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Narrow tetraquarks at large N

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Following a recent suggestion by Weinberg, we use the large-N expansion in QCD to discuss the decay amplitudes of tetraquarks into ordinary mesons as well as their mixing properties. We find that the flavor structure of the tetraquark is a crucial ingredient to determine both this mixing as well as the decays. Although in some cases tetraquarks should be expected to be as narrow as ordinary mesons, they may get to be even narrower, depending on this flavor structure.

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In a recent paper, Weinberg [1] pointed out that tetraquark mesons (i.e. those formed by two quarks and two antiquarks), if they survive the large-N limit [2], would have decay rates proportional to $1/N$ and would, therefore, be as narrow as ordinary mesons. In this paper we would like to point out that the flavor structure in a tetraquark meson is crucial to determine the behavior of its decay amplitude at large N and, although Weinberg's result is generically true, there are tetraquark mesons for which there is an extra suppression resulting in a decay width, which goes as $1/N^2$. Complications like the effects of mixing between ordinary mesons and tetraquarks need to be discussed in general, although they are not always an essential ingredient to reach this conclusion.

Let us start, following Ref. [1], by considering tetraquark interpolating fields as

$$\mathcal{Q}(x)^{\alpha;\beta}_{AB;CD} = \mathbf{B}^{\alpha}_{AB}(x)\mathbf{B}^{\beta}_{CD}(x), \tag{1}$$

where

$$\mathbf{B}^{\alpha}_{AB}(x) = \sum_a \bar{q}^a_A(x)\Gamma^\alpha q^a_B(x) \tag{2}$$

and $q^a_A(x)$ is the quark field with flavor A and color a [in $SU(N)$] and Γ^α are colorless matrices containing spin information. We will always assume that the flavor indices A, B in these quark bilinears are different, so that the vacuum expectation value $\langle \mathbf{B}^{\alpha}_{AB}(x)\rangle_0$ identically vanishes [3]. This simplifies the following discussion somewhat. Since the flavor structure in the first bilinear is such that $A \neq B$, there are only three nontrivial possibilities for the other bilinear $\bar{q}_C q_D$ (note that $C \neq D$ as well): either $C = B$ or $D = B$ or, finally, $A \neq B \neq C \neq D$; i.e. all the flavor indices are different [4]. These three possibilities will determine the possible quark contractions in the relevant Green's function, which in turn will determine its large-N behavior.

Before entering the discussion of the different possible decay amplitudes for a tetraquark into ordinary mesons, it is of the utmost importance to identify the tetraquark as a physical state in a Green's function, e.g. as a pole in the correlator [5]:

$$\langle \mathcal{Q}(x)\mathcal{Q}^\dagger(0)\rangle_0. \tag{3}$$

Therefore, as emphasized above, we now need to look at all the possible quark contractions in this correlator, and this is where the flavor structure comes in.

We will call the "type-A" diagram a diagram like that in Fig. 1, where the flavor in the tetraquark operator $\mathcal{Q}(x)$ is of the form $\bar{q}_A q_B \bar{q}_B q_D$, which allows an internal quark contraction between the two quarks with flavor index B. This diagram is of order N. An example of this type of tetraquark is $\bar{u}d\bar{d}s$. Cutting vertically through this diagram immediately reveals the presence of a potential four-quark intermediate state. Without knowing the solution to

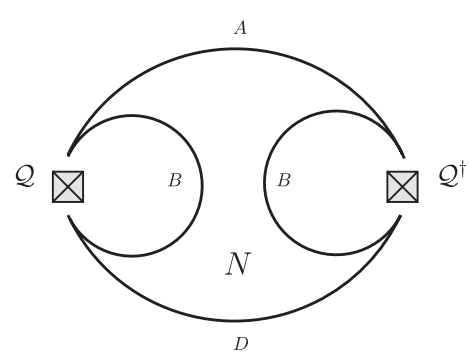

FIG. 1. Type-A diagram. In this figure, and in all the ones that follow, it will be always understood that any number of gluon lines not changing the large-N behavior of the diagram should be added. This diagram is of order N, and this will be explicitly depicted with an "N" in the diagram. The flavor labels A, B, D of the quark lines are also indicated.

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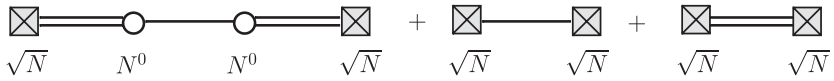

FIG. 2. Contribution of intermediate states to the type-A correlator of Fig. 1. A double line represents a tetraquark, a single line represents an ordinary meson, a boxed cross represents the insertion of the $\mathcal{Q}$ operator, and an empty circle represents a vertex between a tetraquark and an ordinary meson (which, in this case, implies mixing). The corresponding large-N behavior for each of these terms in the diagram is explicitly written.

large-N QCD, it is not possible to know whether this four-quark intermediate state really exists and forms the necessary pole in this Green's function. But if we assume that the state exists, large-N can be used to predict how narrow it is and how it mixes with ordinary $\bar{q}q$ mesons. One can see in Fig. 2 how to interpret the possible contributions to this Green's function from all the possible intermediate states which are obtained by vertically cutting the diagram. This figure shows the large-N behavior of the different contributions, as determined by matching the final value of N obtained in the diagram of Fig. 1. For instance, looking at the last term in Fig. 2, one finds that the amplitude for the operator $\mathcal{Q}$ to create this tetraquark is of order $N^{1/2}$ since this amplitude appears squared. This matching has assumed that no cancellations between the different contributions in Fig. 2 take place, as it is customarily done when following large-N reasoning [6]. Similarly, one also infers that the amplitude for $\mathcal{Q}$ to create an ordinary meson (second term in Fig. 2) is also of order $N^{1/2}$. Using these results, one finally obtains that the mixing between this tetraquark and an ordinary meson (first contribution in Fig. 2) is of order N^0, and it is, therefore, not suppressed in the large-N limit. Because of this, it may be very difficult to disentangle these tetraquarks from ordinary mesons, unless very precise information on the position of the tetraquark pole is known. Furthermore, since this tetraquark is of the form $\bar{q}_A q_B \bar{q}_B q_D$, with the flavor index B contracted, it will fill out the same flavor representation as ordinary $\bar{q}_A q_D$ mesons, e.g. a whole nonet in flavor $SU(3)$.

In order to determine the width of a type-A tetraquark into two ordinary mesons, one may look at the three-point function depicted in Fig. 3 [8],

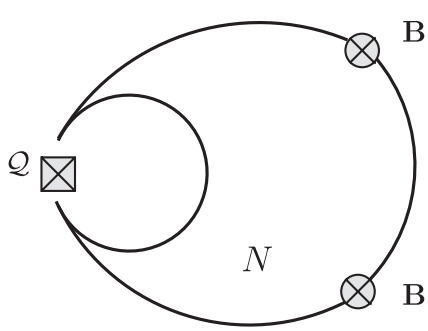

FIG. 3. Three-point function determining the decay of a type-A tetraquark into two ordinary mesons. A circled cross signifies the insertion of the $\bar{q}q$ operator.

$$\langle \mathcal{Q}(x)\bar{q}(y)q(y)\bar{q}(z)q(z)\rangle_0, \tag{4}$$

which is of order N, and look for the tetraquark pole. Its interpretation in terms of physical states is done in Fig. 4. One concludes from this figure that the vertex between this tetraquark and two ordinary $\bar{q}q$ mesons is of order $N^{-1/2}$ (depicted as an empty circle in Fig. 4). Therefore, the decay width of a type-A tetraquark is of order $1/N$, i.e. as narrow as an ordinary meson. This is the result obtained in Ref. [1].

Figure 5 helps one understand why a type-A tetraquark behaves like an ordinary meson in its decay: one may reinterpret this process as the mixing of the tetraquark with an ordinary meson (with an unsuppressed amplitude of order N^0) plus the subsequent decay of the ordinary meson.

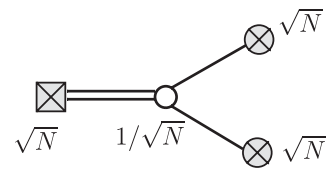

FIG. 4. Interpretation in terms of physical states of the contributions in Fig. 3. Symbols are like in Fig. 2.

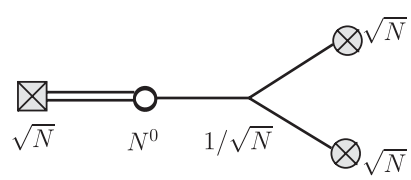

FIG. 5. Reinterpretation in terms of mixing between a tetraquark and an ordinary meson of the decay in Fig. 3.

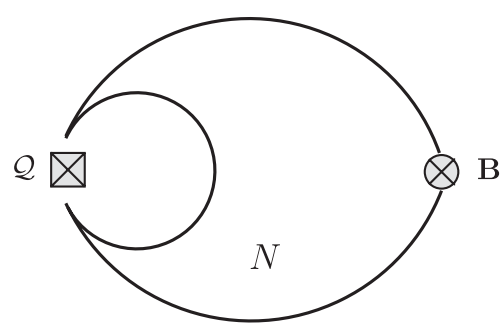

FIG. 6. Mixed correlator $\langle \mathcal{Q}(x)\bar{q}(y)q(y)\rangle_0$.

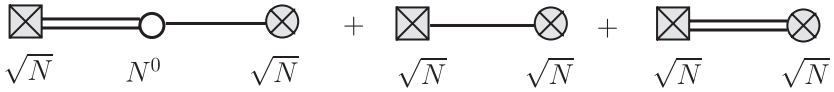

FIG. 7. Contributions from the intermediate physical states to the mixed correlator in Fig. 6.

The consistency of the conclusions drawn before may be cross-checked by looking at the mixed correlator in Fig. 6, which is of order N, and its interpretation in terms of physical states, as is done in Fig. 7. From the first term in Fig. 7, one again obtains that mixing between a tetraquark and an ordinary meson is of order N^0.

Tetraquarks with the flavor structure $\bar{q}_A q_B \bar{q}_C q_B$ (recall that $A \neq B$ and $C \neq B$) do not allow the contractions of Fig. 1. An example of this tetraquark is $\bar{u}d\bar{s}d$. These tetraquarks will be called of "type A′" because, after a simple rerun of the arguments presented for the previous diagrams in Figs. 1–4, the same final conclusion about the behavior of the width as for type-A tetraquarks is reached, namely, of order $1/N$. An important difference is, however, that these tetraquarks cannot mix with ordinary mesons because any intermediate state always contains 4 quarks, as Fig. 8 shows. So, the first two contributions in Fig. 2 are absent in this case.

We now come to the second type of tetraquarks, which we will call "type B," i.e. with all flavor indices different. This fact prevents any internal quark contraction within the tetraquark operator $\mathcal{Q}$. An example for this type of tetraquark is $\bar{u}d\bar{c}s$. The corresponding diagram is depicted in Fig. 9. In order to make this contribution connected, a minimum number of two gluons is necessary between the two quark loops; otherwise, the diagram factorizes into two $\bar{q}q$ meson propagators, which can contain no tetraquark pole at leading order (see also below). Of course, in the diagram in Fig. 9, any number of gluons should be added as long as they do not alter the large-N counting, which is of order N^0. This diagram may also be present in the cases of the type-A and type-A′ tetraquarks, but its contribution remains subleading, which is why we have not discussed it until now. When all the four flavor indices in $\mathcal{Q}$ are different, the contribution in Fig. 9 becomes leading and needs to be discussed separately.

Any intermediate state in Fig. 9 always contains four quarks, which means that there is no mixing with ordinary mesons. Since the diagram is of order N^0, the amplitude for $\mathcal{Q}$ to create one of these tetraquarks is also of order N^0 (see Fig. 10). Fig. 11 shows the three-point function describing the decay of this tetraquark into ordinary mesons, which is of order N^0 as well. Its interpretation in terms of physical

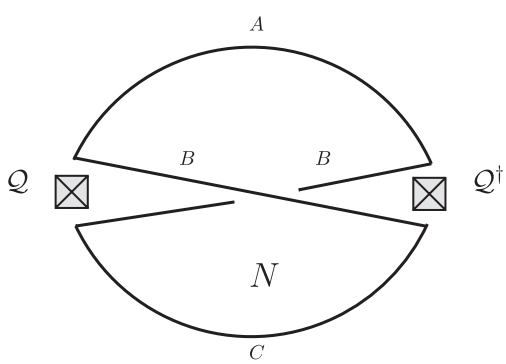

FIG. 8. Diagram for the two-point correlator of a tetraquark operator of type A′. The labels A, B, C on the quark lines denote distinct flavors.

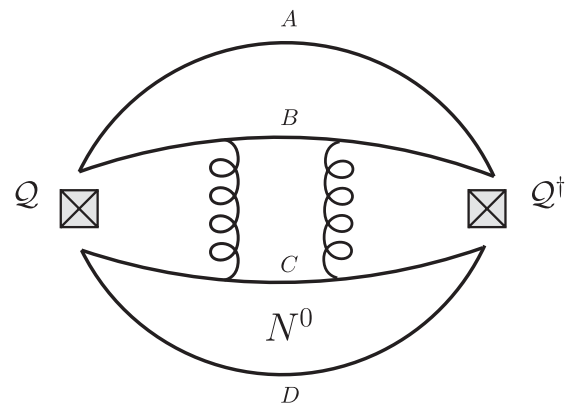

FIG. 9. Diagram for the two-point correlator of a tetraquark operator of type B. A minimum of two gluons is necessary to make this contribution connected. The labels A, B, C, D on the quark lines denote distinct flavors.

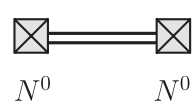

FIG. 10. Interpretation of the diagram in Fig. 9 in terms of intermediate states.

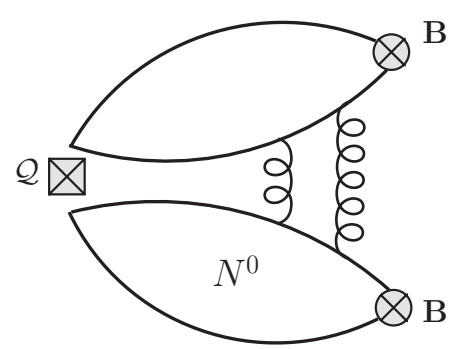

FIG. 11. Diagram for correlator governing the decay of a tetraquark operator of type B.

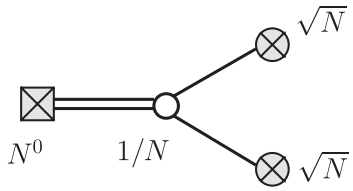

FIG. 12. Interpretation of the diagram in Fig. 11 in terms of physical states.

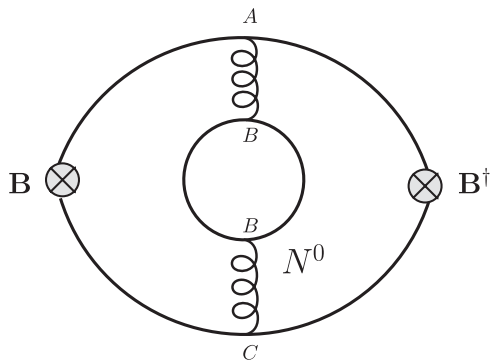

FIG. 13. Diagram showing the presence of a four-quark state in the $\bar{q}q$ two-point correlator. The labels A, B, C on the quark lines denote distinct flavors.

intermediate states in given in Fig. 12. One concludes, therefore, that the width for this type-B tetraquark is of order $1/N^2$ and is narrower than the type-A and type-A′ tetraquarks discussed previously. This concludes our discussion of the tetraquarks which may be excited from the vacuum by the action of the four-quark operator $\mathcal{Q}$.

We would like to point out, however, about the existence of another logical possibility. This consists of a $\bar{q}q$ bilinear exciting a tetraquark meson through mixing, like in Fig. 13, which shows a diagram of order N^0. We will call these tetraquarks "type C." Clearly, a vertical cut of the diagram may contain a four-quark intermediate state (Fig. 14 shows all the possible intermediate states). Can this state be a tetraquark; i.e. can this singularity become a pole? Although it is true that the color flow in the diagram shows that the intermediate state can be split in the product of two $\bar{q}q$ color singlets, this does not imply that these two singlets necessarily have to become two separate meson states. We think it is not possible, on the sole basis of large-N counting arguments, to conclusively argue one way or the other without a more detailed dynamical knowledge of the QCD solution in this limit, but we see no reason that would forbid the presence of a tetraquark pole in

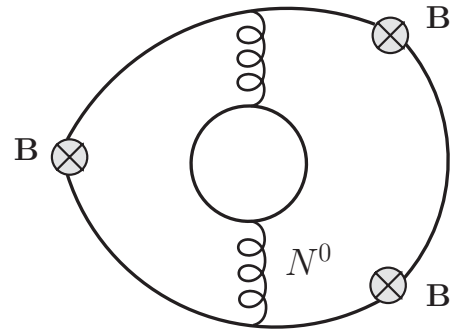

FIG. 15. Diagram depicting the three-point function governing the decay of a tetraquark of type C.

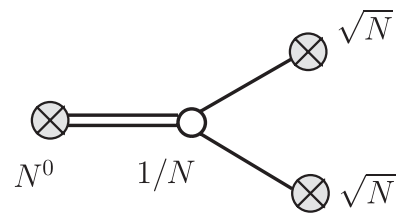

FIG. 16. Interpretation of the diagram in Fig. 15 in terms of physical states.

Fig. 13. If this is the case, these tetraquarks will have the flavor structure of the type $\sum_B \bar{q}_A q_B \bar{q}_B q_D$; i.e. they will also form a nonet, like type-A tetraquarks do.

As to the decay properties of a type-C tetraquark, one may look at the three-point function depicted in Fig. 15, which is of order N^0, and its corresponding interpretation in terms of physical states in Fig. 16. Since the vertex for this type of tetraquark to decay into ordinary mesons is of order $1/N$, its width will be of order $1/N^2$, i.e. as narrow as type-B tetraquarks.

In conclusion, we have found that tetraquarks are narrow objects in the large-N limit [1]. They are at least as narrow as ordinary mesons, but they may get to be even narrower if the flavor structure is the appropriate one. There have been long discussions in the literature trying to resolve the dichotomy between the description in terms of a four-quark state or a two-meson molecule [9]. Our conclusion is that, if the large-N expansion is a good guidance, a tetraquark state should be narrow. If the state is broad, it is more likely to be a two-meson bound state (molecule) resulting from an infinite chain of $1/N$-suppressed meson-meson interactions. This is plausibly the way states like the $f_0(500)$ may be formed [10].

Perhaps the most promising tetraquark states from a theory point of view are those we called type B, with all

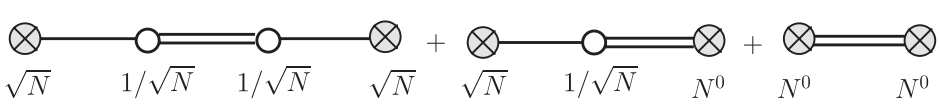

FIG. 14. Interpretation of the diagram in Fig. 13 in terms of physical states.

the quark flavors different. For these tetraquarks the complications brought about by mixing will not be an issue since they cannot mix with ordinary mesons and, furthermore, they are expected to be long-lived as their width goes like $1/N^2$. It would be very interesting to see if the lattice could reveal their existence [11]. On the phenomenology side, there is mounting evidence about the existence of mesons with a four-quark content [12]. It remains to be seen whether the large-N expansion can be helpful for explaining this phenomenology and how it fares as compared to existing alternatives [13].

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[4] The cases $A = C$ and $A = D$ are analogous to the cases $B = D$ and $B = C$ (respectively) and need not be discussed separately.
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