

The effect of the timing of selection on the mutation load, inbreeding depression and population size

Diala Abu Awad, Sylvain Billiard, Viet Chi Tran

▶ To cite this version:

Diala Abu Awad, Sylvain Billiard, Viet Chi Tran. The effect of the timing of selection on the mutation load, inbreeding depression and population size. 2014. hal-00987288v1

HAL Id: hal-00987288 https://hal.science/hal-00987288v1

Submitted on 5 May 2014 (v1), last revised 5 May 2014 (v2)

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers. L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

The effect of the timing of selection on the mutation load, inbreeding depression and population size

Diala Abu Awad*, Sylvain Billiard* and Viet Chi Tran^\dagger

^{*}Génétique et évolution des populations végétales, UFR de Biologie, UMR CNRS 8198,

Université Lille 1, 59655 Villeneuve d'Ascq Cedex, France

 $^{^{\}dagger} {\rm Laboratoire}$ Paul Painlevé, UFR de Mathématiques, UMR CNRS 8524, Université

Lille 1, 59655 Villeneuve d'Ascq Cedex, France

Running Head: Demography and the timing of selection

Keywords: Genetic load, Population size, Inbreeding depression, Selection, Deleterious mutations, Demography

Corresponding Author:

Diala Abu Awad

UMR-CNRS 8198

Laboratoire Génétique et évolution des populations végétales

Bâtiment SN2

Université Lille 1

F-59655 Villeneuve d'Ascq

France

 $+33\ 3\ 20\ 33\ 63\ 97$

diala.abu-awad@ed.univ-lille1.fr

ABSTRACT

1

If and how the genetic load affects population size has been long debated 2 as on one hand it has been suggested that a high genetic load leads to ex-3 tinction, and on the other hand, if selection during the life-cycle takes place 4 before density dependence has acted, then population size should not be af-5 fected by its genetic load. Explicitly considering the life cycle and the timing 6 of selection is therefore a factor that cannot be ignored when quantifying the 7 effect of deleterious mutations on population size. In addition, population 8 genetics models calculating the expected genetic loads and levels of inbreedg ing depression ignore the potential effects of demography on these variables. 10 Here we propose a deterministic model in continuous time where deleterious 11 mutations affect individual fitness in one of four ways: by decreasing mating 12 success, fecundity and adult or zygote survival. The genetic load, inbreeding 13 depression and population size are variables that emerge from the model. 14 Our results are compared to the expectations from the fundamental model 15 of natural selection. We find that changing the timing of selection mostly 16 affects population size, but also leads to genetic loads and inbreeding depres-17 sion that diverge from the fundamental model. Our results emphasize the 18

¹⁹ importance of integrating both population demography and genetics in order
²⁰ to study the demographic impact and, more generally, the fate of deleterious
²¹ mutations.

INTRODUCTION

22

Deleterious mutations are constantly introduced into populations at rela-23 tively high rates (Keightley and Lynch, 2003). In spite of their deleterious 24 effect, these alleles are not always immediately eliminated by selection from 25 the genetic pool but can persist for several generations and, in the case of 26 stochasticity, can even go to fixation. These mutations decrease mean popu-27 lation fitness by engendering a genetic or mutational load, and are responsible 28 for inbreeding depression. In the field of population genetics, the evolution of 29 load and inbreeding depression as a function of population size and structure 30 has been greatly explored (Bataillon and Kirkpatrick, 2000; Roze and Rous-31 set, 2004; Glémin, 2003). However, these models do not consider an explicit 32 interaction between the two, as population size is considered to be a parame-33 ter. If and how these mutations affect population size remains unclear; while 34 inbreeding depression is a major concern in conservation biology, whether the 35 genetic load of populations affects population size and viability is debatable. 36 In the case of very small populations, it is widely accepted that they can be 37 at risk of a mutational meltdown (extinction due to the fixation of deleterious 38 alleles at an accelerating rate; Lande, 1994; Lynch et al., 1995; Coron et al., 39

5

2013), but there is little understanding of how the genetic load affects the 40 demography of populations that are not at risk of extinction. We find two 41 opposing views in literature (reviewed in Agrawal and Whitlock, 2012); some 42 authors argue that populations cannot persist with high mutation or genetic 43 loads (Kondrashov, 1995), whereas others have insisted that load has little or 44 no ecological consequences (Turner and Williams, 1968; Wallace, 1970). In 45 the latter case, authors have argued that due to density dependence, deaths 46 of individuals due to selection simply replace the unavoidable deaths due to 47 a lack of resources (soft selection), whereas in the former case, the genetic 48 load is expected to directly decrease population size, independently of density 49 dependent factors (hard selection). The main difference between these two 50 types of selection is the timing of the elimination of individuals via selection, 51 either before resource consumption (soft selection) or after these resources 52 have been used and rendered inaccessible (hard selection). In a model pro-53 posed by Agrawal and Whitlock (2012), the authors came to the conclusion 54 that if individuals are eliminated by selective death before having consumed 55 any resources (i.e. at the zygote stage), then the genetic load would not af-56 fect population size, as the loss of juveniles would be "masked by ecological 57 compensation". 58

The life stage at which selection takes place, or as we refer to it, the 59 timing of selection, is therefore non-negligible when considering the effect 60 of the genetic load on population size (Wallace, 1970; Charlesworth, 1971; 61 Clarke, 1973). However, population size is not only affected by the timing 62 of resource consumption but more generally by the ecological trait affected 63 (Clarke, 1973). Models that have studied the ecological consequences of 64 differences in fitness related traits between individuals and/or species (such 65 as competitive abilities, death rates and reproductive rates) have come to 66 the conclusion that they can affect population size (for example Abrams, 67 2003; Abrams et al., 2003; Schreiber and Rudolf, 2008). However, these 68 models consider only phenotypic traits and do not take into account the 69 genetics that could be behind the differences in phenotypes. The only work 70 to our knowledge to explicitly consider ecology and genetics is a paper by 71 Clarke (1973) in which the author coined the term "numerical load" (the 72 demographic equivalent of the genetic load) the decrease of population size 73 due to the presence of deleterious mutations, a term which we will be using 74 throughout this paper. Clarke (1973) proposed a model where population size 75 is a consequence of the effect of the genetic load on different life traits, and 76 where selection takes place either before or after density-dependent factors 77

come into play. He found that in both cases, the genetic load leads to a 78 numerical load, but in the former case density-dependent factors partially 79 compensate for density-independent mortality, leading to a smaller numerical 80 load. His overall conclusions agree with the results of ecological models; if 81 the genetic load affects ecological traits, then population size is also affected. 82 The magnitude of the effect of the genetic load on the numerical load in 83 turn depends on the traits affected by selection (i.e. a same genetic load on 84 different traits can lead to different numerical loads). 85

A limitation of the model proposed by Clarke (1973) is that it consid-86 ers a genetic load that evolves independently of the demographic model and 87 the timing of selection. The relative selective values he proposed were con-88 structed so as to ensure that the genetic load remains equal to that expected 89 from the fundamental model of population genetics (Gillespie, 1998, p.61). 90 The genetic load is expected to directly affect a measure of individual or pop-91 ulation fitness and fitness has been defined as being an individual's (or geno-92 type's) mating success, fecundity or survival (Agrawal and Whitlock, 2012). 93 While expected to be of great importance in demographic models (Haldane, 94 1957; Agrawal and Whitlock, 2012), the timing of selection is considered to 95 be of no importance when calculating the genetic load or inbreeding depres-96

sion in population genetics models, and selection has been interpreted as 97 affecting either zygote survival (e.g. Gillespie, 1998) or reproductive success 98 (e.g. Roze and Rousset, 2004). However, explicitly considering the life cycle 99 could affect how selection acts, leading to a genetic load and an amplitude 100 of inbreeding depression that depend on the timing of selection. Indeed, it 101 has been observed in several species that the amplitude of inbreeding depres-102 sion expressed can vary between traits (Frankham et al. 2010, Chapter 13; 103 Angeloni et al. 2014). 104

It has been suggested that the ambiguity of what is found in literature 105 concerning the effect of the genetic load on population size could be due 106 to a lack of theoretical works that attempt to address this question using 107 "explicit ecological models" (Agrawal and Whitlock, 2012). In this present 108 work we attempt to address the question of how the introduction of recurrent 109 deleterious mutations into a population (whose size is affected by selection) 110 influences the genetic load, the numerical load and inbreeding depression. We 11: propose a deterministic model where the number of individuals carrying each 112 genotype is considered explicitly. In the presence of selection, genotypes differ 113 in their selective values at a single trait, and there is no density or frequency-114 dependent effect on fitness. We consider selection at four traits considered 115

to represent fitness: mating success, fecundity, zygote survival and adult survival. Population size, the genetic load and inbreeding depression are all emerging properties of the model.

119

ANALYTICAL MODEL

We consider the evolution of a population with a varying population size and 120 a single bi-allelic locus, where A is the wild-type and a the mutant allele. The 12 population is pannictic and made up of sexually reproducing hermaphrodite 122 individuals. The environment is stable, and the population is isolated and 123 spatially unstructured. Three genotypes can be found in the population, aa, 124 Aa and AA, which, from here onwards, are denoted X, Y, and Z respectively. 125 At a given time t, the population is made up of three kinds of individuals, 126 X_t, Y_t and Z_t representing the number of individuals carrying the respective 127 genotype. We denote the population size $N_t = X_t + Y_t + Z_t$. In a large 128 population setting, these quantities can be considered as continuous, and 129 the evolution of the number of individuals of each genotype is described in 130 continuous time using ordinary differential equations. Three processes affect 131 the change in the number of individuals of each genotype, births (occurring 132

with rate R_t^V , where V can be either X, Y or Z), deaths (at a rate M_t^V) and mutation. Selection and density dependence are introduced in these processes. We consider that the mutation from A to a is unidirectional and occurs with a probability μ at the gamete stage.

We first introduce the demographic and mutational properties of the 137 model without considering selection and show that this model respects the 138 genotypic frequencies predicted by the Hardy-Weinberg model for neutral 139 alleles (and no mutation). Selection is then introduced during different mo-140 ments of the life cycle and we define the variables measured in order to esti-14 mate the effect of the recurrent introduction of deleterious mutations on the 142 numerical load, the genetic load and inbreeding depression. In order to facil-143 itate the reading of the following sections, all the notations used throughout 144 the text have been summarized in Table 1. 145

¹⁴⁶ Model without selection

As we consider mutations occurring during gamete formation, the proportion of *a* gametes produced per genotype are 1, $\frac{1+\mu}{2}$ and μ for *X*, *Y* and *Z* individuals respectively. Mutational events are therefore integrated into the birth rate R_t^V ; for example, as *Z* individuals produce a proportion μ of *a*

Table 1: Notations.

- Λ, L, δ The numerical load, the genetic load and inbreeding depression.
- N_t, N_{eq} Population size at time t with or with selection, and population size at equilibrium when there is no selection.
- V_t, \tilde{V}_t The total number of either X, Y or Z individuals (with genotypes aa, Aa and AA respectively) at time t, and the number of individuals at time t that contribute to the genetic pool.
- N_{mut}, V_{mut} Population size and number of V individuals at mutation-selection balance.
- R_t, M_t The total birth and death rates of the population at time t.
- R_t^V, M_t^V The birth and death rates of individuals of genotype V at time t.

b, d The intrinsic birth and death rates of individuals.

- s,h The coefficient of selection and dominance of allele a. The relative fitnesses of X, Y and Z individuals at a given trait are (1 s), (1 hs) and 1 respectively.
- μ, μ_{fix} The mutation rate from A to a and the threshold value of μ for which there is deterministic fixation of a.

gametes and $(1-\mu)$ of A gametes. When two Z individuals are crossed, they 151 produce X, Y and Z offspring with proportions μ^2 , $2\mu(1-\mu)$ and $(1-\mu)^2$ 152 respectively. For each reproductive pair, the parents contribute both via the 153 male and the female functions. We consider that the total number of female 154 gametes produced by all individuals in the population is limited and, when 155 there is no selection, depends only on the number of individuals, whereas male 156 gametes are produced in very large quantities and are subject to competition. 15 The probability that an individual reproduces via the male function depends 158 on the proportion of male gametes contributed compared to the total amount 159 of male gametes available. For example, when the X and Y individuals cross 160 to give X individuals, X individuals contribute X_t ovules and a proportion 161 of $\frac{X_t}{N_t}$ male gametes, while Y individuals contribute $(1+\mu)\frac{Y_t}{2}$ ovules and a 162 proportion of $(1+\mu)\frac{Y_t}{2N_t}$ male gametes (as only $\frac{(1+\mu)}{2}$ of the gametes produced 163 carry an a allele). 164

Generally, the equation describing the change in the number of individuals for each genotype is given by

$$\frac{dV_t}{dt} = R_t^V - M_t^V.$$
(1)

For each of the genotypes, when there is mutation and no selection, the

birth rate R_t^V is given by

$$\begin{aligned} R_t^X &= \frac{b}{N_t} \left(X_t^2 + 2X_t Z_t \mu + Z_t^2 \mu^2 + X_t Y_t (1+\mu) + Y_t Z_t \mu (1+\mu) + \frac{1}{4} Y_t^2 (1+\mu)^2 \right) \\ R_t^Y &= \frac{b}{N_t} \left(X_t Y_t (1-\mu) + 2X_t Z_t (1-\mu) + 2Z_t^2 (1-\mu) \mu + \frac{1}{2} Y_t^2 (1-\mu^2) + Y_t Z_t \left(1+\mu-2\mu^2\right) \right) \\ R_t^Z &= \frac{b}{N_t} \left(\frac{1}{4} Y_t^2 (1-\mu)^2 + Y_t Z_t (1-\mu)^2 + Z_t^2 (1-\mu)^2 \right). \end{aligned}$$

The birth rate depends on an intrinsic birth rate b, which, by default, holds the same value for all genotypes, on the reproductive events that lead to the production of new individuals with genotype V and on the mutation rate μ . The death rate M_t^V depends on an intrinsic death rate d and is density dependent (we consider a carrying capacity K). The equation for M_t^V is given by

$$M_t^V = d\frac{N_t}{K}V_t.$$
 (2)

When solving $\frac{dN}{dt} = \frac{dX}{dt} = \frac{dY}{dt} = \frac{dZ}{dt} = 0$ we find the optimal population size is given by (see File S4 for the proof)

$$N_{eq} = \frac{bK}{d}.$$
(3)

If we consider that there is neither selection nor mutation ($\mu = 0$), then we find that the frequencies of X, Y and Z are at Hardy-Weinberg equilibrium (see File S3). Explicitly considering the demography of a population leads to the same genotypic frequencies at a neutral locus as those predicted by
deterministic population genetics models. This implies that, once selection is
introduced, any differences observed between our model and the fundamental
model of natural selection are due to the interaction between the timing of
selection and demography.

173 Timing of selection

174

Fitness can be defined as being an individual's relative mating success, fe-175 cundity or survival either at the zygote or adult stages. We consider all four 176 definitions of fitness. Selection can occur at different times during the life 17 cycle, affecting either reproduction or survival. As a is deleterious, Z individ-178 uals always have the maximal fitness. The relative fitness of each genotype 179 at a given trait (i.e. its reproductive rate or survival) is equal to (1 - s), 180 (1 - hs) and 1 for genotypes X, Y and Z respectively, where s is the selec-18 tion coefficient and h the dominance of the mutant allele a. If we consider 182 that a affects the inherent birth rate, then if the inherent birth rate of Z183 individuals is b^Z and the inherent birth rate of X individuals is b^X , then the 184 relative fitness of X individuals is $\frac{b^X}{b^Z} = (1 - s)$. 185

Selection on reproduction: In order to model the effect of the deleterious 186 allele a on the reproductive success of individuals, we introduce a new term 187 \widetilde{V}_t instead of V_t in the R_t^V function. This term represents the contribution 188 of V individuals to the genetic pool, which is proportional to their fitness 189 and can reduce their reproductive success (i.e. $\widetilde{X}_t = (1-s)X_t$). There are 190 two ways in which carrying a can affect reproductive success; it can reduce 19 the mating success of individuals (i.e. for X individuals only a proportion 192 of (1-s) matings are successful or lead to fertilization) or by reducing the 193 fecundity of individuals (i.e. X individuals produce a proportion (1 - s)194 gametes compared to Z individuals). 195

Mating success: When mating success is reduced, all individuals produce the same quantity of gametes and the proportion of male gametes an individual V contributes to the next generation is $\frac{\tilde{V}_t}{N_t}$. The probability of a successful reproductive event is proportional to the parental fitnesses. For example, R_t^X for this model of selection is given by

$$R_t^X = \frac{b}{N_t} \left(\widetilde{X}_t^2 + 2\widetilde{X}_t \widetilde{Z}_t \mu + \widetilde{Z}_t^2 \mu^2 + \widetilde{X}_t \widetilde{Y}_t (1+\mu) + \widetilde{Y}_t \widetilde{Z}_t \mu (1+\mu) + \frac{1}{4} \widetilde{Y}_t^2 (1+\mu)^2 \right)$$

$$\tag{4}$$

Fecundity: When fecundity is affected by selection, an individual V contributes \tilde{V}_t female gametes and a proportion of $\frac{\tilde{V}_t}{\tilde{X}_t+\tilde{Y}_t+\tilde{Z}_t}$ male gametes to the next generation (the proportion of male gametes produced by V depends on the total amount of male gametes produced and not on the number of individuals in the population). For example

$$R_t^X = \frac{b}{\widetilde{X}_t + \widetilde{Y}_t + \widetilde{Z}_t} \left(\widetilde{X}_t^2 + 2\widetilde{X}_t \widetilde{Z}_t \mu + \widetilde{Z}_t^2 \mu^2 + \widetilde{X}_t \widetilde{Y}_t (1+\mu) + \widetilde{Y}_t \widetilde{Z}_t \mu (1+\mu) + \frac{1}{4} \widetilde{Y}_t^2 (1+\mu)^2 \right).$$
(5)

The full equations for the change in the number of individuals of each genotype for these models can be found in File S2. Note that in both models with selection on reproduction the probability of reproduction via the female function remains unaffected as we consider that there is no competition between the female gametes.

Selection on survival: Selection can also occur during the life cycle, independently of reproductive success, affecting either zygote or adult survival.
Zygote survival can be translated as the proportion of germinating seeds, or,
more generally, viable offspring. Selection on adult survival is considered to
occur before reproduction.

Zygote survival: The probability of zygote survival is decreased by considering a birth rate R_t^V that is genotype dependent. This can be done by introducing a term b^V , an intrinsic birth rate that is proportional to the genotype's fitness. For example, $b^X = (1 - s)b$ and

$$R_t^X = \frac{b(1-s)}{N_t} \left(X_t^2 + 2X_t Z_t \mu + Z_t^2 \mu^2 + X_t Y_t (1+\mu) + Y_t Z_t \mu (1+\mu) + \frac{1}{4} Y_t^2 (1+\mu)^2 \right).$$
(6)

The full equations for the change in the number of individuals of each genotype can be found in the File S2.

Adult survival: We consider that the number of adults that survive selection before reproduction of genotype type V is \tilde{V} , hence proportional to their fitness. As only surviving individuals reproduce and compete for resources, V is replaced by \tilde{V} in the birth rate R_t^V and in the death rate M_t^V . Therefore we obtain the same expression for R_t^V as for selection on fecundity and M_t^V is given by

$$M_t^V = d \frac{\widetilde{X}_t + \widetilde{Y}_t + \widetilde{Z}_t}{K} q V_t.$$
(7)

The full equations for the change in the number of individuals of each genotype can be found in File S2.

210 Mutation-selection balance

In order to understand how the interaction between selection and population demography affects population size and the frequency of a recurrent deleterious mutation, we derive the deterministic expectations at mutation-selection balance for each of the models of selection described above (mating success, fecundity and zygote and adult survival) by solving $\frac{dX_t}{dt} = \frac{dY_t}{dt} = \frac{dZ_t}{dt} = 0$. This allows us to obtain the number of individuals carrying each genotype at mutation-selection balance (X_{mut} , Y_{mut} and Z_{mut}), the sum of which gives us the population size at equilibrium N_{mut} . Using N_{mut} we obtain the expression for the numerical load Λ (the decrease of population size due to the presence of deleterious mutations), a term defined by Clarke (1973) and given by

$$\Lambda = \frac{N_{eq} - N_{mut}}{N_{eq}},\tag{8}$$

where N_{eq} is the population size at equilibrium when there is no selection (s = 0, see equation 3). We also use the expressions for X_{mut} , Y_{mut} and Z_{mut} to derive the expressions for the genetic load L and inbreeding depression δ , which we then compare We also compare the genetic load L and inbreeding depression δ that emerge from our model to those expected from the fundamental model of natural selection. In order to compare our results to the fundamental model of natural selection, we replace X_{mut} , Y_{mut} and Z_{mut} with q^2 , 2q(1-q) and $(1-q)^2$ respectively, where q is the frequency of the deleterious mutant a at mutation-selection balance. Using the equations given by Gillespie (1998, p.71), the explicit expression for q is

$$q = \frac{2\mu}{h(s+2s\mu) + \sqrt{s\left(4\mu(1+\mu) - 8h\mu(1+\mu) + s(h+2h\mu)^2\right)}}.$$
 (9)

The genetic load L is defined as the decrease in population fitness due to the presence of deleterious mutations and is calculated by transforming the equation for population fitness of the fundamental model (Gillespie, 1998, p.61):

$$L = 1 - \frac{(1-s)X_{mut} + (1-hs)Y_{mut} + Z_{mut}}{N_{mut}}$$
(10)

Inbreeding depression δ is defined as the difference in fitness between offspring produced via selfing and via outcrossing. We calculate it using equation 3 in Roze and Rousset (2004):

$$\delta = 1 - \frac{(1-s)X_{mut} + \left(\frac{1}{4} + \frac{1-hs}{2} + \frac{1-s}{4}\right)Y_{mut} + Z_{mut}}{(1-s)X_{mut} + (1-hs)Y_{mut} + Z_{mut}}.$$
 (11)

For all four models of selection, there exists a solution where the population is made entirely of X individuals. There is therefore a threshold value of the mutation rate μ , as a function of the selection coefficient s and the dominance h, which leads to the deterministic fixation of a. This threshold value is noted μ_{fix} and is calculated by solving the equations for μ when considering that N_{mut} is equal to X_{mut} . In order to calculate the fixation threshold in the fundamental model, we solve for μ when q (equation 9) is equal to 1.

219

RESULTS

Our purpose is to find explicit solutions for population size N_{mut} , the nu-220 merical load Λ , the genetic load L and inbreeding depression δ at mutation-22 selection balance for all four models of selection (mating success, fecundity 222 and zygote and adult survival). The full expressions (valid for all parameter 223 values when $h \neq 0.5$) can be found in Table S1 of the Supporting Information, 224 as for the sake of legibility, we present only the expressions for recessive and 225 co-dominant mutations (h = 0 and h = 0.5 respectively) in the main text in 226 Table 2. For $h \neq 0$, the expressions were found by using Wolfram's Mathe-22 matica 9.0 (Wolfram Research, 2012), whereas the proofs for population size 228 for h = 0 can be found in File S4. The expressions for the frequencies of each 229 genotype at mutation selection balance are also in the File S3. The domain 230

| Model | h | N_{mut} | Λ | L | δ | μ_{fix} |
|-------------------|-----|--------------------------------------|--------------------------|----------------------------------|--|--------------------|
| Mating success | 0 | $N_{eq}(1-\mu)^2$ | $2\mu - \mu^2$ | μ | $\frac{\sqrt{\mu s} - \mu}{2(1 - \mu)}$ | s |
| | 0.5 | $N_{eq} \frac{(1-\mu)^2}{(1+\mu)^2}$ | $\frac{4\mu}{(1+\mu)^2}$ | $\frac{2\mu}{1+\mu}$ | 0 | $\frac{s}{2-s}$ |
| Fecundity | 0 | $N_{eq}(1-\mu)$ | μ | μ | $\frac{\sqrt{\mu s} - \mu}{2(1 - \mu)}$ | s |
| | 0.5 | $N_{eq} \frac{1-\mu}{1+\mu}$ | $\frac{2\mu}{1+\mu}$ | $\frac{2\mu}{1+\mu}$ | 0 | $\frac{s}{2-s}$ |
| Adult survival | 0 | N_{eq} | 0 | μ | $\frac{\sqrt{\mu s} - \mu}{2(1 - \mu)}$ | s |
| | 0.5 | N_{eq} | 0 | $\frac{2\mu}{1+\mu}$ | 0 | $\frac{s}{2-s}$ |
| Zygote survival | 0 | $N_{eq}(1-\mu)$ | μ | $\frac{\mu(1-s)}{1-\mu}$ | $\frac{\sqrt{\mu s} - \mu}{2 - 2\mu(2 - s)}$ | s |
| | 0.5 | $N_{eq} \frac{1-\mu}{1+\mu}$ | $\frac{2\mu}{1+\mu}$ | $\frac{\mu(2-\mu s-s)}{1+\mu^2}$ | 0 | $\frac{2}{2-s}$ |
| Fundamental model | 0 | _ | _ | $rac{\mu}{1+\mu}$ | $\frac{\sqrt{(1+\mu)\mu s}-\mu}{2}$ | $\frac{s}{1-s}$ |
| | 0.5 | _ | _ | $\frac{2\mu}{1+2\mu}$ | 0 | $\frac{s}{2(1-s)}$ |

Table 2: Expressions for population size N_{mut} , numerical load Λ , genetic load L, inbreeding depression δ and the threshold value of the mutation rate for deterministic fixation μ_{fix} at mutation-selection balance for h = 0 and 0.5 for selection on mating success, fecundity and zygote and adult survival, as well as the fundamental model of natural selection. Exact expressions or any $h \neq 0.5$ obtained using Wolfram's Mathematica 9 are given in Table S1.

of validity of these expressions holds only when the deterministic fixation of *a* is not possible (see the expressions for μ_{fix} in Table 2 and File S1 for the domain of validity for the equations in Table S1). If the conditions for fixation are met, there is only one valid solution, where $N_{mut} = X_{mut}$.

²³⁵ The numerical load

236

Introducing selection can lead to a decrease in population size at mutation-237 selection balance N_{mut} compared to the population size when there is no 238 selection N_{eq} (equation 3). The only model that does not lead to a numerical 239 load Λ is when selection affects adult survival. Concerning the other three 240 models, Λ increases with the coefficient of selection s (Figure 1), the dom-24 inance h (Figure 1) and the mutation rate μ . When selection is on zygote 242 survival or on fecundity, we find the same expression for Λ , whereas selection 243 on mating success leads to a different expression and a higher numerical load 244 (see Table 2 and Table S1). 245



Figure 1: Numerical load Λ at mutation-selection balance as a function of the coefficient of selection s (dashed lines, for h = 0.1 and $\mu = 10^{-3}$) and as a function of dominance h (full lines,with s = 0.1 and $\mu = 10^{-3}$). Selection on mating success is represented in red, whereas selection on fecundity or on zygote survival are in yellow. Lines are plotted using the expressions in Table S1.

²⁴⁶ The timing of selection and the genetic load

247

The timing of selection affects the genetic load L at mutation-selection bal-248 ance both quantitatively and qualitatively (Figure 2). We find the same ex-249 pressions for L when selection is on mating success, fecundity and on adult 250 survival. For these three models L increases with the coefficient of selection 25 s (Figure 2) and with dominance h (Figure 2). When selection affects zygote 252 survival, there is a non-monotonic effect of both s and h on L (this effect 253 becomes smaller with increasing mutation rates μ , results not shown); L is 254 therefore maximal for intermediate values of s and/or h. 25

When s is small the timing of selection has very little effect on the value 256 of L. However, increasing s increases the difference between the models, with 257 a lower L at mutation-selection balance when selection is on zygote survival. 258 In spite of the differences between the expressions for L when selection is on 259 mating success, fecundity and adult survival and the expression for L from 260 the fundamental model (see Table 2 and Supporting Information Table S1), 261 numerically, they are very close as long as the mutation rate μ is low. For 262 higher values of μ , the fundamental model predicts lower values of L than 263

our models. This can be deduced from the equations for L when mutations are completely recessive and codominant (h = 0 and 0.5, see Table 2), the difference between the equations for L with selection on reproduction and Lfor the fundamental model is that the numerator is greater by a factor μ for the fundamental model. As μ increases, the difference between selection on reproduction and on survival becomes smaller until L at mutation-selection balance is slightly greater for selection on survival than on reproduction.

From the fundamental model, the frequency of each genotype at equilib-271 rium is expected to be q^2 , 2q(1-q) and $(1-q)^2$, for X, Y and Z individuals 272 respectively, where q is the frequency of a after selection. This relation-273 ship is no longer true in our model, except when mutations are co-dominant 274 (h = 0.5) and selection is not on zygote survival (see frequencies of geno-275 types in Supporting Information Section C.2.). When selection is on zygote 276 survival, the frequency of X individuals is reduced by a factor (1 - s) as 27 selection directly affects the introduction of a alleles via mutation, and so 278 the introduction of new X and Y individuals. 279

The timing of selection not only affects the numerical load Λ and the genetic load L, but also the interaction between the two at mutation-selection balance. When plotting Λ as a function of L (see Figure 3), selection on



Figure 2: Genetic load L at mutation-selection balance at mutation-selection balance as a function of the coefficient of selection s (dashed lines, for h = 0.1 and $\mu = 10^{-3}$) and as a function of dominance h (full lines, with s = 0.1 and $\mu = 10^{-3}$). Selection on mating success, fecundity or adult survival are represented in red, selection on zygote survival in yellow and the fundamental model in black. Lines are plotted using the expressions in Table S1.

mating success or on fecundity leads to a positive linear relationship between 283 Λ and L; population size decreases with increasing genetic load. When selec-284 tion is on fecundity there is a direct and simple relationship between L and 28 $\Lambda (L = \Lambda)$ as can clearly be seen when comparing these expressions in Table 286 2. When selection is on mating success or on survival, the relationship is not 28 as simple. For example, in the latter case, we observe the same genetic load 288 for small values of s and very large values of s, whereas the numerical load 289 continually increases with s. In this case, the genetic load is not enough to 290 explain population size, the deleterious effect of a must also be taken into 29 account (see equations for L in Table 2 for selection on zygote survival). 292

²⁹³ Inbreeding depression

294

Selection on mating success, fecundity and adult survival all lead to the same expression for inbreeding depression δ . For small values of the coefficient of selection *s* and small mutation rates μ , all models lead to a level of inbreeding depression δ that is numerically close. Increasing *s* leads to a smaller δ when selection is on zygote survival compared to the other models (see Figure 4). As seen for the genetic load *L*, selection on zygote survival leads to a non-



Figure 3: The numerical load Λ (full lines) and inbreeding depression δ (dashed lines) as a function of the genetic load L at mutation-selection balance, for h = 0.1, $\mu = 10^{-3}$ and s between 0 and 1. Λ for selection on mating success, fecundity and zygote survival is represented in red, blue and yellow respectively. δ for selection on mating success, fecundity and adult survival are in red, zygote survival in yellow and the expressions from the fundamental model in black. Lines are plotted using the expressions in Table S1 of the Supporting Information.

monotonic relationship between s and δ . At higher values of μ , the tendencies 301 change; the fundamental model predicts the highest δ , followed by selection 302 on zygote survival (Figure 4). As can be expected from the expressions 303 presented in Table 2, increasing dominance h decreases δ , whereas increasing 304 the coefficient of selection s increases δ . The relationship between inbreeding 305 depression and the genetic load depends on what parameter is modified, as 306 increasing s or h lead to opposite tendencies. Increasing h leads to a decrease 30 in δ while it increases L, whereas increasing s leads to higher δ and an increase 308 in L (see Figure 3), except in the case of selection on zygote survival, where 309 increasing s has a non-monotonic effect on both L and δ . 310

311 Deterministic fixation

The threshold value of the mutation rate for deterministic fixation μ_{fix} is the same whether selection is on reproduction or on survival. The expected μ_{fix} for the fundamental model is lower than that calculated for our models of selection. A lower threshold implies that the deterministic fixation of a deleterious mutation occurs at lower mutation rates. In the equations for μ_{fix} (see Table 2 and Supporting Information), we note that in our models the dominance of the mutations plays a greater role in defining the thresh-



Figure 4: Inbreeding depression δ at mutation-selection balance as a function of the coefficient of selection s (dashed lines, for h = 0.1 and $\mu = 10^{-3}$) and as a function of the mutation rate μ (full lines, with s = 0.1 and h = 0.1. Selection on mating success, fecundity or adult survival are represented in red, selection on zygote survival in yellow and the fundamental model in black. Lines are plotted using the expressions in Table S1.

old value than in the fundamental model, due to the presence of h in the numerator.

321

DISCUSSION

Whether the genetic load affects population size remains an open question. 322 In a recent review on the genetic load L, Agrawal and Whitlock (2012) came 323 to the conclusion that even though a population's mean genetic load might 324 not affect population size (as fitter individuals in the population could make 325 up for the lack of reproductive success of less fit individuals i.e. soft selec-326 tion), if all individuals within the population have a high L, population size 327 is expected to be affected (i.e. hard selection). The latter instance agrees 328 with the generally accepted existence of the mutational meltdown (Lande, 329 1994; Lynch et al., 1995; Coron et al., 2013), whereas the former suggests 330 that segregating and polymorphic mutations should have little or no effect 331 on population demography. But can soft selection completely mask the de-332 mographic consequences of the genetic load? For this to be the case, two 333 conditions need to be met: 1) An infinite or very large number of juveniles 334 (much greater that the environment's carrying capacity) are produced at ev-335

ery time step and 2) Selection must occur only at the juvenile or gametic 336 stages, i.e. before any resources are consumed (reviewed in Agrawal and 337 Whitlock, 2012). The first condition is not always met; not all organisms 338 produce a very large number of offspring, and as for the second condition, 339 selection occurs throughout the life-cycle, considering it affects only the juve-340 nile and gametic stages is a strong hypothesis. In our model, we have chosen 34 a simplified representation of the life cycle, with selection affecting gamete 342 production (fecundity), the success of a mating event, the survival of either 343 zygotes before they have had time to consume any of the available resources 344 or adults before they have had a chance to reproduce. Our results show 345 that deleterious mutations can indeed lead to a numerical load Λ and that 346 it depends on the timing of selection as predicted by Clarke (1973), but our 347 results also indicate that the genetic load can also be affected by the timing 348 of selection. 349

³⁵⁰ Population size as a consequence of selection

If not all individuals in a population have the optimal reproductive capacity, all the while consuming the same amount of resources, then this automatically decreases the population's mean reproductive capacity. For the

numerical load to be compensated, resources used by poorly reproducing in-354 dividuals need to be freed. When selection is on adult survival, all surviving 355 individuals reproduce at the same rate as less fit adults are eradicated be-356 fore they reproduce and resources for new individuals are freed immediately. 35 When selection is on mating success a resource-consuming adult does not 358 always have efficient reproductive encounters, either because its genotype is 359 counter-selected, its partner's genotype is counter-selected, or the offspring 360 eventually produced from this encounter are not of an optimal genotype. 36 The numerical load Λ observed for selection on fecundity and zygote survival 362 can be explained by the same reasoning as for selection on mating success 363 (i.e. all adults consume the same quantity of resources, but not all of them 364 produce the same proportion of gametes or viable offspring), however the 365 effect of selection on population size is smaller. With selection on fecundity 366 and on zygote survival all reproductive encounters are successful, it is only 36 the number of potential offspring that decreases (either because fewer ga-368 metes are produced or because not all offspring are viable). With selection 369 on mating success not only do individuals carrying a alleles have a lower 370 success, gametes of poor quality are produced at the same rate as those of 371 better quality, and the competition between the two further decreases the 372

³⁷³ probability of viable gametes meeting and engendering an offspring.

³⁷⁴ The importance of the timing of selection

When comparing our four models of selection we observe two distinct behav-375 iors concerning the frequency and state (homozygous or heterozygous) of the 376 deleterious allele a, and consequently the genetic load L and amplitude of 371 inbreeding depression δ at mutation-selection balance. Even though we find 378 different numerical loads when selection affects mating success, fecundity or 379 adult survival, these models all lead to the same frequency of AA, Aa and aa 380 individuals, and hence the same expressions for L and δ , at mutation selec-38 tion balance, whereas selection on zygote survival leads to different behaviors 382 for these variables. For simplicity, we will refer solely to selection on fecun-383 dity and not to all three models in order to facilitate the comparison between 384 these three model and selection on zygote survival. The expressions for both 385 L and δ when selection is on zygote survival lead to a non-monotonic effect 386 of the coefficient of selection s on these variables, as well as a non-monotonic 38 effect of dominance h on L, something which is not observed for selection on 388 fecundity. This non-monotonic effect is due to differences between both the 389 frequency and the state of the deleterious allele a (File S3). If we imagine a 390

population in which we find no individuals carrying the a allele, the rate of 39 introduction of individuals carrying a depends only on the mutation rate μ 392 when selection is on fecundity. However, when selection is on zygote survival, 393 this rate depends on μ , s and h as the birth rate R_t for zygote survival directly 394 depends on these parameters (equation 6 and File S2). The birth rate of aa 395 and Aa individuals is therefore lower when selection is on zygote survival 396 than when it affects fecundity. We would therefore expect that if a affects 39 zygote survival it would segregate at lower frequencies at mutation selection 398 balance than if a affects fecundity. As the rate of introduction of individuals 399 carrying a also depends on s and h, we would also expect that of the muta-400 tions affecting zygote survival strongly deleterious mutations would tend to 401 be recessive, and slightly deleterious mutations would have a larger variance 402 of dominance associated. On the other had, if selection affects fecundity, 403 then we should observe a more random association between the coefficient 404 of selection and the dominance of a. This reasoning is also applicable to the 405 estimation of the levels of inbreeding depression δ within populations, as our 406 results agree with the empirical observation that levels of δ vary between 40 traits (Frankham et al. 2010, Chapter 13; Angeloni et al. 2014). In our 408 model, we find that inbreeding depression is lower for zygote survival than 400

for fecundity. A better understanding of how δ evolves in populations is key for the understanding of evolutionary processes (e.g. the transition between outcrossing and self-fertilization in plants (Charlesworth and Charlesworth, 1987; Porcher and Lande, 2005) and for conservation efforts.

⁴¹⁴ How considering the life cycle affects predictions compared to the ⁴¹⁵ fundamental model

When considering neutral alleles, we find that whether the life cycle is taken 416 into account or not, the genotypic frequencies will be at Hardy-Weinberg 41 equilibrium. However, in the case of deleterious mutations, explicitly con-418 sidering the life-cycle and the timing of selection can have an effect on the 419 fate of such alleles. The genetic load L and the amplitude of inbreeding 420 depression are both affected by how selection acts against deleterious muta-421 tions. Historically, L has been estimated ignoring the potential effects of the 422 life cycle, which can be relatively (numerically) small when selection affects 423 reproductive success, fecundity and adult survival or relatively important as 424 is the case when the deleterious allele a affects zygote survival. Selection 425 on zygote survival leads to an unexpected non-monotonic relationship be-426 tween the effects of the deleterious mutations (its coefficient of selection s42

and dominance h) which has never been observed, to our knowledge, in mod-428 els that have estimated load using the fundamental model, not even when 429 taking stochasticity into account (Bataillon and Kirkpatrick, 2000; Roze and 430 Rousset, 2004; Glémin, 2003). The predictions of the fundamental model 43 imply that all mutations, independently of their properties (s and h) lead to 432 L proportional to the rate at which they are introduced, the mutation rate μ . 433 This is not the case when selection affects zygote survival: very deleterious 434 mutations that are observed in the population have to be recessive. If they 435 are not recessive, the zygotes carrying them never survive to become a part 43f of the population, and hence such mutations (that are both very deleterious 43 and dominant) cannot be detected. Considering the life cycle can therefore 438 provide further insight into how selection can affect the fate of newly intro-439 duced mutations, as well as perhaps understanding the distribution of their 440 dominance and selective coefficient as well as the interaction between the 441 two. 442

443 Towards more accurate models

The timing of selection is not the only demographic factor with a potential effect on the numerical load, the genetic load and inbreeding depression; the

intrinsic birth and death rates could also play an important role. In our 446 deterministic model these parameters have no effect, as we find neither the 44 intrinsic birth nor death rates (b and d) in the equations for the numerical 448 load, the genetic load or inbreeding depression. Results of demographically 440 explicit and stochastic models have suggested that b and d can affect the fate 450 of a deleterious allele, or more specifically its probability of fixation Coron 45 et al. (2013). Developing a stochastic version of this model would allow us to 452 evaluate the importance of these parameters; as we could imagine that higher 453 birth rates lead to less efficient selection against deleterious mutations and 454 a higher genetic load at mutation-selection balance, without necessarily in-45 creasing the numerical load. In this paper, we consider a specific demographic 456 model; other demographic models may lead to different relationships between 45 the genetic and numerical loads. Considering density and/or frequency de-458 pendent selection ((Wallace, 1970); (Charlesworth, 1971)) may also modify 459 the present predictions. Our results highlight the importance of considering 460 the interaction between selection against deleterious mutations and the de-461 mographic evolution of populations. The modern tools that are now available 462 allow us to find explicit, and in some cases simple, equations for calculating 463 load and inbreeding depression while considering demography. Using these 464

tools could provide us with more accurate and detailed predictions of how
these two aspects (demography and genetics) that have often been considered
as separate things, both affect the evolution of populations.

468

ACKNOWLEDGEMENTS

We would like to thank Sylvain Glémin, Emmanuelle Porcher, François Massol, Camille Coron and Xavier Vekemans for their input and helpful comments. This work benefited from the support of the Chair "Modélisation
Mathémathique et Biodiversité" of Veolia Environnement - École Polytechnique - Muséum National d'Histoire Naturelle - Fondation X and the French
Agence Nationale de la Recherche Modéles Aléatoires en Écologie, Génétique
et Évolution (MANEGE; ANR-09-BLAN-0215).

476

References

⁴⁷⁷ Abrams, P., 2003 Effects of altered resource consumption rates by one con⁴⁷⁸ sumer species on a competitor. Ecol. Lett. 6: 550–555.

479 Abrams, P., C. Brassil, and R. Holt, 2003 Dynamics and responses to mor-

- tality rates of competing predators undergoing predator-prey cycles. Theor.
 Pop. Biol. 64: 163–176.
- Agrawal, A. F., and M. C. Whitlock, 2012 Mutation load: The fitness of
 individuals in populations where deleterious alleles are abundant. Annu.
 Rev. Ecol. Syst. 43: 115–135.
- Angeloni, F., P. Vergeer, C. Wagemaker, and N. Ouborg, 2014 Within and
 between population variation in inbreeding depression in the locally threatened perennial Scabiosa columbaria. Conserv. Genet. 15: 331–342.
- Bataillon, T., and M. Kirkpatrick, 2000 Inbreeding depression due to mildly
 deleterious mutations in finite populations: size does matter. Genet. Res. 75:
 75–81.
- ⁴⁹¹ Charlesworth, B., 1971 Selection in density-regulated populations. Ecol⁴⁹² ogy 52: 469–474.
- ⁴⁹³ Charlesworth, D., and B. Charlesworth, 1987 Inbreeding depression and its
 ⁴⁹⁴ evolutionary consequences. Annu. Rev. Ecol. Syst. 18: 237–268.
- ⁴⁹⁵ Clarke, B., 1973 Mutation and population size. Heredity 31: 367–379.

- ⁴⁹⁶ Coron, C., E. Porcher, S. Méléard, and A. Robert, 2013 Quantifying the
 ⁴⁹⁷ mutational meltdown in diploid populations. Am. Nat. in press (accepted).
- Frankham, R., J. D. Ballou, and D. A. Briscoe, 2010 Introduction to Conservation Genetics (2nd Edition). Cambridge University Press, Cambridge,
 UK.
- Gillespie, J. H., 1998 Population Genetics: A Concise Guide (2nd Edition).
 The John Hopkins University Press, Baltimore.
- Glémin, S., 2003 How are deleterious mutations purged? Drift versus nonrandom mating. Evolution 57: 2678–2687.
- ⁵⁰⁵ Haldane, J., 1957 The cost of natural selection. Jour. Genet. 55: 511–524.
- Keightley, P., and M. Lynch, 2003 Toward a realistic model of mutations
 affecting fitness. Evolution 57: 683–685.
- Kondrashov, A., 1995 Contamination of the genome by very slightly deleterious mutations Why have we not died a thousand times over. J. Theor.
 Biol. 175: 583–594.
- Lande, R., 1994 Risk of population extinction from fixation of new deleterious
 mutations. Evolution 48: 1460–1469.

⁵¹³ Lynch, M., J. Conery, and R. Burger, 1995 Mutational meltdowns in sexual ⁵¹⁴ populations. Evolution 49: 1067–1080.

- ⁵¹⁵ Porcher, E., and R. Lande, 2005 Loss of gametophytic self-incompatibility ⁵¹⁶ with evolution of inbreeding depression. Evolution 59: 46–60.
- ⁵¹⁷ Roze, D., and F. Rousset, 2004 Joint effects of self-fertilization and popula⁵¹⁸ tion structure on mutation load, inbreeding depression and heterosis. Ge⁵¹⁹ netics 167: 1001–1015.
- Schreiber, S., and V. H. W. Rudolf, 2008 Crossing habitat boundaries: coupling dynamics of ecosystems through complex life cycles. Ecol. Lett. 11:
 576–587.
- Turner, J. R. G., and M. H. Williams, 1968 Population size, natural selection and genetic load. Nature 218: 700.
- ⁵²⁵ Wallace, B., 1970 Genetic Load. Its Biological and Conceptual Aspects.
 ⁵²⁶ Prentice-Hal, Englewood Cliffs, New Jersey.
- Wolfram Research, I., 2012 Mathematica (Version 9.0). Champaign, Illinois:
 Wolfram Research, Inc.