Mathematical modeling of human behaviors during catastrophic events.

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MATHEMATICAL MODELING OF HUMAN BEHAVIORS DURING CATASTROPHIC EVENTS.

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Abstract. In this paper, we introduce a new approach for modeling the human collective behaviors in the specific scenario of a sudden catastrophe, this catastrophe can be natural (i.e. earthquake, tsunami) or technological (nuclear event). The novelty of our work is to propose a mathematical model taking into account different concurrent behaviors in such situation and to include the processes of transition from one behavior to the other during the event. Thus, in this multidisciplinary research included mathematicians, computer scientists and geographers, we take into account the psychological reactions of the population in situations of disasters, and study their propagation mode. We propose a SIR-based model, where three types of collective reactions occur in catastrophe situations: reflex, panic and controlled behaviors. Moreover, we suppose that the interactions among these classes of population can be realized through imitation and emotional contagion processes. Some simulations will attest the relevance of the proposed model.
Keywords. Modeling, catastrophic event, human behavior

1 Introduction

Nowadays, management of disasters has become a major issue, due to their huge financial and human costs. In fact, our societies, independently from their development level, are still not sufficiently prepared to a natural or anthropic catastrophe and to possible domino effects. However, there is an increasing trend concerning the number of disasters, ranking from a hundred in 1960 to over 800 in 2000, and the trend is not expected to be inverted in the future, due to the population growth and densification in the risk zones [1].

A fundamental lever for reducing human vulnerability in the face of such events is definitely the population training, in order to adapt their behaviors to extreme situations. In fact, during several catastrophic events, controlled and uncontrolled behaviors in either individuals, small groups or crowds have been observed. These reactions depend not only on the event and its temporality (nature, unexpectedness, presence of alert), but also on the population characteristics (density, composition, preparation level) [5].

In this paper, our aim is to model the collective behaviors that take place in a crowd during a catastrophe, in order to better apprehend and handle the collective reactions. In the literature, several models at different scales have been proposed for modeling crowd dynamics, also in extreme scenarios as in a panic situation [3]. At microscopic level we encounter cellular automata or agent based models [14], where each individual of the population is modeled as single entity. Moreover, especially for the study of pedestrian flows, some microscopic models consider the pedestrians as particles subject to a mixture of socio-psychologically and physical forces [11]. This approach permits to take into account the heterogeneity of the population but this means also high computational requirements and sometimes a difficulty in transferring the microscopic properties at a macroscopic level [16]. At macroscopic level, the models of crowd dynamics consist in partial differential equations that describe the evolution in time and space of the density and mean velocity of the crowd flow. In particular, interactions of crowds and structures in panic situations have been considered [2, 9]. Finally, at mesoscopic level, between the microscopic and the macroscopic ones, we have the models that exploit the approach of the kinetic theory, through the Boltzmann or Vlasov equations, depending on the different range of interactions [3].

However, these mathematical models consider only the panic reaction, and do not take into account what is well-known by now in human sciences [7, 15]: in a catastrophe, the population can exhibit different concurrent reactions, not only the panic one, and each individual does not keep the

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same behavior during all the event. It is what we propose to do in this paper in exploiting the potential of SIR models [13] that are widely used in epidemics. Indeed, in these models, one can decompose the population in several subpopulations categorized in compartments. Furthermore, different types of transition between these compartments can be easily considered.
In this paper, we omit the spatial dynamics in order to focus first of all on the different psychological behaviors in a crowd, and the processes of transition from one behavior to an other during a disaster.

The collective behaviors that have been observed in the impact and in the destruction zone [5] can be classified in two main categories:

- the instinctive behaviors, managed by the reptilian zone of the brain, that handle with the impulsive and urged behaviors [12]

- the controlled behaviors, where the pre-frontal cortex adapts in a more reflexive way the reactions to an external perturbation [8].

In particular, in the first group we have all the behaviors of instinctive escape and fight, the panic, but also the behaviors as a sort of automaton [17], while in the second one we have all the persons that keep calm and self-control.

In our case, we have subdivided the population in situation of catastrophe in three groups corresponding to the three following collective reactions: reflex behaviors except the panic one, panic and controlled behaviors. Indeed, according to the specific status of the panic, this reaction has been differentiated from the others reflex behaviors. Moreover, according to [10], we suppose that the interactions among these classes of population can be realized through imitation and emotional contagion processes. In fact, it is well-known that in a crowd the perception of an emotional state causes in an observer an automatic imitation of this expression.

The paper is structured as follows. In part 2, our choice of the thee groups of compartmental reactions is discussed and from this discussion, the mathematical model is deduced. In part 3, the available data present in the literature are presented and our strategy for calibrating the model is given. In part 4, numerical simulations attest the relevance of our suggested approach.

2 The mathematical model

2.1 Choice of three groups of compartmental reactions

In this paper, we consider the human behaviors in the impact zone of a catastrophe, with a fast dynamic and no alert to the population. We suppose that the effect of surprise is total and there are no precursor signs or warnings that allow the population to adopt preventive behaviors. To give an example, there may be an earthquake or a local tsunami. We have distinguished three different types of behaviors in such situation.

The first type consists in the reflex behaviors and concerns the reptilian brain. In our case, it corresponds to the set of instinctive behaviors except panic. This mechanism permits to react quickly, either by running away as fast as possible or by being flabbergasted and being physically unable to move in space. It can take the form of sideration and automate behaviors for example [5]. In our model, we have decided to globalize all these reflex behaviors, despite their diversity.

The second one corresponds the panic behavior. Panic has a particular status since, even if it is not always adopted (as, for example, during an earthquake in prepared regions as Japan), this behavior is the most feared. Indeed, this mechanism is difficult to stop once started [6] and can provoke dangerous situations in a crowd, due to trampling and crushing. Moreover, the extinction of collective panic is more linked to internal dynamics than to the remoteness of the danger [5]. Thus, even if it belongs to reflex behaviors, we consider it apart due to its particular nature. Furthermore, in our model, the collective panic can propagate via imitation and contagion mechanisms [10].

Finally, the third type includes all the controlled behaviors. They are governed by the prefrontal cortex, which takes over the reptilian brain. Thus, reflex reactions are substituted by controlled, intelligent and reasoned reactions. They can take different forms in a catastrophe, as, for example, evacuation, leak, containment, sheltering, research for help, pillage, theft... As for the first type, we have decided to globalize all these controlled behaviors, despite their variety.

It is worth noting that the three previous behaviors do not all occur at the same time and respect a certain order. Indeed, the first behavior of an individual in the face of danger is a reflex one followed, in a second step, by controlled or panic behavior [8].

2.2 Formalization of the human behavior

In this paper, we propose a SIR-based mathematical model composed of four classes, one constitutes daily behaviors, and the three others correspond to the three previous behaviors described at Section 2.1. Thus, first of all, we suppose to have a class named Q composed of individuals in a daily behavior and that, during the event, no death nor birth takes place. Hence, globally the population is constant and composed by N individuals. Moreover, during the catastrophe, Q is the sum of two subpopulations:

- $Q_1(t)$: it designs the number of individuals with routine behaviors. Clearly, just before the catastrophic event occurs, all the population is in this state, therefore $Q_1(0) = N$,

- $Q_2(t)$: it designs the number of individuals who come back to normal lifestyle after the outbreak of the disaster. We expect that at the end of the event, all the individuals will be in this state, thus $Q_2(t_{end}) = N$.

According to Section 2.1, the population during the catastrophe is decomposed into 3 subpopulations who are represented by the following variables:

- $x(t)$ = number of persons with reflex behaviors,

- $y(t)$ = number of persons with controlled behaviors,

- $z(t)$ = number of persons with panic behaviors.

Since we suppose to be in presence of a sudden and unpredictable event, all the involved population will have firstly a reflex reaction, corresponding to instinctive comportments. Thus, the routine behaviors, represented here with the variable $Q_1(t)$, can only be transformed in reflex behaviors, that is in $x(t)$. Hereafter, reflex behaviors can become controlled or panic behaviors. Since $Q_2(t)$ represents the number of individuals who come back to normal lifestyle, it can be alimented only by the controlled behaviors $y(t)$. In fact, an individual needs to recover self-control in order to regain the everyday routine. Moreover, we suppose that, once they have come back to normality, they maintain their habitual behaviors. Thus, the individuals in Q_2 cannot pass in Q_1 and re-enter in the loop.

Furthermore, according to our psychological and geographical references [7, 10, 6, 4], during catastrophic events we have interactions and transitions between the different behaviors, as represented in Figure 1. The exterior event, that is the catastro-

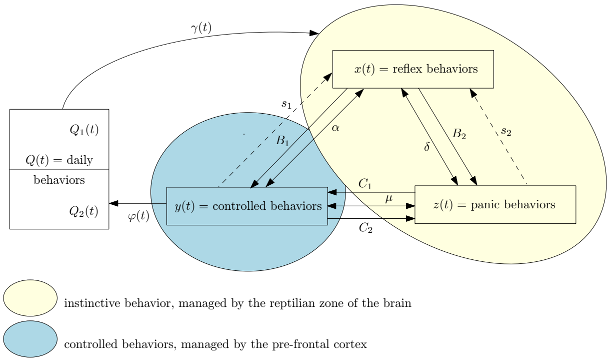

Figure 1: Graphic modeling of three types of human behaviors in context of disasters

phe onset, is represented by a forcing function γ which can be discrete or continuous, depending on the type of the event under study. For example, an event such as a local tsunami can be modeled by a discrete function, whereas an inundation can be modeled by a continuous function since it can be announced fewer hours before its start. In our case, we suppose to be in the first situation and the brutality and the speed of the catastrophic event is modeled through a logistic function.

For taking into account the possible continuation or repetition of the catastrophe perception stress, the arrows labeled s_1 and s_2 are added, where s_1 and s_2 are supposed to be constant parameters. Once the population is in a reflex behavior, they can evolve in a controlled or panic one according to the parameters B_1 and B_2, respectively. In the same way, a part of the controlled population can evolve to a panic behavior and reciprocally according to the coefficients C_1 and C_2, respectively.
All the previous transitions are causality links. However, some processes of imitation and contagion exist and are modeled by the arrows labeled α, μ, δ. In the graphic, α transposes the process of imitation between x and y which is realized in both directions. This process is modeled as an epidemiological propagation and has the following form: $\alpha \cdot f_1(x(t)) \cdot y(t)$. This modeling permits to favor the imitation in one direction, in particular from x to y. Indeed, in our numerical tests, we have assumed that there must be at least 55% of reflex behaviors for that controlled individual behaviors imitate reflex ones.
In the same manner, the constant δ traduces the imitation processes between x and z and is mod-

eled by the function $\delta \cdot f_2(x(t)) \cdot z(t)$. Finally, the constant μ traduces the imitation processes between controlled and panic individuals behavior, knowing that the imitation is essentially in the sens panic towards controlled individuals behavior. It is modeled by the term $\mu \cdot g(y(t)) \cdot z(t)$.
From the graphical modeling in Figure 1, the mathematical model is deduced:

$$\left\{\begin{array}{l} \dfrac{dx}{dt} = \gamma(t)Q_1(t)\left(1-\dfrac{x(t)}{x_m}\right) - (B_1+B_2)x(t) \\ +\alpha f_1(x(t))y(t) + \delta f_2(x(t))z(t) + s_1y(t) + s_2z(t), \\ \dfrac{dy}{dt} = B_1x(t) - \alpha f_1(x(t))y(t) + C_1z(t) - s_1y(t) \\ \quad -C_2y(t) - \varphi(t)y(t)\left(1-\dfrac{Q_2(t)}{Q_2m}\right) + \mu g(y(t))z, \\ \dfrac{dz}{dt} = B_2x(t) - s_2z(t) - \delta f_2(x(t))z(t) - C_1z(t) \\ \quad +C_2y(t) - \mu g(y)z, \\ \dfrac{dQ_1}{dt} = -\gamma(t)Q_1(t)\left(1-\dfrac{x(t)}{x_m}\right), \\ \dfrac{dQ_2}{dt} = \varphi(t)y(t)\left(1-\dfrac{Q_2(t)}{Q_{2m}}\right) \end{array}\right. \tag{1}$$

Since the concerned population is supposed to be constant, that is the equality $Q_1(t)+Q_2(t)+x(t)+y(t)+z(t)=N$ for all $t \in [0,T]$ is verified, system (1) can be reduced to four equations and rewritten as:

$$\left\{\begin{array}{l} \dfrac{dx}{dt} = \gamma(t)Q_1(t)\left(1-\dfrac{x(t)}{x_m}\right) - (B_1+B_2)x(t) \\ +\alpha f_1(x(t))y(t) + \delta f_2(x(t))z(t) + s_1y(t) + s_2z(t), \\ \dfrac{dy}{dt} = B_1x(t) - \alpha f_1(x(t))y(t) + C_1z(t) - s_1y(t) \\ \quad -C_2y(t) - \varphi(t)y(t)\left(1-\dfrac{N-Q_1(t)-x(t)-y(t)-z(t)}{Q_2m}\right) + \mu g(y(t))z(t), \\ \dfrac{dz}{dt} = B_2x(t) - s_2z(t) - \delta f_2(x(t))z(t) - C_1z(t) \\ \quad +C_2y(t) - \mu g(y)z(t), \\ \dfrac{dQ_1}{dt} = -\gamma(t)Q_1(t)\left(1-\dfrac{x(t)}{x_m}\right). \end{array}\right. \tag{2}$$

3 Calibration of the model

Unfortunately, in the literature, the available data to calibrate the model are scarce. However, one can distinguish two groups of quantitative data. The first one concerns the percentages of the population adopting a certain type of behavior and the second one relates on the duration of such behaviors.

3.1 The percentages of population adopting a certain type of behavior

The different types of human behaviors described previously can manifest in variable proportions, in function of the considered catastrophe, the suddenness of the threat, the composition of the group, the individual aptitudes for understanding the danger and the knowledge of the environment. Moreover, [4] considers that in most of the catastrophes, "15% of individuals manifest obvious pathological reactions, 15% keep their cool and 70% manifest an apparently calm behavior but answer in fact to a certain degree of emotional sideration and lost of initiative which reports to a pathological register". These percentage have to be modulated according to the different parameters of our model, which leads us to consider:

- $x(t) = 50$ to 75% of the population
- $y(t) = 12$ to 25% of the population
- $z(t) = 12$ to 25% of the population

At our knowledge, no data are available for quantifying transition mechanisms from one state to an other.

3.2 The duration of the behavior

The three different reactions have different duration [17]. The duration of the reflex and panic behaviors varies from few minutes to one hour. Most of the time, these two types of behavior do not exceed 15 minutes. However, for the first one, it may take longer especially if it corresponds to a delay of evacuation in a disaster area. In this case, support and research behaviors for relatives and victims gradually appears [5]. For the second one, the collective panic behaviors resolves generally spontaneously. However, sometimes, an external intervention permits to the panic population $z(t)$ to come back to an automate behavior $x(t)$, before adopting a controlled behavior $y(t)$.
In general, the duration of the uncontrolled behavior $x(t)+z(t)$ does not last more than 1h30. In this model, we suppose that an individual cannot stay 1 hour in a reflex behavior and another hour in a panic state. The duration of the controlled behavior $y(t)$ varies from few minutes to fewer hours,

according to the intervention of the emergency response.
The choice of the parameters will be done in order to find these data.

4 Numerical examples

For the numerical simulations, we have transformed the model in a dimensionless form, that is, population numbers correspond to fractions of the total population. In the following subsection, the functions intervening in the dimensionless model are specified.

4.1 The functions γ, φ, f_1, f_2, f_3

In the case of a sudden catastrophe, as a local tsunami, modeled by the function γ, we suppose that the population begins to be rapidly informed that is to say after 1 minute, and that all the concerned population is informed in the 3 following minutes, hence, the shape of the function γ in Figure 2. Clearly, the return to the normality, corresponding to the function φ, can not be immediate. We have supposed that it is done after 5 minutes from the outbreak and this return is done very slowly, which leads us to consider the function φ as in Figure 2. As we have said before, the form of the curves has to be modulated according to the type of catastrophe event (depending if it is announced or not).

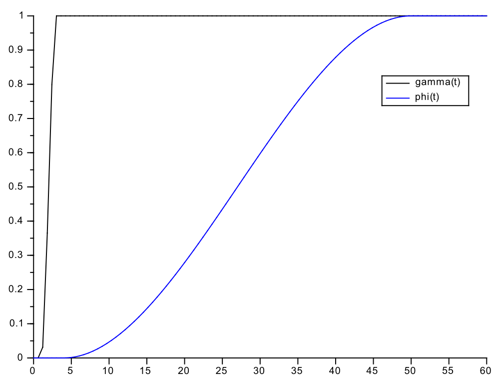

Figure 2: Functions φ and γ

The terms $\alpha f_1(x(t))y(t)$ and $\delta f_2(x(t))z(t)$ are the terms of imitation between $x(t)$ and $y(t)$, and $x(t)$ and $z(t)$, respectively. We promote the imitation from $x(t)$ to $y(t)$ in assuming that there must be at least 55% of reflex behaviors for that controlled individuals imitate reflex behavior (see Figure 3).

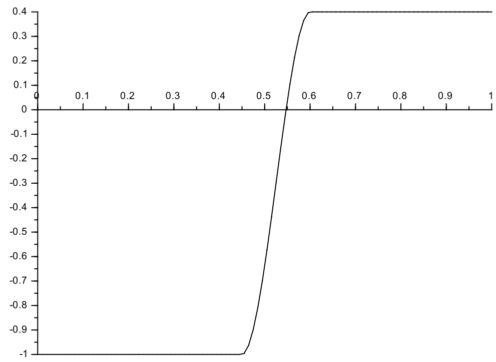

Figure 3: Function f_1 and f_2

For the imitation term $\mu g(y(t))z$, we suppose that the imitation is essentially in the sense from $z(t)$ to $y(t)$ (see Figure 4).

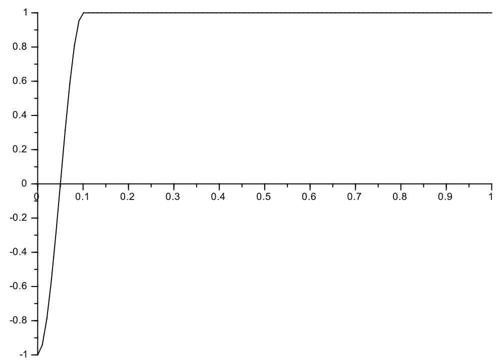

Figure 4: Function g

Functions f_1, f_2, g, φ and γ are modelised through the function

$$h(s) = \begin{cases} h_{min} \text{ for } s < s_{min} \\ h_{max} \text{ for } s > s_{max} \\ -\dfrac{h_{max} - h_{min}}{2} \cos\left(\dfrac{s - s_{min}}{s_{max} - s_{min}}\pi\right) \\ +\dfrac{h_{min} + h_{max}}{2} \text{ else} \end{cases} \tag{3}$$

where $[s_{min}, s_{max}]$ is the support of the function h and h_{min} (resp. h_{max}) is its minimum value (resp. maximum value).

4.2 Numerical simulations

Different scenarios were made and correspond to different values of parameters.

4.2.1 Simulation 1

The first one corresponds to Figure 5. The chosen values of the parameters permit to find the calibration data presented at Section 3. The areas between each curve and the horizontal axis gives the global percentage of the corresponding population. The global percentage of reflex behaviors ($x(t)$) equal to 61,41% is included between 50 and 75% otherwise the global percentages of panics and controlled behaviors respectively equal to 18,42% and 20,17% are included between 12% and 25%. Furthermore, the model gives the evolution of these global behaviors distributions.

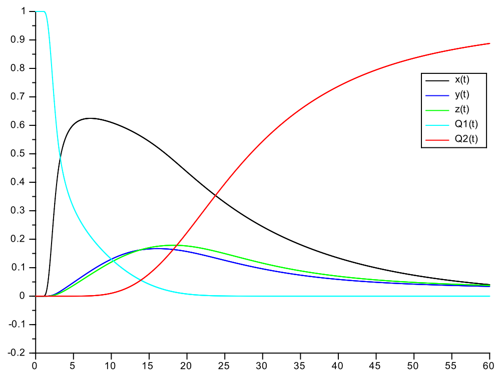

Figure 5: Simulation of the model with the parameters values: $x_m = 0.75$, $Q_{2m} = 1$, $B_1 = 0.04$, $B_2 = 0.02$, $s_1 = 0.01$, $s_2 = 0.01$, $C_1 = 0.5$, $C_2 = 0.5$, $\alpha = \delta = \mu = 0.01$.

4.2.2 Simulation 2

In this section, we are interested in the evolution of the model when the parameter B_2 varies, in particular, the possibility to evolve from reflex to panic behaviors. The numerical simulations at Figure 6 shows that for a low value of B_2, the density of population having a reflex behavior remains important during all the simulation. However, for a high value of this parameter (Figure 7), the density of this population decreases to extinguish rapidly. Furthermore, the densities of panic and controlled populations grow significantly between Figure 6 and 7.

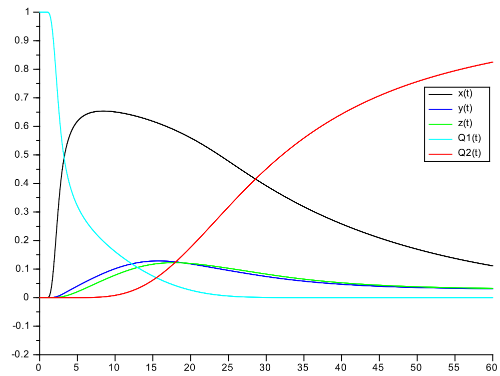

Figure 6: Simulation of the model with the parameters values: $x_m = 0.75$, $Q_{2m} = 1$, $B_1 = 0.04$, $B_2 = 0.002$, $s_1 = 0.01$, $s_2 = 0.01$, $C_1 = 0.5$, $C_2 = 0.5$, $\alpha = \delta = \mu = 0.01$.

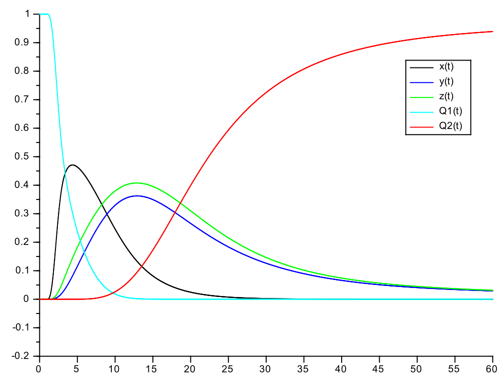

Figure 7: Simulation of the model with the parameters values: $x_m = 0.75$, $Q_{2m} = 1$, $B_1 = 0.04$, $B_2 = 0.2$, $s_1 = 0.01$, $s_2 = 0.01$, $C_1 = 0.5$, $C_2 = 0.5$, $\alpha = \delta = \mu = 0.01$.

4.2.3 Simulation 3

In the following simulations, we force the emergence of panic and controlled behaviors in acting on the parameters B_1, B_2, C_1 and C_2. Figure 8 shows that the return from controlled to daily behavior can be furthered, while Figure 9 induces a high proportion of panic behaviors and a return more difficult to normality as remarked by [6].

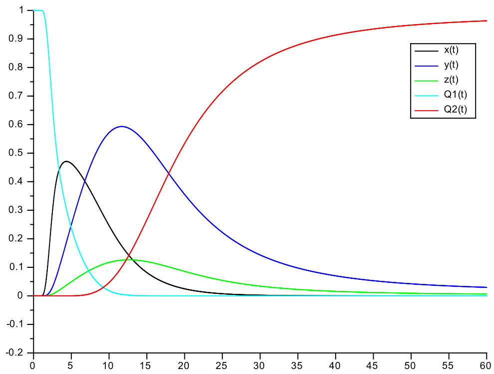

Figure 8: Simulation of the model with the parameters values: $x_m = 0.75$, $Q_{2m} = 1$, $B_1 = 0.2$, $B_2 = 0.04$, $s_1 = 0.01$, $s_2 = 0.01$, $C_1 = 0.5$, $C_2 = 0.1$, $\alpha = \delta = \mu = 0.01$.

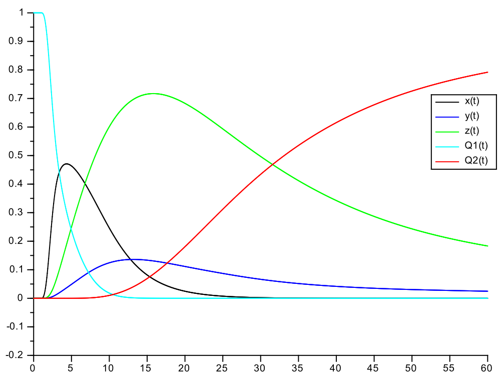

Figure 9: Simulation of the model with the parameters values: $x_m = 0.75$, $Q_{2m} = 1$, $B_1 = 0.04$, $B_2 = 0.2$, $s_1 = 0.01$, $s_2 = 0.01$, $C_1 = 0.1$, $C_2 = 0.5$, $\alpha = \delta = \mu = 0.01$.

5 Conclusion

This paper introduces a new step in the modeling of the crowd dynamics in catastrophic events. Indeed, it considers three concurrent behaviors and includes the processes of transition from one behavior to the other. Up to now, the main models consist in modeling the panic which is a fear behavior but it is not always adopted. Furthermore, panic does not necessarily lasts during all the event and, on the contrary, the global behavior of the crowd can change. In this work, two other behaviors have been integrated in the modeling: the reflex one and the controlled one. As seen in human sciences, our simulations show that they can influence the crowd behavior and a return to normality. The next step of this work will consist in doing a mathematical study of this model and integrating it in a diffusion process.

References

[1] D. Provitolo A. Dauphiné. *Risques et catastrophes. Observer, spatialiser, comprendre, gérer.* Paris : Armand Colin, 2013.

[2] J. Venel B. Maury. A mathematical framework for a crowd motion model. *C.R. Acad. Sci. Paris*, 346:1245–1250, 2008.

[3] N. Bellomo and C. Dogbé. On the modeling of traffic and crowds: A survey of models, speculations, and perspectives. *SIAM review*, 53(3):409–463, 2011.

[4] S.T. Boyd. Réactions psychologiques des victimes de catastrophes. *South african Medicine Jal*, 60:744–748, 1981.

[5] L. Crocq. *La psychologie des catastrophes et les blessés psychiques.* Noto R. Médecine de catastrophe, Paris, Milan, Barcelone, Masson, 1994.

[6] L. Crocq. *Les paniques collectives.* Paris : Odile Jacob, 2013.

[7] E. Dubos-Paillard D. Provitolo and J.P. Müller. Emergent human behaviour during a disaster: thematic versus complex systems approaches. In *EPNACS 2011 within European Conference of Complex System*, Septembre 2011.

[8] N. George and L. Gamond. Premières impressions. *L'essentiel Cerveaux et Psycho: Les émotions au pouvoir Science*, 7, 2011.

[9] P. Goatin, R.M. Colombo, and M.D. Rosini. A macroscopic model for pedestrian flows in panic situations. In *GAKUTO International Series Mathematical Sciences and Applications*, volume 32, pages 255–272, 2010.

[10] E. Hatfield, J.T. Cacioppo, and R.L.Rapson. *Emotional Contagion.* Cambridge, Cambridge University Press, 1994.

[11] D. Helbing, I. Farkas, and T. Vicsek. Simulating dynamical features of escape panic. *Nature*, 407(6803):487–490, 2000.

[12] H. Laborit. *La légende des comportements.* Paris: Flammarion, 1994.

[13] James D Murray. *Mathematical Biology I: An Introduction*. Springer, New York, NY, USA,, 2002.

[14] X. Pan, C. S. Han, K. Dauber, and K.H. Law. A multi-agent based framework for the simulation of human and social behaviors during emergency evacuations. *AI & Society*, 22(2):113–132, 2007.

[15] D. Provitolo. *Structural and dynamical complexities of risk and catastrophe systems: an approach by System Dynamics Modelling*. Springer (Ed.), 2009.

[16] H. Rahmandad and J. Sterman. Heterogeneity and network structure in the dynamics of diffusion: Comparing agent-based and differential equation models. *Management Science*, 54(5):998–1014, 2008.

[17] E. Vermeiren. *Psychologie des foules : éléments de compréhension et de prévention en situation de catastrophe collective*. Urgences, 2007.