Leibniz's Manuscripts on Perspective

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Asbtract

In 1677, Leibniz wrote his first essay about geometric characteristic, a new geometry of situation and space, without magnitudes, figures or quantities. Although it is mathematically and philosophically central, the geometry of situations is nonetheless not so accurately known, because of the lack of the editions of it, during Leibniz's life and after it. However, the published texts of the *Analysis Situs* between 1676 and 1682, in the French edition and translation of J. Echeverría and M. Parmentier in 1995, reveal that its invention could be connected to Leibniz's discovery of the perspective works of Pascal and Desargues. The issue is then to determine the nature of the relation between perspective and geometric characteristic, in order to clarify the true origin of the Leibnizian invention, to define the nature of that space presented as the object of the new geometry, and to decide wether perspective is for Leibniz a "particular example" of a more general science, or a "general model" for more particular geometrical specimen.
This last question becomes really problematic when we consider that Leibniz himself wrote some texts about perspective. This set of six mansucritps conserved at the Leibniz-Archiv in Hannover (which are almost all in latin and present a compact and crossed out texts with annotated margins and drawn shapes) is totally unpublished, except for a transcribed paragraph (by J. Echeverría). However, we suppose they might contain some answers to the previous questions and perhaps even some other important elements to shed light on Leibniz's general theory of geometry.
Then, my purpose is, first, to develop the reasons why the understanding of the published texts of the Leibniz's invention of the geometry of situations necessarily requires the reading of the manuscripts of perspective. Secondly, I will present the relationship between this reading and a possible new understanding of Leibnizian mathematics as well as of Leibnizian metaphysics, since geometry and perspective play a central role in the general Leibnizian doctrine. Third, I will conclude by exposing the theorical and methodological conditions of a possible transcription of these uneasily readable mansucripts.

Introduction

My paper will not present an achieved work on Leibniz's manuscripts on perspective, but only the project of such a work for the application I made to the Humboldt Foundation for a post-doctoral position at the Max Planck Institute for the History of Science in Berlin and at the Lebniz-Archiv in Hanover. This work is an interesting project for two reasons. First, Leibniz wrote a few manuscripts on perspective which are unpublished and even not transcribed, except two extracts. Some precious papers of Javier Echeverría[1] give us some crucial details of them, but they only are the first systematic attempt to read and comment them. As such, they shed some light as well as they raise questions, because of the ambiguity of the still incomplete knowledge of Leibniz's geometrical perspective they leave us. For example, the place of perspective in Leibniz's general project of renewing geometry is not clear and the relation between perspective and Leibniz's new geometry called *analysis situs*, as two innovative geometries of space, has still to be specified. To clarify such points, we need to transcribe and analyse this set of six manuscripts, which are for the most drafts in Latin.

Besides, as the unique production of a great mathematician and philosopher, these few manuscripts obviously are important. Indeed, and here is the second reason to have an interest in them, perspective is a fundamental example in the philosophy of Leibniz, maybe even a model which appears in his very early texts, around 1666, as well as in the later ones in 1714. This leads me to a paradoxical note: the use of perspective in Leibniz's philosophy is as frequent as Leibniz's mathematical attempts at elaborating a geometrical perspective are parsimonious. Thus, several issues come. Why did Leibniz renounce to produce a complete science of perspective, while he used to mention perspective in his whole philosophy? Did nonetheless his few attempts at elaborating such a science influence the meaning of the philosophical uses of perspective? Or, on the contrary, did his philosophy suggest him some trails for a mathematical conception of perspective? Finally, what kind of geometrical difficulties or preferences did reveal Leibniz's relative abandonment of perspective in geometry, but not in philosophy?

In the following presentation, I will evidently not answer all these questions. I will rather develop the reasons why such questions are interesting. In the first part, I will expound why the understanding of Leibniz's project of a new geometry devoted to relations and to quantities, the *analysis situs*, requires a better knowledge of his geometrical essays on perspective. In the second and third parts, I will present the issues of the reading of the geometrical texts on perspective, for the understanding, first, of the historical genesis of Leibniz's geometry and, secondly, of the philosophical meaning of his most metaphysical conceptions. Finally, I will conclude with the methodological conditions of the transcription of these manuscripts.

[1] Echeverría, J. (1983) "*Recherches inconnues de Leibniz sur la géométrie perspective*," in *Leibniz et la Renaissance*, *SL Supplementa* 23, 191-201 ; Echeverría, J. (1994) "*Leibniz, interprète de Desargues*," in *Desargues en son temps*, Dhombres, J. and Sakarovitch, J. eds, Paris, Blanchard, 1994, 283-93.

I. The reasons of the necessity to read Leibniz's manuscripts on perspective for the understanding of his general project of a new geometric science

Because of the lack of a complete edition of Leibniz's mathematical texts, the geometrical ones are pretty unknown, except those about the geometry of infinitesimals which are a lot commented. Nonetheless, in 1995, Javier Echeverría and Marc Parmentier[2] produced a French edition of the first texts of geometric characteristic, a new purely qualitative geometry, based on non-quantitative situational relations, and endowed with a symbolic and combinatorial calculus. This edition enables us to have a more precise idea of the origin of the project of a new geometry as Leibniz presented it in 1677-1679. They namely reveal a connection between the project of this geometry of situations and the perspective of the 17th century, mostly Desargues' and Pascal's ones[3]. This suggests three issues. The first is "genetic" or "heuristic" and concerns the mutual influence of these two geometric sciences on Leibniz when he invented them. The second is "theoretical:" what kind of conceptual relations do these both geometries have in common? And the third issue is "generic:" which one is the most general science and, consequently, which one is the species?

Indeed, Leibniz was looking for the most general science of geometry, a geometric science which would include his own geometry of infinitesimals as well as the Euclidean geometry or the Cartesian one. Furthermore, such a geometry had to be a geometry of space itself, and not anymore of objects in space as it used to be. On one hand, in 1679, Leibniz claimed that the geometry of situations was the most general geometric science[4]. Nonetheless, on the other hand, in Echeverría's paper "*Recherches inconnues de Leibniz sur la géométrie perspective*," we can learn that, in a manuscript entitled "*Scientia Perspectiva*," Leibniz wrote that the geometry of situations is included in the more general science of perspective. This apparent contradiction naturally puts into consideration the problem of the manuscripts' dating. As a matter of fact, Echeverría approximately estimates that Leibniz wrote this text around 1695, almost 20 years after his first attempts of geometric characteristic. The issue becomes then the following : what has changed between 1679 and 1695 so that Leibniz considered that the perspective geometry has gone up in the order of the general science?

Such a question is important in order to understand the conceptual content of Leibniz's general geometry as the geometry of space what was the second "theoretical" issue I mentioned. Actually, the issue is not only to describe the order of the geometric sciences in Leibniz's project of universal mathematics, but also to clarify some of the main concepts of the qualitative geometry, like space of course, and also *situs* or position or *locus*, transformation and preservation or correspondence, similarity and congruence, continuity, and infinity. The most revealing difficulty about the theoretical relation between perspective and the geometry of situations concerns the nature of transformation. Indeed, both geometries are

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[2] Echeverría, J. and Parmentier, M. eds. (1995) *G. W. Leibniz, La caractéristique géométrique*, Paris, Vrin.

[3] Desargues, G. (1636*) Exemple de l'une des manieres universelles du S. G. D. Leibniz touchant la pratique de la perspective sans emploier aucun tiers point, de distance ny d'autre nature, qui soit hors du champ de l'ouvrage*, Paris ; Desargues, G. (1639) *Brouillon project d'une atteinte aux événements des rencontres du cône avec un plan*, Paris, reprint in Taton, R. (1951) *L'œuvre mathématique de G. Desargues*, Paris PUF, reprint: Paris, Vrin, 1988, 99-180.
Pascal, B. (1964-1992) *Œuvres complètes*, Mesnard, J. ed. Paris, Desclée de Brouwer ; Pascal, B. (1640) *Essai pour les coniques*, in Pascal, B. (1964-1992), t. 2, 220-35 ; Pascal B. (1654) *Generatio Conisectionum*, in Pascal, B. (1964-1992), t. 2, 1108-19 ; Pascal B. (1657) *Introduction à la Géométrie*, in Itard, J. (1964) "*L'introduction à la géométrie de Pascal*," 103-7, in Taton, R. (1964), *L'œuvre scientifique de Pascal*, Paris, PUF, 102-19.

[4] Leibniz, G. W. (1679a) *Characteristica Geometrica*, in Echeverría, J. and Parmentier, M. eds. (1995), 148-9 ; Leibniz, G. W. (1679b) *Leibniz für Christiaan Huygens* (Hannover, 18. September 1679), A III 2, 851-2.

based on the possibility of a transformation preserving some properties: perspective consists in a projective transformation whereas the geometry of situations uses congruence, that is to say a motion preserving the mutual positions of the moving points. In both, space is considered as the nature of which enables such transformations. In other terms, perspective produces some corresponding shapes which are very different from the originals. Nonetheless, they have in common some proportional properties based on the involution relation. The dissimilitude between the projected objects and their originals appertains to the relative places of the eye (or the vertex of a cone) and the projection plane. In the geometry of situations, similarity and congruence are fundamental, so that the transformation cannot affect the proportional properties: it is a motion without deformation. But, both geometries are in any case geometries of perception: the perspective perception, or the simultaneous perception of spatial positions. Then, my issue is the following: cannot we consider that the geometric characteristic is like a perspective projection at infinity? In other words, if the perceiving eye is infinitely far from the projection plane, the perspective deformation disappears and only remains congruence, that is to say the preservation of the relative positions of objects in space. But did Leibniz have such a conception? We do not know, even if, thanks to J. Echeverría[5], we know that he has thought that the eye could be at infinity or at an infinitesimal distance of the projection plane.

This leads me to the second part of my presentation which also is about the third issue I mentioned about the "genetic" relation between perspective and the geometry of situations. Indeed, concerning the problem of the nature of space, or of its structure, the question of the influence of Desargues' or Pascal's perspective geometries is crucial, since there is a relation between the *genesis* of the concept of space and its *theoretical* meaning.

II. The necessity of a better knowledge of Leibniz's own perspective for our understanding of his mathematics

I obviously cannot present all the aspects of my topic. This is why I choose to carry on my analyses about the concept of space. In the Leibnizian history of mathematics, it appears that the geometry of situations is not always based on the same primary notions. For instance, in the first texts of geometric characteristic, Leibniz sometimes preferred the concept of congruence to the concept of similarity. But in his later texts devoted to *analysis situs*, Leibniz always preferred the concept of similarity. The difference between them consists in their quantitative dimension: similarity suggests that two objects have the same shape and the same magnitude, they are the same and they are equal. On the contrary, congruence only requires the preservation of shapes, and not of magnitudes: two things are congruent when they can be distinguished only by their co-presence. Thus, if similarity is central, then space is necessary metric or, in other words, it is the structure of metric relations. On the other hand, if congruence is the primary notion, as it could be in the earlier texts, then it seems possible to get rid of quantity in the conception of space and only spatial or positional relations remain. In addition, in Desargues' work, perspective consists in the correspondence between the properties of the original straight lines and the projected ones. As such, it is a quantitative perspective in which the spatial situation can be considered as distance, and space as a reticulated system. On the contrary, in Pascal's work on conics and perspective, the projective transformation is punctual and uses the notion of incidence of the points and the lines, or of the lines with the planes. And Leibniz's concept of congruence can meet Pascal's conception, since congruence can also be defined as the possibility of a motion from a place to another,

[5] Echeverría, J. (1983), 200.

that is to say as a coincidence between a spatial place and a spatial object. Using such an "incident" conception, Leibniz is close to the continuous and dynamical transformation that he discovered when he read Pascal's manuscripts on conics which are known only by Leibniz's copies.

The issue becomes that following. If it is possible to get rid of the concept of magnitude or distance in perspective, is that the reason why Leibniz changed his mind by claiming that perspective is the most general science of geometry? Is there any relation between the conceptual changes in the geometry of situations and the ones in the perspective geometry? Besides, did Leibniz really produce a perspective science without any metric considerations? In other words, did he distinguish between the theory of conics, which involves quantitative properties, and a more general perspective science which would consist in the science of geometrical objects by means of a continuous transformation based on incidence?

All that seems related to the status of the distance and, consequently, of the straight line, which is crucial for Leibniz, namely because of the importance of Euclid's geometry, but not only for that. It is also strongly connected to the problem of the nature of space as previously presented. And both aspects are linked. Indeed, in a 1679 text, Leibniz mentioned the problem of the geodesic lines, that is to say of the shortest lines on a curved surface, in analogy with the straight line which is the shortest line on a plane. According to J. Echeverría, in one of his perspective manuscripts (*Fundamentum perspectivae meo marte investigatum*), Leibniz also referred to the possibility of a perspective on a sphere. Besides, in 1673, inventing his method of metamorphosis for the squaring of the circle, he conceived a corresponding relation between a straight line and a curved line which consisted in a corresponding relation between the points of both lines by means of infinitesimal ordinates. Nonetheless, as far as we know, Leibniz never developed the consequences of such remarks, namely the possibility of a non-Euclidean geometry as a geometry of a curved or spherical space, or the power of a perspective considered not only in the case of the plane, but in the case of any continuous surface and any continuous transformation. This lack of daring is unusual for Leibniz and certainly reveals some important points.

One possible hypothesis is that Leibniz's perspective works could be closer to his works on quadratures or infinitesimal calculus than to the geometry of situations. Then, it could also be the case for the geometry of situations. In such a framework, one could explain the success of the *analysis situs* and the abandonment of the works on geometric perspective through the advantage that Leibniz gave to the general undertaking of the universal characteristic. The second hypothesis consists in the distinction between the perspective as a science of space and the perspective as the theory of conics. J. Echeverría[6] claims that, in his manuscripts, Leibniz stopped working on perspective as soon as he managed to demonstrate Pascal's "mystic hexagram," which is also related to the problem of the conics' tangents. This might prove that Leibniz has not considered perspective as a new geometry of space in general, but as the projective geometry of conics. Thus, this might justify his relative indifference for some powerful innovations as the spherical space or the general projective geometry, and also confirm the relation between Leibniz's perspective and his differential calculus.

Lastly, as we can see, without a direct reading of Leibniz's manuscripts on perspective, the history of his own geometry and its nature remain obscure and uncertain, namely concerning the mutual roles of quantity and relation. Besides, choosing to consider perspective as the theory of conics or as the general science of projection is very significant for the interpretation of the use of perspective in Leibniz's philosophical papers. This leads me to my third and last part devoted to the necessity of the reading of Leibniz's manuscripts for the understanding of his metaphysics.

[6] Echeverría, J. (1994), 286.

III. The necessity of a better knowledge of Leibniz's own perspective for our understanding of his metaphysics

I just presented the difference between the Arguesian conception of perspective and the Pascalian one. The first consists in a projection preserving some properties of the lines, for instance a tangent, projected from a section of a cone to another. The second considers the projection of points by means of the incidence of the lines of the cone with the plane of projection. The first is more conical, static and projective, the second is more punctual, dynamic and perspective. Besides, Pascal also wrote a text, entitled *Introduction à la géométrie*, which we know thanks to the copy that Leibniz made. In this text, Pascal presented the principles of a new science of geometry, devoted to space and situation, without any consideration for projection. In that case, Leibniz's interest in Pascal's *Introduction à la géométrie* suggests that there might be more similarity between Pascal's geometry and Leibniz's conception of a dynamical substance which also is a point of view on the universe, than between the projective method of Desargues and Leibniz's metaphysics of dimensionless, perceptive and active monads.

Indeed, in his philosophical texts, Leibniz uses two kinds of perspective example. The first consists in the perception of a town from a point of view (the top of a tower), and can already be found in his very early texts[7]. The second presents the conic perspective, that is to say the sections of a cone as the projected pictures of the cone's basis on a plane of projection[8]. Both illustrations do not reveal the same ideas: whilst the example of the point of view emphasises the perceptive action and the relation between a single object and the infinity of its possible perspective representations, the conical example is based on the correspondence relation between the properties of conics and the properties of the circle. Then, the first seems less geometric than the second, but more suitable for philosophical considerations. Nonetheless, as we do not know how Leibniz considers perspective in geometry, as a science of conic sections or as a more general geometry of projective representation, we cannot claim that the example of the point of view cannot be also considered as geometric.

Furthermore, one should distinguish between the notions of 'point of view,' 'situation,' and 'centre.' One can intuitively consider that *situs* is both a geometric notion, as the mutual relations of a set of points, and an ordinary notion implying a certain position, in that case of the substance in the world. Applied to monads, 'situation' appears then in two ways which are however not accurately distinguished: on one hand, as the point of the monadic perspective view on the world and, on the other hand, as the 'centre' of its perception. Both conceptions are often intertwined and postulate several identifications which contain the risk of overlooking some differences. Indeed, the notion of 'situation' is more implicit than explicit in Leibniz's philosophy, whilst the notion of the perspective point of view is explicit, but the notion of '*situs'* is very helpful. For example, a clarified knowledge of the geometry of situations helps to understand the difference between the monadic perception and the divine perception which is presented through the (traditional) idea of a centre which is nowhere and yet everywhere. Such a divine centre is obviously not a simple point of view. Nonetheless, Leibniz presents it in relation to the monadic point of view, which is 'somewhere,' in a certain 'place' despite the immateriality of monads. Hence, the geometric *situs* permits to explain how monads can express the world according a certain point of view without being in a place in the world: by being in a certain mutual relation with all the other substances. And it also permits to explicit how divine perception, which is as a non-perspective perception of the universe by God, can be thought in comparison with monadic perception, by means of the

[7] Leibniz, G. W. (1666) *De arte combinatoria*, A VI 1, 172. See also the correspondence with Thomasius, A II 1.
[8] Leibniz, G. W. (1678?) *Quid sit idea*, A VI 4, 1370.

geometric situational definition of a sphere, as explained in my forthcoming paper in the *Journal of the History of Philosophy*, entitled "*L'expression leibnizienne et ses modèles mathématiques*."

Then, reading Leibniz's philosophy in the light of the notion of situation is both appropriate and incautious: incautious because the notion of situation is not clearly used in its geometrical meaning, but appropriate because Leibniz's constant research on *analysis situs* necessarily constitute a source of ideas for his philosophy, since Leibniz notouriously founds his thinking in the principle of the heuristic power of the reason unity. Besides, the previously presentend philosophical ambiguities also unveil some pieces of mathematical imprecision. If they reveal the connection between the geometry of situations and that of perspective, they also emphasise our relative ignorance of Leibniz's mathematical conceptions about perspective, situation, and the correlated notion of geometric 'centricity,' an ignorance that the reading of Leibniz's manuscripts on perspective could remove.

Conclusion

I am now going to conclude about the methodological aspects of the reading of these manuscripts in the framework of all those issues. I wanted to go to Germany, in order to have a first accurate reading of the manuscripts, but it was not possible before this conference. Then, I only can present some aspects relative to the stages of my research project.

First, what appears in my previous analyses is the absolute necessity of dating the manuscripts. Indeed, Echeverría suggests two periods for Leibniz's works on perspective: one between 1678 and 1685, and the other around 1695. But that concerns only three of the six manuscripts. Besides, Leibniz read Desargues', Pascal's and other geometers' essays on perspective around 1675-76, but also later, after he came back from Paris where he stayed from 1672 to 1676. In 1695, Leibniz claimed that perspective is the most general science of geometry. But he pursued his works on the geometry of situations, and not his on perspective. Furthermore, in his philosophical papers, he first mentioned the perspective from the top of a tower, before any mathematical work or reading about perspective, and only later he used the conic example. In his texts of the late metaphysics, we can find both ideas of the point of view and of projected conics. To answer both questions, about the genetic and generic relation between the geometry of situations and perspective, and about the relation between the mathematical conceptions and their uses in philosophy, we hardly need an exact dating of the manuscripts in order to decide if a parallelism exists between the different fields of Leibniz's geometry, and between metaphysics and geometry.

The second stage of my research project will consist in a work that I absolutely do not know: the work of transcription. It will obviously be helpful to confirm the previous dating, as well as it will need it because of the importance of the evolution of symbols, concepts, and their meaning in Leibniz's language. Such a transcription seems to demand skills in geometry and in philosophy, but also maybe in optics. But Leibniz's optics is not well-known and I cannot affirm that there is something like an optical theory, even if several optical examples can be found in Leibniz's texts of philosophy. In any case, this work has to be considered as an inter-disciplinary one, but its main material is undoubtedly geometrical. The need of a solid knowledge in the perspective works which could have influenced Leibniz seems then demanded. Hence, my post-doctoral research at the Max Planck Institute in 2011 constitutes my condition of possibility of such a transcription work with the support of the philological and mathematical experts of the Leibniz-Archiv and the interdisciplinary members of the Max Planck Institute for the History of Science. Nonetheless, it is still only a project and I apologize for the imprecision of my methodological presentation. I thank you.