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► **To cite this version:**

Nathalie Mathieu-Bolh, Xavier Pautrel. Environmental taxation, health and the life-cycle. 2013.  
hal-00990256v1

**HAL Id: hal-00990256**

**<https://hal.science/hal-00990256v1>**

Preprint submitted on 14 Jan 2014 (v1), last revised 13 May 2014 (v2)

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## Environmental taxation, health and the life-cycle

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2014/03

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# Environmental taxation, health and the life-cycle\*

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July 4, 2013

## Abstract

We build a model that takes into consideration the evolution of health over the life cycle and its consequences on individual optimal choices. In this framework, the effect of environmental taxation are not limited to the traditional negative crowding-out and positive productivity effects. We show that environmental taxation generates new general equilibrium effects ignored by previous contributions. Indeed, as the environmental tax improves the health profile over the life-cycle, it influences saving, labor supply, retirement and investment in health. We also show that whether those general equilibrium effects are positive or negative for the economy crucially depends on the degree of substitutability between young and old labor. We complete our theoretical analysis with numerical examples. Within the range of our parameters, it appears that ignoring those general equilibrium effects results in significantly understating the negative of environmental taxation on output per capita and welfare.

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\*Thanks go to the participants of the 2013 European Association of Environmental and Resources Economists conference for their helpful comments.

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# 1 Introduction

What is the economic effect of environmental taxation when pollution impacts health? Whereas a vast number of theoretical contributions address this question taking the effect of pollution on mortality into consideration, fewer include the effect of pollution on morbidity. Furthermore, those contributions do not model the interaction between pollution and health over the life-cycle. By contrast, we build a model that takes into consideration the evolution of health over the life cycle and its consequences on individual optimal choices. In this framework, the effect of environmental taxation are not limited to the traditional negative crowding-out and positive productivity effects. We show that environmental taxation generates new general equilibrium effects ignored by previous contributions. Indeed, as the environmental tax improves the health profile over the life-cycle, it influences investment in health as well as saving, labor supply and retirement choices. We also show that whether those general equilibrium effects are positive or negative for the economy crucially depends on the degree of substitutability between young and old labor. We complete our theoretical analysis with numerical examples. Within the range of our parameters, it appears that ignoring those new general equilibrium effects results in significantly understating the negative effect of environmental taxation on output and welfare.

The effect of pollution on morbidity is well established in the epidemiological literature. Pollution favors the development of certain chronic diseases, especially cancer, cardiovascular disease and respiratory diseases, that have durable detrimental impacts in terms of illness and disability.<sup>1</sup> According to Briggs (2003) about 8-9% of the total disease burden may be attributed to pollution in developed countries. While the direct and indirect impacts of illness on productivity is the object of growing interest,<sup>2</sup> the overall fraction of pollution-related health problems that affect productivity is unknown. Nevertheless, the empirical literature focuses on some specific types of pollution and finds that the negative effect of pollution on productivity is quantitatively significant. Hausman et al. (1984), who estimated that a 1 unit ( $\mu g/m^3$ ) increase in particulate matter pollution increases lost work days by 0.7%. Hansen and Selte (2000) show that sick leaves are significantly linked to particulate matter pollution (PM10). Hanna and Oliva (2011) find that a one percent increase in sulfur dioxide results in a 0.61 percent decrease in the hours worked in Mexico city. Graff Zivin and Neidell (2012) find that a 10 ppb decrease in Ozone concentrations increases worker productivity by 4.2%.

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<sup>1</sup>See Brauer et al. (2011); Ruckerl et al. (2011); Gold and Mittleman (2013); Rajagopalan and Brook (2012); Brook et al. (2010) regarding air pollution, Paulu et al. (1999), Valent et al. (2004) for water pollution and Nadal et al. (2004), Chen and Liao (2005), Schuhmacher and Domingo (2006) for industrial pollution.

<sup>2</sup>See Bloom et al. (2004), Devol and Bedroussian (2007) and Zhang et al. (2011), for example.

Thus, the theoretical literature has explored the effect of environmental policy taking into consideration the link between pollution and health in infinite horizon models, with the idea that productivity gains and decreased medical expenditure related to pollution reduction generally mitigate the costs of environmental policies (See Mayeres and Van Regemorter (2008), Huh-tala and Samakovlis (2007), and Ostblom and Samakovlis (2007)). Williams (2002) proposes a general equilibrium model in which reduced pollution increases health or productivity. By contrast with the previously cited studies, the author finds that the resulting effects on labor supply can magnify or diminish the benefits of reduced pollution. Williams (2003) further shows that interactions with health effects from pollution reduce the optimal environmental tax rather than increasing it as in Schwartz and Repetto (2000). In a growth model with research and development, Aloi and Tournemaine (2011) find that environmental tax has positive effect on growth and welfare through productivity gains and reallocation of resources toward R&D.

By nature, those models ignore the interactions between pollution and morbidity over the life-cycle and are thereby missing some of the channels through which environmental policy affects the economy. It is however crucial to understand this interaction on two levels that have not been studied in the literature on environmental taxation. First, the health profile is susceptible to be modified by pollution. Indeed, pollution contributes to chronic diseases, which primarily affect people age 15 to 59 according to the OMS. The health profile influences the productivity profile (Lakdawalla et al., 2004; Bhattacharya et al., 2008; Perlkowski and Berger, 2004) and also weights on life-cycle saving, labor and retirement (Dwyer and Mitchell, 1999; Deschryvere, 2006, amongst others). Second, as pointed out by Cropper (1981), individuals investment in health during the first part of their lives interacts with pollution, which modifies their health profile. Thus, a decreased investment in health can potentially offset some of the benefits of environmental taxation on health.<sup>3</sup>

Therefore, we propose to study the effect of environmental taxation in a two-period overlapping generations model exhibiting three main features. First, we explicitly model the health status as a stock that increases with investment in health and decreases with pollution. Second, we make the link between health and productivity over the life-cycle explicit. Third, we model retirement decisions, allowing individuals to chose whether to continue to work or retire during the second stage of their lives. Fourth, we allow for labor by the young and the old to be complements or substitutes.<sup>4</sup>

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<sup>3</sup>It is important to note that our focus is on the time individuals derive from leisure to invest in health (rather than on the amount they spend on health services), which further justifies the fact that our model captures investment in health in the first part of individuals lives (rather than the second), and its interaction with environmental taxation.

<sup>4</sup>See Kalwij et al. (2010); Gruber et al. (2010); Gruber and Milligan (2010) for empirical evidence on the imperfect substitutability between young and old labor in developed economies.

Thus, our work also fills a void in the overlapping generations literature, which does not endogenize the link between environment, health and productivity to study environmental taxation. Indeed, previous contributions (Mathieu-Bolh and Pautrel (2011) and Raffin (2012)) assume an ad-hoc link between pollution and productivity and do not model health.<sup>5</sup> Our two-period framework also contrasts with Pautrel (2012) who assumes a constant health profile, does not allow individuals to choose between work or retirement in the second period of their lives, and does not explore various characteristics of young and old labor.

We present our theoretical results using three variants from the simplest to the most complete one. We first describe the steady state effect of environmental taxation without a choice of health expenditure or a PAYG system. Second, we introduce a retirement choice in the model and the PAYG system. Third, we account for the fact that individuals invest in health. Using this approach, we are able to provide a decomposition of the effects of environmental taxation on output. The main results of the paper are as follows.

1. We identify new effects of environmental taxation on output. A first new effect is the "health-saving effect". The environmental tax limits the decline in health over the life-cycle. When old and young labor are substitutes (complements), the tax increases (decreases) the steady state interest rate, decreasing (increases) saving and output. Second, our model captures the effects of environmental taxation on aggregate efficient labor, that are channeled through several new general equilibrium effects, the "young and old labor supply effects" and the "health investment effect". Those effects modify the response of aggregate efficient labor. This result is in sharp contrast with a large fraction of the existing literature, where it is generally assumed that environmental taxation only positively impacts workers' productivity. By contrast, in our model, increased productivity resulting from better health leads agents to modify their labor supply, retirement decisions and investment in health over the life cycle.
2. Furthermore, we compare our results with simpler frameworks and show that if the life-cycle characteristic of the health profile is ignored and investment in health is supposed to be exogenous, the new general equilibrium effects disappear. If investment in health is endogenous and the health profile is flat, the new general equilibrium effects are modified. In both cases, numerical simulations indicate that past models understate the effect of environmental taxation on output per capita

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<sup>5</sup>In Mathieu-Bolh and Pautrel (2011), an exogenous age-productivity profile is introduced but it does not influence individual decisions. It only influences aggregate variables through intergenerational redistribution. By contrast, in our framework, health and retirement decisions are endogenous. Thus, environmental taxation influences the health profile, individual decisions, and thereby the aggregate economy.

and welfare, even if they account for a labor supply response but ignore life-cycle critic elements.

3. We show that the effect of environmental taxation on output crucially depend on the characteristics of old and young labor. If investment in health is exogenous and perfect substitution of young and old labor is assumed, like in the existing literature, the positive effect of environmental taxation on output per capita channeled through aggregate efficient labor are reduced. If, by contrast, old and young labor tend to be complements, the effect of environmental taxation on output are magnified. If investment in health is endogenous, it interacts with labor supply choices and the previous result is modified. Overall, our numerical simulations indicate that when young and old labor are assumed to be perfect substitutes, the negative effects of environmental taxation on output per capita and welfare are overstated.

The paper is organized as follows. In the first section, we present the model. In the second section we describe the steady state for the three variants, discuss our results and present numerical examples.

## 2 The model

We consider an infinite horizon economy where agents live two periods. At each date  $t$ , a population of old individuals of size  $N_{t-1}$  coexist with a population of young individuals of size  $N_t = (1 + n)N_{t-1}$ ,  $n \geq 0$ .

### 2.1 Individuals

Individuals work during the two periods of their lives. Each young agent is endowed with one unit of time, supplying  $\lambda_{1,t} \in ]0, 1[$  in final production, using  $m_t \in ]0, 1[$  as an investment in healthcare activities to improve her health status in the second period of her life, and using the remaining time  $1 - m_t - \lambda_{1,t}$ , as leisure. Therefore, when young, she earns a wage income  $\lambda_{1,t}w_{1,t}$ , where  $w_{1,t}$  is the efficient wage. This income is used to consume  $c_{1,t}$ , to save  $s_t$  or to pay retirement benefits ( $\tau_t^w w_{1,t}$  with  $\tau_t^w \in (0, 1)$ ):

$$(1 - \tau_t^w)\lambda_{1,t}w_{1,t} = c_{1,t} + s_t \quad (1)$$

During the second period, each agent is also endowed with one unit of time, supplying  $\lambda_{2,t+1} \in (0, 1)$  in final production and the remaining time  $1 - \lambda_{2,t+1}$  as retired. When old, she earns a wage income  $\lambda_{2,t+1}w_{2,t+1}$ . She also receives the revenue of her first period saving and retirement benefits  $q_{t+1}$ . Therefore, her second period consumption is:

$$c_{2,t+1} = R_{t+1}s_t + (1 - \tau_{t+1}^w)w_{2,t+1}\lambda_{2,t+1} + (1 - \lambda_{2,t+1})q_{t+1}$$

with  $R_{t+1} \equiv 1 + r_{t+1}$ . Assuming a pay-as-you-go system, the retirement benefits paid to retirees in  $t$  must be equal to the contributions by workers (who include the old born in  $t$  and the young born in  $t + 1$ ):

$$(1 - \lambda_{2,t+1})q_{t+1} = \tau_{t+1}^w [\lambda_{2,t+1}w_{2,t+1} + (1 + n)l_{t+1}w_{1,t+1}]$$

Therefore the budget constraint of an old agent born in  $t$  is :

$$c_{2,t+1} = R_{t+1}s_t + [\lambda_{2,t+1}w_{2,t+1} + \tau_{t+1}^w(1 + n)\lambda_{1,t+1}w_{1,t+1}] \quad (2)$$

Individuals born in  $t$  with a health-status denoted  $h_{1,t}$ . The health status of an agent born in period  $t$  evolves between period  $t$  and period  $t + 1$  depending on two opposing forces (Aisa and Pueyo, 2004). On the one hand, biological processes involve a natural decay in health as time passes (Grossman, 1972). Following Cropper (1981), we further assume that health depreciates over time as a function of the stock of pollution (denoted  $P_t$ ). On the other hand, the health status improves with the investment  $m$  made by the young agent. Therefore, for an agent born in  $t$ , the individual health-status evolves from period  $t$  to period  $t + 1$  according to:

$$h_{2,t+1} - h_{1,t} = H(m_t) - \delta(P_t)h_{1,t} \quad (3)$$

with  $\partial H(m_t)/\partial m_t > 0$  and  $\partial^2 H(m_t)/(\partial m_t)^2 < 0$ . The positive impact of investment in health is captured by the first term in the right-hand side. Therefore, investment in health makes the health profile steeper. The detrimental influence of pollution on health appears in the depreciation rate function  $\delta(P_t)$ . To capture the possible threshold effect of pollution in health, we model the function  $\delta(\cdot)$  as:<sup>6</sup>

$$\delta(P_t) \equiv \frac{d_0 + \xi P_t^\varsigma}{1 + d_1 P_t^\varsigma} \quad (4)$$

with  $\varsigma, d_1 \geq 0, d_0 \geq 0$  such that  $d_0 d_1 < \xi$ ,  $\lim_{P \rightarrow \infty} \delta(P) = d_0$ ,  $\delta'(P) = \frac{\varsigma(\xi - d_0 d_1)P^{\varsigma-1}}{(1 + d_1 P^\varsigma)^2} > 0$ ,  $\delta''(P) < 0$  if  $\varsigma \leq 1$  and  $\delta''(P) \gtrless 0$  for any  $P \gtrless \left[ \frac{\varsigma - 1}{(1 + \varsigma)d_1} \right]^{1/\varsigma}$  if  $\varsigma > 1$ .

The lifetime utility of the representative agent born in  $t$  is:

$$U_t = \log \left( (c_{1,t}(1 - m_t - \lambda_{1,t})^\varphi)^\phi h_{1,t}^{1-\phi} \right) + \beta \left[ \log \left( c_{2,t+1}^\phi h_{2,t+1}^{1-\phi} \right) + \gamma \log(1 - \lambda_{2,t+1}) \right] \quad (5)$$

$\beta$  is the time-preference parameter,  $\varphi$  captures the preference for leisure by the young and  $\gamma$  captures the preference for leisure by the old (or retirement).

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<sup>6</sup>See Fanti and Gori (2011).



The parameter  $\phi \in (0, 1)$  captures the influence of the health-status in utility.

The maximization of (5) subject to (1), (2) and (3) yield saving:

$$s_t = \frac{\beta [(1 - \tau_t^w)\lambda_{1,t}w_{1,t}] - [\lambda_{2,t+1}w_{2,t+1} + \tau_{t+1}^w(1+n)\lambda_{1,t+1}w_{1,t+1}]/R_{t+1}}{1 + \beta}, \quad (6)$$

Saving reflects the difference between the after-tax income available in the economy in the first period and the present value of income in the second period. It reflects that a high income in the second period requires less saving. In the first period, the after tax income represents the income of the young. In the second period, it represents the income of the old, which encompasses labor and retirement income. Retirement income is proportional to the time retirees spent working while they were young. The later the old retire ( $\lambda_2$  high), the higher their income and the lower their saving.

If the presence of a PAYG system, the old receive retirement benefits whereas the young pay retirement contributions. Intergenerational redistribution that takes place through the PAYG influences the saving rate. Contributions to the retirement system are influenced by the choice of labor versus investment in health by the young. Therefore, other things equal, if the young's investment in health decreases over time, retirement benefits decrease and saving decreases.

Utility maximization also give labor supplied by the young in final output:

$$\lambda_{1,t} = \frac{(1 + \beta)(1 - m_t)}{1 + \beta + \varphi} - \frac{\varphi \left[ \lambda_{2,t+1} \frac{w_{2,t+1}/R_{t+1}}{w_{1,t}} + \tau_{t+1}^w(1+n)\lambda_{1,t+1} \frac{w_{1,t+1}/R_{t+1}}{w_{1,t}} \right]}{(1 - \tau_t^w)(1 + \beta + \varphi)}, \quad (7)$$

and labor supplied by the old:

$$\lambda_{2,t+1} = \frac{(1 + \beta)\phi - \gamma\beta \left[ \tau_{t+1}^w(1+n) \frac{\lambda_{1,t+1}w_{1,t+1}}{w_{2,t+1}} + (1 - \tau_t^w) \frac{\lambda_{1,t}w_{1,t}}{w_{2,t+1}/R_{t+1}} \right]}{(\phi + \gamma)(1 + \beta) - \gamma} \quad (8)$$

The time spent working  $\lambda_2$  rather than retiring is determined by the PAYG in the first place. Indeed the term in brackets simply reflects the social security wealth (difference between retirement benefits and contributions). In the second place, the decision to retire depends on relative wages across ages and periods in life. In the third place, if investment by the young is large, the old spends more time working. This is due to the fact that they are healthier and receive less retirement benefits in that case.

Finally, optimal individual health expenditure is given by:

$$\frac{H(m_t) + (1 - \delta(P_t))h_{1,t}}{H'(m_t)} - \frac{\beta(1 - \phi)}{\phi\varphi}(1 - m_t - \lambda_{1,t}) = 0 \quad (9)$$

Health expenditure  $m_t$  is positively related to the level of pollution  $P_t$ , negatively related to labor supply  $\lambda_{1,t}$  and the health status  $h_{1,t}$  of the young.

## 2.2 Firms

There is a continuum of identical firms that operate under perfect competition. They produce a final good  $Y_t$  using the production function:

$$Y_t = BK_t^{\alpha_K} L_t^{\alpha_L} E_t^{1-\alpha_K-\alpha_L}$$

where  $K_t$  is the amount of physical capital,  $L_t$  is aggregate efficient labor,  $E_t$  is the flow of pollution emissions and  $\epsilon, \alpha_K, \alpha_L \in (0, 1)$ .<sup>7</sup>

We assume that efficient units of labor supplied by the young depend on their respective productivity, which is influenced by their health status (denoted by  $h_{1,t}$  for young and  $h_{2,t}$  for old born at  $t - 1$ ). In contrast with previous contributions, we assume that efficiency units of labor provided by the old and the young may be not perfectly substitutes in production. Therefore, aggregate labor in efficiency terms is defined as:

$$L_t = \left[ \psi (h_{1,t}^\pi l_{1,t})^\theta + (1 - \psi) (h_{2,t}^\pi l_{2,t})^\theta \right]^{1/\theta} \quad \theta \leq 1, \psi \in (0, 1)$$

where  $l_{1,t}$  (respectively  $l_{2,t}$ ) is the amount of labor supplied by the young (the old) at time  $t$ , and  $h_{1,t}^\pi l_{1,t}$  (resp.  $h_{2,t}^\pi l_{2,t}$ ) is efficient labor supplied by the young (the old). The parameter  $\pi \in (0, 1)$  captures the effect of health on workers' productivity. The parameter  $\theta$  measures the degree of substitutability between old and young workers in production. The elasticity of substitution between the two types of labor equals  $1/(1 - \theta)$ . When  $\theta = 1$ , young and old labor are perfect substitutes. When  $0 \leq \theta < 1$ , they are imperfect substitutes. When  $\theta = 0$ , they are unitary substitutes. When  $\theta < 0$ , they are complements.

The profit of the firm in period  $t$  is  $Y_t - w_{1,t}l_{1,t} - w_{2,t}l_{2,t} - R_t K_t - \tau E_t$ , where  $R_t$  is the rental rate of capital and  $\tau$  is an environmental tax levied by the government. Profit-maximization yields the following first-order conditions:

$$R_t = \alpha_K Y_t / K_t \tag{10}$$

$$w_{1,t} = \alpha_L Y_t / L_t \frac{\partial L_t}{\partial l_{1,t}} \quad \text{and} \quad w_{2,t} = \alpha_L Y_t / L_t \frac{\partial L_t}{\partial l_{2,t}} \tag{11}$$

$$\tau = (1 - \alpha_K - \alpha_L) Y_t / E_t \tag{12}$$

The last expression enables us to express final output in terms of physical capital, labor and the environmental tax:

$$Y_t = f(\tau) K_t^\alpha L_t^{1-\alpha} \tag{13}$$

with  $L_t = \left[ \psi (h_{1,t}^\pi l_{1,t})^\theta + (1 - \psi) (h_{2,t}^\pi l_{2,t})^\theta \right]^{1/\theta}$ ,  $f(\tau) \equiv B^{1/(\alpha_K + \alpha_L)} \left( \frac{1 - \alpha_K - \alpha_L}{\tau} \right)^{\frac{1 - \alpha_K - \alpha_L}{\alpha_K + \alpha_L}}$  and  $\alpha \equiv \alpha_K / (\alpha_K + \alpha_L)$ .

<sup>7</sup>See Stokey (1998) for a justification of the introduction of polluting emissions as a factor of production.

## 2.3 Government

The government levies an environmental tax on the flow of pollution emissions in each period. The revenues are used to fund public abatement activities, denoted  $A_t$ :

$$A_t = \tau E_t$$

The pollution stock rises with the flow of pollution emissions and is reduced by abatement activities:

$$P_{t+1} = (1 - \sigma)P_t + \Pi(E_t, A_t)$$

where  $\sigma > 0$  is the nature regeneration rate, and  $\Pi(\cdot)$  is the net flow of pollution at date  $t$  with  $\Pi_E > 0$  and  $\Pi_A < 0$ . Because population grows at positive rate  $(1 + n)$ , therefore  $E$  and  $A$  evolve both at the same rate. Nevertheless, in the stationary equilibrium, the stock of pollution must be constant and equal to  $P$ . Therefore, we impose that  $\Pi(E_t, A_t) = g(\tau)$  with  $g_\tau < 0$  for all dates.<sup>8</sup> Therefore:

$$P_{t+1} = (1 - \sigma)P_t + g(\tau) \quad (14)$$

## 2.4 Equilibrium

First, we consider the equilibrium in labor markets. Young agents supply  $\lambda_{1,t}N_t$  units of labor and firms demand  $l_{1,t}$ . Therefore in equilibrium:

$$l_{1,t} = \lambda_{1,t}N_t$$

Old agents supply  $\lambda_{2,t}N_{t-1}$  units of labor and firms demand  $l_{2,t}$ . Therefore:

$$l_{2,t} = \lambda_{2,t}N_{t-1}$$

Aggregate labor expressed in efficiency terms is:

$$L_t = \frac{N_t}{1+n} \left[ \psi \left( (1+n)\lambda_{1,t}h_{1,t}^\pi \right)^\theta + (1-\psi) \left( \lambda_{2,t}h_{2,t}^\pi \right)^\theta \right]^{1/\theta} \quad (15)$$

From equation (11), we obtain:

$$w_{1,t} = \psi \alpha_L f(\tau) \tilde{k}_t^\alpha h_{1,t}^\pi \left[ \psi + (1-\psi) \left( \frac{\lambda_{2,t}h_{2,t}^\pi}{(1+n)\lambda_{1,t}h_{1,t}^\pi} \right)^\theta \right]^{(1-\theta)/\theta} \quad (16)$$

and:

$$w_{2,t} = (1-\psi) \alpha_L f(\tau) \tilde{k}_t^\alpha h_{1,t}^\pi \left( \frac{h_{2,t}}{h_{1,t}} \right)^{\pi\theta} \left( \frac{(1+n)\lambda_{1,t}}{\lambda_{2,t}} \right)^{1-\theta} \left[ \psi + (1-\psi) \left( \frac{\lambda_{2,t}h_{2,t}^\pi}{(1+n)\lambda_{1,t}h_{1,t}^\pi} \right)^\theta \right]^{(1-\theta)/\theta}$$

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<sup>8</sup>See Gradus and Smulders (1993) for a justification.

(17)

Therefore the relative reward of young labor with respect to old labor is:

$$\frac{w_{1,t}}{w_{2,t}} = \frac{\psi}{1-\psi} \left( \frac{h_{1,t}}{h_{2,t}} \right)^{\pi\theta} \left( \frac{\lambda_{2,t}}{(1+n)\lambda_{1,t}} \right)^{1-\theta} \quad (18)$$

The marginal productivity of labor for the young relative to the old logically reflects the relative health status of the young versus the old. If  $\theta = 1$  (young and old labor are perfect substitutes), the relative health status is the only element explaining the wage ratio between the young and the old. In that case, the health profile is positively related to the wage profile. If  $0 < \theta < 1$  (young and old labor are imperfect substitutes), labor supply choices by the young and the old influences the wage ratio. If  $\theta = 0$  (young and old labor are unitary substitutes), the health status becomes irrelevant and labor supply choices at young and old ages are sole determinants of the wage ratio. If  $\theta < 0$  (young and old labor are complements), the health profile is negatively related to the wage profile.

Finally, equations (10) and (13) give us the expression of the interest rate:

$$R_t = \alpha_K f(\tau) \tilde{k}_t^{\alpha-1} \quad (19)$$

Market-clearing in goods and capital markets leads to the equilibrium condition  $K_{t+1} = s_t N_t$  which is expressed in terms of per worker capital stock:

$$\tilde{k}_{t+1} \equiv \frac{K_{t+1}}{L_{t+1}} = \left[ \psi \left( (1+n)\lambda_{1,t+1} h_{1,t+1}^\pi \right)^\theta + (1-\psi) \left( \lambda_{2,t+1} h_{2,t+1}^\pi \right)^\theta \right]^{-1/\theta} s_t \quad (20)$$

Using (13) and (15), output per capita is defined as:

$$y_t = \left( \frac{1+n}{2+n} \right) f(\tau) \tilde{k}_t^\alpha \frac{L_t}{N_t} \quad (21)$$

Therefore, the economy can be summarized by equations (3), (6), (7), (8), (14), (16), (17), (19) and (20).

### 3 The steady-state

In this section, we investigate the influence of the environmental tax on the steady-state equilibrium. For the sake of simplicity, we assume that the health-status of the young  $h_{1,t}$  is exogenous and denoted  $\bar{h}$ , and in the steady-state, we define  $\Delta_h^* \equiv h_2^*/\bar{h}$ .

Furthermore, to keep the presentation of the results tractable, we make two simplifying assumptions that we will relax in the discussion part of the paper:

1. The impact of health-status on production is linear:  $\pi = 1$ .
2. There is no PAYG social-security retirement system, therefore  $\tau_t^w = \tau_{t+1}^w = 0$ . It means that individuals fund their own retirement.

The steady-state equilibrium is such that  $\{m_t, R_t, h_{2,t}, s_t, \tilde{k}_t, w_{1,t}, w_{2,t}, P_t, \lambda_{2,t}\} = \{m^*, R^*, h_2^*, s^*, \tilde{k}^*, w_1^*, w_2^*, P^*, \lambda_2^*\}$ , where variables with a  $*$  are constant. Thus, the steady-state equilibrium is defined by the following equations:

$$\tilde{k}^* = \left[ \psi(1+n)^\theta \lambda_1^{*\theta} + (1-\psi)(\lambda_2^* \Delta_h^*)^\theta \right]^{-1/\theta} s^* / \bar{h} \quad (\text{E1}^*)$$

$$s^* = \frac{\beta \lambda_1^* w_1^* - \lambda_2^* w_2^* / R^*}{1 + \beta} \quad (\text{E2}^*)$$

$$\lambda_2^* = \frac{(1+\beta)\phi - \gamma\beta \frac{\lambda_1^* w_1^*}{w_2^* / R^*}}{(\phi + \gamma)(1+\beta) - \gamma} \quad (\text{E3}^*)$$

$$\lambda_1^* = \frac{(1+\beta)(1-m^*) - \varphi \frac{\lambda_2^* w_2^* / R^*}{w_1^*}}{1 + \beta + \varphi} \quad (\text{E4}^*)$$

$$\frac{H(m^*) + (1 - \delta(P^*))\bar{h}}{H'(m^*)} - \frac{\beta(1-\phi)}{\phi\varphi}(1 - m^* - \lambda_1^*) = 0 \quad (\text{E4}^* \text{bis})$$

$$h_2^* = H(m^*) + [1 - \delta(P^*)]\bar{h} \quad (\text{E5}^*)$$

$$w_1^* = \psi \alpha_L f(\tau) \tilde{k}^{*\alpha} \bar{h} \left[ \psi + (1-\psi) \left( \frac{\lambda_2^*}{(1+n)\lambda_1^*} \Delta_h^* \right)^\theta \right]^{(1-\theta)/\theta} \quad (\text{E6}^*)$$

$$w_2^* = (1-\psi) \alpha_L f(\tau) \tilde{k}^{*\alpha} h_2^* \left( \frac{(1+n)\lambda_1^*}{\lambda_2^* \Delta_h^*} \right)^{1-\theta} \left[ \psi + (1-\psi) \left( \frac{\lambda_2^* \Delta_h^*}{(1+n)\lambda_1^*} \right)^\theta \right]^{(1-\theta)/\theta} \quad (\text{E7}^*)$$

$$R^* = \alpha_K f(\tau) \tilde{k}^{*\alpha-1} \quad (\text{E8}^*)$$

$$P^* = \sigma^{-1} g(\tau) \quad (\text{E9}^*)$$

From (E5<sup>\*</sup>), (E9<sup>\*</sup>) and the definition of  $\Delta_h^*$ , we obtain:

$$\Delta_h^* = H(m^*)\bar{h}^{-1} + 1 - \delta(\sigma^{-1}g(\tau)) \quad (\text{E5}^* \text{-1})$$

Using equations (E1<sup>\*</sup>), (E2<sup>\*</sup>), (E6<sup>\*</sup>) to (E8<sup>\*</sup>), we obtain:

$$\left[ \frac{\lambda_2^*}{(1+n)\lambda_1^*} \Delta_h^* \right]^\theta = \mathcal{D}(R^*) \equiv \frac{\psi \frac{\alpha_L}{\alpha_K} \left[ \beta R^* - \frac{\alpha_K}{\alpha_L} (1+\beta) \right]}{(1-\psi) \left( (1+n) + \frac{\alpha_K}{\alpha_L} (1+\beta) \right)} > 0 \quad (22)$$

with  $\mathcal{D}'(R^*) > 0$ .

**LEMMA 1.** When  $\theta = 0$ , there exists a unique interest rate  $R^*$ , which is independent from  $\tau$ .

**Proof.** Straightforward from (22) and (E5\*-1).  $\square$

Using (15), aggregate labor in efficiency units can be expressed as:

$$L^* = N_t \lambda_1^* \bar{h} \left[ \frac{\psi \frac{\alpha_L}{\alpha_K}}{(1 + \beta)(1 + n) + \frac{\alpha_L}{\alpha_K}} \right]^{1/\theta} [1 + \beta R^*]^{1/\theta} \quad (23)$$

Using (E4\*bis), (E7\*), (E8\*) and (22), we obtain:

$$\lambda_1^* = \Lambda_1(R^*, m^*) \equiv \frac{(1 + \beta)(1 - m^*)}{(1 + \beta + \varphi) + \varphi(1 + n) \left( \frac{1 - \psi}{\psi} \right) \mathcal{D}(R^*)/R^*} \quad (24)$$

with  $\partial \Lambda_1(\cdot)/\partial R^* < 0$  and  $\partial \Lambda_1(\cdot)/\partial m^* < 0$ .<sup>9</sup>

Using (E4\*bis), (E7\*), (E8\*) and (22), we obtain:

$$\lambda_2^* = \Lambda_2(R^*) \equiv \frac{(1 + \beta)\phi}{(\phi + \gamma)(1 + \beta) - \gamma + \left( \frac{\psi}{1 - \psi} \right) \left( \frac{\gamma\beta}{1 + n} \right) R^*/\mathcal{D}(R^*)} \quad (25)$$

with  $\partial \Lambda_2(\cdot)/\partial R^* > 0$ .

This Lemma identifies one special case in which environmental taxation does not affect the equilibrium steady-state: When the elasticity of substitution between young and old labor is one ( $\theta = 0$ ), the health profile has no effect on the wage profile. As shown by equation (18), the wage profile solely depends upon the labor supply profile. Therefore, even if an increase in the pollution tax improves the health profile, it has no effect on the wage profile. As a result, in equilibrium, the income profile, saving or the interest rate are not influenced by the pollution tax. In the next sections, we will therefore distinguish between the cases when  $\theta = 0$ , and  $\theta \neq 0$ .

### 3.1 No retirement and exogenous health-expenditures

To eliminate retirement, we make the assumption that  $\gamma = 0$ . As a consequence, agents who have no preference for leisure in the second period of life do not retire. Therefore  $\lambda_2^* = 1$ . Furthermore, we assume that health-care investment is exogenous, such that  $m^* = \bar{m}$ . Therefore, from equation (E5\*-1),  $\Delta_h^* = \mathcal{H}(\tau) \equiv \frac{H(\bar{m})}{\bar{h}} + 1 - \delta (\sigma^{-1} g(\tau))$ . Because  $g(\tau)$  is decreasing in  $\tau$ ,  $\mathcal{H}(\tau) > 0$ ,  $\forall \theta \leq 1$ .

Using (22), in the steady-state, the interest rate is given by:

$$\left[ \frac{\mathcal{H}(\tau)}{(1 + n)\Lambda_1(R^*, \bar{m})} \right]^\theta = \mathcal{D}(R^*) \quad (26)$$

---

<sup>9</sup> $\mathcal{D}(R^*)/R^*$  is an increasing function of  $R^*$ .

**PROPOSITION 1.** *In the stationary equilibrium, there is a unique and positive interest rate such that:*

$$R^* = \mathcal{R}(\tau)$$

with:

- (i)  $\mathcal{R}'(\tau) < 0$  when old workers and young workers are complement in production ( $\theta < 0$ ).
- (ii)  $\mathcal{R}'(\tau) = 0$  when old workers and young workers are unitary substitutes in production ( $\theta = 0$ , Cobb-Douglas case).
- (iii)  $\mathcal{R}'(\tau) > 0$  when old workers and young workers are non-unitary substitutes in production ( $0 < \theta \leq 1$ ).

**Proof.** (i) When  $\theta < 0$ , because  $\mathcal{H}(\tau)$  is increasing in  $\tau$ , the left-hand side of (26) is decreasing in  $\tau$  and in  $R^*$ . Because  $\mathcal{D}(R^*)$  is increasing in  $\tau$ , (26) defines a unique positive  $R^* = \mathcal{R}(\tau)$  with  $\mathcal{R}'(\tau) < 0$ . (ii) When  $\theta = 0$ , the left-hand side of (26) is independent from  $\tau$  and  $R^*$ . Therefore (26) defines a unique positive  $R^*$  independent from  $\tau$ . (iii) When  $\theta \in ]0, 1]$ , (26) can be written as (using the expression of  $\Lambda_1(R^*, \bar{m})$ )

$$\frac{\mathcal{H}(\tau)}{(1+n)} = \frac{(1+\beta)(1-\bar{m})\mathcal{D}(R^*)^{1/\theta-1}}{(1+\beta+\varphi)/\mathcal{D}(R^*) + \varphi(1+n) \left(\frac{1-\psi}{\psi}\right) / R^*} \quad (27)$$

The LHS of this equation is increasing in  $R^*$  and the RHS is increasing in  $\tau$ . Therefore (26) defines a unique positive  $R^* = \mathcal{R}(\tau)$  with  $\mathcal{R}'(\tau) > 0$ .  $\square$

Proposition 1 states that when health evolves over the life-cycle, environmental taxation impacts the interest rate. Therefore, it influences saving. We call this new effect the *health-saving effect*.<sup>10</sup>

It is explained by the fact that pollution affects the health profile over the life-cycle. The health profile influences the wage profile, which modifies the income profile and influences saving as a result. Indeed, pollution negatively affects  $h_2$ . The lower  $h_2$  relatively to  $\bar{h}$ , the higher the first period wage relative to the second period wage as long as labor in the first and second periods of life are non-unitary substitutes ( $0 < \theta \leq 1$ ). When the decrease in the wage profile is steeper, saving is higher and the interest rate is lower. An increase in the tax decreases pollution, makes the wage profile flatter (it decreases less), thereby decreasing saving and increasing the interest rate. When  $\theta = 0$ , the health profile has no effect on the wage profile and we retrieve Lemma 1. There is no health saving effect. When labor at old and young ages are complements ( $\theta < 0$ ) the health saving effect is reversed because the wage profile is negatively related to the health profile (See equation 18). An increase in the pollution tax accentuates the decrease in wage

<sup>10</sup>In section 4.1, we demonstrate that if the evolution of health over the life cycle is ignored, the health-saving effect disappears.

between the two periods in life, thereby increasing saving and decreasing the interest rate.

**COROLLARY 1.**

$$\lambda_1^* = \mathcal{L}(\tau) \quad \text{with } \mathcal{L}'(\tau) \begin{matrix} \leq \\ \geq \end{matrix} 0 \text{ for } \theta \begin{matrix} \geq \\ \leq \end{matrix} 0.$$

**Proof.** From equation (24) and Proposition 1. □

With Corollary 1, we show that there is another new general equilibrium effect besides the health saving effect. Corollary 1 means that young agents reduce their labor supply when old and young workers are substitute in production. This “*young labor participation effect*” is also linked to health. Indeed, recall that when environmental tax rises and limits the deterioration of health, the decrease in wage over the life-cycle is less steep. Therefore, young agents decrease both their saving and labor supply. Inversely, young agents increase their labor supply when old and young labor are complements in production. Indeed, in that case, the rise in environmental taxation accentuates the decrease in wage over the life-cycle, increasing young agents saving and labor supply.

The expression giving per capita output enables us to identify all the effects of a tighter environmental tax. Per capita output is given by equations (21) and (23), and the results of the previous section:

$$y^* = \left( \frac{1}{2+n} \right) \underbrace{f(\tau)^{1/(1-\alpha)}}_I \underbrace{\left( \frac{\alpha_K}{\mathcal{R}(\tau)} \right)^{\alpha/(1-\alpha)}}_{II} \left[ \psi \underbrace{[(1+n)\bar{h}\mathcal{L}(\tau)]^\theta}_{IIIa} + (1-\psi) \underbrace{\mathcal{H}(\tau)^\theta}_{IIIb} \right]^{1/\theta}$$

when  $\theta \neq 0$ , and:

$$y^* = \left( \frac{1}{2+n} \right) \underbrace{f(\tau)^{1/(1-\alpha)}}_I \underbrace{\left( \frac{\alpha_K}{R^*} \right)^{\alpha/(1-\alpha)}}_{II} \left[ (1+n)\bar{h}\lambda_1^* \right]^\psi \underbrace{\mathcal{H}(\tau)^{1-\psi}}_{IIIb}$$

when  $\theta = 0$ , where *I*, *II* and *III* represents the three effects associated with tighter environmental tax.

Effect *I* is the conventional crowding-out effect of private capital by the environmental tax. It comes from the fact that the financing public abatement activities requires capital input. Effect *II* is the *health-saving effect*. From Proposition 1, we identify a positive (negative) effect of environmental taxation on per capita output when young and old labor tend to be substitutes (complements). Effect *III* is the *health-labor effect*. In our simple case with exogenous retirement and investment in health decisions, it encompasses (*IIIa*) the *young labor participation effect* (Corollary 1) and (*IIIb*), the standard productivity increase due to the positive effect of the environmental tax on health. The *young labor participation effect* indicates a new



positive (negative) effect of output per capita when young and old labor tend to be substitutes (complements). By contrast, the standard productivity increase due to reduced morbidity always has a positive effect on output per capita in the steady state.

The table below provides a qualitative summary of the effects of environmental taxation on steady state output per capita.

Table 1: Effects on output per capita and substitutability of young and old labor

	$\theta > 0$	$\theta < 0$	$\theta = 0$
<i>I</i> (crowding-out effect)	–	–	–
<i>II</i> (health-saving effect)	–	+	0
<i>IIIa</i> (young labor participation effect)	–	+	0
<i>IIIb</i> (productivity effect)	+	+	+

Therefore, consistently with the literature, we retrieve the negative crowding out effect and the positive productivity effect of environmental taxation. However, we show that when the evolution of health over the life-cycle is taken into consideration, reduced morbidity can trigger two additional effects, the health saving effect and the young labor supply effect. Those two effects reinforce (weaken) the positive impact of reduced morbidity on output per capita when young and old labor are complements (substitutes).

### 3.2 Endogenous retirement, exogenous health expenditures

We reintroduce retirement in the model, setting  $\gamma > 0$ . The retirement decision is endogenous, and therefore  $\lambda_2^*$  is given by equation (25).

Using (22), the interest rate at the steady-state is defined by:

$$\left[ \frac{\mathcal{H}(\tau)\Lambda_2(R^*)}{(1+n)\Lambda_1(R^*, \bar{m})} \right]^\theta = \mathcal{D}(R^*) \quad (28)$$

**PROPOSITION 2.** *In the presence of endogenous retirement, Proposition 1 still holds:*

$$R^* = \tilde{\mathcal{R}}(\tau) \quad \tilde{\mathcal{R}}'(\tau) \begin{matrix} \geq \\ \leq \end{matrix} 0 \text{ for } \theta \begin{matrix} \geq \\ \leq \end{matrix} 0.$$

$$h_2^* = \mathcal{H}(\tau) \quad \tilde{\mathcal{H}}'(\tau) > 0 \quad \forall \theta \leq 1.$$

**Proof.** See Appendix A □

Furthermore,

**COROLLARY 2.** *In the presence of endogenous retirement,*

$$(i) \lambda_1^* = \tilde{\mathcal{L}}_1(\tau) \text{ with } \tilde{\mathcal{L}}_1'(\tau) \begin{matrix} \leq \\ \geq \end{matrix} 0 \text{ for } \theta \begin{matrix} \geq \\ \leq \end{matrix} 0;$$

(ii)  $\lambda_2^* = \tilde{\mathcal{L}}_2(\tau)$  with  $\tilde{\mathcal{L}}_2'(\tau) \begin{matrix} \geq \\ \leq \end{matrix} 0$  for  $\theta \begin{matrix} \geq \\ \leq \end{matrix} 0$ .

**Proof.** From (24) and (25) using Proposition 2. □

When  $\theta > 0$ , the tax on pollution limits the decrease in wages over the life cycle. The relatively higher wage in the first period of life increases labor supply (similarly to Corollary 1). The relatively low wage in the second period encourages work over retirement. This new general equilibrium effect is called the *health-retirement effect*. When  $\theta = 0$ , the pollution tax has no effect on the wage profile. Therefore, it has no effect on the retirement choice either. When  $\theta < 0$ , the tax on pollution makes the wage profile flatter. It discourages work in the first period of life and discourages work over retirement in the second period of life.

The influence of the environmental tax on per capita output can be summarized by:

$$y^* = \left( \frac{1}{2+n} \right) \underbrace{f(\tau)^{1/(1-\alpha)}}_I \underbrace{\left( \frac{\alpha_K}{\tilde{\mathcal{R}}(\tau)} \right)^{\alpha/(1-\alpha)}}_{II} \left[ \psi \underbrace{[(1+n)\bar{h}\tilde{\mathcal{L}}_1(\tau)]^\theta}_{IIIa} + (1-\psi) \underbrace{[\mathcal{H}(\tau)\tilde{\mathcal{L}}_2(\tau)]^\theta}_{\substack{IIIb \\ IIIc}} \right]^{1/\theta}$$

when  $\theta \neq 0$ , and:

$$y^* = \left( \frac{1}{2+n} \right) \underbrace{f(\tau)^{1/(1-\alpha)}}_I \underbrace{\left( \frac{\alpha_K}{R^*} \right)^{\alpha/(1-\alpha)}}_{II} \left[ (1+n)\bar{h}\lambda_1^* \right]^\psi \underbrace{[\lambda_2^* \mathcal{H}(\tau)]^{1-\psi}}_{IIIb}$$

when  $\theta = 0$ .

The table below provides a qualitative summary of the effects of environmental taxation on steady state output per capita.

Table 2: Effects on output per capita and substitutability of young and old labor

	$\theta > 0$	$\theta < 0$	$\theta = 0$
<i>I</i> (crowding-out effect)	-	-	-
<i>II</i> (health-saving effect)	-	+	0
<i>IIIa</i> (young labor participation effect)	-	+	0
<i>IIIb</i> (productivity effect)	+	+	+
<i>IIIc</i> (retirement effect)	+	-	0

Compared to the previous case, in the presence of endogenous retirement (with a PAYG system), environmental taxation impacts per capita output growth through an additional channel, the health retirement effect, which is included in the health labor effect. The health retirement effect further reinforces (weakens) the positive impact of reduced morbidity on the growth of output per capita when young and old labor are complements (substitutes).

### 3.3 Endogenous retirement, endogenous health-care investment

We consider the case when both the retirement decision and the decision to invest in health are endogenous. Thus,  $m^*$  is endogenous and in equilibrium, it satisfies:

$$\left( 1 - \frac{1 + \beta}{1 + \beta + \varphi + \varphi(1 + n) \left( \frac{1-\psi}{\psi} \right) \mathcal{D}(R^*)/R^*} \right) \frac{\beta(1 - \phi) [1 - m^*] H'(m^*)}{\phi\varphi} = \frac{H(m^*) + [1 - \delta(\sigma^{-1}g(\tau))] \bar{h}}{H(m^*) + [1 - \delta(\sigma^{-1}g(\tau))] \bar{h}} \quad (29)$$

Therefore equation (29) gives  $m^*$  such that

$$m^* = \Omega(R^*, \tau) \quad (30)$$

with  $\partial\Omega(R^*, \tau)/\partial R^* > 0$  and  $\partial\Omega(R^*, \tau)/\partial\tau < 0$ .

In the previous scenarios, we assumed an exogenous investment in health. Thus the tightening of the environmental tax would always lead to an improvement of the health profile. When the investment in health is endogenous, it interferes with the effect of the pollution tax on the health profile. Indeed, if investment in health is reduced when the pollution tax increases, this potentially can cancel out or reverse the positive effect of the environmental tax on the health profile.

Equation (22) and previous results enable us to obtain the expression of the steady-state interest rate:

$$\left\{ \frac{\Lambda_2(R^*) [H(\Omega(R^*; \tau)) + [1 - \delta(\sigma^{-1}g(\tau))] \bar{h}]}{(1 + n)\Lambda_1(R^*, \Omega(R^*; \tau))} \right\}^\theta = \mathcal{D}(R^*) \quad (31)$$

**PROPOSITION 3.** *When the retirement decision and the decision to invest in health are endogenous, Proposition 1 still holds:*

$$R^* = \tilde{\mathcal{R}}(\tau) \quad \tilde{\mathcal{R}}'(\tau) \underset{\leq}{\geq} 0 \text{ for } \theta \underset{\leq}{\geq} 0$$

under the necessary condition (when  $\theta < 0$ )

$$\epsilon_\tau^{\Omega(\cdot)} \left[ \epsilon_{\Omega(\cdot)}^{\Lambda_1(\cdot)} - \epsilon_{\Omega(\cdot)}^{h_2^*} \right] < \epsilon_\tau^{\delta(\cdot)} \epsilon_{\delta(\cdot)}^{h_2^*} \quad \text{where} \quad \epsilon_j^i \equiv \frac{\partial i / \partial j}{i / j}. \quad (C1)$$

**Proof.** See Appendix B. □

Condition (C'1) means that, *Ceteris paribus*, the contribution of  $\tau$  on  $R^*$  through investment in health  $m^*$  (captured by the LHS in equation C'1) is lower than the contribution of  $\tau$  on  $R^*$  through health depreciation (captured by the RHS in equation C'1). In the following, we will consider that condition (C'1) is verified.

**COROLLARY 3.**

- (i)  $m^* = \mathcal{M}(\tau)$  with  $\mathcal{M}'(\tau) < 0, \forall \theta \leq 1$ ;
- (ii)  $\lambda_1^* = \tilde{\mathcal{L}}_1(\tau)$  with  $\tilde{\mathcal{L}}_1'(\tau) > 0$  for  $\theta \leq 0$ ;
- (iii)  $\lambda_2^* = \tilde{\mathcal{L}}_2(\tau)$  with  $\tilde{\mathcal{L}}_2'(\tau) \begin{smallmatrix} \geq \\ \leq \end{smallmatrix} 0$  for  $\theta \begin{smallmatrix} \geq \\ \leq \end{smallmatrix} 0$
- (iv)  $h_2^* = \tilde{\mathcal{H}}(\tau)$  with  $\tilde{\mathcal{H}}'(\tau) > 0, \forall \theta \leq 1$ ;

*Proof.* See Appendix C. □

In the presence of endogenous retirement and endogenous investment in health, we retrieve the health retirement (iv) and the productivity effects (ii). However, endogenous investment in health has two new consequences. First, environmental taxation impacts per capita output growth through investment in health(i). Investment in health decreases (respectively increases) the level of physical capital in the steady-state when young and old labor are non-unitary substitutes (respectively complements). Second, the young labor effect (iii) is modified. Contrary to previous cases, the environmental tax always results in an increase in young labor whether young and old labor are substitutes or complements. Channels (ii) to (iv) increase per capita aggregate effective labor  $l^*$ .

$$y^* = \left( \frac{1}{2+n} \right) \underbrace{f(\tau)^{1/(1-\alpha)}}_I \underbrace{\left( \frac{\alpha_K}{\tilde{\mathcal{R}}(\tau)} \right)^{\alpha/(1-\alpha)}}_{II} \left[ \psi \underbrace{[(1+n)\bar{h}\tilde{\mathcal{L}}_1(\tau)]^\theta}_{IIIa} + (1-\psi) \underbrace{[\tilde{\mathcal{H}}(\tau)\tilde{\mathcal{L}}_2(\tau)]^\theta}_{IIIb \quad IIIc} \right]^{1/\theta}$$

when  $\theta \neq 0$ , and:

$$y^* = \left( \frac{1}{2+n} \right) \underbrace{f(\tau)^{1/(1-\alpha)}}_I \underbrace{\left( \frac{\alpha_K}{R^*} \right)^{\alpha/(1-\alpha)}}_{II} \left[ (1+n)\bar{h} \underbrace{\tilde{\mathcal{L}}_1(\tau)}_{IIIa} \right]^\psi \underbrace{[\lambda_2^* \tilde{\mathcal{H}}(\tau)]^{1-\psi}}_{IIIb}$$

when  $\theta = 0$ .

The table below provides a qualitative summary of the effects of environmental taxation on steady state output per capita.

Table 3: Effects on output per capita and substitutability of young and old labor

	$\theta > 0$	$\theta < 0$	$\theta = 0$
$I$ (crowding-out effect)	-	-	-
$II$ (health-saving effect)	-	+	0
$IIIa$ (young labor participation effect)	?	+	+
$IIIb$ (productivity effect)	+	+	+
$IIIc$ (retirement effect)	+	-	0

## 4 Discussion

In the previous section, we showed that the environmental tax influences both the interest rate and labor decisions by the young and the old when the evolution of health over the life-cycle is taken into account and young and old workers are not unitary substitutes. By contrast with previous contributions, we showed that effect of the environmental tax on health has general equilibrium implications that can modify the influence of the environmental tax. The purpose of this discussion is twofold. First, we compare the case when the evolution of health over life-cycle is not taken into account or investment in health is exogenous with the cases covered in the previous section. Second, we provide numerical examples to assess the overall impact of the environmental tax and the role of the new general equilibrium effects.

### 4.1 Flat health profile

We modify our model to replicate the case when the evolution of health over the life-cycle is ignored. We consider that health-status is the same for young and old individuals, that is  $h_2 = h_1 = h^*$  and therefore  $\Delta_h^* = 1$ . Because the evolution of the health status is given by equation (3), we obtain the endogenous expression of the health status in the steady-state:

$$h^* = \frac{H(m^*)}{\delta(P^*)} \quad (32)$$

Therefore, equation (22) defining the steady-state interest rate becomes:

$$\left[ \frac{\lambda_2^*}{(1+n)\lambda_1^*} \right]^\theta = \mathcal{D}(R^*) \equiv \frac{\psi \frac{\alpha_L}{\alpha_K} \left[ \beta R^* - \frac{\alpha_K}{\alpha_L} (1+\beta) \right]}{(1-\psi) \left( (1+n) + \frac{\alpha_K}{\alpha_L} (1+\beta) \right)} > 0 \quad (33)$$

$\lambda_1$  and  $\lambda_2$  are always defined respectively by (24) and (25).

**PROPOSITION 4.** *When the evolution of health over the life-cycle is ignored,*

- (i) *if investment in health is exogenous, the steady-state interest rate  $R^*$  is independent from the environmental tax  $\tau$ ;*
- (ii) *if investment in health is endogenous:*

$$R^* = \widehat{\mathcal{R}}(\tau) \quad \text{with } \widehat{\mathcal{R}}'(\tau) \begin{cases} \leq 0 \\ \geq 0 \end{cases} \text{ for } \theta \begin{cases} \geq 0 \\ \leq 0 \end{cases}$$

*under the necessary condition (when  $\theta > 0$ )*

$$\epsilon_\tau^{\Omega(\cdot)} \left[ \epsilon_{\Omega(\cdot)}^{H'(\cdot)} - \epsilon_{\Omega(\cdot)}^{h^*} \right] < \epsilon_\tau^{\delta(\cdot)} \epsilon_{\delta(\cdot)}^{h^*} \quad \text{where} \quad \epsilon_j^i \equiv \frac{\partial i / \partial j}{i/j}. \quad (\text{C}'1)$$

*Proof.* (i) Straightforward from equations (33), (24) and (25). (ii) See Appendix D.  $\square$

Proposition 4 (i) means that when the evolution of health over the life-cycle is ignored and investment in health is exogenous, the steady-state interest rate and the labor supply by the young and the old are not affected by the environmental tax. Therefore the new general equilibrium effects appear only when investment in health is endogenous. However, if the evolution of health over the life-cycle is ignored but investment in health is endogenous, the health saving effect is reversed (Proposition 4 (ii)).

#### COROLLARY 4.

*If investment in health is endogenous:*

(i)  $m^* = \widehat{\mathcal{M}}(\tau)$  with  $\widehat{\mathcal{M}}'(\tau) < 0, \forall \theta \leq 1$ ;

(ii)  $\lambda_1^* = \widehat{\mathcal{L}}_1(\tau)$  with  $\widehat{\mathcal{L}}_1'(\tau) > 0$  for  $\theta \leq 0$ ;

(iii)  $\lambda_2^* = \widehat{\mathcal{L}}_2(\tau)$  with  $\widehat{\mathcal{L}}_2'(\tau) \begin{matrix} \leq \\ \geq \end{matrix} 0$  for  $\theta \begin{matrix} \geq \\ \leq \end{matrix} 0$

(iv)  $h^* = \widehat{\mathcal{H}}(\tau)$  with  $\widehat{\mathcal{H}}'(\tau) > 0, \forall \theta \leq 1$ ;

*If investments in health is exogenous:*

(i')  $m^* = \bar{m}$  ;

(ii')  $\lambda_1^*$  is independent from  $\tau$ ;

(iii')  $\lambda_2^*$  is independent from  $\tau$ ;

(iv')  $h^* = \widehat{\mathcal{H}}(\tau)$  with  $\widehat{\mathcal{H}}'(\tau) > 0, \forall \theta \leq 1$ ;

*Proof.* For (i)-(iv) See Appendix E. (i') is straightforward. (ii'), (iii') and (iv') come respectively from (24), (25) and (32).  $\square$

## 4.2 PAYG system

In our previous study of the steady-state equilibrium, we abstracted from the PAYG system for convenience. In the current section, we investigate how the retirement system and the level of the retirement tax  $t^w$  potentially modify previous results.

When social security system is taken into account, the equilibrium equation (22) becomes:

$$\left[ \frac{\lambda_2^*}{(1+n)\lambda_1^*} \Delta_h^* \right]^\theta = \check{\mathcal{D}}(R^*) \equiv \frac{\psi \frac{\alpha_L}{\alpha_K} \left[ \beta(1-\tau^w)R^* - \frac{\alpha_K}{\alpha_L}(1+\beta) - (1+n)\tau^w \right]}{(1-\psi) \left( (1+n) + \frac{\alpha_K}{\alpha_L}(1+\beta) \right)} > 0$$

(34)

with  $\check{\mathcal{D}}'(R^*) > 0$ . Then:

$$\lambda_1^* = \check{\Lambda}_1(R^*, m^*) \equiv \frac{(1 - \tau^w)(1 + \beta)(1 - m^*)}{(1 - \tau^w)(1 + \beta + \varphi) + \varphi(1 + n) \left[ \left( \frac{1 - \psi}{\psi} \right) \check{\mathcal{D}}(R^*) + \tau^w \right] / R^*} \quad (35)$$

with  $\partial \check{\Lambda}_1(\cdot) / \partial R^* < 0$  (under the realistic assumption  $\alpha_L \geq \alpha_K$ )<sup>11</sup> and  $\partial \check{\Lambda}_1(\cdot) / \partial m^* < 0$ , and:

$$\lambda_2^* = \check{\Lambda}_2(R^*) \equiv \frac{(1 + \beta)\phi}{(\phi + \gamma)(1 + \beta) - \gamma + \gamma\beta \left( \frac{\psi}{1 - \psi} \right) \left[ \tau^w + \frac{1 - \tau^w}{1 + n} R^* \right] / \check{\mathcal{D}}(R^*)} \quad (36)$$

with  $\partial \check{\Lambda}_2(\cdot) / \partial R^* > 0$  because  $\left[ \tau^w + \frac{1 - \tau^w}{1 + n} R^* \right] / \check{\mathcal{D}}(R^*)$  is decreasing in  $R^*$ .

Then, equation (29) becomes:

$$\left( 1 - \frac{1 + \beta}{1 + \beta + \varphi + \varphi(1 + n) \left[ \left( \frac{1 - \psi}{\psi} \right) \check{\mathcal{D}}(R^*) + \tau^w \right] / R^*} \right) \frac{\beta(1 - \phi) [1 - m^*] H'(m^*)}{\phi\varphi} = \frac{H(m^*) + [1 - \delta(\sigma^{-1}g(\tau))] \bar{h}}{H(m^*) + [1 - \delta(\sigma^{-1}g(\tau))] \bar{h}} \quad (37)$$

which defines the investment in health  $m^*$  as:

$$m^* = \check{\Omega}(R^*, \tau) \quad (38)$$

with  $\partial \check{\Omega}(R^*, \tau) / \partial R^* > 0$  and  $\partial \check{\Omega}(R^*, \tau) / \partial \tau < 0$ .

The expression of the steady-state interest rate is given by the following modified equation (31):

$$\left\{ \frac{\check{\Lambda}_2(R^*) \left[ H(\check{\Omega}(R^*; \tau)) + [1 - \delta(\sigma^{-1}g(\tau))] \bar{h} \right]}{(1 + n)\check{\Lambda}_1(R^*, \check{\Omega}(R^*; \tau))} \right\}^\theta = \check{\mathcal{D}}(R^*) \quad (39)$$

As a consequence:

**PROPOSITION 5.** *When the retirement decision and the decision to invest in health are endogenous and there is a Pay-as-you-go system, Proposition 1 still holds:*

$$R^* = \check{\mathcal{R}}(\tau) \quad \check{\mathcal{R}}'(\tau) \gtrless 0 \text{ for } \theta \gtrless 0$$

under the necessary condition (when  $\theta < 0$ ):

$$\epsilon_\tau^{\check{\Omega}(\cdot)} \left[ \epsilon_{\check{\Omega}(\cdot)}^{\check{\Lambda}_1(\cdot)} - \epsilon_{\check{\Omega}(\cdot)}^{h_2^*} \right] < \epsilon_\tau^{\delta(\cdot)} \epsilon_{\delta(\cdot)}^{h_2^*} \quad \text{where} \quad \epsilon_j^i \equiv \frac{\partial i / \partial j}{i / j}. \quad (C''1)$$

<sup>11</sup>Under this assumption  $\left[ \left( \frac{1 - \psi}{\psi} \right) \check{\mathcal{D}}(R^*) + \tau^w \right] / R^*$  is an increasing function of  $R^*$ .

*Proof.* Similar to the proof of Proposition 3. See Appendix B. □

Thus, the introduction of a PAYG system does not modify qualitatively the overall impact of the environmental tax on the steady-state interest rate. Nevertheless, it does not mean that the quantitative impact could not be sizable. Because analytical demonstrations are cumbersome, we use numerical examples in the following section to investigate the quantitative impact of the environmental tax on the per capita steady-state output with different level for the social security tax.

### 4.3 Numerical examples

We simulate the model with endogenous retirement and endogenous health expenditure. We want to stress upon the fact that the objective of this section is simply to get a sense of the magnitude and direction of the cumulated new general equilibrium effects on output and welfare. The numerical simulations are to be taken with caution considering the following facts. The model is very stylized, which is necessary to derive theoretical results but it is limited in its ability to reproduce all the characteristics of the US economy. The value of a number of parameters of the model is uncertain. Therefore, we consider that a reasonable strategy is to adjust the parameter values to reproduce some of the most salient features of the US economy. We present the parameters chosen in Tables 4, 5 and 6.

The social security tax is 12.4% in the US. The population growth rate is set at 1% (annually), which is close to the US growth rate of 0.9% in 2012. The rate of time preference  $\beta$  and the consumption weight  $\phi$  are in line with the range of values considered by French (2005). The parameters of the production function are standard as in (*reference*). The value of  $\theta$  reflects that young and old labor are imperfect complementary factors of production as in Hebbink (1993) (*more references needed*). Since the value of  $\theta$  is uncertain and influences the theoretical results, we provide numerical results for a wide range of values of  $\theta$  (Table 9). The preference parameters are adjusted such that total leisure time is about two thirds of individual time (Prescott, 2004), and that the time spent on investment in health represents less than 10% of individual time. We obtain a time spent on investment in health equal to 9.4% of total time which is within the range of 10% of .....(*reference*) and 3.9 to 7.1% of time in lost leisure solely due to bad health (French, 2005). Welfare is simply measured by utility and we chose the value of  $B$  to obtain a positive welfare in the steady state. Our parameters enable us to obtain a capital output ratio close to 3, in line with the US economy.

There is great uncertainty regarding the values of the parameters of the investment in health and pollution functions . For the numerical simulations, the investment in health function is defined as  $H(m) = \eta m^\epsilon$ . We also define  $g(\tau) = \tau^{-1}$ , and we impose  $d_0 = d_1 = 0$  in  $\delta(P)$  (see equation 4), which



results in the following pollution function:  $\delta(P^*) = \xi(\sigma\tau)^{-\varsigma}$ . The chosen values of the parameters are presented in Table 4. Those values are different from Pautrel (2012), who assumed a linear function for investment in health with  $\eta = 0.8$  (and implicitly  $\epsilon = 1$ ), and a non-linear pollution function, which parameters are equivalent to setting  $\xi = 0.6$ ,  $\sigma = 0.3$ , and  $\varsigma = 1$ . The study by Skinner (2001) indicates that 20% of medical expenditure provide no benefit to health, which justifies the choice of  $\eta$ . Besides, the literature does not provide insights regarding the choice of the parameters of the pollution function. The best we can do is therefore to provide a sensitivity analysis of our numerical results to calibration of the investment in health and pollution functions (Table 13). The health profile depends on the investment in health and the pollution functions. Overall, with our benchmark calibration, we obtain a health profile that decreases over the life-cycle (See Table 7).

There is a wide range of estimates regarding the intertemporal elasticity of substitution between young and old labor. In our simulations, we find that the wage of old workers is 1.4 times the wage of young workers and hours worked decrease by 4.6% between the two periods of life. As discussed by French (2005), this combination of a large variations in wages and small variations in hours worked is consistent with a small intertemporal elasticity of substitution as is standard in models without uncertainty.

Table 4: Preferences and health

$\varphi$	$\phi$	$\beta$	$\gamma$	$h_1$	$\eta$	$\epsilon$
3.5	0.6	0.987	1.5	0.2	0.7	0.5

Table 5: Production and labor substitutability

$\alpha_K$	$\alpha_L$	$B$	$\theta$	$\psi$
0.3	0.6	200	-0.5	0.6

Table 6: Government, population growth, and pollution

$\tau$	$\tau_w$	$n$	$\xi$	$\sigma$	$\varsigma$
2%	12.4%	1%	0.87	0.6	0.1

In the tables below, the values corresponding to the benchmark economy appear in bold.

Table 7: The economy and the environmental tax

	$\tau$				
	2%	4%	6%	8%	10%
$\Delta h$	<b>0.720</b>	0.761	0.785	0.802	0.814
$k^*/y^*$	<b>2.916</b>	2.883	2.866	2.855	2.847
<i>Leisure</i>	<b>0.672</b>	0.677	0.680	0.682	0.684
$R^*$	<b>2.631</b>	2.606	2.593	2.584	2.577
$\lambda_1^*$	<b>0.234</b>	0.236	0.238	0.239	0.240
$\lambda_2^*$	<b>0.216</b>	0.214	0.214	0.213	0.213
$m^*$	<b>0.094</b>	0.0857	0.081	0.078	0.076
$s^*/y^*$	<b>0.135</b>	0.137	0.139	0.139	0.140
$y^*$	<b>59.295</b>	54.706	52.111	50.312	48.943
$W^*$	<b>1.679</b>	1.627	1.592	1.566	1.545

In Table 7, we indicate the steady state values for various levels of the environmental tax. Our numerical result indicates that a higher environmental decreases output per capita and welfare. Indeed, within our choice of parameters, the negative effects of environmental taxation dominate the positive effects. The crowding out effect captured by the decline in  $k^*/y^*$  and the retirement effect captured by the decrease in  $\lambda_2^*$  both decrease output per capita. Those effects dominate the health saving effect captured by the increase in  $s^*$ , the young labor participation effect captured by the increase in  $\lambda_1^*$ , and the productivity effect captured by the larger  $\Delta h$ , which have a positive effect on output per capita. Despite the increase in leisure and the improvement in the health profile, welfare decreases because the health saving effect results in lower consumption.

Table 8: Double environmental tax and the health profile

% change in	Health profile		
	Life-cycle	Flat	
		$m$ endogenous	$m$ exogenous
$\Delta h$	<b>5.72</b>	0	0
$k^*/y^*$	<b>-0.94</b>	-4.22	-6.70
<i>Leisure</i>	<b>0.74</b>	0.68	0
$R^*$	<b>-0.94</b>	-0.11	0
$\lambda_1^*$	<b>1.16</b>	0.56	0
$\lambda_2^*$	<b>-0.54</b>	0.07	0
$m^*$	<b>-8.99</b>	-6.15	0
$s^*/y^*$	<b>1.46</b>	-0.22	0
$y^*$	<b>-7.74</b>	-7.24	-4.52
$W^*$	<b>-3.09</b>	-3.18	0

In Table 8, we double the environmental tax from two to four percent and compare the effect on macroeconomic variables in the benchmark case

and in the case when the health profile is flat, keeping investment in health  $m$  endogenous (first two columns). When the health profile is flat, recall that the health status improves with the environmental tax *over the entire life-cycle*, shifting the health profile. Contrary to the benchmark case, there is no change in the health profile over the life-cycle. Our numerical simulations indicate that the decrease in output per capita is understated by half a percentage point if the change in the health profile between the two periods in life is ignored. Specifically, ignoring the life-cycle changes in the health profile leads to largely overstate the crowding-out effect of environmental taxation. Indeed the decrease in the capital output ratio is overstated by 3.28 percentage points. By contrast with previous analysis, our benchmark case indicates that the decrease in output per capita is not the result of an overwhelmingly important crowding-out effect but the cumulated result of several negative general equilibrium effects related to life-cycle choices. If the net effect on welfare is very similar in both simulated case, the benchmark case indicates that contrary to previous analysis, the decrease in welfare comes from a larger decrease in second period consumption and first period leisure. The larger decrease in second period consumption is partially offset by the improvement in the health profile.

In the last column of the table, we present steady state variables when the health profile is flat and investment in health is exogenous. The health profile does not change between the two periods in life and individuals cannot adjust their investment in health as overall productivity increases. Therefore, the new general equilibrium effects that appeared in the benchmark model disappear. Consistent with the theoretical results, we only retrieve the standard crowding out and productivity effects. The crowding out effect is overstated by 4.86 percentage points and the negative effect on output is understated by 3.22 percentage points compared to the benchmark model. We also find that the negative effect on welfare is understated by 3.09 percentage points compared to the benchmark case. Indeed, in the last column of the table, we find no effect on welfare as the positive effect of the pollution tax hike on health is perfectly offset by the decrease in consumption.

Table 9: Double environmental tax and labor substitutability

$\theta$	-1	<b>-0.5</b>	-0.1	0.1	0.5	1
% change in:						
$\Delta h$	5.66	<b>5.72</b>	5.78	5.82	5.95	6.49
$R^*$	-1.86	<b>-0.94</b>	-0.19	0.19	0.95	2.45
$\lambda_1^*$	1.32	<b>1.16</b>	0.98	0.86	0.57	-0.67
$\lambda_2^*$	-0.78	<b>-0.54</b>	-0.15	0.19	1.73	47.06
$m^*$	-8.98	<b>-8.99</b>	-9.00	-9.04	-8.97	-8.93
$y^*$	-7.17	<b>-7.74</b>	-8.24	-8.51	-9.15	-10.45
$W^*$	-2.66	<b>-3.09</b>	-3.38	-3.51	-3.71	-4.02

In Table 9, we show that when the environmental tax is doubled from

two to four percent, our numerical results change depending on the value assigned to the degree of substitutability  $\theta$  between young and old labor. Consistent with our theoretical section, the numerical results indicate that whether the new general equilibrium effects are positive or negative depends on the degree of substitutability between young and old labor. As shown in the table, this significantly affects the analysis regarding the economic effects of the environmental tax. If young and old labor are supposed to be perfect substitutes ( $\theta = 1$ ) like in past contributions, it appears that the negative effect of environmental taxation on output per capita and welfare are respectively overstated by 2.71 percentage points and 0.93 percentage points, compared to our benchmark case. Indeed, when young and old labor are perfect substitutes, an increase in the environmental tax leads to a large reallocation of labor supply toward the second stage of life as individuals are relatively healthier. On the contrary, when young and old labor are imperfect complements ( $\theta = -0.5$ ), the reallocation of labor supply is toward the first stage of life and much smaller. Thus, when young and old labor are assumed to be perfect substitutes, the distortive effect of environmental taxation are overstated and reversed over the life-cycle compared to our benchmark case.

Table 10: Double environmental tax and population growth

$n$	-1%	0	1%	2%	3%
% change in:					
$\Delta h$	5.82	5.78	<b>5.72</b>	5.68	5.64
$R^*$	-0.77	-0.85	<b>-0.94</b>	-1.04	-1.15
$\lambda_1^*$	1.10	1.13	<b>1.16</b>	1.17	1.17
$\lambda_2^*$	-0.72	-0.63	<b>-0.54</b>	-0.46	-0.39
$m^*$	-9.19	-9.02	<b>-8.99</b>	-8.90	-8.81
$y^*$	-7.97	-7.86	<b>-7.74</b>	-7.61	-7.48
$W^*$	-3.00	-3.04	<b>-3.09</b>	-3.17	-3.28

In Table 10, we simulate the effect of a doubling of the environmental tax from two to four percent for different growth rates  $n$  of the population. The long term effects of environmental taxation exhibit small variations within a wide range of parameters for population growth.

Table 11: Double environmental tax and retirement

	$\lambda_2$ endogenous			$\lambda_2$ exogenous
	$\gamma = 0$	$\gamma = \mathbf{1.5}$	$\gamma = 3$	
% change in:				
$\Delta h$	5.79	<b>5.72</b>	5.71	5.79
$R^*$	-0.70	<b>-0.94</b>	-1.06	-0.70
$\lambda_1^*$	1.18	<b>1.16</b>	1.15	1.18
$\lambda_2^*$	0	<b>-0.54</b>	-0.52	—
$m^*$	-9.10	<b>-8.99</b>	-8.95	-9.12
$y^*$	-8.27	<b>-7.74</b>	-7.47	-8.27
$W^*$	-2.14	<b>-3.09</b>	-3.90	-2.14

In Tables 11, we simulate the doubling of the environmental tax from two to four percent for different degrees of preference for retirement  $\lambda_2$ . We also simulate the case with exogenous retirement. Our numerical result indicates that ignoring retirement choices results in overstating the negative effect of environmental taxation on output per capita by 0.53 percentage points, and understating the negative effect of environmental taxation on welfare by 0.95 percentage points. As the environmental tax improves the health profile, it results in postponing retirement, which is beneficial to output and has a negative effect on welfare.

Table 12: Double environmental tax and Social Security tax

$\tau^w$	0%	10%	<b>12.4%</b>	15%
% change in:				
$\Delta h$	5.73	5.73	<b>5.72</b>	5.72
$R^*$	-1.02	-0.95	<b>-0.94</b>	-0.93
$\lambda_1^*$	1.22	1.19	<b>1.16</b>	1.16
$\lambda_2^*$	0.48	0.5	<b>-0.54</b>	-0.54
$m^*$	-9.04	-9.03	<b>-8.99</b>	-8.97
$y^*$	-7.66	-7.73	<b>-7.74</b>	-7.75
$W^*$	-2.85	-3.03	<b>-3.09</b>	-3.17

In Table 12, we simulate the doubling of the environmental tax for different values of the social security tax. In the absence of a social security tax, the retirement system is not a PAYG but a funded system, which workers save for their own retirement. Our numerical results indicate that the elimination of the PAYG would only moderately decrease the negative impact of environmental taxation on output and welfare respectively by 0.08 and 0.24 percentage points.

Table 13: Double environmental tax, health and pollution functions

	Benchmark	$H(m)$		$\delta(P)$	
		$\epsilon = 0.7$	$\eta = 0.8$	$\varsigma = 0.2$	$\xi = 1$
% change in:					
$\Delta h$	<b>5.72</b>	4.96	4.74	16.38	6.31
$k^*/y^*$	<b>-0.94</b>	-1.65	-0.96	-6.06	-1.54
<i>Leisure</i>	<b>0.74</b>	1.62	0.71	5.03	1.22
$R^*$	<b>-0.94</b>	-0.69	-0.80	-2.80	-1.09
$\lambda_1^*$	<b>1.16</b>	1.82	0.97	5.77	1.54
$\lambda_2^*$	<b>-0.54</b>	-0.35	-0.50	-1.37	-0.60
$m^*$	<b>-8.99</b>	-10.02	-8.64	-29.02	-11.18
$s^*/y^*$	<b>1.46</b>	1.40	1.59	5.41	2.16
$y^*$	<b>-7.74</b>	-7.72	-8.98	1.26	-7.57
$W^*$	<b>-3.09</b>	-3.32	-3.60	20.10	-2.21

In Table 13, we change the parameters of the health investment and the pollution functions to examine the sensitivity of our results.

We set the value of  $\epsilon$  to 0.7 to accentuate the non-linearity of the health investment function, which slightly reinforces the effect of environmental policy on output and welfare. By extension, ignoring the non-linearity of the health investment function would lead to understate the effect of environmental taxation. We also increase the effect of health expenditure on health by setting  $\eta = 0.8$ . This has a more significant effect on our results. As expected, the more health responds to investment in health, the larger the negative effect of environmental taxation on output and welfare.

We set the value of  $\varsigma$  to 0.2 to accentuate the non-linearity of the pollution function. The numerical results are very sensitive to the value of this parameter. When the value of  $\varsigma$  increases, the productivity effect dominates the other effects and the impact of the tax hike on output per capita is positive. Leisure and consumption increase (due to a large health saving effect), resulting in a large welfare gain. We also increase the effect of the tax on pollution by increasing the value of  $\xi$  to 1. The results are moderately sensitive to this parameter. The more pollution responds to the tax, the larger the productivity effect and the smaller the negative effect of the tax hike on output per capita and welfare.

## 5 Conclusion

In this paper, we study the economic effect of environmental taxation when pollution impacts morbidity. In contrast with the existing literature, we propose a model that takes into consideration the interaction between pollution and health over the life-cycle and its consequences on individual optimal choices. We show that when the interaction between pollution and health over the life-cycle is captured, the effect of environmental taxation on health

are not limited to the traditional negative crowding-out effect and positive productivity effect. We identify several new general equilibrium effects which influence output per capita and are channeled through saving, labor supply by young and old workers, and investment in health. We show that if the life-cycle characteristic of the health profile is ignored or investment in health is considered exogenous, the new general equilibrium effects either do not appear or are modified. Numerical examples indicate that those simpler frameworks tend to understate the negative effect of environmental taxation on output per capita and welfare. We also show that the direction of responses of the labor supply and investment in health depends on the degree of complementarity or substitutability between young and old labor. Numerical examples indicate that frameworks assuming that young and old labor are perfect substitutes tend to overstate the negative effects of environmental taxation on output per capita and welfare.

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## A Proof of Proposition 2

When  $\theta < 0$ , the LHS of equation (28) is decreasing in  $\tau$  and in  $R^*$  because  $\lambda_2^* = \Lambda_2(R^*)$  is an increasing function of  $R^*$ . Because  $\mathcal{D}(R^*)$  is increasing in  $\tau$ , (26) defines a unique positive  $R^* = \tilde{\mathcal{R}}(\tau)$  with  $\tilde{\mathcal{R}}'(\tau) < 0$ . When  $\theta = 0$ , the left-hand side of (28) is independent from  $\tau$  and  $R^*$ . Therefore (28) defines a unique positive  $R^*$  independent from  $\tau$ . When  $\theta \in ]0, 1]$ , (26) can be written as (using the expressions of  $\Lambda_1(R^*, \bar{m})$  and  $\Lambda_2(R^*)$ ):

$$\frac{\mathcal{H}(\tau)}{(1+n)} = \frac{(1-\bar{m})\mathcal{D}(R^*)^{1/\theta-1}R^*}{\phi} \times \frac{[\phi(1+\beta) + \beta\gamma] \mathcal{D}(R^*)/R^* + \left(\frac{\psi}{1-\psi}\right) \left(\frac{\gamma\beta}{1+n}\right)}{(1+\beta+\varphi) + \varphi(1+n) \left(\frac{1-\psi}{\psi}\right) \mathcal{D}(R^*)/R^*} \quad (\text{A.1})$$

The second term in the RHS of this equation is increasing in  $\mathcal{D}(R^*)/R^*$ : its derivative with respect to  $\mathcal{D}(R^*)/R^*$  is equal to:

$$\frac{(1+\beta)\phi\psi^2}{[(1+n)(1-\psi)\mathcal{D}(R^*)/R^* + \psi]^2} > 0$$

Because  $\mathcal{D}(R^*)/R^*$  is increasing in  $R^*$ , therefore the RHS of the equation is increasing in  $R^*$  while the LHS is increasing in  $\tau$ . Therefore (28) defines a unique positive  $R^* = \tilde{\mathcal{R}}(\tau)$  with  $\tilde{\mathcal{R}}'(\tau) > 0$ .

## B Proof of Proposition 3

When  $\theta = 0$ , Lemma 1 applies. When  $\theta < 0$ , equation (31) may be written as:

$$\frac{\mathcal{D}(R^*)^{1/\theta}}{\Lambda_2(R^*)} (1+n)\Lambda_1(R^*, \Omega(R^*; \tau)) - H(\Omega(R^*; \tau)) - [1 - \delta(\sigma^{-1}g(\tau))] \bar{h} = 0 \quad (\text{B.1})$$

where the LHS is decreasing in  $R^*$ . Therefore, under the condition that  $\lim_{R^* \rightarrow 1} LHS > 0 > \lim_{R^* \rightarrow +\infty} LHS$ , there exists a unique steady-state interest rate  $R^*$ . The influence of  $\tau$  on the LHS is given by the sign of the derivative of the LHS with respect to  $\tau$ :  $\tau^{-1} \left\{ \underbrace{\epsilon_\tau^{\Omega(\cdot)}}_{-} \left[ \underbrace{\epsilon_{\Omega(\cdot)}^{\Lambda_1(\cdot)}}_{-} - \underbrace{\epsilon_{\Omega(\cdot)}^{h_2^*}}_{+} \right] - \underbrace{\epsilon_\tau^{\delta(\cdot)}}_{-} \underbrace{\epsilon_{\delta(\cdot)}^{h_2^*}}_{-} \right\}$  where  $\epsilon_j^i \equiv \frac{\partial i / \partial j}{i/j}$ .

When this expression is negative (resp. positive), from the theorem of the implicit function,  $R^*$  is a decreasing (resp. increasing) function of  $\tau$ .

When  $\theta \in ]0, 1]$ , equation (31) may be written as (using (29)):

$$\frac{H'(\Omega(R^*; \tau))}{(1+n)} = \frac{\mathcal{D}(R^*)^{1/\theta-1} R^*}{\beta(1-\phi)} \times \frac{[\phi(1+\beta) + \beta\gamma] \mathcal{D}(R^*)/R^* + \left(\frac{\psi}{1-\psi}\right) \left(\frac{\gamma\beta}{1+n}\right)}{1 + (1+n) \left(\frac{1-\psi}{\psi}\right) \mathcal{D}(R^*)/R^*} \quad (\text{B.2})$$

The second term in the RHS of this equation is increasing in  $\mathcal{D}(R^*)/R^*$ : its derivative with respect to  $\mathcal{D}(R^*)/R^*$  is equal to:

$$\frac{(1+\beta)[\phi + \beta(\gamma + \phi) + \phi\varphi]\psi^2}{[(1+n)\varphi(1-\psi)\mathcal{D}(R^*)/R^* + (1+\beta+\varphi)\psi]^2} > 0$$

Because  $\mathcal{D}(R^*)/R^*$  is increasing in  $R^*$ , therefore the RHS of the equation is increasing in  $R^*$  while the LHS is increasing in  $\tau$ . Therefore (28) defines a unique positive  $R^* = \mathcal{R}(\tau)$  with  $\mathcal{R}'(\tau) > 0$ .

## C Proof of Corollary 3

For (i) when  $\theta \leq 0$ , it is straightforward from equation (29). When  $\theta > 0$ , from equation (B.2),  $H'(m^*)$  is equal to an expression increasing in  $R^*$ . Because  $H'(m^*)$  is decreasing in  $m^*$ , it means that  $m^*$  is equal to an expression decreasing in  $R^*$ . From Proposition 3,  $m^*$  is therefore decreasing in  $\tau$ .

For (ii), it is straightforward that  $\lambda_1^*$  is increasing in  $\tau$  when  $\theta < 0$ , from equations (24), (30) and Proposition 3. When  $\theta = 0$ , Lemma 1 applies, therefore from equations (24) and (30)  $\lambda_1^*$  is increasing in  $\tau$ .

equation (29) enables us to write:

$$\lambda_1^* = [1 - m^*] - \phi\varphi \frac{H(m^*) + [1 - \delta(\sigma^{-1}g(\tau))]\bar{h}}{\beta(1-\phi)H'(m^*)} \quad (\text{C.1})$$

For (iii) see Proposition 3 and equation (23).

For (iv), the RHS of equation (29) is  $h_2^*$ . When  $\theta \geq 0$ , from Proposition 3, Corollary 3(i) and the fact that  $\mathcal{D}(R^*)/R^*$  is increasing in  $R^*$ , the RHS is increasing in  $\tau$ , therefore  $h_2^*$  is increasing in  $\tau$  when  $\theta \geq 0$ . When  $\theta < 0$ , from equation (31), Proposition (3) and Corollary 3(ii),  $h_2^*$  is increasing in  $\tau$ .

## D Proof of Proposition 4(ii)

When  $m^*$  is endogenous, with  $\Delta_h^* = 1$  and  $\bar{h}$  is replaced by  $h^*$  defined by (32), we obtain  $m^*$  as an implicit function of  $R^*$  and  $\tau$ :

$$\frac{\phi\varphi}{\beta(1-\phi)} \frac{H(m^*)}{(1-m^*)H'(m^*)} = \delta(\sigma^{-1}g(\tau)) \left( 1 - \frac{(1+\beta)}{1+\beta+\varphi+\varphi(1+n)\left(\frac{1-\psi}{\psi}\right)\mathcal{D}(R^*)/R^*} \right) \quad (\text{D.1})$$

The LHS is an increasing function of  $m^*$  and the RHS is decreasing in  $\tau$  and increasing in  $R^*$ , therefore this expression defines:

$$m^* = \tilde{\Omega}(R^*; \tau) \quad (\text{D.2})$$

with  $\partial\tilde{\Omega}(R^*; \tau)/\partial R^* > 0$  and  $\partial\tilde{\Omega}(R^*; \tau)/\partial\tau < 0$ . From (24) and (25):

$$\frac{\lambda_2^*}{\lambda_1^*} = \frac{\phi \left[ (1 + \beta + \varphi) + \varphi(1 + n) \left( \frac{1-\psi}{\psi} \right) \mathcal{D}(R^*)/R^* \right]}{(1 - \tilde{\Omega}(R^*; \tau)) \left[ (\phi + \gamma)(1 + \beta) - \gamma + \left( \frac{\psi}{1-\psi} \right) \left( \frac{\gamma\beta}{1+n} \right) R^*/\mathcal{D}(R^*) \right]}$$

that is  $\lambda_2^*/\lambda_1^*$  is increasing in  $R^*$  and decreasing in  $\tau$ . Using (33), we obtain the expression of the steady-state interest rate as a function of  $\tau$ :

$$\left[ \frac{\frac{\phi}{1+n} \left[ (1 + \beta + \varphi) + \varphi(1 + n) \left( \frac{1-\psi}{\psi} \right) \mathcal{D}(R^*)/R^* \right]}{(1 - \tilde{\Omega}(R^*; \tau)) \left[ (\phi + \gamma)(1 + \beta) - \gamma + \left( \frac{\psi}{1-\psi} \right) \left( \frac{\gamma\beta}{1+n} \right) R^*/\mathcal{D}(R^*) \right]} \right]^\theta = \mathcal{D}(R^*) \quad (\text{D.3})$$

When  $\theta < 0$ , the LHS is decreasing in  $R^*$  and increasing in  $\tau$  while the RHS is always increasing in  $R^*$ . From the implicit function theorem, we obtain that  $R^*$  is increasing in  $\tau$ . When  $\theta = 0$ , Lemma 1 applies:  $R^*$  is independent from  $\tau$ .

When  $\theta \in ]0, 1]$ , using equation (24) and (25) to replace respectively  $\lambda_1^*$  and  $\lambda_2^*$  in equation (D.1), we obtain:

$$\frac{\phi\beta(1 - \phi)H'(\tilde{\Omega}(R^*; \tau))}{H(\tilde{\Omega}(R^*; \tau))} \delta(\sigma^{-1}g(\tau)) = \frac{[(\phi + \gamma)(1 + \beta) - \gamma] \mathcal{D}(R^*)/R^* + \left( \frac{\psi}{1-\psi} \right) \left( \frac{\gamma\beta}{1+n} \right) \mathcal{D}(R^*)^{\frac{1}{\theta}-1} R^*}{\varphi + \varphi(1 + n) \left( \frac{1-\psi}{\psi} \right) \mathcal{D}(R^*)/R^*} \quad (\text{D.4})$$

Because the ratio in the RHS of this equation is increasing in  $\mathcal{D}(R^*)/R^*$ , because  $\mathcal{D}(R^*)/R^*$  is increasing in  $R^*$  and  $1/\theta - 1 > 0$ , therefore the RHS is an increasing function of  $R^*$ . From (30) and the definition of  $H(\cdot)$ , the LHS is decreasing in  $R^*$ . Therefore there exists a unique steady-state interest rate  $R^*$ .

Under the necessary condition that  $\partial LHS/\partial\tau < 0$ , from the theorem of the implicit function,  $R^*$  is a decreasing function of  $\tau$ . This condition can be written as:

$$\epsilon_\tau^{\Omega(\cdot)} \left[ \epsilon_{\Omega(\cdot)}^{H'(\cdot)} - \epsilon_{\Omega(\cdot)}^{h^*} \right] < \epsilon_\tau^{\delta(\cdot)} \epsilon_{\delta(\cdot)}^{h^*} \quad \text{where} \quad \epsilon_j^i \equiv \frac{\partial i/\partial j}{i/j}. \quad (\text{C'1})$$

## E Proof of Corollary 4

For (i) when  $\theta \in [0, 1]$ , it is straightforward from equation (D.2) and Proposition 4(ii). When  $\theta < 0$ , from equation (D.3), we can write that:

$$m^* = 1 - \frac{\frac{\phi}{1+n} \left[ (1 + \beta + \varphi) + \varphi(1 + n) \left( \frac{1-\psi}{\psi} \right) \mathcal{D}(R^*)/R^* \right]}{\mathcal{D}(R^*)^{1/\theta} \left[ (\phi + \gamma)(1 + \beta) - \gamma + \left( \frac{\psi}{1-\psi} \right) \left( \frac{\gamma\beta}{1+n} \right) R^*/\mathcal{D}(R^*) \right]}$$

where the RHS is decreasing in  $R^*$ . From Proposition 4(ii), it means that  $m^*$  is decreasing in  $\tau$ .

For (iv) when  $\theta \in [0, 1]$ , equation (D.4) can be written as:

$$h^* = \frac{H(m^*)}{\delta(\sigma^{-1}g(\tau))} = \phi\beta(1 - \phi)H'(m^*) \times \left[ \frac{[(\phi + \gamma)(1 + \beta) - \gamma] \mathcal{D}(R^*)/R^* + \left( \frac{\psi}{1-\psi} \right) \left( \frac{\gamma\beta}{1+n} \right) \mathcal{D}(R^*)^{\frac{1}{\theta}-1} R^*}{\varphi + \varphi(1 + n) \left( \frac{1-\psi}{\psi} \right) \mathcal{D}(R^*)/R^*} \right]^{-1} \quad (\text{E.5})$$

where the RHS is decreasing in  $R^*$  from the demonstration of Proposition 4 (see comments below equation D.4) and increasing in  $\tau$  from Corollary 4(i). From Proposition 4(ii)  $R^*$  is decreasing in  $\tau$ , therefore the RHS is increasing in  $\tau$  and  $h^*$  is increasing in  $\tau$ .

When  $\theta < 0$ , equation (D.1) may be written as:

$$h^* = \frac{H(m^*)}{\delta(\sigma^{-1}g(\tau))} = \frac{\beta(1 - \phi)(1 - m^*)H'(m^*)}{\phi\varphi} \left( 1 - \frac{(1 + \beta)}{1 + \beta + \varphi + \varphi(1 + n) \left( \frac{1-\psi}{\psi} \right) \mathcal{D}(R^*)/R^*} \right) \quad (\text{E.6})$$

where the RHS is increasing in  $R^*$  and increasing in  $\tau$  from the definition of  $H(\cdot)$  and Corollary 4(i). From Proposition 4(ii)  $R^*$  is increasing in  $\tau$ , therefore the RHS is increasing in  $\tau$  and  $h^*$  is increasing in  $\tau$ .