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# Non-Local Non-Negative Spherical Deconvolution for Single and Multiple Shell Diffusion MRI

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## I. INTRODUCTION

In diffusion MRI (dMRI), Spherical Deconvolution (SD) is a category of methods which estimate the fiber Orientation Distribution Function (fODF). Existing SD methods, including the widely used Constrained SD [1], normally have two common limitations: 1) the non-negativity constraint of the fODFs is not satisfied in the continuous sphere; 2) many spurious peaks are detected, especially in the regions with low anisotropy; In [2], we proposed a novel SD method, called Non-Negative SD (NNSD), to avoid these two limitations. NNSD guarantees the non-negativity constraint of fODFs in the continuous sphere  $\mathbb{S}^2$ , and it is robust to the false positive peaks. In this abstract, we propose Non-Local NNSD (NLNNSD) which considers non-local spatial information and Rician noise in NNSD, and apply it to the testing data in ISBI contest.

## II. METHOD

We represent the square root of fODF  $\Phi(\mathbf{u})$  as a linear combination of real Spherical Harmonic (SH) basis  $Y_l^m(\mathbf{u})$  with even order, i.e.  $\Phi(\mathbf{u}) = \left( \sum_{l=0}^L \sum_{m=-l}^l c_{lm} Y_l^m(\mathbf{u}) \right)^2 = \sum_{\alpha=0}^{2L} \sum_{\beta=-\alpha}^{\alpha} \left( \sum_{l,m}^L \sum_{l',m'}^L c_{lm} c_{l'm'} Q_{ll'\alpha}^{mm'\beta} \right) Y_{\alpha}^{\beta}(\mathbf{u})$ , where  $\mathbf{u} \in \mathbb{S}^2$ ,  $Q_{ll'\alpha}^{mm'\beta} = \int_{\mathbb{S}^2} Y_l^m(\mathbf{u}) Y_{l'}^{m'}(\mathbf{u}) Y_{\alpha}^{\beta}(\mathbf{u}) d\mathbf{u}$  is the integral constant of three SHs which can be calculated from the Wigner 3-j symbol. Then based on the closed form of spherical convolution using SH basis, for a given axisymmetric fiber response function along  $z$ -axis  $H(q\mathbf{u}|(0,0,1)) = \sum_{l=0}^L h_l(q) Y_l^0(\mathbf{u})$ , the convolved diffusion signal is

$$E(q\mathbf{u}) = \sum_{\alpha=0}^{2L} \sum_{\beta=-\alpha}^{\alpha} \sum_{l,m}^L \sum_{l',m'}^L \sqrt{\frac{4\pi}{2\alpha+1}} c_{lm} c_{l'm'} Q_{ll'\alpha}^{mm'\beta} h_{\alpha}(q) Y_{\alpha}^{\beta}(\mathbf{u}) = \mathbf{c}^T \mathbf{K}(q\mathbf{u}) \mathbf{c} \quad (1)$$

where for any fixed vector  $\mathbf{q} = q\mathbf{u}$ ,  $\mathbf{K}(\mathbf{u})$  is a square matrix with the elements  $\mathbf{K}_{ll'}^{mm'}(q\mathbf{u}) = \sum_{\alpha=0}^{2L} \sum_{\beta=-\alpha}^{\alpha} \sqrt{\frac{4\pi}{2\alpha+1}} Q_{ll'\alpha}^{mm'\beta} h_{\alpha}(q) Y_{\alpha}^{\beta}(\mathbf{u})$ . Then NNSD [2] is to estimate  $\mathbf{c}$  by minimizing

$$J(\mathbf{c}) = \frac{1}{2} \sum_{i=1}^N \left( \mathbf{c}^T \mathbf{K}(q\mathbf{u}) \mathbf{c} - E_i \right)^2 + \frac{1}{2} \mathbf{c}^T \Lambda \mathbf{c}, \quad \text{s.t. } \|\mathbf{c}\| = 1 \quad (2)$$

where  $\Lambda$  is a diagonal matrix with elements  $\Lambda_{lm} = \lambda_{NNSD} l^2 (l+1)^2$  for the Laplace-Beltrami regularization. The constraint  $\|\mathbf{c}\| = 1$  is because of  $\int_{\mathbb{S}^2} \Phi(\mathbf{u}) d\mathbf{u} = 1$ . In this abstract, we propose Non-local NNSD (NLNNSD) which considers the non-local spatial information and Rician noise. Non-local mean has been used in image denoise [3], [4] and regularization [5]. The cost function in NLNNSD is

$$J(\{\mathbf{c}^x\}) = \frac{1}{2} \sum_{x=1}^V \sum_{i=1}^N \left( (\mathbf{c}^x)^T \mathbf{K}(q\mathbf{u}) \mathbf{c}^x - \text{NLM}(E_i^x) \right)^2 + \frac{1}{2} (\mathbf{c}^x)^T \Lambda \mathbf{c}^x + \frac{1}{2} \lambda_{NLM} \|\mathbf{c}^x - \text{NLM}(\mathbf{c}^x)\|^2 \quad (3)$$

where  $\mathbf{c}^x$  and  $E_i^x$  are the coefficient vector and diffusion signal at voxel  $x$ ,  $V$  is the number of voxels,  $\text{NLM}(\mathbf{c}^x) = \arg \min_{\mathbf{c}} \sum_{y \in V} w_y d(\mathbf{c}, \mathbf{c}^y)^2$  is the non-local Riemannian mean of  $\mathbf{c}^x$  [6],  $\text{NLM}(E_i^x) = \sqrt{\sum_{y \in V} p_y (E_i^y)^2 - 2\sigma^2}$  is the non-local mean of  $E_i^x$  considering Rician noise with standard deviation of  $\sigma$ .  $w_y$  is the non-local weights determined by the distance of coefficient vectors, i.e.

$w_y = \frac{1}{Z_y} \exp\left(-\frac{\sum_{j \in N_x, k \in N_y} G_a \|c^j - c^k\|^2}{2h^2}\right)$ , where  $\mathbf{c}^j$  and  $\mathbf{c}^k$  are the coefficient vectors respectively in the neighborhood  $N_x$  of  $x$  and the neighborhood  $N_y$  of  $y$ ,  $G_a$  is the Gaussian weighting with standard deviation of  $a$ , and  $Z_y$  is the normalization factor.  $p_y$  is the non-local weight determined by the distance of  $\{E_i^x\}$  with another set of  $\{a, h\}$ .

To minimize Eq. (3) with the constraint  $\|\mathbf{c}\| = 1$ , we first set  $\lambda_{NLM} = 0$ , and perform a Riemannian gradient descent on the sphere  $\|\mathbf{c}\| = 1$  [6] to minimize  $J(\mathbf{c}^x)$  individually for each voxel  $x$ .

$$(\mathbf{c}^x)^{(k+1)} = \text{Exp}_{(\mathbf{c}^x)^{(k)}} \left( -dt \frac{\nabla J(\mathbf{c}^x)}{\|\nabla J(\mathbf{c}^x)\|} \right), \quad \text{Exp}_{\mathbf{c}}(\mathbf{v}) = \mathbf{c} \cos \|\mathbf{v}\| + \frac{\mathbf{v}}{\|\mathbf{v}\|} \sin \|\mathbf{v}\|$$

The isotropic fODF with  $\mathbf{c} = (1, 0, \dots, 0)^T$  is chosen as the initialization. Then the non-local Riemannian mean is performed to calculate  $\text{NLM}(\mathbf{c}^x)$  at each voxel. Then the Riemannian gradient descent is performed again with  $\lambda_{NLM}$  and the estimated non-local mean  $\text{NLM}(\mathbf{c}^x)$  in Eq. (3). Note that this procedure can be iteratively performed to update  $\text{NLM}(\mathbf{c}^x)$  and  $\mathbf{c}^x$ , however in practice we found the result with just one iteration is enough.

## III. ISBI HARDI RECONSTRUCTION CHALLENGE

In the ISBI reconstruction challenge, the testing data was generated based on Numerical Fibre Generation toolbox [7]. we test the proposed NLNNSD in the data with three kind of sampling schemes: 1) single shell DTI scheme with 32 directions,  $b = 1200 \text{ s/mm}^2$ ; 2) single shell HARDI scheme with 64 directions,  $b = 3000 \text{ s/mm}^2$ ; 3) multiple shell DSI-like scheme with 514 directions,  $b \in (0, 4000] \text{ s/mm}^2$ . For all night datasets (three schemes with three SNR 10, 20, 30), we fixed  $L = 8$ ,  $\lambda_{NNSD} = 0$ ,  $\lambda_{NLM} = 1$ , and used the tensor fiber response function with FA of 0.8, mean diffusivity of 0.8. In the non-local mean of  $\mathbf{c}^x$  and  $E_i^x$ , we uses a  $11 \times 11 \times 11$  search window, and a  $3 \times 3 \times 3$  patch to define the weights, where the parameters  $a$  and  $h$  were tuned respectively for  $\{\mathbf{c}^x\}$  and  $\{E_i^x\}$  to obtain visually good results for each dataset.

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