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Environmental taxation, health and the life-cycle*†

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Abstract

We build a model that takes into consideration the evolution of health over the life cycle and its consequences on individual optimal choices. In this framework, the effects of environmental taxation are not limited to the traditional negative crowding-out and positive productivity effects. We show that environmental taxation generates new general equilibrium effects ignored by previous contributions. Indeed, as the environmental tax improves the health profile over the life-cycle, it influences saving, labor supply, and retirement. We also show that whether those general equilibrium effects are positive or negative for the economy crucially depends on the degree of substitutability between young and old labor. Our numerical examples suggest that ignoring those new effects may result in large overstatement of the negative effect of an increase in environmental taxation on output, and understatement of the positive effect on welfare.

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1 Introduction

What is the economic effect of environmental taxation when pollution impacts health? Whereas a vast number of theoretical contributions address this question taking the effect of pollution on mortality into consideration, fewer include the effects of pollution on morbidity. Furthermore, those contributions do not model the interaction between pollution and health over the life-cycle. By contrast, we build a model that takes into consideration the evolution of health over the life cycle and its consequences on individual optimal choices. In this framework, the effects of environmental taxation are not limited to the traditional negative crowding-out and positive productivity effects. We show that environmental taxation generates new general equilibrium effects ignored by previous contributions. Indeed, as the environmental tax improves the health profile over the life-cycle, it influences investment in health as well as saving, labor supply and retirement choices. We also show that whether those general equilibrium effects are positive or negative for the economy crucially depends on the degree of substitutability between young and old labor.

The effect of pollution on morbidity is well established in the epidemiological literature. Pollution is known as a causal factor for certain chronic diseases, especially cancer, cardiovascular disease and respiratory diseases, that have durable detrimental impacts in terms of illness and disability.[1] According to Briggs (2003) about 8-9% of the total disease burden may be attributed to pollution in developed countries. While direct and indirect impacts of illness on productivity is the object of growing interest,[2] the overall fraction of pollution-related health problems that affect productivity is unknown. Nevertheless, the empirical literature focuses on some specific types of pollution and finds that the negative effect of pollution on productivity is quantitatively significant. Haus-

[1]See Brauer et al. (2011), Ruckerl et al. (2011), Gold and Mittleman (2013), Rajagopalan and Brook (2012), Brook et al. (2010) regarding air pollution; Paulu et al. (1999), Valent et al. (2004) for water pollution and Nadal et al. (2004), Chen and Liao (2005), Schuhmacher and Domingo (2006) for industrial pollution.

[2]See Bloom et al. (2004), Devol and Bedroussian (2007) and Zhang et al. (2011), for example.

man et al. (1984) estimate that a 1 unit ($\mu g/m^3$) increase in particulate matter pollution increases lost work days by 0.7%. Hansen and Selte (2000) show that sick leaves are significantly linked to particulate matter pollution (PM10). Hanna and Oliva (2011) find that a one percent increase in sulfur dioxide results in a 0.61 percent decrease in the hours worked in Mexico city. Graff Zivin and Neidell (2012) find that a 10 ppb decrease in Ozone concentrations increases worker productivity by 4.2%. With respect to the effect of outdoor air pollution on the productivity of indoor workers, Chang et al. (2014) "suggest that nationwide reductions in PM2.5 from 1999 to 2008 generated $19.5 billion in labor cost savings, which is roughly one-third of the total welfare benefits associated with this change."

Thus, the theoretical literature has explored the effect of environmental policy taking into consideration the link between pollution and health in infinite horizon models, with the idea that productivity gains and decreased medical expenditure related to pollution reduction generally mitigate the costs of environmental policies (See Mayeres and Van Regemorter (2008), Huhtala and Samakovlis (2007), and Ostblom and Samakovlis (2007)). Williams (2002) proposes a general equilibrium model in which reduced pollution increases health or productivity. In contrast to the previously cited studies, this author finds that the resulting effects on labor supply can magnify or diminish the benefits of reduced pollution. Williams (2003) further shows that interactions with health effects from pollution reduce the optimal environmental tax rather than increasing it as in Schwartz and Repetto (2000). In a growth model with research and development, Aloi and Tournemaine (2011) find that environmental taxation has a positive effect on growth and welfare through productivity gains and reallocation of resources toward R&D.

The research designs of those models ignore the interactions between pollution and morbidity over the life-cycle, thereby missing some of the channels through which environmental policy affects the economy. It is however crucial to understand this interaction on two levels that have not been studied in the literature on environmental taxation.

First, the health profile is susceptible to modification by pollution. Indeed, pollution contributes to chronic diseases, which primarily affect people age 15 to 59 according to the OMS. The health profile influences the productivity profile (Lakdawalla et al. 2004, Bhattacharya et al. 2008, Perlkowski and Berger 2004) and also weights on life-cycle saving, labor and retirement (Dwyer and Mitchell 1999, Deschryvere 2006, amongst others). Second, as pointed out by Cropper (1981), individuals' investment in health during the first part of their lives interacts with pollution, which modifies their health profile. Thus, a decreased investment in health can potentially offset some of the benefits of environmental taxation on health.

Therefore, we propose to study the effects of environmental taxation in a two-period overlapping generations model with the following features. First, we explicitly model the health status as a stock that increases with investment in health and decreases with pollution. Second, we make the link between health and productivity over the life-cycle explicit. Third, we model retirement decisions, allowing individuals to choose whether to continue to work or to retire during the second stage of their lives. Fourth, we allow for labor by the young and the old to be complements or substitutes.[3] Finally, since an important focus of our paper is on labor choices, we model investment in health as the time individuals derive from leisure (rather than the amount they spend on health services). Indeed, like Domeij and Johannesson (2006), we argue that in publicly financed healthcare systems, health care spending is not an important source of income uncertainty and does not significantly influence the consumption-saving choice of individuals. Thus, investment in health represents an investment in time rather than a consumption expenditure.[4] Additionally, the empirical literature suggests that time spent on activities such as physical exercises or at-home food preparation is an important factor of health outcomes.[5] Furthermore, this modeling choice enables us to

[3]See Kalwij et al. (2010), Gruber et al. (2010), Gruber and Milligan (2010) for empirical evidence on the imperfect substitutability between young and old labor in developed economies.

[4]By contrast, the life-cycle health profile influences consumption saving choice.

[5]See for example Mullahy and Robert (2010), Xu (2013) and the *2008 Physical Activity Guidelines for Americans* edited by the US Department of Health and Human services.

keep the model tractable for providing analytical results. Thus, our work also fills a void in the overlapping generations literature, which does not endogeneize the link between environment, health and productivity to study environmental taxation. Indeed, previous contributions (Mathieu-Bolh and Pautrel (2011) and Raffin (2012)) assume an ad-hoc link between pollution and productivity and do not model health.[6] Our two-period framework also contrasts with Pautrel (2012) who assumes a constant health profile, does not allow individuals to choose between work or retirement in the second period of their lives, and does not explore various characteristics of young and old labor.

We present our theoretical results using two versions of the model: a simplified one and the full model. We first describe the steady state effect of environmental taxation with exogenous labor supply, retirement, and investment in health. Second, we reintroduce labor supply, retirement and investment in health choices. Using this approach, we are able to provide a decomposition of the effects of environmental taxation on output, and their origin. The main results of the paper are as follows.

1. We identify new effects of environmental taxation on output. A first new effect is the"health-saving effect". The environmental tax limits the decline in health over the life-cycle. When old and young labor are substitutes (complements), the tax increases (decreases) the steady state interest rate, decreasing (increasing) saving and output. Second, our model captures the effects of environmental taxation on aggregate efficient labor. Two new general equilibrium effects appear, the "young labor effect" and the "retirement effect". Those effects modify the response of aggregate efficient labor. This result is in sharp contrast with much of the existing literature, which is generally assumes that environmental taxation has a crowding-out effect and only positively impacts aggregate efficient labor through workers' productivity.

[6] In Mathieu-Bolh and Pautrel (2011), an exogenous age-productivity profile is introduced but it does not influence individual decisions. It only influences aggregate variables through intergenerational redistribution. By contrast, in our framework, health and retirement decisions are endogenous. Thus, environmental taxation influences the health profile, individual decisions, and thereby the aggregate economy.

2. We show that if the life-cycle characteristic of the health profile is ignored the "health-saving effect" disappears. Furthermore, we show that the new effects of environmental taxation on output also crucially depend on the degree of substitutability between old and young labor.

3. Our numerical simulations of the model suggest that ignoring those new effects may result in largely overstating the negative effect of an increase in environmental taxation on output and understating the positive effects on welfare.

The paper is organized as follows. In section two, we present the model. In section three, we describe the steady state for the two versions of the model and discuss our results. In section four, we present numerical examples.

2 The economy

We consider an infinite horizon economy where agents live two periods. Population is assumed to be constant and is equal to $2N$.

2.1 Individuals

Individuals work during the two periods of their lives. Each young agent is endowed with one unit of time, supplying $\lambda_{1,t} \in]0,1[$ in final production, using $m_t \in]0,1[$ as an investment in healthcare activities to improve her health status in the second period of her life, and using the remaining time $1 - m_t - \lambda_{1,t}$, as leisure. Therefore, when young, she earns a wage income $\lambda_{1,t} w_{1,t}$, where $w_{1,t}$ is the efficient wage. This income is used to consume $c_{1,t}$, to save s_t or to pay retirement benefits ($\tau_t^w w_{1,t}$ with $\tau_t^w \in (0,1)$):

$$(1 - \tau_t^w)\lambda_{1,t} w_{1,t} = c_{1,t} + s_t \tag{1}$$

During the second period, each agent is also endowed with one unit of time, supplying $\lambda_{2,t+1} \in (0,1)$ in final production and the remaining time $1 - \lambda_{2,t+1}$ as retired. When old, she earns a wage income $\lambda_{2,t+1}w_{2,t+1}$. She also receives the revenue of her first period saving and retirement benefits q_{t+1}. Therefore, her second period consumption is:

$$c_{2,t+1} = R_{t+1}s_t + (1-\tau^w_{t+1})w_{2,t+1}\lambda_{2,t+1} + (1-\lambda_{2,t+1})q_{t+1}$$

with $R_{t+1} \equiv 1 + r_{t+1}$. Assuming a pay-as-you-go system, the retirement benefits paid to retirees in t must be equal to the contributions by workers (who include the old born in t and the young born in $t+1$):

$$(1-\lambda_{2,t+1})q_{t+1} = \tau^w_{t+1}\left[\lambda_{2,t+1}w_{2,t+1} + \lambda_{1,t+1}w_{1,t+1}\right]$$

Therefore the budget constraint of an old agent born in t is :

$$c_{2,t+1} = R_{t+1}s_t + \left[\lambda_{2,t+1}w_{2,t+1} + \tau^w_{t+1}\lambda_{1,t+1}w_{1,t+1}\right] \tag{2}$$

Individuals are born in period t with a health-status denoted $h_{1,t}$. The health status of an agent born in t evolves between period t and period $t+1$ depending on two opposing forces (Aisa and Pueyo 2004). On one hand, the health status improves with the investment m_t made by the young agent. On the other hand, biological processes involve a natural decay in health as time goes by (Grossman 1972). Following Cropper (1981), we also assume that health depreciates over time as a function of the stock of pollution (denoted P_t). Therefore, for an agent born in t, the individual health-status evolves from period t to period $t+1$ according to:

$$h_{2,t+1} - h_{1,t} = H(m_t) - d\left(P_t\right)\ h_{1,t} \tag{3}$$

with $H'(m_t) > 0$, $H''(m_t) < 0$. Therefore, investment in health makes the health profile steeper. The detrimental influence of pollution on health appears in the depreciation rate function $d\left(P_t\right)$ with $d'(P_t) > 0$.[7]

The lifetime utility of the representative agent born in t is:

$$U_t = \log\left(\left(c_{1,t}(1-m_t-\lambda_{1,t})^{\varphi}\right)^{\phi} h_{1,t}^{1-\phi}\right) + \beta\log\left(\left(c_{2,t+1}(1-\lambda_{2,t+1})^{\gamma}\right)^{\phi} h_{2,t+1}^{1-\phi}\right) \tag{4}$$

[7] We do not specify the pollution function because the functional form of $d(\cdot)$ has no effect on theoretical results.

β is the time-preference parameter, φ captures the preference for leisure by the young and γ captures the preference for leisure by the old (or retirement). The parameter $\phi \in (0,1)$ captures the influence of the health-status in utility.

The maximization of (4) subject to (1), (2) and (3) yields saving:

$$s_t = \frac{\beta(1-\tau_t^w)\lambda_{1,t}w_{1,t} - \left[\lambda_{2,t+1}w_{2,t+1} + \tau_{t+1}^w \lambda_{1,t+1}w_{1,t+1}\right]/R_{t+1}}{1+\beta}, \tag{5}$$

Saving reflects the difference between the after-tax income available in the economy in the first period and the present value of income in the second period. In the first period, the after tax income represents the income of the young. In the second period, it represents the income of the old, which encompasses labor and retirement income. Retirement income is proportional to the time retirees spent working while they were young. The later the old retire (λ_2 high), the higher their income and the lower their saving.

In the presence of a PAYG system, the old receive retirement benefits whereas the young pay retirement contributions. Intergenerational redistribution that takes place through the PAYG influences the saving rate. Contributions to the retirement system are influenced by the choice of labor versus investment in health by the young.

Utility maximization also give labor supplied by the young in final output:

$$\lambda_{1,t} = \frac{(1+\beta)(1-m_t)}{1+\beta+\varphi} - \frac{\varphi\left[\lambda_{2,t+1}\frac{w_{2,t+1}/R_{t+1}}{w_{1,t}} + \tau_{t+1}^w\lambda_{1,t+1}\frac{w_{1,t+1}/R_{t+1}}{w_{1,t}}\right]}{(1-\tau_t^w)(1+\beta+\varphi)}, \tag{6}$$

and labor supplied by the old:

$$\lambda_{2,t+1} = \frac{(1+\beta) - \gamma\beta\left[\tau_{t+1}^w\frac{\lambda_{1,t+1}w_{1,t+1}}{w_{2,t+1}} + (1-\tau_t^w)\frac{\lambda_{1,t}w_{1,t}}{w_{2,t+1}/R_{t+1}}\right]}{(1+\gamma)(1+\beta)-\gamma} \tag{7}$$

The time spent working (λ_2) rather than retiring is determined by the presence of a PAYG in the first place. Indeed, the term in brackets simply reflects the social security wealth (difference between retirement benefits and contributions). Second, the decision to retire depends on relative wages across ages and periods in life. Third, if investment by the young is large, the old spend more time working. This is due to the fact that they are healthier and receive less retirement benefits in that case.

Finally, optimal individual health expenditure is given by:

$$\frac{H(m_t) + (1 - d(P_t))h_{1,t}}{H'(m_t)} - \frac{\beta(1-\phi)}{\phi\varphi}(1 - m_t - \lambda_{1,t}) = 0 \tag{8}$$

Heath expenditure m_t is positively related to the level of pollution P_t, negatively related to labor supply $\lambda_{1,t}$ and the health status $h_{1,t}$ of the young.

2.2 Firms

Identical firms operate under perfect competition. They produce a final good Y_t using the production function:

$$Y_t = BK_t^{\alpha_K} L_t^{\alpha_L} E_t^{1-\alpha_K-\alpha_L}$$

where K_t is the amount of physical capital, L_t is aggregate efficient labor, E_t is the flow of pollution emissions and $\alpha_K, \alpha_L \in (0,1)$.[8]

We assume that efficient units of labor supplied by the young depend on their respective productivity, which is influenced by their health status (denoted by $h_{1,t}$ for young and $h_{2,t}$ for old born at $t-1$). In contrast with previous contributions, we assume that efficient units of labor provided by the old and the young may be not perfectly substitutes in production. Therefore, aggregate efficient labor is defined as:

$$L_t = \left[\psi\left(h_{1,t}\; l_{1,t}\right)^{\theta} + (1-\psi)\left(h_{2,t}\; l_{2,t}\right)^{\theta}\right]^{1/\theta} \qquad \theta \leq 1,\; \psi \in (0,1)$$

where $l_{1,t}$ (respectively $l_{2,t}$) is the amount of labor supplied by the young (the old) at time t, and $h_{1,t}l_{1,t}$ (resp. $h_{2,t}l_{2,t}$) is efficient labor supplied by the young (the old).[9]

[8] See Stokey (1998) for a justification of the introduction of polluting emissions as a factor of production.

[9] We assume that individual productivity is only captured by health-status. Even if we are aware that there are other important factors affecting productivity when people get older (like human capital accumulation through experience, for example), we follow (Garibaldi 2010, p.1) considering that human capital is really effective only when it is embodied in healthy people.

The parameter θ measures the degree of substitutability between old and young workers in production. The elasticity of substitution between the two types of labor equals $1/(1-\theta)$. When $\theta = 1$, young and old workers are perfect substitutes. When $0 \leq \theta < 1$, they are imperfect substitutes. When $\theta = 0$, they are unitary substitutes. When $\theta < 0$, they are complements.

The profit of the firm in period t is $Y_t - w_{1,t}l_{1,t} - w_{2,t}l_{2,t} - R_tK_t - \tau E_t$, where R_t is the rental rate of capital and τ is an environmental tax levied by the government. Profit-maximization yields the following first-order conditions:

$$R_t = \alpha_K Y_t / K_t \tag{9}$$

$$w_{1,t} = \alpha_L Y_t / L_t \frac{\partial L_t}{\partial l_{1,t}} \qquad \text{and} \qquad w_{21,t} = \alpha_L Y_t / L_t \frac{\partial L_t}{\partial l_{2,t}} \tag{10}$$

$$\tau = (1 - \alpha_K - \alpha_L) Y_t / E_t \tag{11}$$

Equation (11) enables us to express final output in terms of physical capital, labor and the environmental tax:

$$Y_t = f(\tau) K_t^{\alpha} L_t^{1-\alpha} \tag{12}$$

with $L_t = \left[\psi \left(h_{1,t}l_{1,t}\right)^{\theta} + (1-\psi)\left(h_{2,t}l_{2,t}\right)^{\theta}\right]^{1/\theta}$, $f(\tau) \equiv B^{1/(\alpha_K+\alpha_L)} \left(\frac{1-\alpha_K-\alpha_L}{\tau}\right)^{\frac{1-\alpha_K-\alpha_L}{\alpha_K+\alpha_L}}$ and $\alpha \equiv \alpha_K/(\alpha_K+\alpha_L)$.

2.3 Government

The government levies an environmental tax on the flow of pollution emissions in each period. The revenues are used to fund public abatement activities, denoted A_t:

$$A_t = \tau E_t$$

The pollution stock rises with the flow of pollution emissions and is reduced by abatement activities:

$$P_{t+1} = (1-\sigma) P_t + E_t - A_t$$

where $\sigma > 0$ is the nature regeneration rate, and $E_t - A_t$ is the net flow of pollution at date t. From the expressions of E_t and A_t, it follows:

$$P_{t+1} = (1-\sigma)P_t + (1-\alpha_K-\alpha_L)\left(\frac{1}{\tau}-1\right)f(\tau)K_t^{\alpha}L_t^{1-\alpha} \tag{13}$$

2.4 Equilibrium

First, we consider the equilibrium in labor markets. Young agents supply $\lambda_{1,t}N$ units of labor and firms demand $l_{1,t}$: in equilibrium $l_{1,t} = \lambda_{1,t}N$. Old agents supply $\lambda_{2,t}N$ units of labor and firms demand $l_{2,t}$: in equilibrium $l_{2,t} = \lambda_{2,t}N$. The aggregate labor supply is given by:

$$L_t = N\left[\psi\left(\lambda_{1,t}h_{1,t}\right)^{\theta} + (1-\psi)\left(\lambda_{2,t}h_{2,t}\right)^{\theta}\right]^{1/\theta} \tag{14}$$

From equation (10), we obtain:

$$w_{1,t} = \psi\alpha_L f(\tau)\tilde{k}_t^{\alpha}h_{1,t}\left[\psi + (1-\psi)\left(\frac{\lambda_{2,t}h_{2,t}}{\lambda_{1,t}h_{1,t}}\right)^{\theta}\right]^{(1-\theta)/\theta} \tag{15}$$

where $\tilde{k}$ is the capital stock per efficient worker, and:

$$w_{2,t} = (1-\psi)\alpha_L f(\tau)\tilde{k}_t^{\alpha}h_{1,t}\left(\frac{h_{2,t}}{h_{1,t}}\right)^{\theta}\left(\frac{\lambda_{1,t}}{\lambda_{2,t}}\right)^{1-\theta}\left[\psi + (1-\psi)\left(\frac{\lambda_{2,t}h_{2,t}}{\lambda_{1,t}h_{1,t}}\right)^{\theta}\right]^{(1-\theta)/\theta} \tag{16}$$

Therefore, the relative reward of young labor with respect to old labor is:

$$\frac{w_{1,t}}{w_{2,t}} = \frac{\psi}{1-\psi}\left(\frac{h_{1,t}}{h_{2,t}}\right)^{\theta}\left(\frac{\lambda_{2,t}}{\lambda_{1,t}}\right)^{1-\theta} \tag{17}$$

Finally, equations (9) and (12) give us the expression of the interest rate:

$$R_t = \alpha_K f(\tau)\tilde{k}_t^{\alpha-1} \tag{18}$$

Clearing of goods and capital markets leads to the equilibrium condition $K_{t+1} = s_t N_t$ which is expressed in terms of per worker capital stock:

$$\tilde{k}_{t+1} \equiv \frac{K_{t+1}}{L_{t+1}} = \left[\psi\left(\lambda_{1,t+1}h_{1,t+1}\right)^{\theta} + (1-\psi)\left(\lambda_{2,t+1}h_{2,t+1}\right)^{\theta}\right]^{-1/\theta} s_t \tag{19}$$

Using (12) and (14), output per capita is defined as:

$$y_t = f(\tau)\tilde{k}_t^{\alpha}\frac{L_t}{2N} \tag{20}$$

Therefore, the economy can be summarized by equations (3), (5) to (8), (13) to (16), (18) to (20).

3 The steady-state

In this section, we investigate the influence of the environmental tax on the steady-state equilibrium. The model is quite complex and environmental tax affects output through multiple channels. Therefore, in 3.1 we investigate first a very simple version of the model to understand the basic mechanisms of transmission of the environmental tax and the specific role of the life-cycle health-profile. In 3.2, we investigate the full model.

3.1 Simplified model

We simplify the model by assuming exogenous labor, retirement, and investment in health. In this case, the only decision the young agent makes is about her saving. Thus we set $\varphi = 0$, $\gamma = 0$ and $\tau_t^w = 0$, and $m = \bar{m}$. First period labor supply becomes $\lambda_1 = 1 - \bar{m} = \bar{\lambda}_1$. From equation (7), second period labor becomes $\lambda_2 = 1$. To easily capture life-cycle health-profile, we assume that the health-status of the young $h_{1,t}$ is exogenous and denoted $\bar{h}$, and in the steady-state, we define $\Delta_h^\star \equiv h_2^\star/\bar{h}$. The steady-state equilibrium is such that $\left\{R_t, h_{2,t}, s_t, \tilde{k}_t, w_{1,t}, w_{2,t}, P_t\right\} = \left\{R^\star, h_2^\star, s^\star, \tilde{k}^\star, w_1^\star, w_2^\star, P^\star\right\}$, where variables with a * are constant. Thus, the steady-state equilibrium is defined by the following equations:

$$\tilde{k}^\star = \left[\psi\bar{\lambda}_1^\theta + (1-\psi)\left(\Delta_h^\star\right)^\theta\right]^{-1/\theta} s^\star/\bar{h} \tag{Ea*}$$

$$s^\star = \frac{\beta\bar{\lambda}_1 w_1^\star - w_2^\star/R^\star}{1+\beta} \tag{Eb*}$$

$$h_2^\star = H(\bar{m}) + \left[1 - d(P^\star)\right]\bar{h} \tag{Ec*}$$

$$w_1^\star = \psi \alpha_L f(\tau) \tilde{k}^{\star\alpha} \bar{h} \left[\psi + (1-\psi) \left(\frac{\Delta_h^\star}{\bar{\lambda}_1} \right)^\theta \right]^{(1-\theta)/\theta} \tag{Ed*}$$

$$w_2^\star = (1-\psi) \alpha_L f(\tau) \tilde{k}^{\star\alpha} h_2^\star \left(\frac{\bar{\lambda}_1}{\Delta_h^\star} \right)^{1-\theta} \left[\psi + (1-\psi) \left(\frac{\Delta_h^\star}{\bar{\lambda}_1} \right)^\theta \right]^{(1-\theta)/\theta} \tag{Ee*}$$

$$R^\star = \alpha_K f(\tau) \tilde{k}^{\star\alpha-1} \tag{Ef*}$$

$$P^\star = \sigma^{-1}(1-\alpha_K-\alpha_L) \left(\frac{1}{\tau} - 1 \right) f(\tau) \tilde{k}^{\star\alpha} L^\star \tag{Eg*}$$

$$L^\star = N \bar{\lambda}_1 \bar{h} \left[\psi + (1-\psi) \left(\frac{\Delta_h^\star}{\bar{\lambda}_1} \right)^\theta \right]^{1/\theta} \tag{Eh*}$$

From (Ec*), (Eg*) and the definition of $\Delta_h^\star$, we obtain:

$$\Delta_h^\star = H(\bar{m})\bar{h}^{-1} + 1 - d \left(\sigma^{-1}(1-\alpha_K-\alpha_L) \left(\frac{1}{\tau} - 1 \right) f(\tau) \tilde{k}^{\star\alpha} L^\star \right) \tag{Ec*-1}$$

Using equations (Ea*), (Eb*), (Ed*) to (Ef*), we obtain:

$$\left[\frac{\Delta_h^\star}{\bar{\lambda}_1} \right]^\theta = \tilde{\mathcal{D}}(R^\star) \equiv \left(\frac{\psi}{1-\psi} \right) \frac{\beta R^\star - \frac{\alpha_K}{\alpha_L}(1+\beta)}{1 + \frac{\alpha_K}{\alpha_L}(1+\beta)} > 0 \tag{21}$$

with $\tilde{\mathcal{D}}'(R^\star) > 0$.

Lemma 1. *When $\theta = 0$, there exists a unique interest rate $R^\star$, which is independent from τ.*

Proof. When $\theta = 0$, (21) becomes $\tilde{\mathcal{D}}(R^\star) = 1$. □

This Lemma identifies one special case in which environmental taxation does not affect the equilibrium steady-state: When the elasticity of substitution between young and old labor is one ($\theta = 0$), the health profile has no effect on the wage profile . As shown by equation (17), the wage profile solely depends upon the labor supply profile. In a life-cycle framework, the marginal productivity of labor for the young relative to the old logically reflects the relative health status of the young versus the old. If $\theta = 1$ (young and old labor are perfect substitutes), the relative health status is the only element explaining the wage ratio between the young and the old. In that case, the health profile is positively related to the wage profile. If $0 < \theta < 1$ (young and old labor are imperfect substitutes), labor supply choices by the young and the old influence the wage ratio. If $\theta = 0$ (young and old labor are unitary substitutes), the health status becomes irrelevant and labor supply choices at young and old ages are sole determinants of the wage ratio. If $\theta < 0$ (young and old labor are complements), the health profile is negatively related to the wage profile. Therefore, even if an increase in the pollution tax improves the health profile, it has no effect on the wage profile. As a result, in equilibrium, the income profile, saving, or the interest rate are not influenced by the pollution tax. In what follows, we will distinguish between the cases when $\theta = 0$, and $\theta \neq 0$.

Using (Eh*), aggregate efficient labor can be expressed as:

$$L^\star = N\bar{\lambda}_1\bar{h}\psi^{1/\theta}\left[\frac{1+\beta R^\star}{(1+\beta)\frac{\alpha_K}{\alpha_L}+1}\right]^{1/\theta} \tag{22}$$

From (Eg*), (Ef*) and (22), the stock of pollution at the steady-state is given by:

$$P^\star = \sigma^{-1}(1-\alpha_K-\alpha_L)\left(\frac{1}{\tau}-1\right)f(\tau)^{\frac{1}{1-\alpha}}\left(\frac{R^\star}{\alpha}\right)^{\frac{-\alpha}{1-\alpha}}N\bar{\lambda}_1\bar{h}\psi^{1/\theta}\left[\frac{1+\beta R^\star}{(1+\beta)\frac{\alpha_K}{\alpha_L}+1}\right]^{1/\theta} \tag{23}$$

and using (Ec*-1) and (21), we obtain the expression of the steady-state interest rate:

$$d\left(\left(\frac{1-\alpha_K-\alpha_L}{\sigma}\right)\left(\frac{1}{\tau}-1\right)f(\tau)^{\frac{1}{1-\alpha}}\left(\frac{R^\star}{\alpha}\right)^{\frac{-\alpha}{1-\alpha}}N\bar{\lambda}_1\bar{h}\psi^{1/\theta}\left[\frac{1+\beta R^\star}{(1+\beta)\frac{\alpha_K}{\alpha_L}+1}\right]^{1/\theta}\right) + \bar{\lambda}_1\tilde{\mathcal{D}}(R^\star)^{1/\theta} = 1+\frac{H(\bar{m})}{\bar{h}} \tag{24}$$

with $d'(\cdot) > 0$.

Proposition 1. *In the stationary equilibrium, there is a unique and positive interest rate such that:*

$$R^\star = \mathcal{R}(\tau)$$

with:

(i) $\mathcal{R}'(\tau) < 0$ *when old workers and young workers are complement in production* *(*$\theta < 0$*).*

(ii) $\mathcal{R}'(\tau) = 0$ *when old workers and young workers are unitary substitutes in production (*$\theta = 0$*, Cobb-Douglas case).*

(iii) $\mathcal{R}'(\tau) > 0$ *when old workers and young workers are non-unitary substitutes in production (*$0 < \theta \leq 1$*).*

Proof. (i) When $\theta < 0$, the LHS of equation (24) is decreasing in $R^\star$ and decreasing in τ. When the interest rate exists, it is unique and from the theorem of implicit function, $R^\star$ is decreasing in τ. (ii) When $\theta = 0$, $\mathcal{R}'(\tau) = 0$ is straightforward from Lemma 1. (iii) When $\theta \in]0, 1]$, under the sufficient condition $\alpha < 1/2$, which is a realistic value for this parameter, the LHS of (24) is increasing in $R^\star$ and decreasing in τ. Therefore, when the interest rate exists, it is unique and from the theorem of implicit function, $R^\star$ is increasing in τ. □

Proposition 1 states that when health evolves over the life-cycle, environmental taxation impacts the interest rate. Therefore, it influences saving. This general equilibrium mechanism is at the heart of a new effect of environmental taxation which we will identify as the "*health-saving effect*" in what follows.

Corollary 1.

(i) $P^\star = \mathcal{P}(\tau)$ *with* $\mathcal{P}'(\tau) < 0, \forall \theta \leq 1$*;*

(ii) $h_2^\star = \mathcal{H}(\tau)$ *with* $\mathcal{H}'(\tau) > 0, \forall \theta \leq 1$*;*

Proof. From (Ec*-1) and (21), $P^\star = 1 + H(\bar{m})/\bar{h} - \bar{\lambda}_1 \tilde{\mathcal{D}}(R^\star)^{1/\theta}$. Therefore, using Proposition 1, we find the influence of τ on $P^\star$. The impact of τ on $h_2^\star$ is given by (Ec*) and Corollary 1(i). □

The expression giving per capita output enables us to identify the channels of transmission of a tighter environmental tax, *I*, *II* and *III*. Per capita output is given by equations (20) and (22), and the results of the previous section:

$$y^\star = \left(\frac{1}{2}\right) \underbrace{f(\tau)^{1/(1-\alpha)}}_{I} \underbrace{\left(\frac{\alpha_K}{\mathcal{R}(\tau)}\right)^{\alpha/(1-\alpha)}}_{II} \left[\psi[\bar{\lambda}_1 \bar{h}]^\theta + (1-\psi)\underbrace{\mathcal{H}(\tau)^\theta}_{III}\right]^{1/\theta}$$

when $\theta \neq 0$, and:

$$y^\star = \left(\frac{1}{2}\right) \underbrace{f(\tau)^{1/(1-\alpha)}}_{I} \left(\frac{\alpha_K}{R^\star}\right)^{\alpha/(1-\alpha)} \left[\bar{\lambda}_1 \bar{h}\right]^\psi \underbrace{\mathcal{H}(\tau)^{1-\psi}}_{III}$$

when $\theta = 0$.

I produces the conventional crowding-out effect of private capital by the environmental tax. *II* produces the health-saving effect. It is explained by the fact that pollution affects the health profile over the life-cycle. The health profile influences the wage profile, which modifies the income profile and influences saving as a result. Indeed, pollution negatively affects h_2. A lower h_2 relatively to $\bar{h}$ corresponds to a higher first period wage relative to second period wage, as long as labor in the first and second periods of life are non-unitary substitutes ($0 < \theta \leq 1$). When the decrease in the wage profile is steeper, saving is higher and the interest rate is lower. An increase in the tax decreases pollution, makes the wage profile flatter (it decreases less), thereby decreasing saving and increasing the interest rate. When $\theta = 0$, the health profile has no effect on the wage profile and we retrieve Lemma 1. There is no health saving effect. When labor at old and young ages are complements ($\theta < 0$) the health saving effect is reversed because the wage profile is negatively related to the health profile (See equation 17). An increase in the pollution tax accentuates the decrease in wage between the two periods in life, thereby increasing saving and decreasing the interest rate. From Proposition 1, we identify a positive (negative) effect of environmental taxation on per capita output when young and old labor tend to be substitutes (complements). *III* produces the *productivity effect*. It reflects the standard productivity increase due to the positive effect of the environmental tax on health.

So far, we have identified the mechanisms through which environmental taxation influences the economy when the life-cycle characteristic of the health profile is taken into account. To identify the specific role of the health profile, we now suppose a flat health profile. We assume that the health-status is the same for young and old individuals, that is $h_2 = h_1 = h^\star$ and therefore $\Delta_h^\star = 1$. Because the evolution of the health status is given by equation (3), we obtain the endogenous expression of the health status in the steady-state:[10]

$$h^\star = \frac{H(\bar{m})}{d(P^\star)} \tag{25}$$

Therefore, equation (21) with $\Delta_h^\star = 1$ defines the steady-state interest rate:

$$\bar{\lambda}_1^{-\theta} = \left(\frac{\psi}{1-\psi}\right)\frac{\beta R^\star - \frac{\alpha_K}{\alpha_L}(1+\beta)}{1+\frac{\alpha_K}{\alpha_L}(1+\beta)} > 0 \tag{26}$$

From this expression, we obtain:

Proposition 2. *In the presence of a flat health profile, the steady-state interest rate $R^\star$ is independent from the environmental tax τ whatever the technology of production.*

Proof. Straightforward from (26). □

In this case, there are only two channels of transmission of the environmental tax on final output leading to the crowding-out effect and the productivity effect. Therefore, the key element of our model is the change in health over the life-cycle. Taking into consideration the change in health over the life-cycle enables us to identify the health-saving effect, which is absent from the past models in the literature.

3.2 Full model

We now verify if our results hold in the more realistic setting which includes choices of labor, retirement and investment in health. We investigate whether the full model

[10] Please note that a flat health profile does not mean an exogenous health status in the steady-state because it depends on $P^\star$, which is endogenous.

enables us to identify additional ways through which environmental taxes influence the economy.

The steady-state equilibrium is such that $\left\{m_t, R_t, h_{2,t}, s_t, \tilde{k}_t, w_{1,t}, w_{2,t}, P_t, \lambda_{1,t}, \lambda_{2,t}\right\} = \left\{m^\star, R^\star, h_2^\star, s^\star, \tilde{k}^\star, w_1^\star, w_2^\star, P^\star, \lambda_1^\star, \lambda_2^\star\right\}$, where variables with a * are constant, with $\tau^w = \tau^w_{t+1} = \tau^w$. Thus, the steady-state equilibrium is defined by the following equations:

$$\tilde{k}^\star = \left[\psi \lambda_1^{\star\theta} + (1-\psi)\left(\lambda_2^\star \Delta_h^\star\right)^\theta\right]^{-1/\theta} s^\star/\bar{h} \tag{E1*}$$

$$s^\star = \frac{\beta(1-\tau^w)\lambda_1^\star w_1^\star - \lambda_2^\star w_2^\star/R^\star - \tau^w \lambda_1^\star w_1^\star/R^\star}{1+\beta} \tag{E2*}$$

$$\lambda_2^\star = \frac{(1+\beta) - \gamma\beta \dfrac{\lambda_1^\star w_1^\star}{w_2^\star/R^\star}\left[1-\tau^w+\tau^w/R^\star\right]}{(1+\gamma)(1+\beta)-\gamma} \tag{E3*}$$

$$\lambda_1^\star = \frac{(1+\beta)(1-m^\star) - \varphi \dfrac{\lambda_2^\star w_2^\star/R^\star}{(1-\tau^w)w_1^\star}}{1+\beta+\varphi\left[1+\frac{\tau^w}{(1-\tau^w)R^\star}\right]} \tag{E4*}$$

$$\frac{h_2^\star}{H'(m^\star)} - \frac{\beta(1-\phi)}{\phi\varphi}(1-m^\star-\lambda_1^\star) = 0 \tag{E5*}$$

$$h_2^\star = H(m^\star) + \left[1-d(P^\star)\right]\bar{h} \tag{E6*}$$

$$w_1^\star = \psi\alpha_L f(\tau)\tilde{k}^{\star\alpha}\bar{h}\left[\psi + (1-\psi)\left(\frac{\lambda_2^\star}{\lambda_1^\star}\Delta_h^\star\right)^\theta\right]^{(1-\theta)/\theta} \tag{E7*}$$

$$w_2^\star = (1-\psi)\alpha_L f(\tau)\tilde{k}^{\star\alpha}h_2^\star\left(\frac{\lambda_1^\star}{\lambda_2^\star\Delta_h^\star}\right)^{1-\theta}\left[\psi + (1-\psi)\left(\frac{\lambda_2^\star\Delta_h^\star}{\lambda_1^\star}\right)^\theta\right]^{(1-\theta)/\theta} \tag{E8*}$$

$$R^\star = \alpha_K f(\tau)\tilde{k}^{\star\alpha-1} \tag{E9*}$$

$$P^\star = \sigma^{-1}(1-\alpha_K-\alpha_L)\left(\frac{1}{\tau}-1\right)f(\tau)\tilde{k}^{\star\alpha}L^\star \tag{E10*}$$

$$L^\star = N\lambda_1^\star\bar{h}\left[\psi + (1-\psi)\left(\frac{\lambda_2^\star\Delta_h^\star}{\bar{\lambda}_1}\right)^\theta\right]^{1/\theta} \tag{E11*}$$

From (E6*), (E10*) and the definition of $\Delta_h^\star$, we obtain:

$$\Delta_h^\star = H(m^\star)\bar{h}^{-1} + 1 - d\left(\sigma^{-1}(1-\alpha_K-\alpha_L)\left(\frac{1}{\tau}-1\right)f(\tau)\tilde{k}^{\star\alpha}L^\star\right) \tag{E5*-1}$$

Using equations (E1*), (E2*), (E7*) to (E9*), we obtain:

$$\left[\frac{\lambda_2^\star}{\lambda_1^\star}\Delta_h^\star\right]^\theta = \mathcal{D}(R^\star) \equiv \left(\frac{\psi}{1-\psi}\right)\frac{\left[\beta(1-\tau^w)R^\star - \tau^w - \frac{\alpha_K}{\alpha_L}(1+\beta)\right]}{1+\frac{\alpha_K}{\alpha_L}(1+\beta)} > 0 \tag{27}$$

with $\mathcal{D}'(R^\star) > 0$.[11]

Lemma 2. *In the presence of choices of labor, retirement and investment in health, Lemma 1 still holds.*

Proof. Straightforward from (27) and (E5*-1). □

Using (E11*), aggregate labor in efficiency units can be expressed as:

$$L^\star = N_t\lambda_1^\star\bar{h}\psi^{1/\theta}\left[\frac{1-\tau^w+\beta(1-\tau^w)R^\star}{(1+\beta)\frac{\alpha_K}{\alpha_L}+1}\right]^{1/\theta} \tag{28}$$

Using (E3*), (E8*), (E9*) and (27), we obtain:

$$\lambda_1^\star = \Lambda_1(R^\star, m^\star) \equiv \frac{(1-\tau^w)(1+\beta)(1-m^\star)}{(1-\tau^w)(1+\beta+\varphi)+\varphi\left[\tau^w + \left(\frac{1-\psi}{\psi}\right)\mathcal{D}(R^\star)\right]/R^\star} < 1-m^\star \tag{29}$$

with $\partial\Lambda_1(\cdot)/\partial R^\star < 0$ and $\partial\Lambda_1(\cdot)/\partial m^\star < 0$.[12]

Using (E5*), (E8*), (E9*) and (27), we obtain:

$$\lambda_2^\star = \Lambda_2(R^\star) \equiv \frac{(1+\beta)}{(1+\gamma)(1+\beta)-\gamma+\gamma\beta\left[1-\tau^w+\tau^w/R^\star\right]\left(\frac{\psi}{1-\psi}\right)R^\star/\mathcal{D}(R^\star)} \tag{30}$$

with $\partial\Lambda_2(\cdot)/\partial R^\star > 0$.[13]

Using previous results, we can express the stock of pollution in the long run:

$$P^\star = \Pi(R^\star, m^\star;\tau) \equiv \frac{(1-\alpha_K-\alpha_L)\alpha_K^{\frac{\alpha}{1-\alpha}}N\bar{h}\psi^{1/\theta}}{\sigma}\left[(1+\beta)\frac{\alpha_K}{\alpha_L}+1\right]^{-1/\theta}$$
$$\times\left(\frac{(1-\tau)\,f(\tau)^{\frac{1}{1-\alpha}}}{\tau}\right)R^{\star\frac{-\alpha}{1-\alpha}}\left[1-\tau^w+\beta(1-\tau^w)R^\star\right]^{1/\theta}\Lambda_1(R^\star, m^\star) \tag{31}$$

[11] Under a realistic choice of parameters, the term into brackets in the right-hand side of equation (27) is positive.

[12] Note that at the denominator of (29), $\left[\tau^w + \left(\frac{1-\psi}{\psi}\right)\mathcal{D}(R^\star)\right]/R^\star = \frac{\beta(1-\tau^w)-(1-\tau^w)(1+\beta)\frac{\alpha_K}{\alpha_L}/R^\star}{(1+\beta)\alpha_K/\alpha_L+1}$ is an increasing function of $R^\star$.

[13] Note that, at the denominator of (30), $R^\star/\mathcal{D}(R^\star)$ is a decreasing function of $R^\star$.

with $\Pi_1(R^\star, m^\star; \tau) < 0$ for $\theta < 0$ (respectively $\Pi_1(R^\star, m^\star; \tau) > 0$ for $\theta > 0$[14]), $\Pi_2(R^\star, m^\star; \tau) < 0$ and $\Pi_3(R^\star, m^\star; \tau) < 0$.

In the steady-state equilibrium, using (27), the allocation of time for health investment $m^\star$ is defined by equation (E5*):

$$\frac{\mathcal{D}(R^\star)^{1/\theta}}{\Lambda_2(R^\star)} = \frac{\beta(1-\phi)H'(m^\star)}{\phi\varphi\bar{h}} \left[\frac{(1-\tau^w)\varphi + \varphi\left[\tau^w + \left(\frac{1-\psi}{\psi}\right)\mathcal{D}(R^\star)\right]/R^\star}{(1-\tau^w)(1+\beta)} \right] \tag{32}$$

The following relation defines $m^\star$ as a function of $R^\star$:

$$m^\star = \Omega(R^\star), \tag{33}$$

with $\partial\Omega(R^\star)/\partial R^\star > 0$ when $\theta < 0$ (resp. $\partial\Omega(R^\star)/\partial R^\star < 0$ when $\theta > 0$).[15]

Equation (32) gives the expression of $\Delta_h^\star$ with respect to $R^\star$ and τ:

$$\Delta_h^\star = \mathcal{H}(R^\star; \tau) \equiv H\left(\Omega(R^\star)\right)/\bar{h} + \left[1 - d\left(\Pi(R^\star, \Omega(R^\star); \tau)\right)\right] \tag{34}$$

with $\mathcal{H}_1(R^\star; \tau) > 0$ when $\theta < 0$ (resp. $\mathcal{H}_1(R^\star; \tau) < 0$ when $\theta > 0$) and $\mathcal{H}_2(R^\star; \tau) > 0$ $\forall\theta \leq 1$.

Using equation (27) and previous results we can express the steady-state interest rate with respect to τ:

$$\left\{ \frac{\Lambda_2(R^\star)}{\Lambda_1(R^\star, \Omega(R^\star))} \mathcal{H}(R^\star; \tau) \right\}^\theta = \mathcal{D}(R^\star) \tag{35}$$

As a result, we obtain the following proposition:

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[14] Under the sufficient condition $\alpha \leq 1/2$, which is a realistic value for this parameter.

[15] The RHS is a decreasing function of $m^\star$ and is increasing in $R^\star$. When $\theta < 0$, the LHS is decreasing in $R^\star$ and therefore, $m^\star$ is unique and it increases in $R^\star$. Equation (32) may be written as:

$$\mathcal{D}(R^\star)^{1/\theta - 1} = \frac{\beta(1-\phi)H'(m^\star)}{(1-\tau^w)\phi\varphi\bar{h}} \frac{\left[(1-\tau^w)\varphi + \varphi\left[\tau^w + \left(\frac{1-\psi}{\psi}\right)\mathcal{D}(R^\star)\right]/R^\star\right]}{\left[(1+\gamma)(1+\beta) - \gamma\right]\mathcal{D}(R^\star) + \gamma\beta\left[1 - \tau^w + \tau^w/R^\star\right]\left(\frac{\psi}{1-\psi}\right)R^\star}.$$

The RHS is decreasing in $R^\star$ and the LHS is increasing in $R^\star$ when $0 < \theta \leq 1$. In this case, $m^\star$ is unique and decreasing in $R^\star$.

PROPOSITION 3. *In the presence of choices of labor, retirement and investment in health, Proposition 1 still holds. In the stationary equilibrium, there is a unique and positive interest rate such that:*

$$R^\star = \mathcal{R}(\tau)$$

with:

(i) $\mathcal{R}'(\tau) < 0$ *when old workers and young workers are complement in production* $(\theta < 0)$.

(ii) $\mathcal{R}'(\tau) = 0$ *when old workers and young workers are unitary substitutes in production* $(\theta = 0$, *Cobb-Douglas case).*

(iii) $\mathcal{R}'(\tau) > 0$ *when old workers and young workers are non-unitary substitutes in production* $(0 < \theta \leq 1)$.

Proof. See Appendix A. □

COROLLARY 2.

(i) $m^\star = \mathcal{M}(\tau)$ *with* $\mathcal{M}'(\tau) < 0, \forall\theta \leq 1;$

(ii) $\lambda_1^\star = \mathcal{L}_1(\tau)$ *with* $\mathcal{L}_1'(\tau) > 0$ *for* $\theta \leq 0;$

(iii) $\lambda_2^\star = \mathcal{L}_2(\tau)$ *with* $\mathcal{L}_2'(\tau) \gtreqless 0$ *for* $\theta \gtreqless 0$

(iv) $h_2^\star = \mathcal{H}(\tau)$ *with* $\mathcal{H}'(\tau) > 0, \forall\theta \leq 1;$

(v) $P^\star = \mathcal{P}(\tau)$ *with* $\mathcal{P}'(\tau) < 0, \forall\theta \leq 1;$

Proof. See Appendix B. □

In the presence of endogenous retirement and endogenous investment in health, we retrieve the effect of environmental taxation on pollution and the health profile (Corollary 1). However, endogenous investment in health has two new consequences. First, environmental taxation impacts per capita output through investment in health (Corollary 2i). Investment in health decreases (respectively increases) the level of physical capital in the steady-state when young and old labor are non-unitary substitutes (respectively complements). Second, it influences the labor choice by the young (Corollary 2ii). Third, it influences retirement decision by the old (Corollary 2iii). The environmental tax always results in an increase in young labor whether young and old labor are unitary substitutes or complements.

The expression giving per capita output enables us to identify the transmission mechanisms I, II, IIIa, IIIb, IIIc of a tighter environmental tax.

$$y^\star = \left(\frac{1}{2}\right) \underbrace{f(\tau)^{1/(1-\alpha)}}_{I} \underbrace{\left(\frac{\alpha_K}{\mathcal{R}(\tau)}\right)^{\alpha/(1-\alpha)}}_{II} \left[\psi[\bar{h}\underbrace{\mathcal{L}_1(\tau)}_{IIIb}]^\theta + (1-\psi)[\underbrace{\mathcal{H}(\tau)}_{IIIa}\underbrace{\mathcal{L}_2(\tau)}_{IIIc}]^\theta\right]^{1/\theta}$$

when $\theta \neq 0$, and:

$$y^\star = \left(\frac{1}{2}\right) \underbrace{f(\tau)^{1/(1-\alpha)}}_{I} \left(\frac{\alpha_K}{R^\star}\right)^{\alpha/(1-\alpha)} [\bar{h}\underbrace{\mathcal{L}_1(\tau)}_{IIIb}]^\psi [\lambda_2^\star \underbrace{\mathcal{H}(\tau)}_{IIIa}]^{1-\psi}$$

when $\theta = 0$.

In the general case, we retrieve the standard transmission mechanisms of environmental taxation (*I* and *IIIa*) respectively producing the crowding-out and productivity effects. We also retrieve mechanism *II* producing the health-saving effect, in the presence of endogenous investment in health. When labor supply and retirement choices are included in the model, we identify two new channels through which environmental taxation affects per capita output: Channel *IIIb* produces the "young labor effect" and channel *IIIc* the "retirement effect".

The table below provides a qualitative summary of the effects of environmental taxation on steady state output per capita.

	$\theta > 0$	$\theta < 0$	$\theta = 0$
$I \multimap$ crowding-out effect	$-$	$-$	$-$
$II \multimap$ health-saving effect	$-$	$+$	0
$IIIa \multimap$ young labor effect	?	$+$	$+$
$IIIb \multimap$ productivity effect	$+$	$+$	$+$
$IIIc \multimap$ retirement effect	$+$	$-$	0

Table 1: Effects on output per capita and substitutability of young and old labor

4 Numerical examples

We simulate the model in the general case. The first objective of this section is to get a sense of the magnitude and direction of the overall effects of environmental taxation on output and welfare. The second objective is to provide a decomposition of the effect on output into crowding-out, health-saving, productivity, young labor and retirement effects, such that:

$$\frac{\partial y^\star}{\partial \tau} = \underbrace{\frac{\partial y^\star}{\partial f(\tau)} f'(\tau)}_{crowding\text{-}out} + \underbrace{\frac{\partial y^\star}{\partial \mathcal{R}(\tau)} \mathcal{R}'(\tau)}_{health\ saving} + \underbrace{\frac{\partial y^\star}{\partial \mathcal{L}_1(\tau)} \mathcal{L}_1'(\tau)}_{young\ labor} + \underbrace{\frac{\partial y^\star}{\partial \mathcal{H}(\tau)} \mathcal{H}'(\tau)}_{productivity} + \underbrace{\frac{\partial y^\star}{\partial \mathcal{L}_2(\tau)} \mathcal{L}_2'(\tau)}_{retirement} \quad (36)$$

The third objective is to compare our results with those of the existing literature, which assumes a flat health profile, or perfect substitutability of young and old labor.

The numerical simulations are to be taken with caution considering the following facts. The model is very stylized, which is necessary to derive theoretical results, but it is limited in its ability to reproduce all the characteristics of the US economy. The value of a number of parameters of the model is uncertain. Therefore, we consider that a reasonable strategy is to adjust the parameter values to reproduce some of the most salient features of the US economy. We present the parameters in Tables 2, 3 and 4.

4.1 Calibration

The social security tax is 12.4% in the US. The rate of time preference β and the consumption weight ϕ are in line with the range of values considered by French (2005).

The parameters of the production function are standard. The value of θ reflects that young and old labor are imperfect complementary factors of production as in Hebbink (1993). Since the value of θ is uncertain and influences theoretical results, we provide numerical results for a wide range of values of θ (Table 9). The preference parameters are adjusted such that total leisure time is close to two thirds of individual time (Prescott 2004), and that the time spent on investment in health is within the range of 3.9 to 7.1%, which represents time of lost leisure solely due to bad health (French 2005). Welfare is simply measured by utility and we chose the value of B to obtain a positive level of welfare in the steady state.

There is great uncertainty regarding the values of the parameters of the investment in health and pollution functions . For the numerical simulations, the investment in health function is defined as $H(m) = \eta m^{\epsilon}$. We also define the pollution function as $d(P^{\star}) = \xi (P^{\star})^{-\varsigma}$. The health profile depends on investment in health and pollution functions. Our choice of parameters yields a decreasing initial health profile, indicating that second period health is two third of first period health. The literature does not provide guidance regarding the choice of the parameters of health and pollution functions. The best we can do is therefore to provide a sensitivity analysis of our numerical results to calibration of the investment in health and pollution functions (Table 13). In tables 2 to 4, we present the parameters and in Table 5, we show the steady-state values of the economy. The column in bold represents our base case.

Table 2: Preferences and health

φ	ϕ	β	γ	h_1	η	ϵ
2	0.6	0.3	0.01	0.3	1.3	0.5

Table 3: Production and labor substitutability

α_K	α_L	B	θ	ψ
0.3	0.6	500	-0.5	0.9

Table 4: Government, population growth, and pollution

τ	τ_w	ξ	σ	ς
2%	12.4%	0.02	0.9	0.5

Table 5: The economy and the environmental tax

τ	**2%**	4%	6%
$\Delta_h^\star$	**0.66**	0.85	0.96
Leisure*	**0.60**	0.613	0.615
$R^\star$	**4.82**	4.66	4.59
$\lambda_1^\star$	**0.35**	0.36	0.365
$\lambda_2^\star$	**0.0745**	0.0707	0.0690
m^*	**0.04**	0.02	0.019
s^*	**0.12**	0.129	0.131
y^*	**402.76**	386.95	372.65

4.2 Results

Table 6: Effects of environmental taxation (%)

	τ	**2%** $\rightarrow$ 4%	4% $\rightarrow$ 6%	6% $\rightarrow$ 8%
	Welfare	0.76	-0.23	-0.38
	Crowding-out	-11.12	-6.61	-4.72
+	Health-saving	1.71	0.74	0.42
+	Productivity	6.65	2.53	1.40
+	Young labor	1.32	0.43	0.22
+	Retirement	-1.18	-0.512	-0.30
=	*Output per capita*	*-2.62*	*-3.43*	*-2.98*

Our numerical results are consistent with the theoretical results of Table 1. Furthermore, with our choice of parameters, the negative effects of environmental taxation dominate the positive effects, resulting in a decrease in output. The crowding out effect and the retirement effect decrease output per capita. The health saving, young labor and retirement effects are smaller than the crowding-out and productivity effects, but they are quantitatively significant. The effect of environmental taxation on welfare depend on the initial tax level. When the environmental tax is increased from 2 to 4 percent, the effect on welfare is positive. Indeed, the positive marginal effect of the tax hike on health is large. As a consequence, the productivity effect is large and individuals significantly decrease labor supply in the second stage of life. By contrast, when the environmental tax is increased further, the effect on welfare becomes negative. The marginal effect on health and labor supply are much smaller relatively to the negative effect on consumption.

Table 7: Effects of environmental tax with flat health profile (%)

	τ	**2**% → 4%	4% → 6%	6% → 8%
	Welfare	4.97	2.77	1.94
	Crowding-out	-11.12	-6.61	-4.72
+	Health-saving	-0.06	-0.03	-0.02
+	Productivity	3.73	2.14	1.53
+	Young labor	0.73	0.33	0.21
+	Retirement	0.04	0.02	0.01
=	*Output per capita*	-6.69	-4.14	-2.99

In Table 7, we simulate the effect on macroeconomic variables in the case when the health profile is flat (keeping investment in health m endogenous). When the health profile is flat, recall that the health status improves with the environmental tax *over the entire life-cycle*, shifting the health profile. Contrary to the base case, there is no change in the health profile over the life-cycle. Our numerical simulations indicate that, with the tax increasing from 2 to 4 percent, if the change in health profile between the two periods in life is ignored, the decrease in output per capita is overstated by about four percentage points compared to the base case. Additionally, the effect on welfare is overstated by about four percentage points compared to the base case when the change in the health profile is ignored. When the tax is increased further, the difference between the base case (with a life cycle health profile) and the case with a flat health profile becomes smaller as all marginal effects become smaller. Specifically, ignoring the life-cycle changes in the health profile leads to a large understatement of all the positive effects of environmental taxation. Comparing Table 6 and Table 7, we find that the decrease in output per capita is not the result of an overwhelmingly important crowding-out effect but the cumulated result of several general equilibrium effects related to life-cycle choices.

Table 8: Double environmental taxation and labor substitutability

	τ: 2%→ 4% θ	-1	**-0.5**	0.5	2/3
	Welfare	2.02	0.76	1.14	1.08
	Crowding-out	-11.12	-11.12	-11.12	-11.12
+	Health-saving	3.84	1.71	-0.29	-0.05
+	Productivity	9.47	6.65	0.32	0.02
+	Young labor	1.06	1.32	4.08	4.12
+	Retirement	-2.27	-1.18	0.27	0.04
=	*Output per capita*	1.00	-2.62	-6.75	-6.98

In Table 8, we show that when the environmental tax is doubled from two to four percent, our numerical results change depending on the value assigned to the degree of substitutability θ between young and old labor. Consistent with our theoretical section, the numerical results indicate that whether health saving and retirement effects are positive or negative depends on the degree of substitutability between young and old labor. As shown in the table, this significantly affects the analysis regarding the economic effects of the environmental tax. If young and old labor are close to be perfect substitutes, the negative effect of environmental taxation on output per capita is overstated by more than four percentage points compared to our base case. By assuming perfect substitution of young and old labor, past contributions are likely to significantly overstate the negative effect of environmental taxation.

Indeed, when young and old labor are perfect substitutes, an increase in the environmental tax leads to a large reallocation of labor supply toward the second stage of life as individuals are relatively healthier. At the same time, individuals increase labor supply during this stage of their lives, resulting in positive retirement effect. On the contrary, when young and old labor are imperfect complements, the elderly decrease their labor supply significantly and individuals compensate for the decline in income over the life-cycle by increasing saving. Overall productivity gains are large because individuals invest in health to be more productive as old workers. The inability of workers to reallocate labor supply over different stages of life leads to other compensation mechanisms. Therefore, when young and old labor are assumed to be perfect substitutes, the literature tends to overstate the negative effects of environmental taxation on output per capita, understate the productivity effect and ignore other important general equilibrium effects.

4.3 Robustness

In this section, we study the sensitivity of our results to changes in parameters. With our initial calibration, the obtained level of health in the second period of life is 66% of the level of health in the first period of life. In Table 9, we simulate the model with different values of $\bar{h}$, to obtain a steeper ($\Delta_h = 0.33$) or flatter ($\Delta_h = 0.99$) health profile in the initial equilibrium. We find that in the health profile is initially steeper (flatter), the increase in environmental taxation has a large positive (negative) effect on output and welfare. Indeed, in that case, environmental taxation strongly (slightly) improves the health profile, enhancing (minimizing) all benefits of environmental taxation on the economy. As expected, the effect on productivity is increased the most (least) and all other new life-cycle effects are magnified (reduced). The crowding-out effect is unchanged since it does not relate life-cycle elements.

Table 9: Environmental taxation and health profile

	Initial Δh $\rightharpoonup$	0.33	0.66	0.99
	Welfare	3.74	0.76	-0.64
	Crowding-out	-11.12	-11.12	-11.12
+	Health-saving	3.14	1.71	1.07
+	Productivity	14.33	6.65	3.53
+	Young labor	3.89	1.32	0.84
+	Retirement	-2.09	-1.18	-0.76
=	Output per capita	8.16	-2.62	-6.45

Because of the lack of empirical knowledge about the value of the parameters of health and pollution functions, we simulate the model for different values of those parameters. In Table 10 and 11, we show that numerical results are very sensitive to the assessment of pollution and health functions. Following an increase in environmental taxation, when ς is small, the positive effects of pollution reduction on health become smaller. In the same way, when ϵ is small, investment in health has a smaller effect on health. As a consequence, in both cases, the positive effects of pollution reduction become much smaller. Life-cycle effects of environmental taxation decrease and the dominant effect becomes the crowding-out effect, resulting in a drop in output and welfare.

Table 10: Environmental taxation and pollution function

	τ: 2%$\rightharpoonup$ 4% ς	0.01	**0.5**	0.8
	Welfare	-2.60	0.76	200.39
	Crowding-out	-11.12	-11.12	-11.12
+	Health-saving	0.0006	1.71	2.32
+	Productivity	0.0018	6.65	36.64
+	Young labor	0.0002	1.32	43.00
+	Retirement	-0.0004	-1.18	-1.36
=	*Output per capita*	-11.12	-2.62	69.48

In Table 10, we increase the value of ς to accentuate the non-linearity of the pollution function. The numerical results are very sensitive to the value of this parameter. When the value of ς is high, the productivity and young labor effects dominate the other effects and the impact of the tax hike on output per capita and welfare are large and positive.

Table 11: Environmental taxation and health function

	τ: 2%→ 4%			
	ϵ	0.1	**0.5**	0.9
	Welfare	-1.2	0.76	2.57
	Crowding-out	-11.12	-11.12	-11.12
+	Health-saving	0.85	1.71	1.27
+	Productivity	2.86	6.65	6.07
+	Young labor	0.18	1.32	3.71
+	Retirement	-0.64	-1.18	-0.80
=	*Output per capita*	-7.87	-2.62	-0.87

In Table 12, we simulate the doubling of the environmental tax for different values of the social security tax. In the absence of a social security tax, the retirement system is not a PAYG but a funded system, in which workers save for their own retirement. Our numerical results indicate that the elimination of the PAYG would attenuate the negative impact of environmental taxation on output and would enhance its positive effect on welfare respectively by about 1 and 0.38 percentage points. This result is consistent with the decrease in the overall tax burden resulting from the decrease in the social security tax.

Table 12: Environmental taxation and Social Security tax

	τ: 2%→ 4% τ^w	0%	10%	12.4%
	Welfare	1.04	0.81	0.76
	Crowding-out	-11.12	-11.12	-11.12
+	Health-saving	2.00	1.76	1.71
+	Productivity	7.00	6.72	6.65
+	Young labor	1.65	1.38	1.32
+	Retirement	-1.17	-1.19	-1.18
=	*Output per capita*	-1.65	-2.45	-2.62

5 Conclusion

In this paper, we study the economic effect of environmental taxation when pollution impacts morbidity. In contrast with the existing literature, we propose a model that takes into consideration the interaction between pollution and health over the life-cycle and its consequences on individual optimal choices. We show that when the interaction between pollution and health over the life-cycle is captured, the effect of environmental taxation on health are not limited to the traditional negative crowding-out effect and positive productivity effect. We identify several new general equilibrium effects, the health saving effect, the young labor effect and the retirement effect. We also show that those new effects can positively or negatively influence output per capita depending on the degree of complementarity or substitutability between young and old labor. We also show that the health saving effect does not appear when models ignore the life-cycle characteristics of the health profile.

Our numerical simulations suggest that ignoring those life-cycle effects may lead to large overstatement of the negative effects of environmental taxation on output and understatement of the positive effect on welfare. The sensitivity of our results to the specifications of pollution, health functions and degree of substitutability of labor across periods of life calls for further empirical investigation to accurately assess the effect of environmental taxes on output and welfare.

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A Proof of Proposition 3

First,

$$\frac{\Lambda_2(R^\star)}{\Lambda_1(R^\star, m^\star)} = \frac{[(1-\tau^w)(1-m^\star)]^{-1}\left\{(1-\tau^w)(1+\beta+\varphi)+\varphi\left[\tau^w+\left(\frac{1-\psi}{\psi}\right)\mathcal{D}(R^\star)\right]/R^\star\right\}}{(1+\gamma)(1+\beta)-\gamma+\gamma\beta\left[1-\tau^w+\tau^w/R^\star\right]\left(\frac{\psi}{1-\psi}\right)R^\star/\mathcal{D}(R^\star)} \tag{A.1}$$

is increasing in $R^\star$ $\forall \theta \leq 1$.

When $\theta = 0$, Lemma 1 applies. When $\theta < 0$, it is straightforward that the LHS of equation (35) is decreasing in $R^\star$ and decreasing in τ. Because the RHS of equation (35) is increasing in $R^\star$, when the steady-state $R^\star$ exists, it is unique and from the theorem of implicit function it is decreasing in τ.

When $\theta \in]0,1]$, equation (35) may be written as:

$$\mathcal{H}(R^\star;\tau) = \mathcal{D}(R^\star)^{1/\theta}\frac{\Lambda_1(R^\star, m^\star)}{\Lambda_2(R^\star)} \tag{A.2}$$

and from equation (E5*), using (27) and (A.1), we have

$$\mathcal{D}(R^\star)^{1/\theta}\frac{\Lambda_1(R^\star, m^\star)}{\Lambda_2(R^\star)} = \frac{\beta(1-\phi)H'(m^\star)(1-m^\star)}{\phi\varphi\bar{h}}\times \left[1-\frac{(1-\tau^w)(1+\beta)}{(1-\tau^w)(1+\beta+\varphi)+\varphi\left[\tau^w+\left(\frac{1-\psi}{\psi}\right)\mathcal{D}(R^\star)\right]/R^\star}\right]$$

which is decreasing in $m^\star$ and increasing in $R^\star$. From equation (33) we know that $m^\star$ is a decreasing function of $R^\star$ when $\theta \in]0,1]$. It comes that the RHS of equation (A.2) is increasing in $R^\star$. Because the LHS of equation (A.2) is decreasing in $R^\star$ and increasing in τ, when the steady-state $R^\star$ exists, it is unique and from the theorem of implicit function it is increasing in τ.

B Proof of Corollary 2

For *(i)* when $\theta \leq 0$, it is straightforward from equation (32) that $H'(m^\star)$ is equal to an expression decreasing function of $R^\star$. Because $H'(m^\star)$ is decreasing in $m^\star$, it means that $m^\star$ is equal to an expression increasing in $R^\star$. Therefore, from Proposition 3, $m^\star$ is decreasing in τ. When $\theta > 0$, from equation (A.2), $H'(m^\star)$ is equal to an expression increasing in $R^\star$. Because $H'(m^\star)$ is decreasing in $m^\star$, it means that $m^\star$ is equal to an expression decreasing in $R^\star$. From Proposition 3, $m^\star$ is therefore decreasing in τ.

For *(ii)*, it is straightforward that $\lambda_1^\star$ is increasing in τ when $\theta < 0$, from equation (29), Corollary 2(i) and Proposition 3. When $\theta = 0$, Lemma 1 applies, therefore from equation (29) and Corollary 2(i) $\lambda_1^\star$ is increasing in τ. When $\theta > 0$, equation (E5*) enables us to write:

$$\lambda_1^\star = [1 - m^\star] - \phi\varphi \frac{H(m^\star) + [1 - d(P^\star)]\,\bar{h}}{\beta(1-\phi)H'(m^\star)}$$

$\lambda_1^\star$ is negatively influenced by $m^\star$ which is decreasing in τ and is positively influenced by $P^\star$ which is decreasing in τ (see below). Without further assumptions about parameters it is not possible to find analytically the influence of τ on $\lambda_1^\star$ when $\theta > 0$.

For *(iii)* see Proposition 3 and equation (28).

For *(iv)*, the RHS of equation (32) is $h_2^\star$. When $\theta \geq 0$, from Proposition 3, Corollary 2(i) and the fact that $\mathcal{D}(R^\star)/R^\star$ is increasing in $R^\star$, the RHS is increasing in τ, therefore $h_2^\star$ is increasing in τ when $\theta \geq 0$. When $\theta < 0$, from equation (35), Proposition (3) and Corollary 2(ii)-(iii), $h_2^\star$ is increasing in τ.

For *(v)*, from equation (E6*), we have $d(P^\star) = H(m^\star) - h_2^\star$. Because $H(\cdot)$ is increasing in $m^\star$, from Corollary 2(i)-(iv) and the fact that $d'(\cdot) > 0$, $P^\star$ is decreasing in τ $\forall\theta \leq 1$.