numerical Implementation." *International journal for numerical methods in eng.*, vol. 42, no. 8, pp. 1499, 1998.

[40] Ibrahimbegovic A. and Chorfi L., "Viscoplasticity model at finite deformation with combined isotropic and kinematic hardening", *Computer and Structures*, vol. 77, pp. 509-525, 2000.

[41] Bathe K.J., "Finite element procedures in engineering analysis" Prentice-Hall (eds). Inc. Englewood Cl.iffs: New Jersey, 1982.

[42] Queval. JC., Combescure D., Sollogoub P., Coin A. and Mazars J., "CAMUS experimental program. In-plane tests of 1/3 scaled R/C bearing walls", Proc. XIth ECEE-98, CD-ROM eds Bisch P., Labbé P. & Pecker A., Paris - CNIT La defense, 1998.

[43] Mroz Z., "On the description of anisotropic workhardening", *J. Mech. Phys. Solids*, 15, 163, 1967.

[44] Armstrong P.J. and Frederick C.O., "A Mathematical Representation of the Multiaxial Bauschinger Effect", G.E.G.B., Report RD/B/N, 731, 1966.

[45] Chaboche J.L., "Cyclic Viscoplastic Constitutive Equations, Part I: A Thermodynamically Consistent formulation", *J. Applied Mechanics*, Vol 60, pp. 813 - 821, 1993.

[46] Ragueneau F., La Borderie Ch. and Mazars J., "Damage Model for Concrete Like Materials Coupling Cracking and Friction, Contribution

towards Structural Damping: First Uniaxial Application", *Mechanics Cohesive Frictional Materials*, Vol. 5, pp 607-625, 2000.

[47] Whirley R.G. and Hallquist J.O.,"DYNA3D: A Nonlinear Explicit Three-Dimensional Finite Element Code for Solid and Structural Mechanics", User Manual, Report UCRL-MA-10254, Lawrence Livermore National Laboratory, Livermore, USA, 1991.

[48] Hallquist J.O.,"LS-DYNA 3D –Theoretical Manual", Livermore Software Technology Corporation, Livermore, USA, 1995.

[49] Lubarda V.A., Mastilovic S. and Knap J., "Brittle–Ductile transition in porous Rock by Cap Model", *J. of Engrg. Mech. ASCE*, **122**, 1996.

[50] Burlion N., Gatuingt F., Pijaudier-Cabot G. and Daudeville L., "Compaction and tensile damage in concrete: constitutive modelling and application to dynamics", *Computer Methods in Applied Mechanics and Engineering*, **183**, 291–308 2000.

[51] Bazant Z.P., Bishop F.C. and Chang T.P.,"Confined Compression tests of Cement Paste and Concrete up to 300 Ksi", *ACI Journal*, **33**, 553–560, 1986.

[52] Burlion N., Pijaudier-Cabot G., and Dahan N., "Experimental Analysis of Compaction of Concrete and Mortar", *International Journal for Numerical and Analytical Methods in Geomechanics*, vol. 25, no. 15, pp. 1467-1486, 2001

[53] Gary G., Bailly P. and Gatuingt F., "Testing concrete at high strains and high rates of strain", *Impact response of Materials and Structures, Proc. 3rd International Symposium on Impact Engineering'98, Singapore-Japan*, 241–246, 1998.

[54] Feenstra P.H., "Computational Aspects of Biaxial Sress in Plain and Reinforced Concrete", *Doct. Dissertation*, Delft University of Technology, The Netherlands, 1993.

[55] Yazdani S. and Schreyer H.L.,"Combined Plasticity and Damage Mechanics Model for Plain Concrete", *J. of Engrg. Mech. ASCE*, **116**, 1435–1450, 1990.

[56] Donze F., Magnier S.A., and Daudeville L., "Study of the Behavior of Concrete at High Strain Rate Compressions by a Discrete Element Method", *J. of Engrg. Mech. ASCE*, **125**, 1154–1163, 1999.

[57] Mazars J., *Application de la mecanique de l'endommagement au comportement non lineaire et a la rupture du beton de structure*, These de Doctorat d'Etat, Universite Paris 6, Paris, France, 1984.

[58] Dube J.F., Pijaudier-Cabot G., and La Borderie C., "A Rate Dependent Damage Model for Concrete in Dynamics", *J. of Engrg. Mech ASCE*, **122**, 939–947 (1996).

[59] Sluys L.J. and de Borst R., "Analysis of Impact Fracture in a Double –Notched Specimen Including Rate Effects", *Fracture Mechanics of Concrete Structures*, Z.P. Bazant Ed., Elsevier Pub., 610–615, 1992.

[60] Bazant Z.P., Carol I., Adley M. and Akers, "Microplane Model M4 for Concrete: I Formulation with Work-Conjugate Deviatoric Stress", *J. of Engrg. Mech. ASCE*, **126**, 944–954, 2000.

[61] Sluys L.J., *Wave Propagation, Localisation and Dispersion in Softening Solids*, Doct. Dissertation, Delft University of Technology, The Netherlands, 1992.

[62] Needleman A. and Tvergaard V., "An analysis of ductile rupture in notched bars", *Journal of Physics and mechanics of Solids*, **32**, 461–490, 1984.

[63] Mazars J., "A description of micro and macroscale damage of concrete structures", *Journal of Eng. Fract. Mechanics*, **25**, 729–737, 1986.

[64] Colantonio L. and Stainier L., "Numerical Integration of Visoplastic Constitutive Equations for Porous Materials", *Numerical Methods in*

Engineering96, J.-A. Desideri, P. Le Tallec, E. Onate, J. Periaux, and E. Stein, Wiley Pubs, 28–34, 1996.

[65] Bailly P., "Rapport final, première partie, expérimentation", *Comportement des ouvrages en dynamique rapide–Comportement dynamique du béton*, Reseau de laboratoire GEO, ENI Bourges, France 1999.

[66] Denoual C. and Hild F., "A damage model for the dynamic fragmentation of brittle solids", *Computer Methods in Applied Mechanics and Engineering*, **183**, (3/4), 247–258, 2000.

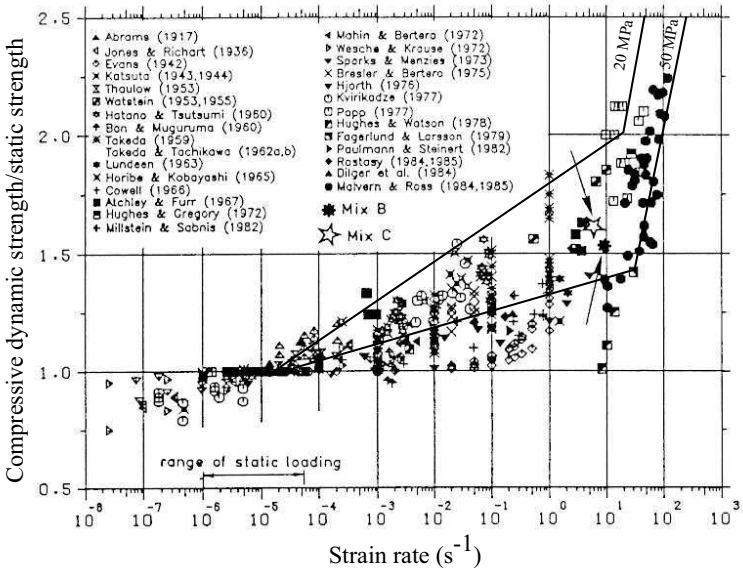

Fig. 1. Compressive strength increase versus strain rate (from [10])

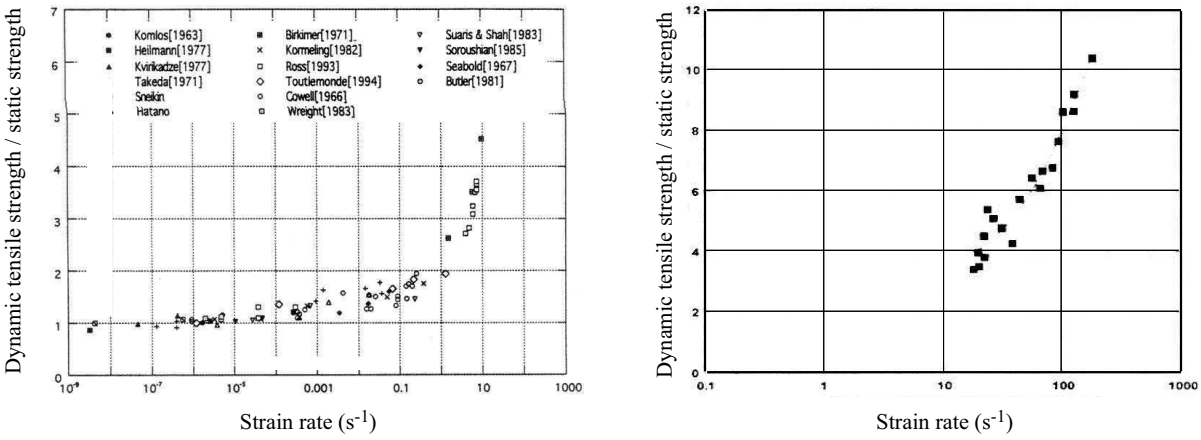

Fig. 2. Tensile strength increase versus strain rate (from [19])

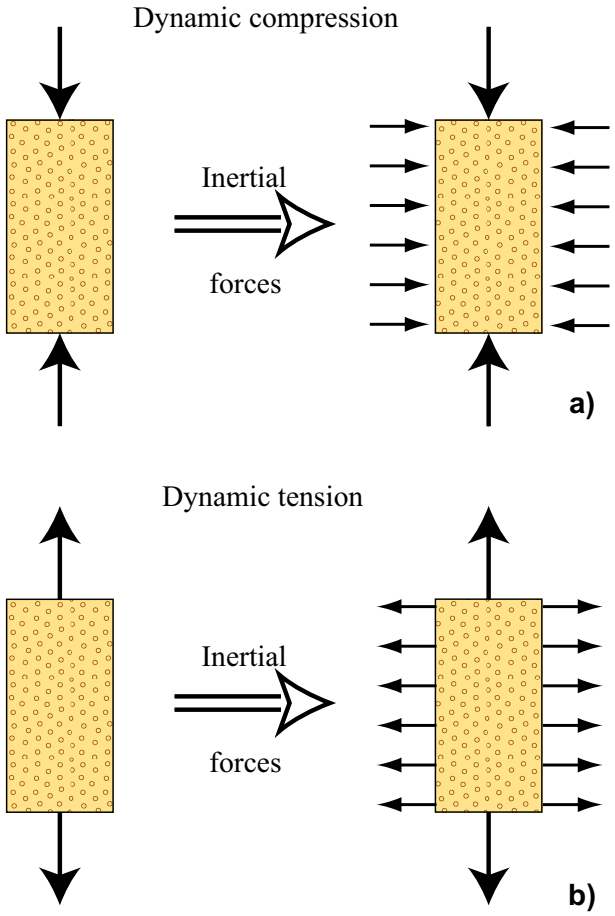

Fig. 3. Effect of inertial confinement in uniaxial compression (a) and uniaxial tension (b)

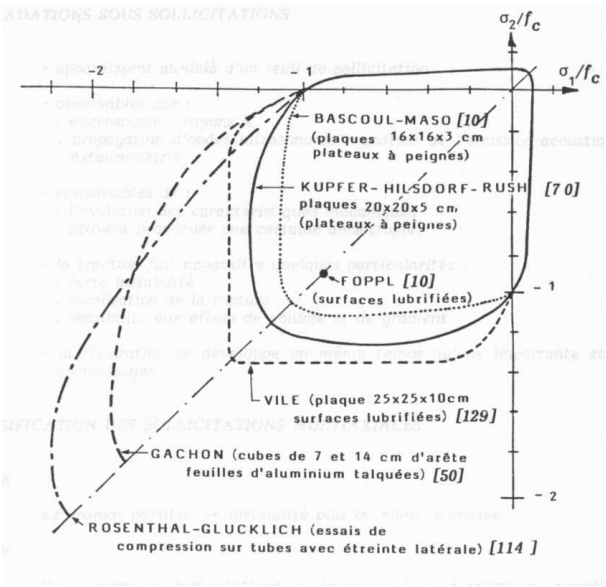

Fig. 4. Experimental rupture criteria for biaxiale loading (from [57])

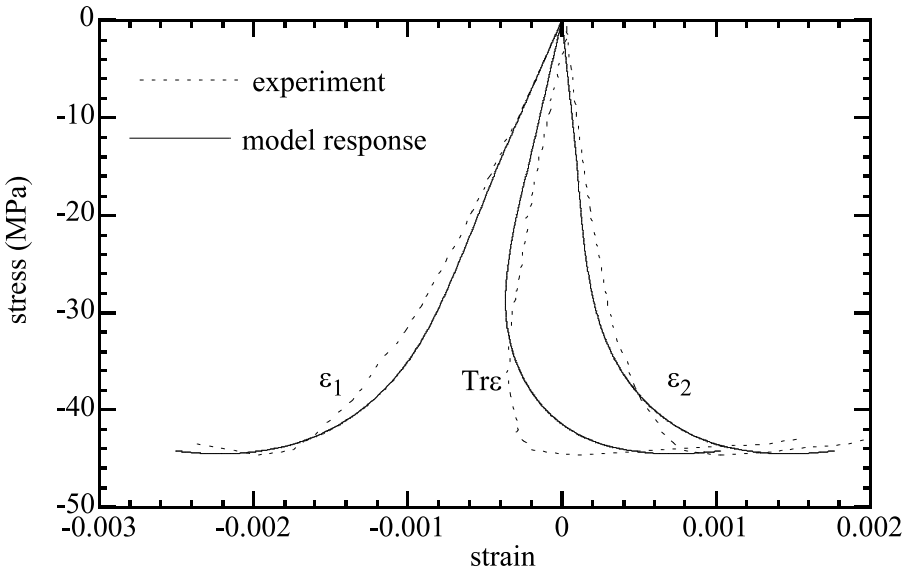

Fig. 5. Uniaxial compression test: 3D response

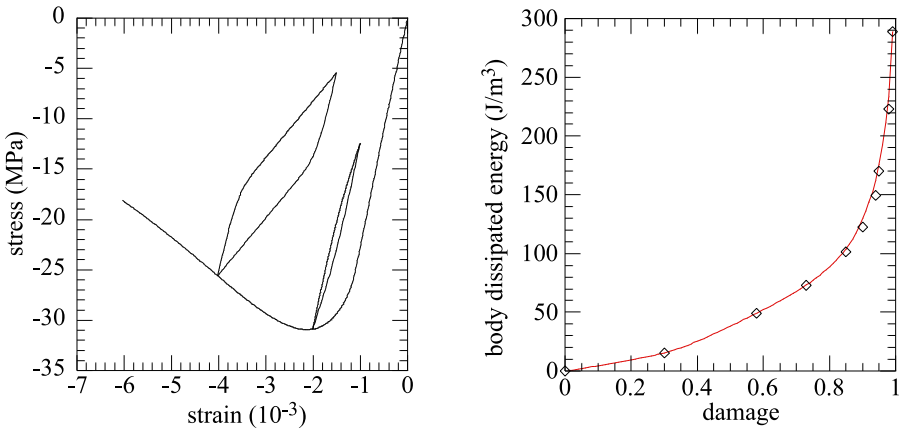

Fig. 6. Compression response: inelastic strains and loops of hysteresis coupled to damage. Dissipated energy versus Damage

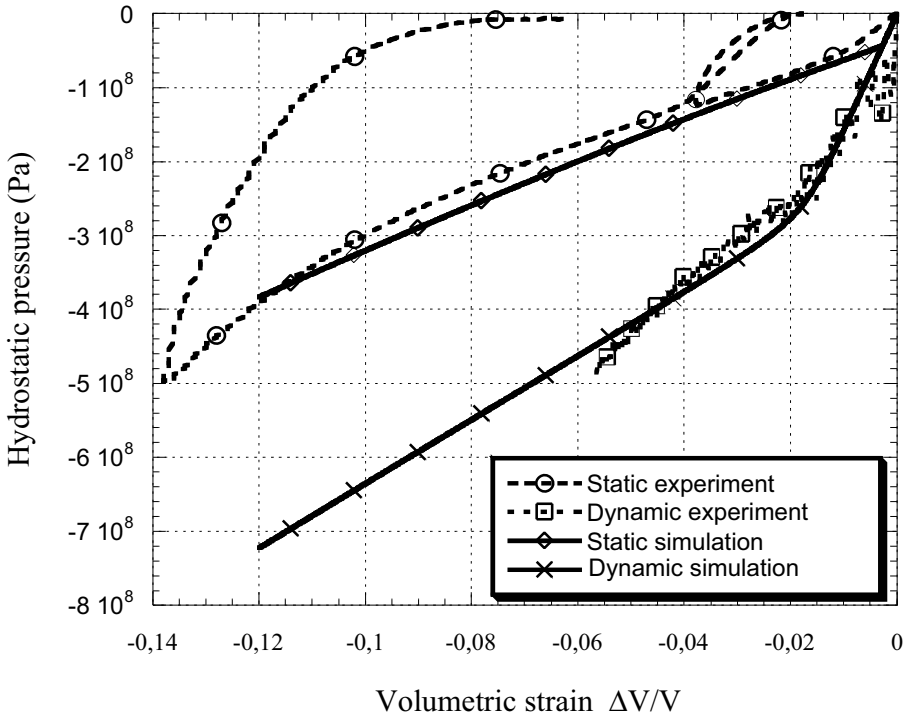

Fig. 7. Calibration of the parameter of viscosity on the hydrostatic response of the model.

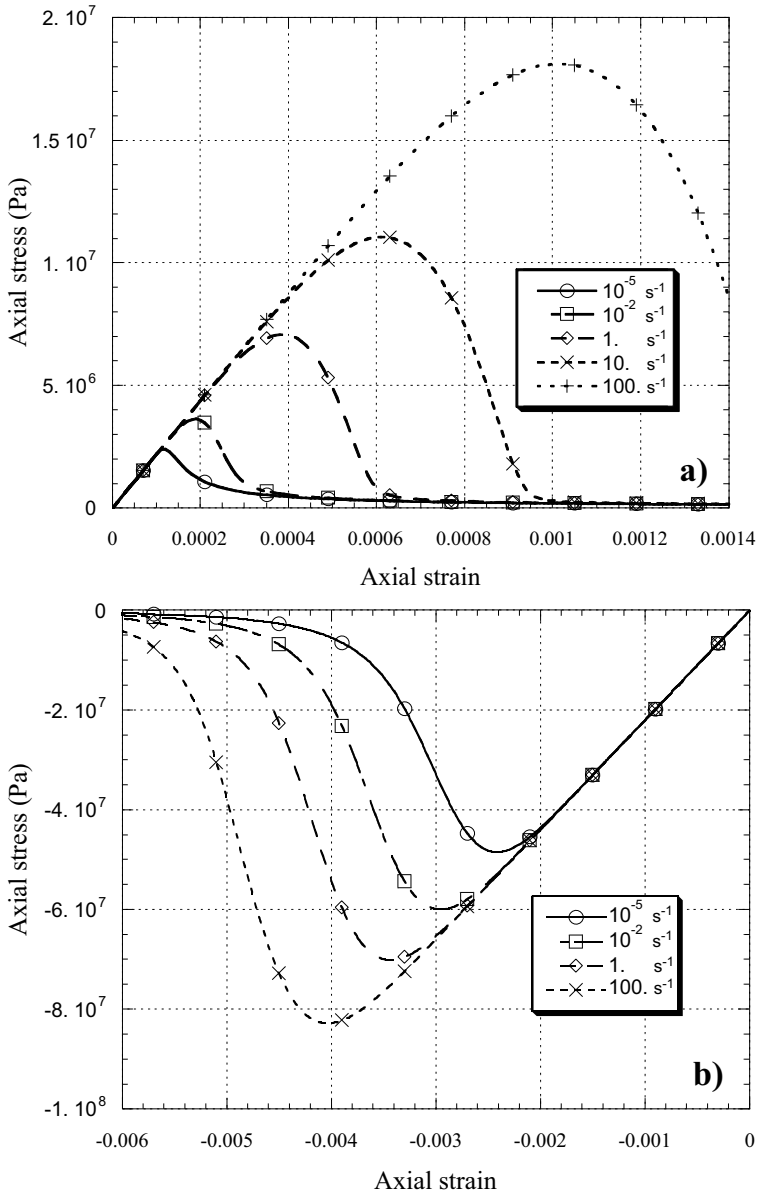

Fig. 8. Model response for several strain rate: a) uniaxial tension test, b) uniaxial compression test.

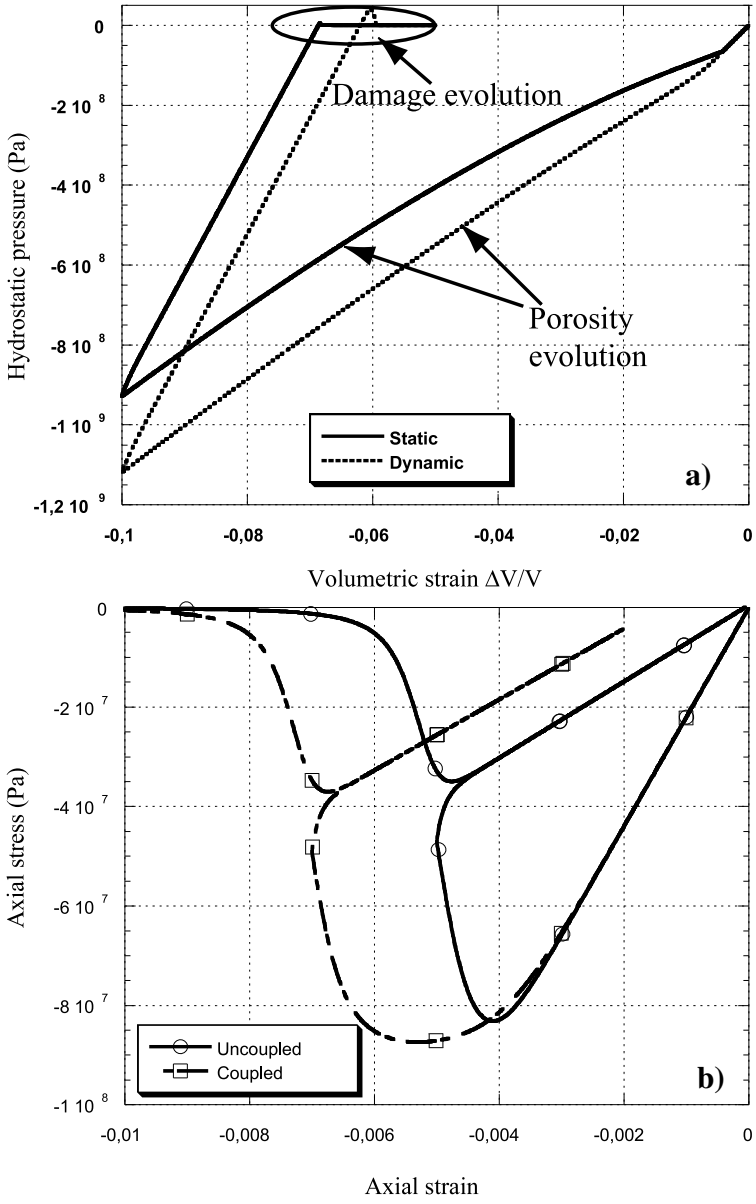

Fig. 9. Response of the model coupled in the case of: a) hydrostatic compression followed by hydrostatic tension for a static loading ($\dot{\varepsilon} = 10^{-5}\ s^{-1}$) and a dynamic loading ($\dot{\varepsilon} = 1\ s^{-1}$), b) uniaxial (coupled and uncoupled) compression.