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THE MULTIPLICITIES OF THE EQUIVARIANT INDEX OF TWISTED DIRAC OPERATORS

PAUL-EMILE PARADAN, MICHÈLE VERGNE

RÉSUMÉ. In this note, we give a geometric expression for the multiplicities of the equivariant index of a Dirac operator twisted by a line bundle.

1. Introduction

This note is an announcement of work whose details will appear later.

Let $M$ be a compact connected manifold. We assume that $M$ is even dimensional and oriented. We consider a spin$^c$ structure on $M$, and denote by $\mathcal{S}$ the corresponding irreducible Clifford module. Let $K$ be a compact connected Lie group acting on $M$, and preserving the spin$^c$ structure. We denote by $D : \Gamma(M, \mathcal{S}^+) \to \Gamma(M, \mathcal{S}^-)$ the corresponding twisted Dirac operator. The equivariant index of $D$, denoted $Q_{K}^{\text{spin}}(M)$, belongs to the Grothendieck group of representations of $K$,

$$Q_{K}^{\text{spin}}(M) = \sum_{\pi \in \hat{K}} m(\pi) \pi.$$

An important example is when $M$ is a compact complex manifold, $K$ a compact group of holomorphic transformations of $M$, and $L$ any holomorphic $K$-equivariant line bundle on $M$ (not necessarily ample). Then the Dolbeault operator twisted by $L$ can be realized as a twisted Dirac operator $D$. In this case $Q_{K}^{\text{spin}}(M) = \sum_{q} (-1)^{q} H^{0,q}(M, L)$.

The aim of this note is to give a geometric description of the multiplicity $m(\pi)$ in the spirit of the Guillemin-Sternberg phenomenon $[Q, R] = 0 [3, 7, 8, 11, 9]$.

Consider the determinant line bundle $L = \det(\mathcal{S})$ of the spin$^c$ structure. This is a $K$-equivariant complex line bundle on $M$. The choice of a $K$-invariant hermitian metric and of a $K$-invariant hermitian connection $\nabla$ on $L$ determines an abstract moment map

$$\Phi_{\nabla} : M \to \mathfrak{k}^*$$

by the relation $\mathcal{L}(X) - \nabla_{X_M} = \frac{i}{2} \langle \Phi_{\nabla}, X \rangle$, for all $X \in \mathfrak{k}$. We compute $m(\pi)$ in term of the reduced “manifolds” $\Phi_{\nabla}^{-1}(f)/K_{f}$. This formula extends the result of [10].
However, in this note, we do not assume any hypothesis on the line bundle $L$, in particular we do not assume that the curvature of the connection $\nabla$ is a symplectic form. In this pre-symplectic setting, a (partial) answer to this question has been obtained by [6, 4, 5, 1] when $K$ is a torus. Our method is based on localization techniques as in [9], [10].

2. Admissible coadjoints orbits

We consider a compact connected Lie group $K$ with Lie algebra $\mathfrak{k}$. Consider an admissible coadjoint orbit $O$ (as in [2]), oriented by its symplectic structure. Then $O$ carries a $K$-equivariant bundle of spinors $S^O$, such that the associated moment map is the injection $O \subset \mathfrak{k}^*$. We denote by $Q_K^{\text{spin}}(O)$ the corresponding equivariant index.

Let us describe the admissible coadjoint orbits with their spin$^c$ index.

Let $T$ be a Cartan subgroup of $K$ with Lie algebra $\mathfrak{t}$. Let $\Lambda \subset \mathfrak{t}^*$ be the lattice of weights of $T$ (thus $e^{i\lambda}$ is a character of $T$). Choose a positive system $\Delta^+ \subset \mathfrak{t}^*$, and let $\rho = \frac{1}{2} \sum_{\alpha \in \Delta^+} \alpha$. Let $\mathfrak{t}^*_\geq 0$ be the closed Weyl chamber and we denote by $\mathcal{F}$ the set of the relative interiors of the faces of $\mathfrak{t}^*_\geq 0$. Thus $\mathfrak{t}^*_\geq 0 = \bigsqcup_{\sigma \in \mathcal{F}} \sigma$, and we denote $\mathfrak{t}^*_\geq 0 \in \mathcal{F}$ the interior of $\mathfrak{t}^*_\geq 0$.

We index the set $\hat{K}$ of classes of finite dimensional irreducible representations of $K$ by the set $(\Lambda + \rho) \cap \mathfrak{t}^*_\geq 0$. The irreducible representation $\pi_\lambda$ corresponding to $\lambda \in (\Lambda + \rho) \cap \mathfrak{t}^*_\geq 0$ is the irreducible representation with infinitesimal character $\lambda$. Its highest weight is $\lambda - \rho$.

Let $\sigma \in \mathcal{F}$. The stabilizer $K_\xi$ of a point $\xi \in \sigma$ depends only of $\sigma$. We denote it by $K_\sigma$, and by $\mathfrak{k}_\sigma$ its Lie algebra. We choose on $\mathfrak{t}_\sigma$ the system of positive roots contained in $\Delta^+$, and let $\rho_\sigma$ be the corresponding $\rho$.

When $\mu \in \sigma$, the coadjoint orbit $K \cdot \mu$ is admissible if and only if $\mu - \rho + \rho_\sigma \in \Lambda$. The spin$^c$ equivariant index of the admissible orbits is described in the following lemma.

Lemma 2.1. Let $K \cdot \mu$ be an admissible orbit : $\mu \in \sigma$ and $\mu - \rho + \rho_\sigma \in \Lambda$. If $\mu + \rho_\sigma$ is regular, then $\mu + \rho_\sigma \in \rho + \overline{\sigma}$. Thus we have

$$Q_K^{\text{spin}}(K \cdot \mu) = \begin{cases} 0 & \text{if } \mu + \rho_\sigma \text{ is singular}, \\ \pi_{\mu + \rho_\sigma} & \text{if } \mu + \rho_\sigma \text{ is regular}. \end{cases}$$

In particular, if $\lambda \in (\Lambda + \rho) \cap \mathfrak{t}^*_\geq 0$, then $K \cdot \lambda$ is admissible and $Q_K^{\text{spin}}(K \cdot \lambda) = \pi_\lambda$. 

Let $\mathcal{H}_f$ be the set of conjugacy classes of the reductive algebras $\mathfrak{t}_f$, $f \in \mathfrak{t}^*$. We denote by $S_f$ the set of conjugacy classes of the semi-simple parts $[h, h]$ of the elements $(h) \in \mathcal{H}_f$. The map $(h) \mapsto ([h, h])$ induces a bijection between $\mathcal{H}_f$ and $S_f$.

The map $F \rightarrow \mathcal{H}_f$, $\sigma \mapsto (\mathfrak{t}_\sigma)$, is surjective and for $(h) \in \mathcal{H}_f$ we denote by

- $F^*_h \subset \mathfrak{t}^*$ the set of elements $f \in \mathfrak{t}^*$ with infinitesimal stabilizer $\mathfrak{t}_f$ belonging to the conjugacy class $(h)$.

We have $\mathfrak{t}^*_\sigma = K \left( \bigcup_{\sigma \in F(h)} \sigma \right)$. In particular all coadjoint orbits contained in $\mathfrak{t}^*_\sigma$ have the same dimension. We say that such a coadjoint orbit is of type $(h)$. If $(h) = (t)$, then $\mathfrak{t}^*_\sigma$ is the open subset of regular elements.

We denote by $A(h)$ the set of admissible coadjoint orbits of type $(h)$. This is a discrete subset of orbits in $\mathfrak{t}^*_\sigma$.

**Example 1**: Consider the group $K = SU(3)$ and let $(h)$ be the conjugacy class such that $\mathfrak{k}^*_f$ is equal to the set of subregular element $f \in \mathfrak{k}^*$ (the orbit of $f$ is of dimension $\dim(K/T) - 2$). Let $\omega_1, \omega_2$ be the two fundamental weights. Let $\sigma_1, \sigma_2$ be the half lines $\mathbb{R}_{>0} \omega_1, \mathbb{R}_{>0} \omega_2$. Then $\mathfrak{t}^*_0 \cap \mathfrak{t}^*_\sigma = \sigma_1 \cup \sigma_2$. The set $A(h)$ is equal to the collection of orbits $K \cdot (\frac{1}{2} + 2n \omega_i), n \in \mathbb{Z}_{>0}, i = 1, 2$. The representation $Q^{spin}\mathcal{K}(K \cdot (\frac{1}{2} + 2n \omega_i))$ is $0$ if $n = 0$, otherwise it is the irreducible representation $\pi_{\rho + (n-1)\omega_i}$. In particular, both representations associated to the admissible orbits $\frac{3}{2} \omega_1$ and $\frac{3}{2} \omega_2$ are the trivial representation $\pi_{\rho}$.

### 3. The theorem

Consider the action of $K$ in $M$. Let $(\mathfrak{t}_M)$ be the conjugacy class of the generic infinitesimal stabilizer. On a $K$-invariant open and dense subset of $M$, the conjugacy class of $\mathfrak{t}_n$ is equal to $(\mathfrak{t}_M)$. Consider the (conjugacy class) $(\mathfrak{t}_M, \mathfrak{t}_M)$.

We start by stating two vanishing lemmas.

**Lemma 3.1.** If $(\mathfrak{t}_M, \mathfrak{t}_M)$ does not belong to the set $S_\mathfrak{t}$, then $Q^{spin}_\mathcal{K}(M) = 0$ for any $K$-invariant spin$^c$ structure on $M$.

If $(\mathfrak{t}_M, \mathfrak{t}_M) = ([h, h])$ for some $h \in \mathcal{H}_f$, any $K$-invariant map $\Phi : M \rightarrow \mathfrak{t}^*$ is such that $\Phi(M)$ is included in the closure of $\mathfrak{t}^*_h$.

**Lemma 3.2.** Assume that $(\mathfrak{t}_M, \mathfrak{t}_M) = ([h, h])$ with $h \in \mathcal{H}_f$. Let us consider a spin$^c$ structure on $M$ with determinant bundle $\mathcal{L}$. If there exists a $K$-invariant hermitian connection $\nabla$ on $\mathcal{L}$ such that $\Phi\nabla(M) \cap \mathfrak{t}^*_h = \emptyset$, then $Q^{spin}_\mathcal{K}(M) = 0.$
Thus from now on, we assume that the action of $K$ on $M$ is such that $([\mathfrak{t}_M, \mathfrak{t}_M]) = ([\mathfrak{h}, \mathfrak{h}])$ for some $\mathfrak{h} \in \mathcal{H}_k$. Let us consider a spin$^c$ structure on $M$ with determinant bundle $L$ and a $K$-invariant hermitian connection with moment map $\Phi_{\nabla} : M \to \mathfrak{t}^*$. We extend the definition of the index to disconnected even dimensional oriented manifolds by defining $Q_{K}^{\text{spin}}(M)$ to be the sum over the connected components of $M$. If $K$ is the trivial group, $Q_{K}^{\text{spin}}(M) \in \mathbb{Z}$ and is denoted simply by $Q^{\text{spin}}(M)$.

Consider a coadjoint orbit $O = K \cdot f$. The reduced space $M_O$ is defined to be the topological space $\Phi_{\nabla}^{-1}(O)/K = \Phi_{\nabla}^{-1}(f)/Kf$. We also denote it by $M_f$. This space might not be connected.

In the next section, we define a $\mathbb{Z}$-valued function $O \mapsto Q_{K}^{\text{spin}}(M_O)$ on the set $A(\mathfrak{h})$ of admissible orbits of type $(\mathfrak{h})$. We call it the reduced index:

- if $M_O = \emptyset$, then $Q_{K}^{\text{spin}}(M_O) = 0$,
- when $M_O$ is an orbifold, the reduced index $Q_{K}^{\text{spin}}(M_O)$ is defined as an index of a Dirac operator associated to a natural “reduced” spin$^c$ structure on $M_O$.

Otherwise, it is defined via a limit procedure. Postponing this definition, we have the following theorem.

**Theorem 3.3.** Assume that $([\mathfrak{t}_M, \mathfrak{t}_M]) = ([\mathfrak{h}, \mathfrak{h}])$ with $(\mathfrak{h}) \in \mathcal{H}_k$. Then

$$Q_{K}^{\text{spin}}(M) = \sum_{O \in A(\mathfrak{h})} Q_{K}^{\text{spin}}(M_O) Q_{K}^{\text{spin}}(O).$$

In the expression above, when $\mathfrak{h}$ is not abelian, $Q_{K}^{\text{spin}}(O)$ can be 0, and several orbits $O \in A(\mathfrak{h})$ can give the same representation.

Theorem 3.3 is in the spirit of the $[Q, R] = 0$ theorem. However it has some radically new features. First, as $\Phi_{\nabla}$ is not the moment map of a Hamiltonian structure, the definition of the reduced space requires more care. For example, the fibers of $\Phi_{\nabla}$ might not be connected, and the Kirwan set $\Phi_{\nabla}(M) \cap \mathfrak{t}_g^*$ is not a convex polytope. Furthermore, this Kirwan set depends of the choice of connection $\nabla$. Second, the map $O \in A(\mathfrak{h}) \to Q_{K}^{\text{spin}}(O)$ is not injective, when $\mathfrak{h}$ is not abelian. Thus the multiplicities $m_\lambda$ of the representation $\pi_\lambda$ in $Q_{K}^{\text{spin}}(M)$ will be eventually obtained as a sum of reduced indices involving several reduced spaces.

We explicit this last point.

**Theorem 3.4.** Assume that $([\mathfrak{t}_M, \mathfrak{t}_M]) = ([\mathfrak{h}, \mathfrak{h}])$ with $(\mathfrak{h}) \in \mathcal{H}_k$. Let $m_\lambda \in \mathbb{Z}$ be the multiplicity of the representation $\pi_\lambda$ in $Q_{K}^{\text{spin}}(M)$. We have

$$(1) \quad m_\lambda = \sum_{\sigma \in \mathcal{F}(\mathfrak{h}) \lambda - \rho_\sigma \in \sigma} Q_{\mathfrak{t}}^{\text{spin}}(M_{\lambda - \rho_\sigma}).$$
More explicitly, the sum is taken over the (relative interiors of) faces $\sigma$ of the Weyl chamber such that

$$([\xi_M, \xi_M]) = ([\xi_\sigma, \xi_\sigma]), \quad \Phi_\nabla(M) \cap \sigma \neq \emptyset, \quad \lambda \in \{\sigma + \rho_\sigma\}.$$ 

If $\xi_M$ is abelian, we have simply $m_\lambda = Q_{\text{spin}}^\xi(\Phi_\nabla^{-1}(\lambda)/T)$. In particular, if the group $K$ is the circle group, and $\lambda$ is a regular value of the moment map $\Phi_\nabla$, this result was obtained in [1].

If $\xi_M$ is not abelian, and the curvature of the connection $\nabla$ is symplectic, Kirwan convexity theorem implies that the image $\Phi_\nabla(M) \cap \mathfrak{t}_{\geq 0}$ is contained in the closure of one single $\sigma$. Thus there is a unique $\sigma$ satisfying Conditions (2). In this setting Theorem 3.4 is obtained in [10].

Let us give an example where several $\sigma$ contribute to the multiplicity of a representation $\pi_\lambda$.

We take the notations of Example 1. We label $\omega_1, \omega_2$ so that $\xi_{\omega_i}$ is the group $S(U(2) \times U(1))$ stabilizing the line $\mathbb{C}e_3$ in the fundamental representation of $SU(3)$ in $\mathbb{C}^3 = \mathbb{C}e_1 \oplus \mathbb{C}e_2 \oplus \mathbb{C}e_3$.

Let $P = \{0 \subset L_2 \subset L_3 \subset \mathbb{C}^4\}$ be the partial flag manifold with $L_2$ a subspace of $\mathbb{C}^4$ of dimension 2 and $L_3$ a subspace of $\mathbb{C}^4$ of dimension 3. Denote by $L_1, L_2$ the equivariant line bundles on $P$ with fiber at $(L_2, L_3)$ the one-dimensional spaces $\wedge^2 L_2$ and $L_3/L_2$ respectively. Let $M$ be the subset of $P$ where $L_2$ is assumed to be a subspace of $\mathbb{C}^3$. Thus $M$ is fibered over $P_2(\mathbb{C})$ with fiber $P_1(\mathbb{C})$. The group $SU(3)$ acts naturally on $M$, and the generic stabilizer of the action is $SU(2)$. We denote by $L_{a,b}$ the line bundle $L_1^a \otimes L_2^b$ restricted to $M$. This line bundle is equipped with a natural holomorphic and hermitian connection $\nabla$. Consider the spin$^c$ structure with determinant bundle $L = L_{2a+1, 2b+1}$, where $a, b$ are positive integers. If $a \geq b$, the curvature of the line bundle $L$ is non degenerate, and we are in the symplectic case. Let us consider $b > a$. It is easy to see that, in this case, the Kirwan set $\Phi_\nabla(M) \cap \mathfrak{t}_{\geq 0}$ is the non convex set $[0, b - a]\omega_1 \cup [0, a + 1]\omega_2$. We compute the character of the representation $Q_{\text{spin}}^K(M)$ by the Atiyah-Bott fixed point formula, and find

$$Q_{\text{spin}}^K(M) = \sum_{j=0}^{b-a-2} \pi_{\rho+j\omega_1} \oplus \sum_{j=0}^{a-1} \pi_{\rho+j\omega_2}.$$
In particular the multiplicity of $\pi_\rho$ (the trivial representation) is equal to 2. We use now Theorem 3.3 and the discussion of Example 1, and obtain (reduced multiplicities are equal to 1)

$$Q^K_{spin}(M) = \sum_{j=0}^{b-a-1} Q^K_{spin}(K \cdot \left(1 + \frac{2j}{\omega_1}\right)) \oplus \sum_{j=0}^{a} Q^K_{spin}(K \cdot \left(1 + \frac{2j}{\omega_2}\right)).$$

Using the formulae for $Q^K_{spin}(K \cdot \left(1 + \frac{2n}{2} \omega_i\right))$ given in Example 1, these two formulae (fortunately) coincide. Furthermore we see that both faces $\sigma_1, \sigma_2$ give a non zero contribution to the multiplicity of the trivial representation.

4. Definition of the reduced index

We start by defining the reduced index for the action of an abelian torus $H$ on a connected manifold $Y$. Denote by $\Lambda$ the lattice of weights of $H$. We do not assume $Y$ compact, but we assume that the set of stabilizers $H_m$ of points in $Y$ is finite. Let $h_Y$ be the generic infinitesimal stabilizer of the action $H$ on $Y$, and $H_Y$ be the connected subgroup of $H$ with Lie algebra $h_Y$. Thus $H_Y$ acts trivially on $Y$. Let us consider a spin$^c$ structure on $Y$ with determinant bundle $L$, and a $H$ invariant connection $\nabla$ on $L$. The image $\Phi_{\Delta}(Y)$ spans an affine space $I_Y$ parallel to $h_Y$. We assume that the fibers of the map $\Phi_{\Delta}$ are compact. We can easily prove that there exists a finite collection of hyperplanes $W_1, \ldots, W_p$ in $I_Y$ such that the group $H/H_Y$ acts locally freely on $\Phi_{\Delta}^{-1}(f)$, when $f$ is in $\Phi_{\Delta}(Y)$, but not on any of the hyperplanes $W_i$.

Proposition 4.1. • When $\mu \in I_Y \cap \Lambda$ is a regular value of $\Phi_{\Delta} : Y \to I_Y$, the reduced space $Y_\mu$ is an oriented orbifold equipped with an induced spin$^c$ structure : we denote $Q^{spin}(Y_\mu)$ the corresponding spin$^c$ index.

• For any connected component $C$ of $I_Y \setminus \cup_{k=1}^{p} W^k$, we can associate a periodic polynomial function $q^C : \Lambda \cap I_Y \to \mathbb{Z}$ such that

$$q^C(\mu) = Q^{spin}(Y_\mu)$$

for any element $\mu \in \Lambda \cap C$ which is a regular value of $\Phi : Y \to I_Y$.

• If $\mu \in \Lambda$ belongs to the closure of two connected components $C_1$ and $C_2$ of $I_Y \setminus \cup_{k=1}^{p} W^k$, we have

$$q^{C_1}(\mu) = q^{C_2}(\mu).$$

We can now state the definition of the “reduced” index on $\Lambda$ :

• $Q^{spin}(Y_\mu) = 0$ if $\mu \notin \Lambda \cap I_Y$, 

- for any $\mu \in \Lambda \cap I_Y$, we define $Q^{\text{spin}}(Y_\mu)$ as being equal to $q^C(\mu)$ where $C$ is any connected component containing $\mu$ in its closure. In fact $Q^{\text{spin}}(Y_\mu)$ is computed as an index of a particular spin$^c$ structure on the orbifold $\Phi^{-1}(\mu + \epsilon)/H$ for any $\epsilon$ small and such that $\mu + \epsilon$ is a regular value of $\Phi$.

If $Y$ is not connected, we define the reduced index at a point $\mu \in \Lambda$ as the sum of reduced indices over all connected components of $Y$.

More generally, let $H$ be a compact connected group acting on $Y$ and such that $[H, H]$ acts trivially on $Y$. Let $S_Y$ be an equivariant spin$^c$ structure on $Y$ with determinant bundle $L$. For any $\mu \in \mathfrak{h}^*$ such that $\mu([\mathfrak{h}, \mathfrak{h}]) = 0$, and admissible for $H$, it is then possible to define $Q^{\text{spin}}(Y_\mu)$. Indeed eventually passing to a double cover of the torus $H/[H, H]$ and translating by the square root of the action of $H/[H, H]$ on the fiber of $L$, we are reduced to the preceding case of the action of the torus $H/[H, H]$, and a $H/[H, H]$-equivariant spin$^c$ structure on $Y$.

Consider now the action of a connected compact group $K$ on $M$. Let $\sigma$ be a (relative interior) of a face of $t_{\varepsilon,0}^*$ which satisfies the following conditions

\[(\{t_M, t_M^\varepsilon\}) = (\{t_{\sigma}, t_{\sigma}^\varepsilon\}), \quad \Phi^{-1}_C(\sigma) \neq \emptyset.\]

Let us explain how to compute the “reduced” index map $\mu \rightarrow Q^{\text{spin}}(M_\mu)$ on the set $\sigma \cap \{\Lambda + \rho - \rho_\sigma\}$ that parameterizes the admissible orbits intersecting $\sigma$. We work with the “slice” $Y$ defined by $\sigma$. The set $U_\sigma := K_\sigma(\cup_{\tau \subset \sigma} \tau)$ is an open neighborhood of $\sigma$ in $\mathfrak{t}_\varepsilon^*$ such that the open subset $KU_\sigma \subset \mathfrak{t}_\varepsilon^*$ is isomorphic to $K \times_{K_\sigma} U_\sigma$. We consider the $K_\sigma$-invariant subset $Y = \Phi^{-1}_C(U_\sigma)$. The following lemma allows us to reduce the problem to the abelian case.

**Lemma 4.2.** $Y$ is a non-empty submanifold of $M$ such that $KY$ is an open subset of $M$ isomorphic to $K \times_{K_\sigma} Y$.

- The Clifford module $S_M$ on $M$ determines a Clifford module $S_Y$ on $Y$ with determinant line bundle $L_Y = L_M|_Y \otimes \mathbb{C}_{-2(\rho - \rho_\sigma)}$. The corresponding moment map is $\Phi_C|_Y - \rho + \rho_\sigma$.

- The group $[K_\sigma, K_\sigma]$ acts trivially on $Y$ and on the bundle of spinors $S_Y$.

We thus consider $Y$ with action of $K_\sigma$, and Clifford bundle $S_Y$. If $\mu \in \sigma$ is admissible for $K$, then $\mu - \rho + \rho_\sigma \in \Lambda$ is admissible for $K_\sigma$. The reduced space $M_\mu = \Phi^{-1}_C(\mu)/K_\sigma$ is equal to the reduced space $Y_{\mu - \rho + \rho_\sigma}$. As $[K_\sigma, K_\sigma]$ acts trivially on $(Y, S_Y)$, we are in the abelian case, and we define $Q^{\text{spin}}(M_\mu) := Q^{\text{spin}}(Y_{\mu - \rho + \rho_\sigma})$. 


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Références


Institut de Mathématiques et de Modélisation de Montpellier (I3M), UMR CNRS 5149, Université Montpellier 2
E-mail address: Paul-Emile.Paradan@math.univ-montp2.fr

Institut de Mathématiques de Jussieu, UMR CNRS 7586, Université Paris-Diderot
Paris 7
E-mail address: Vergne@math.jussieu.fr