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To cite this version:

Elie Awwad, Ghaya Rekaya-Ben Othman, Yves Jaouën. Design Criterion of Polarization-Time Codes for Optical Fiber Channels. IEEE International Conference on Communications, Jun 2013, Budapest, Hungary. pp.3428 - 3432. hal-00991719

HAL Id: hal-00991719

https://hal.science/hal-00991719v1

Submitted on 15 May 2014

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Design Criterion of Polarization-Time Codes for Optical Fiber Channels

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***Abstract*—Coherent detection with Polarization Multiplexing (PolMux) is the most promising technique for future optical fiber transmission systems. However, the optical channel suffers from non-unitary impairments known as Polarization Dependent Loss (PDL). Space-Time coding, originally designed for wireless Rayleigh fading channels, was proven to be capable of mitigating PDL. Coding gains of ST codes were evaluated through simulations and experiments that showed differences in their performance on the optical channel and on the wireless channel. In this paper, we derive an upper bound of the pairwise error probability of an optical channel considering the PDL effect. This upper bound explains the performance of ST codes used to mitigate PDL and yields the design criterion that a code should satisfy in order to completely mitigate PDL.**

***Index Terms*—Polarization Dependent Loss, Error Probability, Polarization-Time Coding, Optical Fiber Communication, MIMO**

I. Introduction

From long-haul links to metropolitan and access networks, optical fibers are today's indispensable carriers of data traffic over the world. They are able to transport light over long distances with a low attenuation. Due to this property and to the THz bandwidth in the near infrared region of the electromagnetic spectrum, binary modulation schemes such as On-Off Keying carrying 1 bit/symbol were first used to send data on fiber-optic systems. However, the rapidly growing Internet traffic urged the search for techniques that increase the capacity of the optical fiber channel. For this purpose, coherent detection started replacing conventional direct detection schemes. The main advantages of coherent detection are the enhanced receiver sensitivity and the introduction of multilevel modulation formats that achieve high spectral efficiency. Moreover, research in Digital Signal Processing (DSP) algorithms and development of high-speed electronics offered the possibility of compensating for fiber impairments in the electrical domain at the receiver rather than optically [1]–[3], allowing the implementation of forward error correction codes and advanced equalization techniques in a DSP-based emitter and receiver.

Apart from the amplitude and the phase of an electromagnetic field, the polarization state is an available degree of freedom that is not yet used. With Polarization Multiplexing (PolMux) [4], [5], we can double the spectral efficiency of an optical fiber link by sending independent information on two orthogonal polarization states. However, during propagation through the optical channel, the transmitted signal suffers several impairments. We focus on the linear impairments that include dispersive effects: deterministic Chromatic Dispersion (CD) and random Polarization Mode Dispersion (PMD), as well as fiber attenuation and random fading effects known as Polarization Dependent Loss (PDL). To mitigate the linear impairments of the optical channel and recover the transmitted data, a polarization diversity coherent receiver along with DSP algorithms are needed [3]–[5]. While dispersive effects have been thoroughly studied and completely compensated, mitigation of PDL was still not considered and system margins were left in order to absorb the induced penalties.

A PolMux scheme can be described as a 2×2 Multi-Input-Multi-Output (MIMO) system where 2 polarization states are used at the emission and the reception [6]. Instead of the spatial dimension (antennas) in a wireless scheme, we use here the polarization dimension. Hence, Space-Time (ST) coding can be applied to mitigate PDL by taking advantage of the diversity of the MIMO configuration. The motivation behind the use of ST codes, or more appropriately Polarization-Time (PT) codes in this context, is the random nature of PDL. Indeed, some optical elements attenuate unequally the polarization states of the transmitted signal. This along with random polarization rotations results in breaking the orthogonality of these states. Therefore, it is more likely to recover the original data symbols by sending linear combinations of these symbols at different time slots and on two different polarizations.

The performance of ST codes such as the Golden code [7], Silver code [8] and Alamouti code [9] was evaluated on an optical channel with PDL through simulations [10] and experimental demonstrations [11]. The Silver code performed better than the Golden code, unlike the case of the wireless Rayleigh fading channel. Besides, it was found that the performance of Alamouti code is independent of the amount of PDL in the link [12]. In this work, we aim to explain these observations through error probability calculus. We also check whether the Silver code is the optimal code for an optical channel with PDL or better coding schemes can be found.

Accordingly, the paper is organized as follows: in Section II, we recall the main linear impairments of an optical fiber link, then we present in Section III a mathematical model of an optical channel with PDL that will be used in Section IV to compute an upper bound of the error probability expression. Afterwards, in Section V, we derive the design criterion of codes that minimize the error probability of a transmission on an optical fiber channel with PDL and we investigate in

Section VI the performance obtained with the existing codes. Finally, we conclude and set forth the perspectives of our work.

II. Optical fiber effects

A. Fiber attenuation & dispersive effects

The loss coefficient of Standard Single Mode Fibers (SSMF) is equal to 0.2dB/km. For long-haul links spanning thousands of kilometers, optical amplifiers are periodically inserted to raise the signal power to its initially injected value at the transmitter. As for chromatic dispersion (CD) and polarization mode dispersion (PMD), both time-domain and frequency-domain equalizations exist [3]. However, a special multicarrier format, Orthogonal Frequency Division Multiplexing (OFDM) has proven to be capable of eliminating all interference caused by dispersive effects using a cyclic prefix [6].

B. Polarization Dependent Loss

While the dispersion compensation was thoroughly studied, PDL effects were not being considered until recently. In fact, PDL is mainly introduced by inline components such as amplifiers, isolators, couplers and not the fiber itself. The individual PDL value of each component is kept as low as possible. However, many of these components are found in long-haul optical links leading to a large accumulated PDL. Unlike unitary dispersive effects, PDL causes Optical Signal-to-Noise Ratio (OSNR) distortions. The information-carrying polarizations experience unequal loss and crosstalk [13].

III. Optical channel model

In order to proceed with an error probability calculus, we will first mathematically define the studied optical channel.

A. Optical channel with PDL

An overview of previous works on PDL, especially by A. Mecozzi, M. Shtaif [14] and N. Gisin [15], shows that PDL must be described statistically. However, we consider in our work a simple representation, commonly found in literature [16], [17], that takes into account the OSNR inequality between two polarization states as well as their crosstalk. This representation had been used in [10] to simulate the performance of PT coding in mitigating PDL and an experimental demonstration [11] validated the obtained numerical results. PDL can be described by the following transmission matrix:

$$\begin{aligned} \mathbf{H}_{PDL} &= \mathbf{R}_\alpha \begin{bmatrix} \sqrt{1-\gamma} & 0 \\ 0 & \sqrt{1+\gamma} \end{bmatrix} \mathbf{R}_\alpha^{-1} \\ with\ \mathbf{R}_\alpha &= \begin{bmatrix} \cos(\alpha) & -\sin(\alpha) \\ \sin(\alpha) & \cos(\alpha) \end{bmatrix} \end{aligned} \tag{1}$$

The diagonal matrix gives the imbalanced attenuation values of the least and most attenuated polarization states. The rotation matrix $\mathbf{R}_\alpha$ describes a mismatch between the polarization states of the incident signal and the axes of the PDL component. γ is defined through $\Gamma_{dB} = 10log_{10}\frac{1+\gamma}{1-\gamma}$ where $\Gamma_{dB} \geq 0$ is the PDL coefficient in dB and consists of the ratio between the highest and the lowest gains of the link. Hence, $0 \leq \gamma \leq 1$ with $\gamma = 0$ being a PDL-free case and $\gamma = 1$ corresponding to a perfect polarizer. We consider in this work constant values of Γ_{dB} and the angles of the rotation matrices are drawn from a uniform distribution over $[0 : 2\pi]$.

B. PolMux OFDM channel model

In a PolMux OFDM transmission, an OFDM signal is sent on each polarization. At the transmitter, the information symbols are assigned to different subcarriers in the frequency domain then the signal is converted to time domain by an inverse Fast Fourier Transform (iFFT). In order to absorb the interference induced by CD and PMD, a suitable cyclic prefix (CP) is added to the beginning of each OFDM symbol. At the receiver, the CP is removed and an FFT operation converts the signal back into the frequency domain. Despite the redundancy introduced by the CP, this prefix eliminates all Inter-Symbol-Interference and allows to correct any distortion caused by a linear dispersive channel with a single-tap frequency equalizer. Each subcarrier sees a flat channel and the received symbols are given by [6]:

$$\mathbf{Y}_{k,i} = \exp(j\phi_i)\exp(j\phi_D(\omega_k))\mathbf{H}_k\mathbf{X}_{k,i} + \mathbf{N}_{k,i} \tag{2}$$

where $\mathbf{X}_{k,i}$ is the symbol on the kth subcarrier of the ith OFDM symbol. $\mathbf{Y}_{k,i}$ is the received symbol. ϕ_i is the laser phase noise, $\phi_D(\omega_k)$ is the phase shift owing to chromatic dispersion common to both polarizations. $\mathbf{H}_k$ is the 2×2 Jones matrix of the link, resulting from a concatenation of PMD and PDL components. $\mathbf{N}_{k,i}$ represents the additive noise.

We consider next that the CP is larger than the delay spread induced by CD and PMD. Hence, it absorbs all resulting interference and the remaining effect is the non-unitary PDL that we shall regard in the error probability calculus.

IV. Derivation of an upper bound of the error probability

To get an insight on the behavior of PT codes on an optical link with PDL, we consider a PolMux OFDM transmission and use $\mathbf{H}_{PDL}$ defined in (1) as the channel matrix. Therefore, each subcarrier can be seen as a slow fading and frequency non-selective channel:

$$\mathbf{Y}_{2\times T} = \mathbf{H}_{PDL}\mathbf{X}_{2\times T} + \mathbf{N}_{2\times T} \tag{3}$$

T is the temporal length of the code. $\mathbf{X}$ is the PT codeword matrix. $\mathbf{Y}$ is the received matrix. When $T = 1$, $\mathbf{X}$ and $\mathbf{Y}$ are vectors and $\mathbf{X}$ contains two QAM symbols (No Coding). When $T > 1$, PT coding is used ($T = 2$ for the codes investigated in this paper). The noise $\mathbf{N}$ is modeled as additive white with independent and identically distributed circular complex Gaussian components $\mathcal{CN}(0, 2\sigma^2)$.

According to the channel model in (3), a maximum likelihood (ML) decoder estimates $\mathbf{X}'$ with $\mathbf{X}''$ as:

$$\mathbf{X}'' = \underset{\mathbf{X}' \in \mathcal{C}}{argmin} \|\mathbf{Y} - \mathbf{H}_{PDL}\mathbf{X}'\|^2 \tag{4}$$

where $\mathcal{C}$ is the set of all possible codewords and $\|\cdot\|$ is the Euclidian norm. We suppose that we have perfect channel

state information, i.e. $\mathbf{H}_{PDL}$ is known. The error probability is defined as:

$$\begin{aligned} P_{error} &= Pr\{\mathbf{X}'' \neq \mathbf{X}'\} \\ &= \sum_{\mathbf{X}' \in \mathcal{C}} Pr\{\mathbf{X}'\} Pr\{\mathbf{X}'' \neq \mathbf{X}' | \mathbf{X}'\} \end{aligned} \tag{5}$$

For equiprobable codewords, the error probability can be upper-bounded by [18]:

$$P_{error} \leq \frac{1}{\text{card}(\mathcal{C})} \sum_{\substack{\mathbf{X}', \mathbf{X}'' \in C, \\ \mathbf{X}'' \neq \mathbf{X}'}} Pr(\mathbf{X}' \to \mathbf{X}'') \tag{6}$$

where card($\mathcal{C}$) is the cardinality of $\mathcal{C}$ and $Pr(\mathbf{X}' \to \mathbf{X}'')$ is the pairwise error probability obtained by averaging the conditional pairwise error probability defined as:

$$\begin{aligned} Pr(\mathbf{X}' \to \mathbf{X}'' | \mathbf{H}_{PDL}) = Pr\{\|\mathbf{Y} - \mathbf{H}_{PDL}\mathbf{X}''\|^2 \leq \\ \|\mathbf{Y} - \mathbf{H}_{PDL}\mathbf{X}'\|^2 | \mathbf{X}', \mathbf{H}_{PDL}\} \end{aligned} \tag{7}$$

When computing the pairwise error probability, we suppose that $\mathbf{X}'$ and $\mathbf{X}''$ are the only possible codewords in the codeword space. Hence, we can write the pairwise error probability in terms of the Gaussian tail function Q:

$$Pr(\mathbf{X}' \to \mathbf{X}'' | \mathbf{H}_{PDL}) = Q(\frac{\|\mathbf{H}_{PDL}(\mathbf{X}' - \mathbf{X}'')\|}{2\sigma}) \tag{8}$$

$$with\ Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{+\infty} \exp^{-t^2/2}\, dt \tag{9}$$

Using Chernoff's bound $Q(x) \leq \frac{1}{2}\exp^{-\frac{x^2}{2}}\ \forall x$, and averaging over $\mathbf{H}_{PDL}$, we get:

$$Pr(\mathbf{X}' \to \mathbf{X}'') \leq \mathbb{E}_H[\exp(-\frac{\|\mathbf{H_{PDL}}(\mathbf{X}' - \mathbf{X}'')\|^2}{8\sigma^2})] \tag{10}$$

where $\mathbb{E}_H[]$ is the averaging operation over all possible channel realizations.

Let $\mathbf{X} = \mathbf{X}' - \mathbf{X}''$ be the difference of two codewords and $\mathbf{D} = \begin{bmatrix} \sqrt{1-\gamma} & 0 \\ 0 & \sqrt{1+\gamma} \end{bmatrix}$, we have:

$$\begin{aligned} \|\mathbf{H}_{PDL}(\mathbf{X}' - \mathbf{X}'')\|^2 = \|\mathbf{H}_{PDL}\mathbf{X}\|^2 = \|\mathbf{R}_\alpha \mathbf{D} \mathbf{R}_\alpha^{-1} \mathbf{X}\|^2 \\ = \|\mathbf{D}\mathbf{R}_\alpha^{-1}\mathbf{X}\|^2 \end{aligned} \tag{11}$$

Now, let $\beta = -\alpha$. We can write $\mathbf{R}_\beta \mathbf{X} = \mathbf{U} = \begin{pmatrix} \vec{v}_1 \\ \vec{v}_2 \end{pmatrix}$, $\vec{v}_{1,2}$ being row vectors, we get:

$$\|\mathbf{DU}\|^2 = \|\vec{v}_1\|^2 + \|\vec{v}_2\|^2 + \gamma(\|\vec{v}_2\|^2 - \|\vec{v}_1\|^2) \tag{12}$$

However, $\|\vec{v}_1\|^2 + \|\vec{v}_2\|^2 = \|\mathbf{U}\|^2 = \|\mathbf{X}\|^2$ because a rotation matrix is unitary. Besides, we develop $\|\vec{v}_2\|^2 - \|\vec{v}_1\|^2$ and obtain:

$$\begin{aligned} \|\vec{v}_2\|^2 - \|\vec{v}_1\|^2 = f(\beta) = (\|\vec{x}_2\|^2 - \|\vec{x}_1\|^2)\cos(2\beta) \\ + 2Re(\langle \vec{x}_1, \vec{x}_2 \rangle)\sin(2\beta) \end{aligned} \tag{13}$$

with $\mathbf{X} = \begin{pmatrix} \vec{x}_1 \\ \vec{x}_2 \end{pmatrix}$, $\vec{x}_{1,2}$ being row vectors. Let:

$$a = \|\vec{x}_2\|^2 - \|\vec{x}_1\|^2 \tag{14}$$

$$b = 2Re(\langle \vec{x}_1, \vec{x}_2 \rangle) \tag{15}$$

Substituting (12) to (15) in (10), we get:

$$\begin{aligned} Pr(\mathbf{X}' \to \mathbf{X}'') = \mathbb{E}_H[Pr(\mathbf{X}' \to \mathbf{X}'' | \mathbf{H}_{PDL})] \\ \leq \exp(-\frac{\|\mathbf{X}\|^2}{8\sigma^2})\mathbb{E}_H[\exp(-\frac{\gamma}{8\sigma^2}(a\cos(2\beta) + b\sin(2\beta)))] \end{aligned} \tag{16}$$

The last step consists of averaging over $\mathbf{H}_{PDL}$. Considering constant values of Γ_{dB}, we are left with one random variable β that varies uniformly on the interval $[0 : 2\pi]$. Using the following trigonometric identity:

$$\begin{aligned} acos(2\beta) + bsin(2\beta) = \sqrt{a^2 + b^2}\cos(2\beta + \phi) \\ with\ \phi = \left\{ \begin{array}{c} -arctan(\frac{b}{a})\ if\ a > 0 \\ \pi - arctan(\frac{b}{a})\ if\ a < 0 \end{array} \right\} \end{aligned} \tag{17}$$

We obtain:

$$\begin{aligned} Pr(\mathbf{X}' \to \mathbf{X}'') \leq \\ \exp(-\frac{\|\mathbf{X}\|^2}{8\sigma^2})\frac{1}{2\pi}\int_\phi^{2\pi+\phi} \exp(-\frac{\gamma}{8\sigma^2}\sqrt{a^2+b^2}cos(u))du \end{aligned} \tag{18}$$

The function inside the integral is periodic of period 2π and has an axis of symmetry $u = \pi$ in the interval $[0 : 2\pi]$. Using these two properties, (18) becomes:

$$\begin{aligned} Pr(\mathbf{X}' \to \mathbf{X}'') \leq \\ \exp(-\frac{\|\mathbf{X}\|^2}{8\sigma^2})\frac{1}{\pi}\int_0^{\pi} \exp(-\frac{\gamma}{8\sigma^2}\sqrt{a^2+b^2}cos(u))du \end{aligned} \tag{19}$$

Hence,

$$Pr(\mathbf{X}' \to \mathbf{X}'') \leq \exp(-\frac{\|\mathbf{X}\|^2}{8\sigma^2})I_0(\frac{\gamma}{8\sigma^2}\sqrt{a^2+b^2}) \tag{20}$$

where $I_0(k)$ is the 0th order modified Bessel function of the first kind.

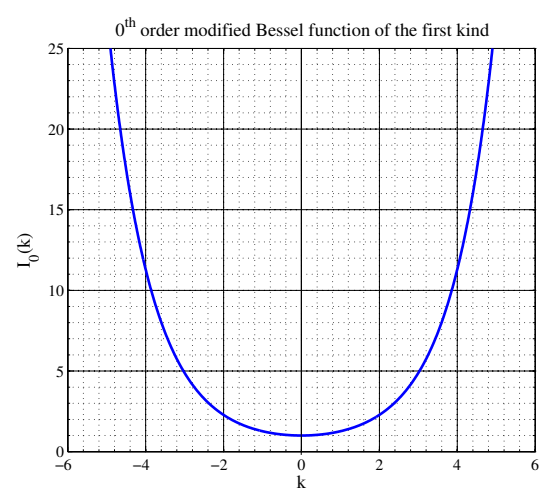

Fig. 1. Plot of the 0th order modified Bessel function of the first kind.

We can approximate the error probability expression in (20) for high SNR values by using a first order approximation of $I_0(k)$ when $k \to \infty$:

$$I_0(k) \approx \frac{\exp(k)}{\sqrt{2\pi k}} \tag{21}$$

Then, we get:

$$\begin{aligned} &\exp(-\frac{\|\mathbf{X}\|^2}{8\sigma^2}) I_0(\frac{\gamma}{8\sigma^2}\sqrt{a^2+b^2}) \\ &\approx \exp(-\frac{\|\mathbf{X}\|^2 - \gamma\sqrt{a^2+b^2}}{8\sigma^2}) \end{aligned} \tag{22}$$

V. Design criterion

In Fig. 1, we see that $I_0(k)$ is monotonously increasing for $k \geq 0$ and hence has its minimum for $k = 0$. This corresponds to a and b in (20) equal to zero and as a result, the obtained error probability expression will be independent of PDL. Consequently, we define the following design criterion:

Proposition 1. *A Polarization-Time code completely mitigates PDL if and only if all codeword differences satisfy:*

1) $a = \|\vec{x}_2\|^2 - \|\vec{x}_1\|^2 = 0$ *and*
2) $b = 2Re(\langle \vec{x}_1, \vec{x}_2 \rangle) = 0$.

When the design criterion is met, we recover the performance for two parallel additive white Gaussian channels which is the best performance we can achieve:

$$p_{error,AWGN} \approx \exp^{-\frac{\|\mathbf{X}\|^2}{8\sigma^2}} \tag{23}$$

We clearly see that the resulting criteria are completely different from the rank and the minimum determinant criteria for a Rayleigh fading channel [19], defining respectively a diversity gain and a coding gain. If we compare the approximation of the error probability expression at high SNR (22) to the one obtained in the case of a 2×2 MIMO Rayleigh fading channel, we notice different behaviors: the error probability of the Rayleigh fading channel decays as SNR^{-2r} [19] where r is the rank of the matrix $\mathbf{X}$, whereas the error probability of the PDL channel decays exponentially as a function of the SNR. Hence, Space-Time codes bring no diversity gain to the optical channel with PDL. Furthermore, if one of the design criterion is not satisfied, (22) points out that the minimum value of $\|\mathbf{X}\|^2 - \gamma\sqrt{a^2+b^2}$ should be maximized in order to minimize the error probability.

VI. Code performance analysis

In this section, we analyze the performance of both coded and uncoded schemes using the derived upper bound of the pairwise error probability expression in (22). All the reported performance in the next subsections match with the performance presented in [10].

In Table I, we report the minimum values of $\|\mathbf{X}\|^2 - \gamma\sqrt{a^2+b^2}$ for each investigated ST code at four different PDL values. To fill the table, we set a spectral efficiency of 4 bits per channel use (/cu) for all coding schemes. Hence, we use 4-QAM information symbols to build the codewords of the Silver and the Golden codes (full-rate codes with 2 symbols/cu), and 16-QAM symbols for the Alamouti code because of the redundancy in its structure (1 symbol/cu). We consider an average symbol energy $E_S = 1$ for all constellations.

TABLE I
Minimum value of $\|\mathbf{X}\|^2 - \gamma\sqrt{a^2+b^2}$ for different coding schemes at different PDL values

	$\Gamma = 0dB$	$\Gamma = 3dB$	$\Gamma = 6dB$	$\Gamma = 10dB$
Silver Code	2	2	2	1.23
Golden Code	2	1.7	1.46	1.07
No Coding	2	1.34	0.8	0.38
Alamouti Code	0.8	0.8	0.8	0.8

A. Alamouti code

In Fig. 2, we report the performance of Alamouti code for different PDL values ($\Gamma = 3, 6, 10dB$) obtained through Monte Carlo simulations. We notice that the performance of the Alamouti code is independent of the amount of PDL in the link. The orthogonality of the Alamouti codeword matrix induces $a = b = 0$ for all possible codeword differences. This explains why it performs the same independently of PDL. However, its performance is affected by the use of 16-QAM symbols giving a squared minimal distance $\|\mathbf{X}\|^2 = 0.8$.

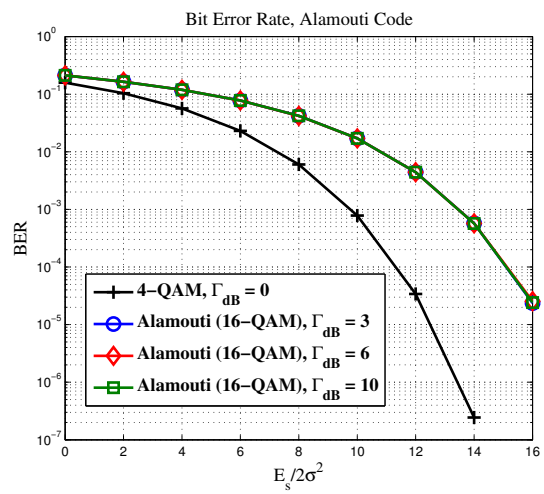

Fig. 2. Bit Error Rate as a function of SNR for the Alamouti code, obtained through Monte Carlo simulations.

B. Silver code

The Silver code is not optimal in mitigating PDL since it does not satisfy the derived design criterion. Unlike the Alamouti code, the Silver code has only some codeword differences having $a = b = 0$. In Fig. 3, we plot the performance of the Silver code for different PDL values. We see that the code mitigates almost all PDL when the PDL coefficient is equal to 3 and 6dB. A slight difference in performance is observed for these two PDL values. Whereas for a PDL of 10dB, the code is not able to completely palliate PDL. Looking at Table I, we find that the minimum value of $\|\mathbf{X}\|^2 - \gamma\sqrt{a^2+b^2}$ for the Silver code is the same at a

PDL of 3 and 6dB, and is equal to 2. Whereas at a PDL of 10dB, this minimum falls to 1.23 which explains the obtained performance.

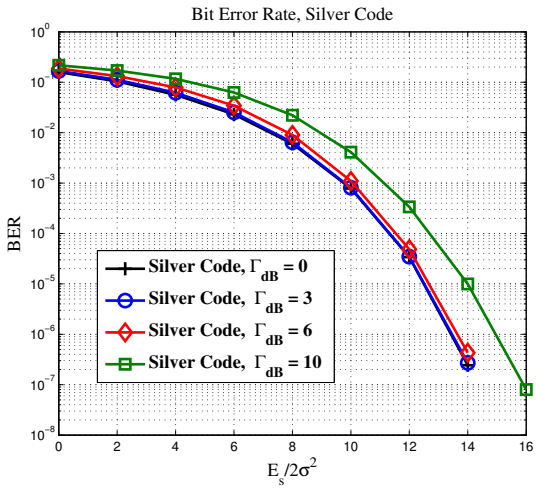

Fig. 3. Bit Error Rate as a function of SNR for the Silver code, obtained through Monte Carlo simulations.

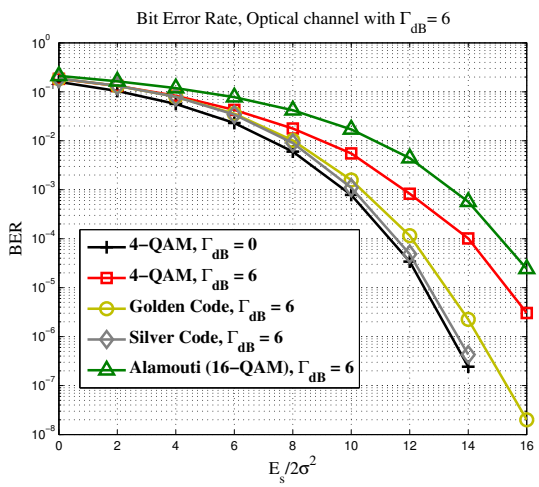

Fig. 4. Performance comparison of 4-QAM (No coding), Silver, Golden and Alamouti coding schemes when $\Gamma_{dB} = 6$.

C. Golden code and uncoded scheme

From Fig. 4, we can see that the Silver code outperforms the Golden code for $\Gamma = 6dB$, and both reduce the penalty that PDL causes to the uncoded scheme. Again, this result can be explained by looking at Table I. Indeed, the minimum value of $\|\mathbf{X}\|^2 - \gamma\sqrt{a^2+b^2}$ is the greatest for the Silver code followed by the Golden code and then the uncoded scheme.

In conclusion, we were able to explain, in terms of error probability bounds, the performance of the Alamouti, Silver and Golden codes when applied on an optical channel with PDL. The Golden and the Silver codes were designed to satisfy the rank and the minimum determinant criteria for a wireless channel that are no more relevant for the optical channel.

VII. Conclusion and Perspectives

We have established an upper bound of the error probability expression for an optical channel with PDL that allows to predict and compare the performance of any Polarization-Time code used to mitigate PDL by computing a certain distance that depends on the codewords and the PDL value. This expression also yields the design criterion required to construct codes that guarantee a PDL-independent error probability matching the performance for an additive white Gaussian channel. In our future work, we look forward to elaborate the error probability calculus by including the statistics of the PDL coefficient Γ_{dB} that is, under some conditions, Maxwellian distributed [14] and design specific codes for the optical channel.

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