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## ► To cite this version:

Margaux Nattaf, Christian Artigues, Pierre Lopez. A polynomial satisfiability test using energetic reasoning for energy-constraint scheduling. International Conference on Project Management and Scheduling (PMS 2014), Mar 2014, Munich, Germany. pp.169-172. hal-00978706

**HAL Id: hal-00978706**

**<https://hal.science/hal-00978706v1>**

Submitted on 14 Apr 2014

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# A polynomial satisfiability test using energetic reasoning for energy-constraint scheduling

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**Keywords:** continuous resource, energy constraints, energetic reasoning, satisfiability test.

## 1 Introduction

A famous global constraint is the cumulative one, which leads to the Cumulative Scheduling Problem (CuSP). In this problem, given a resource with a limited capacity and a set of activities each one having a release date, a due date, a duration and a resource requirement, we want to schedule all activities in their time windows and without exceeding the capacity limit of the resource.

For this NP-complete problem, several solution methods exist and, recently, techniques using satisfiability formulas have been developed. Considering more particularly constraint-based scheduling, which can be seen as a way to solve scheduling problems using constraint programming (Baptiste *et. al.* 2001), a technique using energetic reasoning provides a strong (although incomplete) polynomial satisfiability test known as the "left-shift/right-shift" conditions (Baptiste *et. al.* 1999).

This test is part of an approach of consistency test named "Interval Consistency Test" (Dorndorf *et. al.* 1999). It consists in adding some hypothetical constraints to the problem to find a contradiction and deduce activity domain reductions. As our paper focuses on satisfiability test, we discuss this only briefly in the last section.

In this paper, the idea is to use the "left-shift/right-shift" test and energetic reasoning propagation algorithms (Erschler and Lopez 1990), (Lopez and Esquirol 1996) in order to compute a polynomial satisfiability test for a generalization of CuSP, the Continuous Energy-Constrained Scheduling Problem (CECSP). The CECSP has the following particularity: activities use a continuously-divisible resource and each of them can take any shape bounded by its time window, a minimum and maximum resource requirement and a fixed energy requirement that has to be brought via a power processing rate function.

As the CECSP deals with a continuous resource, the already developed propagation techniques can not be applied directly. Thus, it is an interesting question to know whether a polynomial satisfiability test can be found for CECSP. In our study, we adapt the "left-shift/right-shift" test for CuSP to CECSP.

## 2 Problem statement

In the CECSP problem, we have as input a set  $A = \{1, \dots, n\}$  of activities and a continuous resource, which is available in a limited capacity  $B$ . Each activity has to be performed between its release date  $r_i$  and its deadline  $\tilde{d}_i$ . Instead of being defined by its duration and resource requirement, in CECSP an activity is defined by an energy requirement  $W_i$ , a minimal and maximal resource requirement  $b_i^{min}$  and  $b_i^{max}$ .

To solve the CECSP, we have to find for each activity its starting time  $st_i$ , its finishing time  $ft_i$  and a function  $b_i(t)$ , for all  $t \in \mathcal{T}$  (where  $\mathcal{T} = [\min_{i \in A} r_i, \max_{i \in A} \tilde{d}_i]$ ), representing

the resource amount allocated to this activity. These variables have to satisfy the following constraints:

$$r_i \leq st_i \leq ft_i \leq \tilde{d}_i \quad (i \in A) \quad (1)$$

$$b_i^{min} \leq b_i(t) \leq b_i^{max} \quad (i \in A; t \in [st_i, ft_i]) \quad (2)$$

$$b_i(t) = 0 \quad (i \in A; i \in \mathcal{T} \setminus [st_i, ft_i]) \quad (3)$$

$$\int_{st_i}^{ft_i} f_i(b_i(t))dt = W_i \quad (i \in A) \quad (4)$$

$$\sum_{i \in A} b_i(t) \leq B \quad (t \in \mathcal{T}) \quad (5)$$

where  $f_i(b)$  is a continuous non-decreasing power processing rate function.

We define  $p_i = ft_i - st_i$  as the activity duration. We remark that if  $b_i^{min} = b_i^{max} = b_i$  and  $f_i(b_i(t)) = b_i(t)$ ,  $\forall i \in A$ , then we can set  $p_i$  to  $\frac{W_i}{b_i}$  and, if all inputs are integers, we obtain an instance of CuSP. Thus, the CECSP is NP-complete.

We consider the case where function  $f_i(b)$  is linear or piecewise linear. In these cases, we have adapted a satisfiability test for the CECSP, where  $f_i(b_i(t)) = b_i(t)$ ,  $\forall i \in A$  (Artigues *et. al.* 2009), to this more complex case.

### 3 Energetic reasoning for linear function

We first present the case where the function  $f_i(b)$  has the form  $a_i b + c_i$  with  $a_i > 0$  and  $c_i > 0$ .

Before explaining how energetic reasoning yields a polynomial satisfiability test for CECSP, we present an elementary satisfiability condition to check whether the activity data is consistent. This condition is the following: if we can find an activity  $i \in A$  such that  $f_i(b_i^{max})(\tilde{d}_i - r_i) < W_i$  then the CECSP can not have a solution. This comes from the fact that, since  $f_i(b)$  is a non-decreasing function, scheduling  $i$  at its maximum resource requirement  $b_i^{max}$  inside  $[r_i, \tilde{d}_i]$  gives the largest amount of energy.

In order to apply energetic reasoning to our problem, we have considered the minimum energy requirement and resource consumption of an activity  $i$  over an interval  $[t_1, t_2]$ . These values are denoted by  $\underline{w}(i, t_1, t_2)$  and  $\underline{b}(i, t_1, t_2)$  respectively and defined by:

$$\underline{w}(i, t_1, t_2) = \min \int_{t_1}^{t_2} f_i(b_i(t))dt \quad \text{subject to (1)-(4)}$$

$$\underline{b}(i, t_1, t_2) = \min \int_{t_1}^{t_2} b_i(t)dt \quad \text{subject to (1)-(4)}$$

We have used these values to compute the slack of interval  $[t_1, t_2]$  which is defined by:  $SL(t_1, t_2) = B(t_2 - t_1) - \sum_{i \in A} \underline{b}(i, t_1, t_2)$ . The satisfiability test consists of determining whether there exists an interval  $[t_1, t_2]$ , with  $t_1 < t_2$ , such that  $SL(t_1, t_2) < 0$ . If such an interval exists then the CECSP has no solution.

This proposition is at the core of the "left-shift/right-shift" necessary condition. In (Baptiste *et. al.* 1999), it was shown that this test can be performed only on a polynomial number of intervals for CuSP. For CECSP, we have to check whether a polynomial number of intervals is sufficient to perform the satisfiability test.

To achieve this, we have analyzed possible configurations of the minimum resource consumption. First, since  $f_i(b)$  is a non-decreasing function, we can observe that, given an interval  $[t_1, t_2]$ , the minimum consumption always corresponds to a configuration where activity  $i$  is either left-shifted (the activity starts at  $r_i$  and is scheduled at its maximum requirement

between  $r_i$  and  $t_1$ ) or right-shifted (the activity ends at  $\tilde{d}_i$  and is scheduled at its maximum requirement between  $t_2$  and  $\tilde{d}_i$ ) or both, or scheduled at  $b_i^{min}$  during  $[t_1, t_2]$ . We will denote by  $I$  the interval over which the activity is scheduled at  $b_i^{max}$  outside interval  $[t_1, t_2]$ . For example, if the activity  $i$  is left-shifted, then  $I$  is of the form  $[r_i, t]$  with  $t \leq t_1$ . So the minimum energy requirement in  $[t_1, t_2]$  is:  $\underline{w}(i, t_1, t_2) = \min(b_i^{min}(t_2 - t_1), W_i - |I| * f_i(b_i^{max}))$ .

We still have to compute the minimum required resource consumption. For this, let  $J$  be the interval over which we have to bring an energy  $\underline{w}(i, t_1, t_2)$  to the activity  $i$ . Obviously,  $J$  is either  $[r_i, d_i]$  or  $[r_i, t_2]$  or  $[t_1, t_2]$ , or  $[t_1, d_i]$ . We have two cases to consider :

- the remaining interval is sufficiently large to schedule the activity at its minimum requirement, i.e.  $|J| \geq \frac{\underline{w}(i, t_1, t_2)}{f_i(b_i^{min})}$ , and then  $\underline{b}(i, t_1, t_2) = b_i^{min}(\frac{\underline{w}(i, t_1, t_2)}{f_i(b_i^{min})})$
- the remaining interval is not large enough to schedule the activity at its minimum requirement and finding  $\underline{b}(i, t_1, t_2)$  is equivalent to solving:

$$\begin{aligned} & \text{minimize } \int_J b_i(t) dt \\ & \text{subject to } \int_J f_i(b_i(t)) dt \geq \underline{w}(i, t_1, t_2) \end{aligned}$$

$$\text{Then } \underline{b}(i, t_1, t_2) = \frac{1}{a_i}(\underline{w}(i, t_1, t_2) - |J|c_i).$$

The function  $\underline{b}(i, t_1, t_2)$  defined in this way is a bivariate continuous piecewise linear function. This remark allows us to establish a theorem which states that we can perform the satisfiability test only on a polynomial number of intervals. Indeed, we want to check whether an interval  $[t_1, t_2]$  over which the slack function is negative exists. Since the slack function is a two dimensional piecewise linear function, we only have to check whether this occurs at the extreme point of one of the convex polygon on which it is linear.

The break line segments of the slack function is the same as the ones of the sum of the minimum consumption for each activity. Thus, each extreme point of the slack function is the intersection of two segments, each segment corresponding to the break line segment of an individual minimum consumption function. Thus, we only have to perform the satisfiability test on these intersection points whose number is quadratic in the number of activities.

#### 4 Piecewise linear function

Consider now the case where  $f_i(b)$  is a continuous non-decreasing piecewise linear function. As the function is non-decreasing we can perform the same test as the one for linear functions to check whether the activity data is consistent. The minimum required energy and resource consumption  $\underline{w}(i, t_1, t_2)$  and  $\underline{b}(i, t_1, t_2)$  are defined in the same way.

To compute the slack function of an interval  $[t_1, t_2]$ , we need an analytical expression of function  $\underline{b}(i, t_1, t_2)$ . To achieve this, the function  $\underline{w}(i, t_1, t_2)$  is computed. Actually, possible configurations of the minimum resource consumption are the same as for the case of linear functions (left-shifted activity, right-shifted or both). Thus, we can compute  $\underline{w}(i, t_1, t_2)$  in the same way.

For a piecewise linear function, the difficulty lies in the computation of  $\underline{b}(i, t_1, t_2)$ . In this case, we are not able to derive the expression of minimum resource consumption from a linear program. However, we can analyze the function  $f_i(b)$  to find the point of best energetic efficiency, i.e. the point for which  $\frac{f_i(b)}{b}$  is maximal for  $b_i^{min} \leq b \leq b_i^{max}$ . Let  $\gamma$  be this point. Once  $\gamma$  is calculated, we can use it to exhibit a lower bound for  $\underline{b}(i, t_1, t_2)$ . Indeed, we know that to provide the required energy to the activity, the minimum resource

consumption is obtained by allocating this amount of energy to it during a sufficiently large time. Thus,  $\underline{b}(i, t_1, t_2) \geq \gamma \frac{w(i, t_1, t_2)}{f_i(\gamma)}$ .

We can perform the satisfiability test by setting  $\underline{b}(i, t_1, t_2)$  to  $\gamma \frac{w(i, t_1, t_2)}{f_i(\gamma)}$  and (as  $\underline{b}(i, t_1, t_2)$  is also a bivariate continuous piecewise linear function) a polynomial number of intervals is sufficient. However, the time required to compute  $\gamma$  depends on the number of definition intervals of the function  $f_i(b)$ . However, it seems difficult to compute  $\underline{b}(i, t_1, t_2)$  in a time independent of this number.

## 5 Conclusion

We have presented a polynomial satisfiability test for two variants of the CECSP. This work is still in progress, especially for finding the exact minimum required resource consumption expression for piecewise linear functions.

Although activity domain reductions in (Dorndorf *et. al.* 1999) seem to apply easily to CECSP, an interesting question is to know whether they are still relevant and/or whether we can improve them by using the structure of our problem.

For future research, in order to provide better applications to actual scheduling problems under energy constraints, it will be interesting to study the case where function  $f_i(b)$  is no longer linear. Another interesting problem is to integrate energetic reasoning in other bounding techniques such as linear programming or network flows.

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