



HAL
open science

Continuous fields of properly infinite C^* -algebras

Etienne Blanchard

► **To cite this version:**

| Etienne Blanchard. Continuous fields of properly infinite C^* -algebras. 2014. hal-00974653v5

HAL Id: hal-00974653

<https://hal.science/hal-00974653v5>

Preprint submitted on 4 Aug 2014 (v5), last revised 22 Aug 2014 (v6)

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

CONTINUOUS FIELDS OF PROPERLY INFINITE C*-ALGEBRAS

ETIENNE BLANCHARD

ABSTRACT. Any unital separable continuous $C(X)$ -algebra with properly infinite fibres is properly infinite as soon as the compact Hausdorff space X has finite topological dimension. We study conditions under which this is still the case in the infinite dimensional case.

1. INTRODUCTION

One of the basic C*-algebras studied in the classification programme launched by G. Elliott ([Ell94]) of nuclear C*-algebras through K-theoretical invariants is the Cuntz C*-algebra \mathcal{O}_∞ generated by infinitely many isometries with pairwise orthogonal ranges ([Cun77]). This C*-algebra is pretty rigid in so far as it is a strongly self-absorbing C*-algebra ([TW07]): Any separable unital continuous $C(X)$ -algebra A the fibres of which are isomorphic to the same strongly self-absorbing C*-algebra D is a trivial $C(X)$ -algebra provided the compact Hausdorff base space X has finite topological dimension. (Indeed, the strongly self-absorbing C*-algebra D tensorially absorbs the Jiang-Su algebra \mathcal{Z} ([Win09]). Hence, this C*-algebra D is K_1 -injective ([Rør04]) and the $C(X)$ -algebra A satisfies $A \cong D \otimes C(X)$ ([DW08]).) But I. Hirshberg, M. Rørdam and W. Winter have built a non-trivial unital continuous C*-bundle over the infinite dimensional compact product $\prod_{n=0}^\infty S^2$ such that all its fibres are isomorphic to the strongly self-absorbing UHF algebra of type 2^∞ ([HRW07, Example 4.7]). More recently, M. Dădărlat has constructed in [Dăd09, §3] for all pair (Γ_0, Γ_1) of discrete countable torsion groups a unital separable continuous $C(X)$ -algebra A such that:

- the base space X is the compact Hilbert cube $X = \mathfrak{X}$ of infinite dimension,
- all the fibres A_x ($x \in \mathfrak{X}$) are isomorphic to the strongly self-absorbing Cuntz C*-algebra \mathcal{O}_2 generated by two isometries s_1, s_2 satisfying $1_{\mathcal{O}_2} = s_1 s_1^* + s_2 s_2^*$,
- $K_i(A) \cong C(Y_0, \Gamma_i)$ for $i = 0, 1$, where $Y_0 \subset [0, 1]$ is the canonical Cantor set.

These K-theoretical conditions imply that the $C(\mathfrak{X})$ -algebra A is not a trivial one. But these arguments does not anymore work when the strongly self-absorbing algebra D is the Cuntz algebra \mathcal{O}_∞ ([Cun77]), in so far as $K_0(\mathcal{O}_\infty) = \mathbb{Z}$ is a torsion free group.

We study in this article whether the Pimsner-Toeplitz algebra ([Pim95]) of the nontrivial Dixmier-Douady Hilbert $C(X)$ -module E_{DD} ([DD63]) is a nontrivial unital $C(X)$ -algebra with fibres \mathcal{O}_∞ . This would imply that there exists a properly infinite C*-algebra A which is not K_1 -injective, *i.e.* the mapping $\mathcal{U}(A)/\mathcal{U}_0(A) \rightarrow K_1(A)$ is not

2010 *Mathematics Subject Classification.* Primary: 46L80; Secondary: 46L06, 46L35.

Key words and phrases. C*-algebra, Classification, Proper Infiniteness.

injective, and there exist separable unital continuous $C([0, 1])$ -algebras with properly infinite fibres which are not properly infinite C^* -algebras.

I especially thank E. Kirchberg and a referee for a few inspiring remarks.

2. NOTATIONS

We present in this section the main notations which are used in this article. We denote by $\mathbb{N} = \{0, 1, 2, \dots\}$ the set of positive integers and we denote by $[S]$ the closed linear span of any subset S in a Banach space.

Definition 2.1. ([Dix69], [Kas88], [Blan97]) *Let X be a compact Hausdorff space and let $C(X)$ be the C^* -algebra of continuous function on X .*

- *A unital $C(X)$ -algebra is a unital C^* -algebra A endowed with a unital morphism of C^* -algebra from $C(X)$ to the centre of A .*
- *For all closed subset $F \subset X$ and all element $a \in A$, one denotes by $a|_F$ the image of a in the quotient $A|_F := A/C_0(X \setminus F) \cdot A$. If $x \in X$ is a point in X , one calls fibre at x the quotient $A_x := A|_{\{x\}}$ and one write a_x for $a|_{\{x\}}$.*
- *The $C(X)$ -algebra A is said to be continuous if the upper semicontinuous map $x \in X \mapsto \|a_x\| \in \mathbb{R}_+$ is continuous for all $a \in A$.*

Remarks 2.2. a) ([Cun81], [BRR08]) For all integer $n \geq 2$, the C^* -algebra $\mathcal{T}_n := \mathcal{T}(\mathbb{C}^n)$ is the universal unital C^* -algebra generated by n isometries s_1, \dots, s_n satisfying the relation

$$s_1 s_1^* + \dots + s_n s_n^* \leq 1. \quad (2.1)$$

b) A unital C^* -algebra A is *properly infinite* if and only if one the following equivalent conditions holds ([Cun77], [Rør03, Proposition 2.1]):

- the C^* -algebra A contains two isometries with mutually orthogonal range projections, *i.e.* A unittally contains a copy of \mathcal{T}_2 ,
- the C^* -algebra A contains a unital copy of the simple Cuntz C^* -algebra \mathcal{O}_∞ generated by infinitely many isometries with pairwise orthogonal ranges.

3. GLOBAL PROPER INFINITENESS

Proposition 2.5 of [BRR08] and section 6 of [Blan13] entail the following stable proper infiniteness for continuous $C(X)$ -algebras with properly infinite fibres.

Proposition 3.1. *Let X be a second countable perfect compact Hausdorff space, *i.e.* without any isolated point, and let A be a separable unital continuous $C(X)$ -algebra with properly infinite fibres.*

1) *There exist:*

- (a) *a finite integer $n \geq 1$,*
- (b) *a covering $X = \overset{\circ}{F}_1 \cup \dots \cup \overset{\circ}{F}_n$ by the interiors of closed balls F_1, \dots, F_n ,*
- (c) *unital embeddings of C^* -algebra $\sigma_k : \mathcal{O}_\infty \hookrightarrow A|_{F_k}$ ($1 \leq k \leq n$).*

2) *The tensor product $M_p(\mathbb{C}) \otimes A$ is properly infinite for all large enough integers p .*

Proof. 1) For all point $x \in X$, the semiprojectivity of the C^* -subalgebra $\mathcal{O}_\infty \hookrightarrow A_x$ ([Blac04, Theorem 3.2]) entails that there are a closed neighbourhood $F \subset X$ of the point x and a unital embedding $\mathcal{O}_\infty \otimes C(F) \hookrightarrow A|_F$ of $C(F)$ -algebra. The compactness of the topological space X enables to conclude.

2) Proposition [BRR08, Proposition 2.7] entails that $M_{2^{n-1}}(A)$ is properly infinite and [Rør97, Proposition 2.1] implies that $M_p(A)$ for all integer $p \geq 2^{n-1}$. \square

Remark 3.2. If X is an ordinary second countable compact Hausdorff space and A is a separable unital continuous $C(X)$ -algebra, then $\tilde{X} := X \times [0, 1]$ is a perfect compact space, $\tilde{A} := A \otimes C([0, 1])$ is a unital continuous $C(\tilde{X})$ -algebra and any unital morphism $\mathcal{O}_\infty \rightarrow \tilde{A}$ induces a unital morphism $\mathcal{O}_\infty \rightarrow A$ by composition with the projection map $\tilde{A} \rightarrow A$ coming from the injection $x \in X \mapsto (x, 0) \in \tilde{X}$.

The proper infiniteness of the tensor product $M_p(\mathbb{C}) \otimes A$ does not always imply that the C^* -algebra A is properly infinite ([HR98]). Indeed, there exists a unital C^* -algebra A which is not properly infinite, but such that the tensor product $M_2(\mathbb{C}) \otimes A$ is properly infinite ([Rør03, Proposition 4.5]). We nevertheless have the following corollary.

Corollary 3.3. *Let j_0, j_1 denote the two canonical unital embeddings of the Cuntz extension \mathcal{T}_2 in the full unital free product $\mathcal{T}_2 *_\mathbb{C} \mathcal{T}_2$ and let $\tilde{u} \in \mathcal{U}(\mathcal{T}_2 *_\mathbb{C} \mathcal{T}_2)$ be a K_1 -trivial unitary satisfying $j_1(s_1) = \tilde{u} \cdot j_0(s_1)$ ([BRR08, Lemma 2.4]).*

Then the following conditions are equivalent:

- (a) *The full unital free product $\mathcal{T}_2 *_\mathbb{C} \mathcal{T}_2$ is K_1 -injective.*
- (b) *The unitary \tilde{u} belongs to the connected component $\mathcal{U}_0(\mathcal{T}_2 *_\mathbb{C} \mathcal{T}_2)$ of $1_{\mathcal{T}_2 *_\mathbb{C} \mathcal{T}_2}$.*
- (c) *Every separable unital continuous $C(X)$ -algebra A with properly infinite fibres is a properly infinite C^* -algebra.*

Proof. (a) \Rightarrow (b) A unital C^* -algebra A is called K_1 -injective if and only if all K_1 -trivial unitaries $v \in \mathcal{U}(A)$ are homotopic to the unit 1_A in $\mathcal{U}(A)$ (see *e.g.* [Roh09]). Thus, (b) is a special case of (a) since $K_1(\mathcal{T}_2 *_\mathbb{C} \mathcal{T}_2) = \{1\}$ (see *e.g.* [Blan10, Lemma 4.4]).

(b) \Rightarrow (c) Let A be a separable unital continuous $C(X)$ -algebra with properly infinite fibres. Take a finite covering $X = \overset{\circ}{F}_1 \cup \dots \cup \overset{\circ}{F}_n$ such that there exist unital embeddings $\sigma_k : \mathcal{T}_2 \rightarrow A|_{\overset{\circ}{F}_k}$ ($1 \leq k \leq n$). Set $G_k := F_1 \cup \dots \cup F_k \subset X$ for all $1 \leq k \leq n$ and let us construct by induction isometries $w_k \in A|_{G_k}$ such that the two projections $w_k w_k^*$ and $1_{G_k} - w_k w_k^*$ are properly infinite and full in the restriction $A|_{G_k}$:

– If $k = 1$, the isometry $w_1 := \sigma_1(s_1)$ has the requested properties.

– If $k \in \{1, \dots, n-1\}$ and the isometry $w_k \in A|_{G_k}$ is already constructed, then Lemma 2.4 of [BRR08] implies that there exist an homomorphism of unital C^* -algebra $\pi_k : \mathcal{T}_2 *_\mathbb{C} \mathcal{T}_2 \rightarrow A|_{G_k \cap F_{k+1}}$ satisfying:

$$\begin{aligned} - \quad \pi_k(j_0(s_1)) &= w_k|_{G_k \cap F_{k+1}}, \\ - \quad \pi_k(j_1(s_1)) &= \sigma_{k+1}(s_1)|_{G_k \cap F_{k+1}} = \pi_k(\tilde{u}) \cdot w_k|_{G_k \cap F_{k+1}}. \end{aligned} \tag{3.1}$$

If the unitary \tilde{u} belongs to $\mathcal{U}_0(\mathcal{T}_2 *_\mathbb{C} \mathcal{T}_2)$, then $\pi_k(\tilde{u})$ is homotopic to $1_{A|_{G_k \cap F_{k+1}}} = \pi_k(1_{\mathcal{T}_2 *_\mathbb{C} \mathcal{T}_2})$ in $\mathcal{U}(A|_{G_k \cap F_{k+1}})$, so that $\pi_k(\tilde{u})$ admits a unitary lifting z_{k+1} in $\mathcal{U}_0(A|_{F_{k+1}})$

(see *e.g.* [LLR00, Lemma 2.1.7]). The only isometry $w_{k+1} \in A_{|G_{k+1}}$ satisfying the two constraints:

$$w_{k+1|G_k} = w_k \quad \text{and} \quad w_{k+1|F_{k+1}} = (z_{k+1})^* \cdot \sigma_{k+1}(s_1) \quad (3.2)$$

verifies that the two projections $w_{k+1}w_{k+1}^*$ and $1_{|G_{k+1}} - w_{k+1}w_{k+1}^*$ are properly infinite and full in $A_{|G_{k+1}}$.

The proper infiniteness of the projection $w_n w_n^* \in A_{|G_n} = A$ implies that the unit $1_A = w_n^* w_n = w_n^* \cdot w_n w_n^* \cdot w_n$ is also a properly infinite projection in A , *i.e.* the C^* -algebra A is properly infinite.

(c) \Rightarrow (a) The C^* -algebra $\mathcal{D} := \{f \in C([0, 1], \mathcal{T}_2 *_{\mathbb{C}} \mathcal{T}_2); f(0) \in \mathcal{J}_0(\mathcal{T}_2) \text{ and } f(1) \in \mathcal{J}_1(\mathcal{T}_2)\}$ is a unital continuous $C([0, 1])$ -algebra the fibres of which are all properly infinite. Thus, condition (c) implies that the C^* -algebra \mathcal{D} is properly infinite, a statement which is equivalent to the K_1 -injectivity of $\mathcal{T}_2 *_{\mathbb{C}} \mathcal{T}_2$ ([Blan10, Proposition 4.2]). \square

4. THE PIMSNER-TOEPLITZ ALGEBRA OF A HILBERT $C(X)$ -MODULE

We look in this section at the special case of unital continuous $C(X)$ -algebras with fibres \mathcal{O}_∞ corresponding to the Pimsner-Toeplitz $C(X)$ -algebras of Hilbert $C(X)$ -modules with infinite dimension fibres.

Definition 4.1. ([Pim95]) *Let X be a compact Hausdorff space and E a full Hilbert $C(X)$ -module E , *i.e.* without any zero fibre.*

a) *The full Fock Hilbert $C(X)$ -module $\mathcal{F}(E)$ of E is the direct sum of Hilbert $C(X)$ -module*

$$\mathcal{F}(E) := \bigoplus_{m \in \mathbb{N}} E^{(\otimes_{C(X)} m)}, \quad (4.1)$$

where $E^{(\otimes_{C(X)} m)} := \begin{cases} C(X) & \text{if } m = 0, \\ E \otimes_{C(X)} \dots \otimes_{C(X)} E \text{ (} m \text{ terms)} & \text{if } m \geq 1. \end{cases}$

b) *The Pimsner-Toeplitz $C(X)$ -algebra $\mathcal{T}(E)$ of E is the unital subalgebra of the $C(X)$ -algebra $\mathcal{L}_{C(X)}(\mathcal{F}(E))$ of adjointable $C(X)$ -linear operator acting on $\mathcal{F}(E)$ generated by the creation operators $\ell(\zeta)$ ($\zeta \in E$), where:*

$$\begin{aligned} - \ell(\zeta)(f \cdot \hat{1}_{C(X)}) &:= f \cdot \zeta = \zeta \cdot f & \text{for } f \in C(X) & \text{and} \\ - \ell(\zeta)(\zeta_1 \otimes \dots \otimes \zeta_k) &:= \zeta \otimes \zeta_1 \otimes \dots \otimes \zeta_k & \text{for } \zeta_1, \dots, \zeta_k \in E & \text{if } k \geq 1. \end{aligned} \quad (4.2)$$

c) *Let $(C^*(\mathbb{Z}), \Delta)$ be the compact quantum group generated by a unitary \mathbf{u} with spectrum the unit circle and with coproduct $\Delta(\mathbf{u}) = \mathbf{u} \otimes \mathbf{u}$. Then, there is a unique coaction α of the Hopf C^* -algebra $(C^*(\mathbb{Z}), \Delta)$ on the Pimsner-Toeplitz $C(X)$ -algebra $\mathcal{T}(E)$ such that $\alpha(\ell(\zeta)) = \ell(\zeta) \otimes \mathbf{u}$ for all $\zeta \in E$, *i.e.**

$$\begin{aligned} \alpha : \mathcal{T}(E) &\rightarrow \mathcal{T}(E) \otimes C^*(\mathbb{Z}) = C(\mathbb{T}, \mathcal{T}(E)) \\ \ell(\zeta) &\mapsto \ell(\zeta) \otimes \mathbf{u} = (z \mapsto \ell(z\zeta)) \end{aligned} \quad (4.3)$$

The fixed point $C(X)$ -subalgebra $\mathcal{T}(E)^\alpha = \{a \in \mathcal{T}(E); \alpha(a) = a \otimes 1\}$ under this coaction is the closed linear span

$$\mathcal{T}(E)^\alpha = \left[C(X) \cdot 1 + \sum_{k \geq 1} \ell(E)^k \cdot (\ell(E)^k)^* \right]. \quad (4.4)$$

Besides, the projection $P \in \mathcal{L}(\mathcal{F}(E))$ onto the submodule E induces a quotient morphism of $C(X)$ -algebra $a \in \mathcal{T}(E)^\alpha \mapsto \bar{\mathbf{q}}(a) := P \cdot a \cdot P \in \mathcal{K}(E) + C(x) \cdot 1 \subset \mathcal{L}(E)$.

Proposition 4.2. *Let X be a second countable compact Hausdorff perfect space and let E be a separable Hilbert $C(X)$ -module with infinite dimensional fibres.*

1) *There exist a covering $X = \overset{\circ}{F}_1 \cup \dots \cup \overset{\circ}{F}_m$ by the interiors of closed subsets F_1, \dots, F_m and m sections ζ_1, \dots, ζ_m in E such that $\mathcal{T}(E) = C^* \langle \mathcal{T}(E)^\alpha, \ell(\zeta_1), \dots, \ell(\zeta_m) \rangle$ and $\|(\zeta_k)_x\| = 1$ for all $k \in \{1, \dots, m\}$ and $x \in F_k$.*

2) *If for all $k \in \{1, \dots, m-1\}$ and all norm 1 section $\xi \in E$ with $\|\xi_y\| = 1$ for all point y in a closed subset $\bar{G}_k \subset F_{k+1}$, there is a unitary $z_{k+1} \in \mathcal{T}(E)^\alpha|_{F_{k+1}}$ such that $z_{k+1} \cdot \ell(\xi(k))|_{G_k \cap F_{k+1}} = \ell(\zeta_{k+1})|_{G_k \cap F_{k+1}}$, then there exists a section $\xi \in E$ satisfying $\|\xi_x\| = 1$ for all $x \in X$, so that $\mathcal{T}(E)$ is properly infinite by [Blan13, Lemma 6.1].*

Proof. 1) For all point $x \in X$, there exists a section $\zeta \in E$ satisfying $\|\zeta_x\| = 1$, whence an isomorphism of C^* -algebra $\mathcal{T}(E)_x \cong \mathcal{T}(E_x) = C^* \langle \mathcal{T}(E_x)^\alpha, \ell(\zeta_x) \rangle$. The semiprojectivity of the C^* -algebra $\mathcal{O}_\infty \cong \mathcal{T}(E)_x$ and the compactness of the space X imply that there exist a finite covering $X = \overset{\circ}{F}_1 \cup \dots \cup \overset{\circ}{F}_m$ by the interiors of closed subsets F_1, \dots, F_m and m contractions ζ_1, \dots, ζ_m in E such that $\|(\zeta_k)_x\| = 1$ for all index $k \in \{1, \dots, m\}$ and all point $x \in F_k$.

2) Set $G_k := F_1 \cup \dots \cup F_k$ for all $k \in \{1, \dots, m\}$ (as in Corollary 3.3) and let us construct inductively sections $\xi(k) \in E|_{G_k}$ such that $\|\xi(k)_x\| = 1$ for all $x \in G_k$.

– If $k = 1$, the section $\xi(1) := (\zeta_1)|_{F_1}$ has the requested properties.

– If $k \in \{1, \dots, m-1\}$ and a convenient section $\xi(k)$ in $E|_{G_k}$ is already constructed, then there exists a unital $*$ -homomorphism $\pi_k : \mathcal{T}_2 *_{\mathbb{C}} \mathcal{T}_2 \rightarrow \mathcal{T}(E)|_{G_k \cap F_{k+1}}$ such that $\pi_k(j_0(s_1)) = \ell(\xi(k))|_{G_k \cap F_{k+1}}$ and $\pi_k(j_1(s_1)) = \ell(\zeta_{k+1})|_{G_k \cap F_{k+1}}$. There is by assumption a unitary $z_{k+1} \in \mathcal{T}(E)^\alpha|_{F_{k+1}}$ such that

$$z_{k+1}|_{G_k \cap F_{k+1}} \cdot \ell(\xi(k))|_{G_k \cap F_{k+1}} = \ell(\zeta_{k+1})|_{G_k \cap F_{k+1}} \quad (4.5)$$

The only section $\xi(k+1) \in E|_{G_{k+1}}$ such that $\xi(k+1)|_{G_k} = \xi(k)$ and $\xi(k+1)|_{F_{k+1}} = \bar{q}(z_{k+1})^* \cdot (\zeta_{k+1})|_{F_{k+1}}$ satisfies $\|\xi(k+1)_x\| = 1$ for all point $x \in G_{k+1}$. \square

Remark 4.3. The complex Hilbert cube $\mathfrak{X} := \{x \in \mathbb{C}; |x| \leq 1\}^{\mathbb{N}}$ is a compact space when equipped with the distance $d((x_p), (y_p)) = \sum_p 2^{-p-2} |x_p - y_p|$. The nontrivial separable Hilbert $C(\mathfrak{X})$ -module E_{DD} constructed by J. Dixmier and A. Douady ([DD63], [BK04a, Proposition 3.6]) has infinite dimensional fibres and every section $\zeta \in E_{DD}$ satisfies $\zeta_x = 0$ for at least one point $x = (x_p) \in \mathfrak{X}$. Thus, it does not verify the assumptions for the assertion 2) of Proposition 4.2.

Question 4.4. The Pimsner-Toeplitz algebra $\mathcal{T}(E_{DD})$ is locally purely infinite ([BK04b, Definition 1.3]) since all its simple quotients are isomorphic to the Cuntz algebra \mathcal{O}_∞ ([BK04b, Proposition 5.1]). But is $\mathcal{T}(E_{DD})$ properly infinite?

REFERENCES

- [Blac04] B. Blackadar, *Semiprojectivity in simple C^* -algebras*, Adv. Stud. Pure Math. **38** (2004), 1–17.
- [Blan97] E. Blanchard, *Subtriviality of continuous fields of nuclear C^* -algebras*, with an appendix by E. Kirchberg, J. Reine Angew. Math. **489** (1997), 133–149.
- [Blan10] E. Blanchard, *K_1 -injectivity for properly infinite C^* -algebras*, Clay Math. Proc. **11** (2010), 49–54.
- [Blan13] E. Blanchard, *Continuous fields with fibres \mathcal{O}_∞* , to appear in Math. Scand.
- [BK04a] E. Blanchard, E. Kirchberg, *Global Glimm halving for C^* -bundles*, J. Op. Th. **52** (2004), 385–420.
- [BK04b] E. Blanchard, E. Kirchberg, *Non-simple purely infinite C^* -algebras: the Hausdorff case*, J. Funct. Anal. **207** (2004), 461–513.
- [BRR08] E. Blanchard, R. Rohde, M. Rørdam, *Properly infinite $C(X)$ -algebras and K_1 -injectivity*, J. Noncommut. Geom. **2** (2008), 263–282.
- [Cun77] J. Cuntz, *Simple C^* -Algebras Generated by Isometries*, Commun. Math. Phys. **57** (1977), 173–185.
- [Cun81] J. Cuntz, *K -theory for certain C^* -algebras*, Ann. of Math. **113** (1981), 181–197.
- [DD63] J. Dixmier, A. Douady, *Champs continus d’espaces hilbertiens et de C^* -algèbres*, Bull. Soc. Math. France **91** (1963), 227–284.
- [Dix69] J. Dixmier, *Les C^* -algèbres et leurs représentations*, Gauthiers-Villars Paris (1969).
- [Dăd09] M. Dădărlat, *Fiberwise KK -equivalence of continuous fields of C^* -algebras*, J. K-Theory **3** (2009), 205–219.
- [DW08] M. Dădărlat, W. Winter, *Trivialization of $C(X)$ -algebras with strongly self-absorbing fibres*, Bull. Soc. Math. France **136** (2008), 575–606.
- [Ell94] G. Elliott, *The classification problem for amenable C^* -algebras*, Proc. Internat. Congress of Mathematicians (Zurich, Switzerland, 1994), Birkhauser Verlag, Basel (1995), 922–932.
- [HRW07] I. Hirshberg, M. Rørdam, W. Winter, *$C_0(X)$ -algebras, stability and strongly self-absorbing C^* -algebras*, Math. Ann. **339** (2007), 695–732.
- [HR98] J. Hjelmborg, M. Rørdam, *On stability of C^* -algebras*. J. Funct. Anal. **155** (1998), 153–170.
- [Kas88] G.G. Kasparov, *Equivariant KK -theory and the Novikov conjecture*, Invent. Math. **91** (1988), 147–201.
- [LLR00] F. Larsen, N. J. Laustsen, M. Rørdam, *An Introduction to K -theory for C^* -algebras*, London Mathematical Society Student Texts **49** (2000) CUP, Cambridge.
- [Pim95] M. V. Pimsner, *A class of C^* -algebras generalizing both Cuntz-Krieger algebras and crossed products by \mathbb{Z}* , Free probability theory, Fields Inst. Commun. **12** (1997), 189–212.
- [Roh09] R. Rohde, *K_1 -injectivity of C^* -algebras*, Ph.D. thesis at Odense (2009)
- [Rør97] M. Rørdam, *Stability of C^* -algebras is not a stable property.*, Doc. Math. J. **2** (1997), 375–386.
- [Rør03] M. Rørdam, *A simple C^* -algebra with a finite and an infinite projection*, Acta Math. **191** (2003), 1091–1142.
- [Rør04] M. Rørdam, *The stable and the real ranks of \mathcal{Z} -absorbing C^* -algebras*, Internat. J. Math. **15** (2004), 1065–1084.
- [TW07] A. Toms, W. Winter, *Strongly self-absorbing C^* -algebras*, Trans. Amer. Math. Soc. **359** (2007), 3999–4029.
- [Win09] W. Winter, *Strongly self-absorbing C^* -algebras are \mathcal{Z} -stable*, J. Noncommut. Geom. **5** (2011), 253–264.

Etienne.Blanchard@imj-prg.fr

IMJ-PRG, Bâtiment Sophie Germain, Case 7012,
5 rue Thomas Mann 75205 Paris CEDEX 13