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Comment on "Phase transition of a two-dimensional generalized XY model"

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Comment on the paper by
Y. Komura and Y. Okabe, *J.Phys. A* **44**,
015002, 2011

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Abstract

We point out that the claim of strong universality of [1] is incorrect, as it contradicts known rigorous results.

AMS 2000 subject classification: 82B20, 82B26, 60K35.

Keywords: Generalised XY-model, Kosterlitz-Thouless versus first-order transition, nonuniversality.

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The generalised lattice ferromagnetic XY -model was introduced by two of us in [2]. Its simplest version involves 3-component unit spins on a D -dimensional lattice, parameterized by standard angles in the spherical coordinates and interacting according to the ferromagnetic nearest-neighbour Hamiltonian

$$H = -J \sum_{\langle i,j \rangle} (\sin\theta_i \sin\theta_j)^q \cos(\varphi_i - \varphi_j) , \quad J > 0 .$$

When $D = 2$, the model yields orientational disorder at all finite temperatures, and produces a transition to a low-temperature Berezinskiĭ–Kosterlitz–Thouless (BKT) phase, possessing slow decay of magnetic correlations and infinite susceptibility [2]. In turn, the transition is expected, and was found by simulation [3], to exhibit the usual BKT scenario, at least for small values of q . On the other hand, this scenario does not hold for *all* values of q , i.e. the BKT transitions are *not universal*.

Indeed, we *proved* in [4] that the named transition turns first-order for sufficiently large q . Recall that a hint of this behaviour was already suggested by numerical analysis [3].

Notice that the above *rigorous* results entail that both existence and the type of transition *depend* on the parameter q , i.e. they exclude universality.

The above-mentioned generalized XY -model was recently addressed by simulation for various values of q in Ref. [1]. Although the authors found only a certain evidence of the BKT criticality, they claim that this behaviour holds for *all values* of q . This claim can not be correct since it contradicts to our rigorous results [4], which seem to have been unknown to the authors of the Ref. [1].

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