Non-Local Non-Negative Spherical Deconvolution for Single and Multiple Shell Diffusion MRI

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I. Introduction

In diffusion MRI (dMRI), Spherical Deconvolution (SD) is a category of methods which estimate the fiber Orientation Distribution Function (fODF). Existing SD methods, including the widely used Constrained SD [1], normally have two common limitations: 1) the non-negativity constraint of the fODFs is not satisfied in the continuous sphere; 2) many spurious peaks are detected, especially in the regions with low anisotropy; In [2], we proposed a novel SD method, called Non-Negative SD (NNSD), to avoid these two limitations. NNSD guarantees the non-negativity constraint of fODFs in the continuous sphere $\mathbb{S}^2$, and it is robust to the false positive peaks. In this abstract, we propose Non-Local NNSD (NLNNSD) which considers non-local spatial information and Rician noise in NNSD, and apply it to the testing data in ISBI contest.

II. Method

We represent the square root of fODF $\Phi(\mathbf{u})$ as a linear combination of real Spherical Harmonic (SH) basis $Y_l^m(\mathbf{u})$ with even order, i.e. $\Phi(\mathbf{u}) = \left(\sum_{l=0}^L \sum_{m=-l}^l c_{lm} Y_l^m(\mathbf{u})\right)^2 = \sum_{\alpha=0}^{2L} \sum_{\beta=-\alpha}^{\alpha} \left(\sum_{l,m}^L \sum_{l',m'}^L c_{lm} c_{l'm'} Q_{ll'\alpha}^{mm'\beta}\right) Y_\alpha^\beta(\mathbf{u})$, where $\mathbf{u} \in \mathbb{S}^2$, $Q_{ll'\alpha}^{mm'\beta} = \int_{\mathbb{S}^2} Y_l^m(\mathbf{u}) Y_{l'}^{m'}(\mathbf{u}) Y_\alpha^\beta(\mathbf{u}) d\mathbf{u}$ is the integral constant of three SHs which can be calculated from the Wigner 3-j symbol. Then based on the closed form of spherical convolution using SH basis, for a given axisymmetric fiber response function along z-axis $H(q\mathbf{u}|(0,0,1)) = \sum_{l=0}^L h_l(q) Y_l^0(\mathbf{u})$, the convolved diffusion signal is

$$E(q\mathbf{u}) = \sum_{\alpha=0}^{2L} \sum_{\beta=-\alpha}^{\alpha} \sum_{l,m}^{L} \sum_{l',m'}^{L} \sqrt{\frac{4\pi}{2\alpha+1}} c_{lm} c_{l'm'} Q_{ll'\alpha}^{mm'\beta} h_\alpha(q) Y_\alpha^\beta(\mathbf{u}) = \boldsymbol{c}^T \mathbf{K}(q\mathbf{u}) \boldsymbol{c} \tag{1}$$

where for any fixed vector $\mathbf{q} = q\mathbf{u}$, $\mathbf{K}(\mathbf{u})$ is a square matrix with the elements $\mathbf{K}_{ll'}^{mm'}(q\mathbf{u}) = \sum_{\alpha=0}^{2L} \sum_{\beta=-\alpha}^{\alpha} \sqrt{\frac{4\pi}{2\alpha+1}} Q_{ll'\alpha}^{mm'\beta} h_\alpha(q) Y_\alpha^\beta(\mathbf{u})$. Then NNSD [2] is to estimate $\boldsymbol{c}$ by minimizing

$$J(\boldsymbol{c}) = \frac{1}{2} \sum_{i=1}^{N} \left(\boldsymbol{c}^T \mathbf{K}(q\mathbf{u}) \boldsymbol{c} - E_i\right)^2 + \frac{1}{2} \boldsymbol{c}^T \Lambda \boldsymbol{c}, \qquad \text{s.t.} \quad \|\boldsymbol{c}\| = 1 \tag{2}$$

where Λ is a diagonal matrix with elements $\Lambda_{lm} = \lambda_{NNSD} l^2 (l+1)^2$ for the Laplace-Beltrami regularization. The constraint $\|\boldsymbol{c}\| = 1$ is because of $\int_{\mathbb{S}^2} \Phi(\mathbf{u}) d\mathbf{u} = 1$. In this abstract, we propose Non-local NNSD (NLNNSD) which considers the non-local spatial information and Rician noise. Non-local mean has been used in image denoise [3], [4] and regularization [5]. The cost function in NLNNSD is

$$J(\{\boldsymbol{c}^x\}) = \frac{1}{2} \sum_{x=1}^{V} \sum_{i=1}^{N} \left((\boldsymbol{c}^x)^T \mathbf{K}(q\mathbf{u}) \boldsymbol{c}^x - \mathrm{NLM}(E_i^x)\right)^2 + \frac{1}{2} (\boldsymbol{c}^x)^T \Lambda \boldsymbol{c}^x + \frac{1}{2} \lambda_{NLM} \|\boldsymbol{c}^x - \mathrm{NLM}(\boldsymbol{c}^x)\|^2 \tag{3}$$

where $\boldsymbol{c}^x$ and E_i^x are the coefficient vector and diffusion signal at voxel x, V is the number of voxels, $\mathrm{NLM}(\boldsymbol{c}^x) = \arg\min_{\boldsymbol{c}} \sum_{y \in V} w_y d(\boldsymbol{c}, \boldsymbol{c}^y)^2$ is the non-local Riemannian mean of $\boldsymbol{c}^x$ [6], $\mathrm{NLM}(E_i^x) = \sqrt{\sum_{y \in V} p_y (E_i^y)^2 - 2\sigma^2}$ is the non-local mean of E_i^x considering Rician noise with standard deviation of σ. w_y is the non-local weights determined by the distance of coefficient vectors, i.e. $w_y = \frac{1}{Z_y} \exp(-\frac{\sum_{j \in N_x, k \in N_y} G_a \|\boldsymbol{c}^j - \boldsymbol{c}^k\|^2}{2h^2})$, where $\boldsymbol{c}^j$ and $\boldsymbol{c}^k$ are the coefficient vectors respectively in the neighborhood N_x of x and the neighborhood N_y of y, G_a is the Gaussian weighting with standard deviation of a, and Z_y is the normalization factor. p_y is the non-local weight determined by the distance of $\{E_i^x\}$ with another set of $\{a, h\}$.

To minimize Eq. (3) with the constraint $\|\boldsymbol{c}\| = 1$, we first set $\lambda_{NLM} = 0$, and perform a Riemannian gradient descent on the sphere $\|\boldsymbol{c}\| = 1$ [6] to minimize $J(\boldsymbol{c}^x)$ individually for each voxel x.

$$(\boldsymbol{c}^x)^{(k+1)} = \mathrm{Exp}_{(\boldsymbol{c}^x)^{(k)}} \left(-dt \frac{\nabla J(\boldsymbol{c}^x)}{\|\nabla J(\boldsymbol{c}^x)\|}\right), \quad \mathrm{Exp}_{\boldsymbol{c}}(\boldsymbol{v}) = \boldsymbol{c} \cos\|\boldsymbol{v}\| + \frac{\boldsymbol{v}}{\|\boldsymbol{v}\|} \sin\|\boldsymbol{v}\|$$

The isotropic fODF with $\boldsymbol{c} = (1, 0, \ldots, 0)^T$ is chosen as the initialization. Then the non-local Riemannian mean is performed to calculate $\mathrm{NLM}(\boldsymbol{c}^x)$ at each voxel. Then the Riemannian gradient descent is performed again with λ_{NLM} and the estimated non-local mean $\mathrm{NLM}(\boldsymbol{c}^x)$ in Eq. (3). Note that this procedure can be iteratively performed to update $\mathrm{NLM}(\boldsymbol{c}^x)$ and $\boldsymbol{c}^x$, however in practice we found the result with just one iteration is enough.

III. ISBI HARDI Reconstruction Challenge

In the ISBI reconstruction challenge, the testing data was generated based on Numerical Fibre Generation toolbox [7]. we test the proposed NLNNSD in the data with three kind of sampling schemes: 1) single shell DTI scheme with 32 directions, $b = 1200 s/mm^2$; 2) single shell HARDI scheme with 64 directions, $b = 3000 s/mm^2$; 3) multiple shell DSI-like scheme with 514 directions, $b \in (0, 4000] s/mm^2$. For all night datasets (three schemes with three SNR 10, 20, 30), we fixed $L = 8$, $\lambda_{NNSD} = 0$, $\lambda_{NLM} = 1$, and used the tensor fiber response function with FA of 0.8, mean diffusivity of 0.8. In the non-local mean of $\boldsymbol{c}^x$ and E_i^x, we uses a $11 \times 11 \times 11$ search window, and a $3 \times 3 \times 3$ patch to define the weights, where the parameters a and h were tuned respectively for $\{\boldsymbol{c}^x\}$ and $\{E_i^x\}$ to obtain visually good results for each dataset.

References

[1] J. Tournier, F. Calamante, A. Connelly, "Robust determination of the fibre orientation distribution in diffusion MRI: non-negativity constrained super-resolved spherical deconvolution", NeuroImage 35(4) 1459–1472 (2007).
[2] J. Cheng, D. Shen, P.-T. Yap, "Non-Negative Spherical Deconvolution for Fiber Orientation Distribution Estimation", ISMRM (2013).
[3] A. Buades, B. Coll. J.M. Morel, "A non-local algorithm for image denoising" CVPR (2005).
[4] M. Descoteaux, N. Wiest-Daessle, S. Prima, C. Barillot, R. Deriche, "Impact of Rician Adapted Non-local Means Filtering on HARDI", MICCAI (2008).
[5] G. Peyré, S. Bougleux, L. Cohen, "Non-local regularization of inverse problems", ECCV, 2008.
[6] J. Cheng, A. Ghosh,T. Jiang, R. Deriche, "A Riemannian Framework for Orientation Distribution Function Computing", MICCAI (2009).
[7] T. Close, J. Tournier, F. Calamante, L. Johnston, I. Mareels, A. Connelly, "A software tool to generate simulated white matter structures for the assessment of fibre-tracking algorithms", NeuroImage (2009).